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A Robust Aggregation Approach To Simplification Of Manufacturing Flow Line Models

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Outline of Presentation

- What is the Objective of this Research?
- Why Develop an Abstract Simulation Model?
- What are the Techniques for Developing an Aggregate Simulation Model
- How was the Procedure Tested and What Were the Results?
- How can these Techniques be Expanded?

Objective of this Research

In developing a simulation model of a discrete part manufacturing system, a modeler must decide at what level of abstraction to represent the resources of the system. For example, questions about plant capacity can be modeled with a simple model, whereas questions regarding the efficiency of different part scheduling rules can only be answered with a more detailed model. Unfortunately, many claim that the process of building a simulation model is an “intuitive art.” The objective of this research is to introduce aspects of “science” to model development by defining a quantitative methodology for developing an aggregate simulation model of a manufacturing flow line system for estimating part cycle time.

Why Develop an Abstract Simulation Model?

In developing a simulation model, most of the actual features of the system under study must be ignored and an **abstraction** must be developed. If done correctly, this idealization provides a useful approximation of the real system.

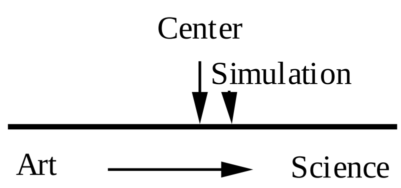
Advantages of an Abstract Simulation Model

- a reduced run length
- a less complex model
- easier to animate
- easier to debug, validate, modify and document
- less demand of programming resources (queues and systems variables)
- less data dependent answers

Process of Developing an Abstract Simulation Model

Many claim that the process of building a simulation model is an “intuitive art”

Another view is that simulation modeling uses aspects of both art and science.



The objective of this research is to a set of “science” tools for model development.

Techniques for Aggregation

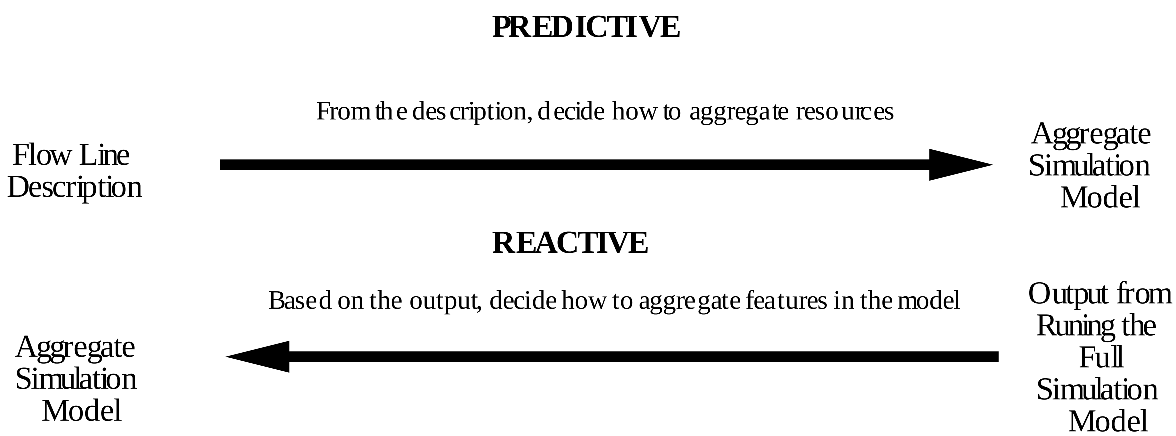
Simplification

Aggregation

Substitution

Research Assumptions

(1) The decision process for aggregating simulation resources is studied from a ***predictive*** point of view.

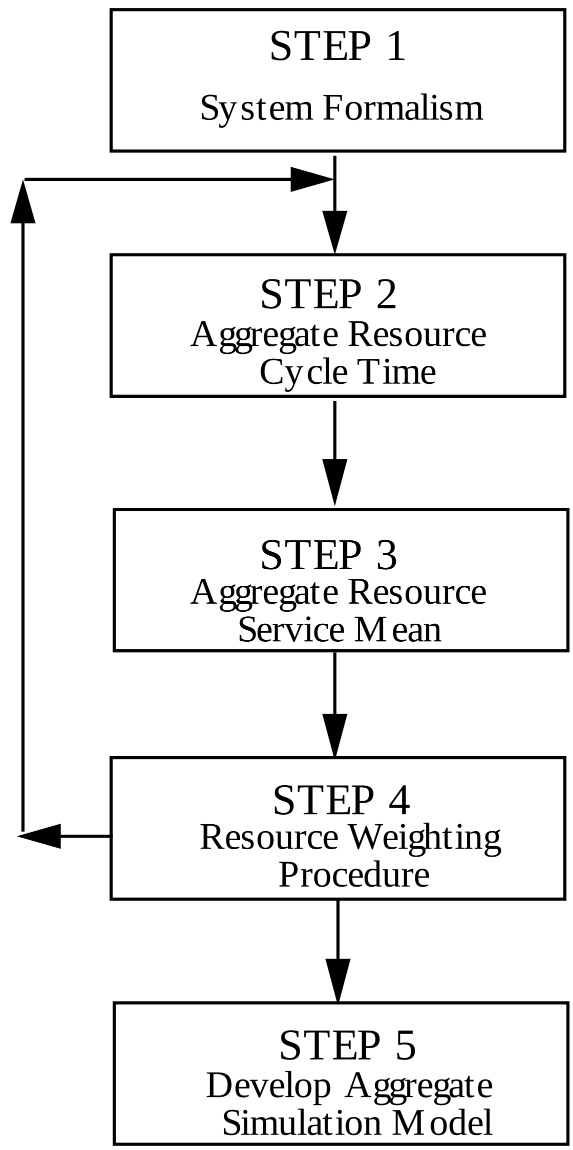


(2) The manufacturing system that this research explores is a production **flow line** (flow shop) system.

(3) All **shop floor data** is readily available

(4) Single performance variable: **average cycle time** (sojourn time) of a part to wait and be serviced by all stations (resources) of the flow line.

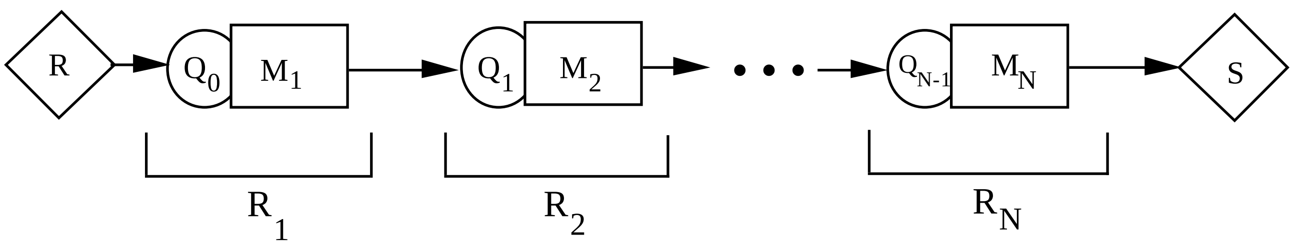
Operation of the Aggregation Methodology



STEP 1: Flow Line Formalism

What is a Flow Line?

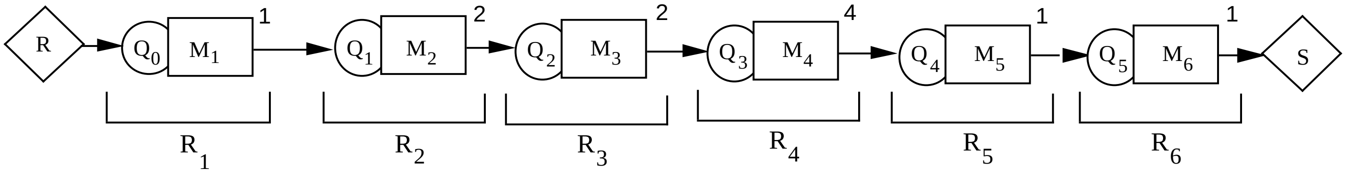
The *single* part type is processed at N production stations (resources) with the order of processing at a production station (resource) being the same for all part.



Flow Line Formalism

$$FL = \left\langle R, R_1, \ldots, R_N, S \right\rangle$$
$$R = \langle 1/\lambda, Z \rangle$$
$$S = \langle U \rangle$$
$$R_i = \left\langle Q_{i-1}, M_i \right\rangle \quad i = 1, \ldots, N$$
$$Q_{i-1} = \left\langle v_{i-1}, x_{i-1} \right\rangle \quad i = 1, \ldots, N$$
$$M_i = \left\langle F_i, m_i, s_i \right\rangle \quad i = 1, \ldots, N$$

Example Flow Line



Flow Line Formalism

$$FL = \langle R, R_1, R_2, R_3, R_4, R_5, R_6, S \rangle$$

$$R = \langle 100, \infty \rangle$$

$$S = \langle \infty \rangle$$

$R_1 = \langle Q_O, M_1 \rangle$

$Q_O = \langle v_O, \infty \rangle$

$M_1 = \langle Uniform(75, 85), 80, 1 \rangle$

$R_2 = \langle Q_1, M_2 \rangle$

$Q_1 = \langle v_1, \infty \rangle$

$M_2 = \langle Normal(130, 15), 130, 2 \rangle$

$R_3 = \langle Q_2, M_3 \rangle$

$Q_2 = \langle v_2, \infty \rangle$

$M_3 = \langle Triangular(120, 150, 180), 150, 2 \rangle$

$R_4 = \langle Q_3, M_4 \rangle$

$Q_3 = \langle v_3, \infty \rangle$

$M_4 = \langle Normal(320, 25), 320, 4 \rangle$

$R_5 = \langle Q_4, M_5 \rangle$

$Q_4 = \langle v_4, \infty \rangle$

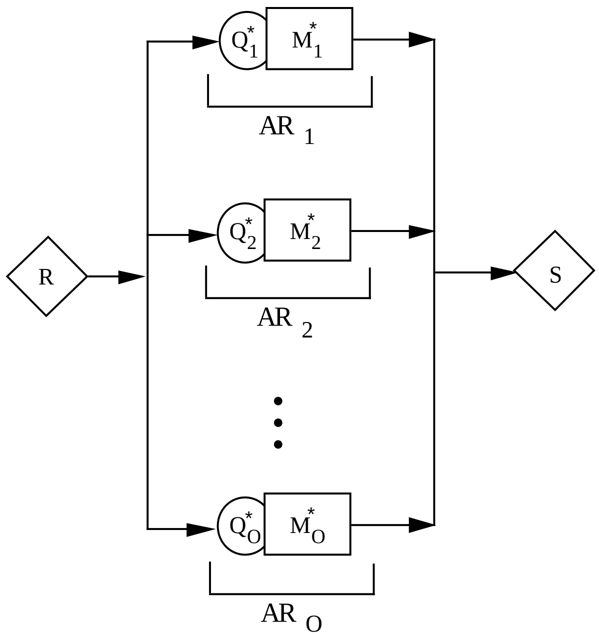
$M_5 = \langle Triangular(32, 43, 60), 45, 1 \rangle$

$R_6 = \langle Q_5, M_6 \rangle$

$Q_5 = \langle v_5, \infty \rangle$

$M_6 = \langle Uniform(64, 80), 72, 1 \rangle$

Aggregate Formalism



Aggregate Flow Line Formalism

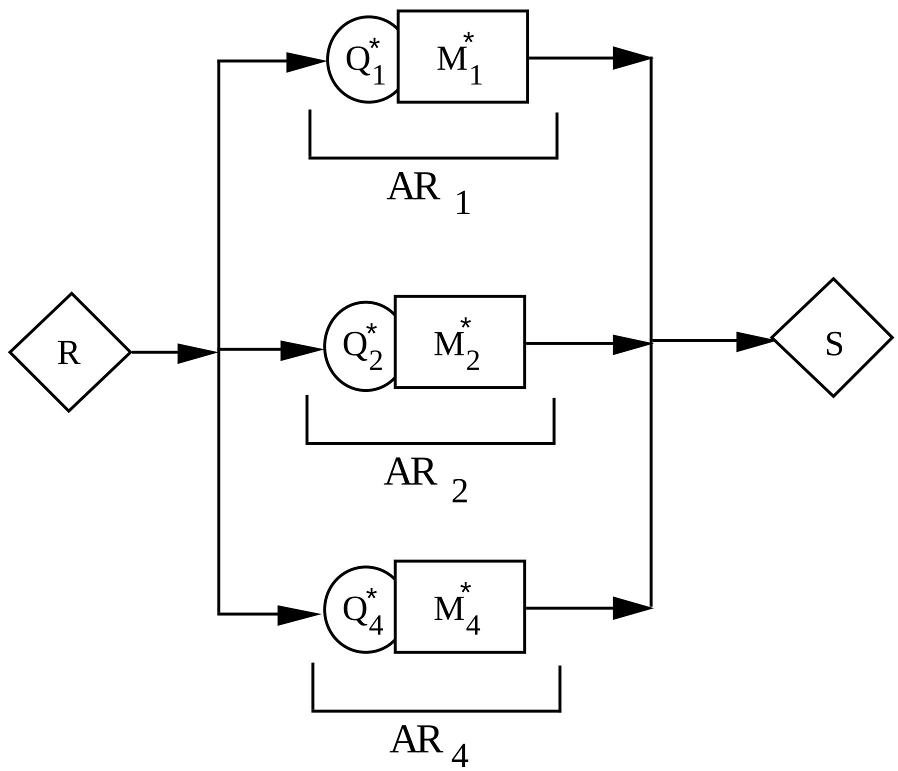
$$AFL = \langle R, AR_1, \ldots, AR_O, S \rangle$$

$$AR_i = \{\varnothing, \langle Q_i^*, M_i^* \rangle\} \quad i = 1, \ldots, O$$

$$Q_i^* = \langle x_i^* \rangle \quad i = 1, \ldots, O$$

$$M_i^* = \langle F_i^*, \delta_i^* \rangle \quad i = 1, \ldots, O$$

Example Flow Line [continued]



Example Aggregate Flow Line Formalism

$$AFL = \langle R, AR_1, AR_2, AR_3, AR_4, S \rangle$$

$AR_1 = \langle Q_1^*, M_1^* \rangle$ $\quad AR_2 = \langle Q_2^*, M_2 \rangle$ $\quad AR_3 = \varnothing$ $\quad AR_4 = \langle Q_4^*, M_3 \rangle$

$Q_1^* = \langle \infty \rangle$ $\quad Q_2^* = \langle \infty \rangle$ $\quad Q_4^* = \langle \infty \rangle$

$M_1^* = \langle F_1^*, \delta_1^* \rangle$ $\quad M_2^* = \langle F_2^*, \delta_2^* \rangle$ $\quad M_4^* = \langle F_4^*, \delta_4^* \rangle$

STEP 2: Computing Cycle Time

$$E[T_j] = \frac{1 + cv_{m_i}^2}{2\lambda(1-\rho_j)}\rho_j C_j + m_j \quad j = 1, \ldots, N$$

where: $E[T_j]$ Expected cycle time of resource j (j = 1,...,N)

$cv_{m_j}^2$ Squared coefficient of variation of the service time of resource j (j = 1,...,N)

λ Arrival rate of parts to the flow line

ρ_j Traffic intensity of resource j (j = 1,...,N): $\rho_j = \dfrac{\lambda m_j}{s_j}$

s_j Number of parallel, identical servers for resource j (j = 1,...,N)

m_j Mean service time of resource j (j = 1,...,N)

C_j The probability that a part arriving to resource j (j = 1,...,N) has to wait for service: $C_j = \dfrac{(\lambda m_j)^{s_j}}{s_j!(1-\rho_j)} P_{0_j}$

P_{0J} The probability that zero service are busy for resource j (j = 1,...N):

$$P_{0_j} = \frac{1}{\left[\sum_{n=0}^{s_j-1} \frac{(\lambda m_j)^n}{n!} + \frac{(\lambda m_j)^{s_j}}{s_j!(1-\lambda m_j/s_j)}\right]}$$

Aggregation Resource Total Cycle Time

$$T_i^* = \sum_{R_j \in AR_i} T_j \qquad \begin{matrix} i = 1, \ldots, O \\ j = 1, \ldots, N \end{matrix}.$$

Aggregation Resource Average Cycle Time

$$\overline{T_i^*} = \frac{T_i^*}{P_i} \qquad i = 1, \ldots, O$$

STEP 3: Service Mean of an Aggregation Resource

Consider an M/M/1 Queueing System

$$\rho = \frac{\lambda}{\mu}$$

$$Po = 1 - \rho$$

$$Lq = \frac{\lambda\rho}{\mu - \lambda}$$

$$Wq = \frac{Lq}{\lambda}$$

$$W = \frac{1}{\mu - \lambda}$$

Solving for the Service Rate

$$\mu = \frac{1}{W} + \lambda$$

Solving for the Service Mean

$$1/\mu = \frac{W}{1 + \lambda W}$$

Consider an M/M/2 Queueing System

$$\rho = \frac{\lambda}{2\mu}$$

$$P_0 = \frac{1}{\left[\frac{(\lambda/\mu)^0}{0!} + \frac{(\lambda/\mu)^1}{1!}\right] + \left[\frac{(\lambda/\mu)^2}{2!(1-\rho)}\right]}$$

$$Lq = \frac{P_0(\lambda/\mu)^2 \rho}{2!(1-\rho)^2}$$

$$W_q = \frac{L_q}{\lambda}$$

$$W = W_q + 1/\mu$$

This simplifies to

$$W = \frac{4\mu}{4\mu^2 - \lambda^2}$$

Solving for the mean service rate

$$\mu = \frac{1 \pm \sqrt{1 + W^2\lambda^2}}{2W}$$

Solving for the mean service time

$$(1/\mu) = \frac{-2 + 2\sqrt{1 + W^2\lambda^2}}{W\lambda^2}.$$

Service Mean for an M/G/S Aggregation Resource

$$E\left[\overline{T_i^*}\right] = \frac{1 + cv_{\delta_i}^2}{2\lambda(1-\rho_i^*)} \rho_i^* C_i^* + \delta_i^* \quad i = 1, \ldots, O$$

where: $E\left[\overline{T_i^*}\right]$ Expected average cycle time of aggregate resource i (i = 1,...,O)

$cv_{\delta_i}^2$ Squared coefficient of variation of the unknown service time δ_i^* for aggregation resource i (i = 1,...,O)

λ Arrival rate of parts to the flow line

ρ_i^* Traffic intensity of aggregation resource i (i = 1,...,O):

$$\rho_i^* = \frac{\lambda \delta_i^*}{i}$$

δ_i^* Mean service time of aggregate resource i (i = 1,...,O)

C_i^* $$\frac{(\lambda \delta_i^*)^i}{i!(1-\rho_i^*)} P_{0_i}^* \quad i = 1, \ldots, O$$

$P_{o_i}^*$ $$P_{o_i}^* = \frac{1}{\left[\sum_{n=0}^{i-1} \frac{(\lambda \delta_i^*)^n}{n!} + \frac{(\lambda \delta_i^*)^i}{i!(1-\lambda \delta_i^*/i)}\right]} \quad i = 1, \ldots, O$$

For a M/G/1 Aggregation Resource

$$\delta_1^* = \frac{\left(1 + \lambda \overline{T_1^*}\right) \pm \sqrt{1 + 2\lambda^2 - 2\lambda \overline{T_1^*} + \lambda^2 \overline{T_1^*}^2 + 2\lambda^2 cv_{\delta_1}^2}}{2\lambda}$$

Estimate the Squared Coefficient of Variation of the Service Time

$$cv_{m_j}^2 = \frac{s_j^2}{m_j^2} \quad j = 1, \ldots, N$$

$$cv_{\delta_i}^2 = \sum_{R_j \in AR_i} \left(\frac{T_j}{T_i^*}\right) cv_{m_j}^2 \quad \begin{array}{l} i = 1, \ldots, O \\ j = 1, \ldots, N \end{array}$$

STEP 4: Determining Distribution Weights

Conditions on the Weights

(1) $$\sum_{R_j \in AR_i} w_j^* m_j = \delta_i^* \qquad \begin{matrix} i = 1, \ldots, O \\ j = 1, \ldots, N \end{matrix}$$

(2) $$\sum_{R_j \in AR_i} w_j^* = 1 \qquad \begin{matrix} i = 1, \ldots, O \\ j = 1, \ldots, N \end{matrix}$$

Aggregation Resource Representing *One* Resource

(1) $w_j^* \times m_j = \delta_j^*$ and (2) $w_j^* = 1.0$

The Resulting Distribution Weight is One

Aggregation Resource Representing *Two* Resources

(1) $(w_2^* \times m_2) + (w_5^* \times m_5) = \delta_2^*,$ and (2) $w_2^* + w_5^* = 1.$

Solve the Two Equations of Two Unknowns to find the Weights

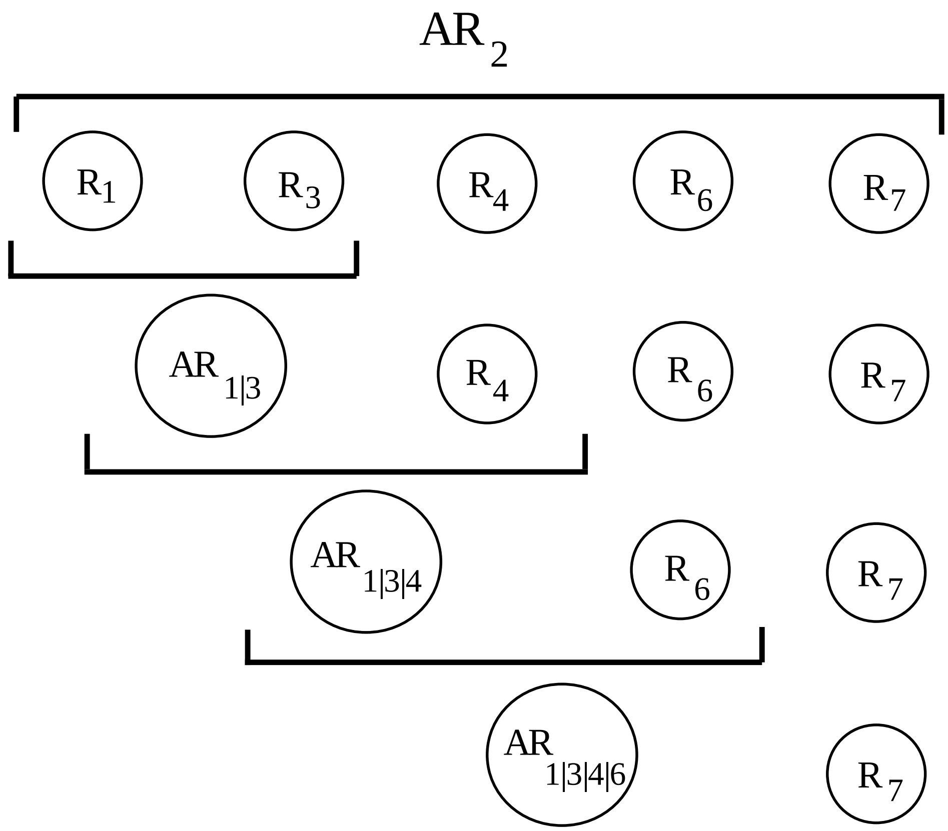
Aggregation Resource Representing *Three* (or more) Resources

(1) $(w_1^* \times m_1) + (w_3^* \times m_3) + (w_5^* \times m_5) = \delta_4^*$

(2) $w_1^* + w_3^* + w_5^* = 1.$

Solving Requires Using a Recursive Algorithm to Solve

Example of Recursive Aggregation



STEP 5: Aggregate Simulation Model

Composite Sampling Requires

The distribution weights $w_1^*, w_2^*, ..., w_N^*$. Subject to:

$$w_j^* \geq 0, \quad R_j \in AR_i \quad j = 1, \ldots, N$$

$$\sum_{R_j \in AR_i} w_j^* = 1 \quad j = 1, \ldots, N$$

and Replicates the Aggregate Service Distribution

$$f_i^*(x) = \sum_{j=1}^{P_i} w_j^* f_j(x) \quad i = 1, \ldots, O$$

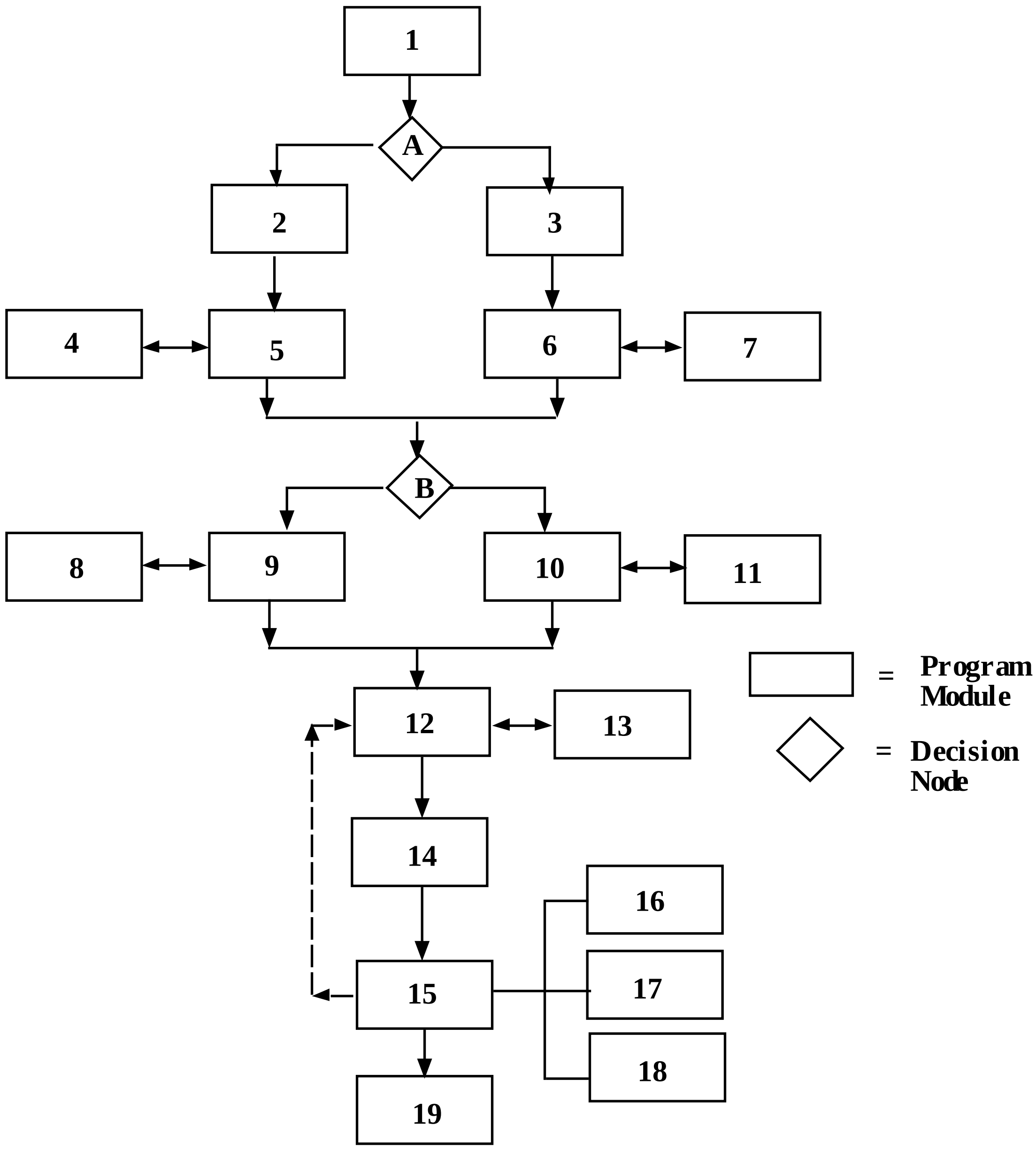
Testing of the Aggregation Methodology

Exponential Flow Line

Single Capacity Server Flow Line

Multiple Capacity Server

MATHEMATICA Aggregation Program



Results of Applying the Aggregation Techniques

A total of 50 simulation models run requiring 7.5 days of CPU time

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Exponential Systems

- 10 models generated and their aggregate simulation model was run

- $$RE = 100\% \times \left[\frac{|\text{Average aggregate cycle time - steady state estimate}|}{\text{steady state estimate}} \right]$$

  was 1.139%

Single Server System

- 10 models generated and their full and aggregate simulation model were run

- $$RE = 100\% \times \left[\frac{|\text{Average aggregate cycle time - Average Full Model Cycle Time}|}{\text{Average Full Model Cycle Time}} \right]$$

  was 4.78%

Multiple Server System

- 10 models generated and their full and aggregate simulation model were run

- $$RE = 100\% \times \left[\frac{|\text{Average aggregate cycle time} - \text{Average Full Model Cycle Time}|}{\text{Average Full Model Cycle Time}} \right]$$

  was 3.5%

- In all test cases, the aggregate simulation model is an upper bound estimate

Contributions of this Research

This research provides an important first step in applying analytical procedures to the process of developing an abstract simulation model. Demonstration that analytical techniques such as queueing analysis can be integrated with simulation to reduce the effort necessary to address simulation questions.

Specific Achievements and Contributions

(1) Development of system formalisms for describing a production flow manufacturing system and its aggregate equivalent.

(2) Identification of procedures for computing the average cycle time of an aggregate resources.

(3) Development of a technique for estimating the mean service time of an aggregate resource.

(4) Creation of a method for describing the mean service time of an aggregation resource in terms of the resource service means that it represents.

(5) Specifications for creating an aggregate simulation model.

(6) Creation of a computer program which implements the aggregation methodology.

Conclusions

Exponential Flow Line System

Small Relative Error

Single Server Flow Line System

Moderate Relative Error

Multiple Server Flow Line System

Small Relative Error

Aggregate Estimate is an Upper Bound Estimate of Cycle Time

Areas for Expansion

(1) Estimating multiple performances characteristics of the flow line system from the aggregate simulation model.

(2) Including procedures for estimating the departure/arrival variability for G/G/S resources.

(3) Incorporate feedback, rework, and scrap into the flow line.

(4) Model resources with limited resource queue capacity.

Anticipated Publications

Conference Papers:

(1) The Science of Simulation Modeling

(2) MATHEMATICA Aggregation Program

(3) Flow Line Formalisms

Journal Articles:

(1) Aggregating an exponential flow line

(2) Aggregating a multiple server flow line

(3) Aggregating a single server flow line