Module 1: How Students Think

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Module 1 How Students Think

Introduction

You have probably been curious at various times in your teaching career about the thinking strategies that students enrolled in physics appear to use to solve problems. It is difficult for most of us to understand that many students do not use reasoning patterns that seem to be obvious. Many students substitute numbers into a formula they remember, even though the formula may not be applicable to the problem at hand. This situation quite naturally leads us to wonder about the reasoning that students utilize when we would employ mental operations such as separating variables, excluding an irrelevant factor, or applying a mathematical relationship such as ratios.

Objectives

To assist you in distinguishing among various patterns of thought used to solve simple problems in physics and mathematics.

Procedure

This module includes three puzzles: the Volume Puzzle (page 1-2), the Ratio Puzzle (page 1-6), and the Islands Puzzle (page 1-10). Each puzzle is followed by several typical student responses to the puzzle. Please complete at least two of the puzzle activities by writing out your own solution to the puzzle and then comparing your ideas with those of the students. The puzzles may be used in any order. Then please answer the review questions before going on to the next module.
Module 1 Instructional Materials

Volume Puzzle Activity

1. Please write your answers in the spaces below.

VOLUME PUZZLE

Here are drawings of two vertical tubes (cylinders) which are filled to the same mark with water: the cylinders are identical in size and shape.

![Cylinder 1 and Cylinder 2 drawings]

Here are two marbles, one made of steel and one made of glass. Both marbles have the same volume (that is, they are the same size). The steel marble is heavier.

Glass  Steel

The steel marble is heavier than the glass one, but both marbles will sink if placed in one of the cylinders. We are going to put one marble into each of the cylinders.

After we have put the glass marble into cylinder 1, both cylinders and their contents look like this:

![Cylinder 1 and Cylinder 2 drawings with glass marble]

If we now put the steel marble into cylinder 2, what will happen to the water level in that cylinder? (Tell whether it will rise, fall, or stay the same; if it rises or falls, tell what the final water level will be in cylinder 2.)

__________________________

__________________________

__________________________

__________________________

__________________________

Explain why you predicted the result above.

__________________________

__________________________

__________________________

__________________________
2. This puzzle is accompanied by a videotape showing three girls working on the Volume Puzzle with the equipment described on the preceding page. The first girl (interviewed by Robert Karplus) is about fourteen years old*, the second (interviewed by John W. Renner) is seventeen, and the third (interviewed by Robert Karplus) is nineteen. Each girl's approach has a unique feature. Try to identify it while you are watching the videotape.

Please view the videotape now, and then describe the unique feature of each girl's thinking in the spaces below.

Fourteen-year-old:

Seventeen-year-old:

Nineteen-year-old:

3. The next pages present written student responses to the Volume Puzzle presented as a paper-and-pencil task. Study the responses and compare Students A with Students B.

**Student A₁ (High School Junior)**

Prediction: "Rise to 8."
Explanation: "Equal volume spheres displace the same volume of water."

**Student A₂ (John Blake - Age 16)**

Prediction: "The water level in cylinder 2 will rise to the same height as in cylinder 1 after the glass marble is put in."
Explanation: "Both marbles had the same volume, therefore the water level, after the marbles were put in, was the same in each cylinder. The weight in no way affected the degree to which the water rose."

**Student A₃ (Barbara Downing - Age 21)**

Prediction: "Cylinder 2's water level will rise to the number 8."
Explanation: "Since the 2 marbles have the same volume they will displace the same amount of water. Eureka! (weight has nothing to do with it)."

**Student A₄ (Harold O'Keefe - Age 20)**

Prediction: "The level in cylinder 2 will also be 8; the same as in cylinder 1."
Explanation: "If both marbles are heavy enough to sink and are the same size they will therefore displace the same amount of water thus raising the water level to the same numbers."

*Scene from PIAGET’S DEVELOPMENTAL THEORY: CONSERVATION. Compliments of Davidson Films, Inc., 3701 Buchanan Street, San Francisco, CA 94123
Student A₅ (College Junior)

Prediction: "The level of the water will be 8."
Explanation: "The reason that the water levels were both the same was because the objects both had the same volume and the cylinders which they were placed in were the same size with the same water level, therefore the objects displaced the same amount of water in both cylinders and their water level remained the same."

Student B₁ (High School Junior)

Prediction: "I think cylinder 2 would be higher to about the number 10 mark."
Explanation: "Because it put more pressure onto the water. This means it would push it upward. The steel ball seems to me like it would be heavier."

Student B₂ (High School Junior)

Prediction: "It will rise. The final water level in cylinder 2 will be 7."
Explanation: "The steel marble is heavier therefore the water will not rise as much."

Student B₃ (College Junior)

Prediction: "The water level in that cylinder will rise. I estimate the metal marble is twice as heavy so the water level will be at 10."
Explanation: "If you put an object that has the slightest weight into water, the level of the water will rise. The result would be the same as if you added water to the cylinder. Add something to something and you get more."

Student B₄ (College Junior)

Prediction: "The level of H₂O in cylinder 2 will rise to higher than 8 - probably 10."
Explanation: "Because the marble in cylinder 2 is heavier than the marble in cylinder 1. It's just like scales, the more weight the higher it goes up."

Student B₅ (David Kenting - Age 19)

Prediction: "The water in cylinder 2 will rise but not as much as in cylinder 1 because the glass marble has more volume."
Explanation: "Since the steel marble is heavier and smaller, it will sink faster but not have as much volume. Therefore the water level would rise, but not as much as the glass marble."

Student B₆ (Norma Kuhn - Age 20)

Prediction: "The steel marble will make it rise to a level of ten or more."
Explanation: "The reason for the increase in rise on the steel marble was because the steel marble is twice as heavy if not more than the glass marble."
Student B7 (Deloris Johnson - Age 19)

Prediction:  "Cylinder level will rise because the marble is heavy. Final water level will be 10."
Explanation:  "Because the steel marble is heavier than the glass marble -- it took up more space than the glass marble."

Student Bg

Prediction:  "I think it will stay the same."
Explanation:  "I don't really know why. But it would seem the steel marble might have the weight to hold it down. The glass marble is lighter so it pushes the water up."

4. What similarities did you find among the responses of Students A?  Please record your analysis here.

5. What similarities did you find among the responses of Students B?  Please record your analysis here.

6. Please look at the responses again briefly and add any comments you may have about the differences between the two types.

Now proceed to another puzzle or to the Review Questions on page 1-14.
THE RATIO PUZZLE

The figure at the left is called Mr. Short. We used large round buttons laid side-by-side to measure Mr. Short's height, starting from the floor between his feet and going to the top of his head. His height was four buttons. Then we took a similar figure called Mr. Tall, and measured it in the same way with the same buttons. Mr. Tall was six buttons high.

Now please do these things:

1. Measure the height of Mr. Short using paper clips in a chain provided. The height is __________

2. Predict the height of Mr. Tall if he were measured with the same paper clips. __________

3. Explain how you figured out your prediction. (You may use diagrams, words, or calculations. Please explain your steps carefully.)

__________________________________________________________________________

__________________________________________________________________________

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__________________________________________________________________________
2. The next pages present written student responses to the Ratio Puzzle. Read these responses and compare them with your own.

**Student A₁ (Age 16)**

Prediction for Mr. Tall: 9 3/6
Explanatıon: "Figured it out by seeing that Mr. Tall is half again as tall as Mr. Short, so I took half of Mr. Short's height in clips and added it on to his present height in clips and came up with my prediction."

**Student A₂ (Age 16)**

Prediction for Mr. Tall: 9 1/2 paperclips
Explanatıon: "I figured that the ratio of paper clips to buttons to be approximately 1 1/2:1 so two more buttons would make approximately 3 more clips. Since it's a little more than 1 1/2:1 he is approximately 9 1/2 clips tall."

**Student A₃ (Age 16)**

Prediction for Mr. Tall: 9.49 clips.
Explanatıon: "I took the relationship of the clips to the buttons on Mr. Short and the unknown clips to buttons of Mr. Tall and found the unknown, algebraically."

**Student A₄ (John Blake - Age 16)**

Prediction for Mr. Tall: 9 1/2 clips
Explanatıon: "Mr. Tall is 1.5 times the height of Mr. Short, as measured with buttons, and if the measurement techniques were identical would be 1.5 times Mr. Short's height with any measurement medium. Assuming that the measurement techniques are identical, Mr. Tall's height in clips is 1.5 x 6 1/3, which is 9 1/2 (I think)."

**Student A₅ (Barbara Downing - Age 21)**

Prediction for Mr. Tall: 9.2 paperclips.
Explanatıon: "The ratio using buttons of height of Mr. Short and Mr. Tall is 2:3. Figuring out algebraically and solving for x:

\[
\frac{2}{3} = \frac{6 \frac{1}{6}}{x}
\]

gives you 9.2 as the height in paper clips."

**Student A₆ (Deloris Johnson - Age 19)**

Prediction for Mr. Tall: 9 paperclips tall.
Explanatıon: "I figured this out by figuring that Mr. Small is 2/3 as tall as Mr. Tall."
Student B₁ (Age 16)

Prediction for Mr. Tall: 8 1/2 clips.
Explanation: "If he is 2 buttons taller I guess he is 2 clips bigger which would make it 8 1/2."

Student B₂ (Age 18)

Prediction for Mr. Tall: 8 clips
Explanation: "Because he is two times as high as Mr. Short."

Student B₃ (David Kenting - Age 19)

Prediction for Mr. Tall: 8 1/2 clips.
Explanation: "I figured the buttons the same size as the clips."

Student B₄ (Age 14)

Prediction for Mr. Tall: 9 clips (pencil marks along Mr. Short)
Explanation: "I estimated the middle and then one fourth of Mr. Short. That's about the size of one button. I measured the button with my clips and found one-and-a-half. So then I counted out six times one-and-a-half buttons and got nine."

Student B₅ (Age 16)

Prediction for Mr. Tall: 12 clips
Explanation: "Mr. Tall was 2 buttons taller than Mr. Short. The buttons must be larger than the paper clips. So I doubled Mr. Short's height in paper clips for Mr. Tall's height."

Student B₆ (Norma Kuhn - Age 20)

Prediction for Mr. Tall: 8 paper clips
Explanation: "Mr. Tall is 8 paper clips tall because when using buttons as a unit of measure he is 2 units taller. When Mr. Short is measured with paper clips as a unit of measurement he is 6 paper clips. Therefore, Mr. Tall is 2 units taller in comparison which totals 8."

Student B₇ (Harold O'Keefe - Age 20)

Prediction for Mr. Tall: 8 paper clips tall
Explanation: "If Mr. Short measures 4 buttons or 6 paper clips (2 pieces more than buttons), then Mr. Tall should be 2 paper clips more than buttons."

Student B₈ (Age 25)

Prediction for Mr. Tall: 8 paper clips tall.
Explanation: "4 buttons reached top of Mr. Short's head. Mr. Tall is 6 buttons tall. 6 paper clips Mr. Short. Mr. Tall is 8 paper clips tall. Paper clips are approximately 1 inch long and the buttons were probably the same."
3. Center your attention on several of those responses which were different from yours. See if you can detect any common elements among them. Record those common elements here.

4. Center your attention on those responses which agree with yours and see if you can detect any common elements among the student responses and your response.

Now proceed to another puzzle or to the review questions on page 1-14.
Islands Puzzle Activity

1. Please write out your answers to the islands puzzle in the spaces below.

THE ISLANDS PUZZLE

The puzzle is about Islands A, B, C, and D in the ocean. People have been traveling among these islands by boat for many years, but recently an airline started in business. Carefully read the clues about possible plane trips at present. The trips may be direct or include stops and plane changes on an Island. When a trip is possible, it can be made in either direction between the islands. You may make notes or marks on the map to help use the clues.

**First Clue:** People can go by plane between Islands C and D.

**Second Clue:** People cannot go by plane between Islands A and B.

Use these two clues to answer Question 1. Do not read the next clue yet.

**Question 1:** Can people go by plane between Island B and D?

Yes____ No____ Can't tell from the two clues____

Please explain your answer.

**Third Clue (do not change your answer to Question 1 now!):** People can go by plane between Island B and D.

Use all three clues to answer Question 2 and 3.

**Question 2:** Can people go by plane between Island B and C?

Yes____ No____ Can't tell from the three clues____

Please explain your answer.

**Question 3:** Can people go by plane between Islands A and C?

Yes____ No____ Can't tell from the three clues____

Please explain your answer.
2. What types of thinking did you do while completing the Islands Puzzle? For example, did you need to recall or review some of the initial instructions while answering the questions? Did you use combinations of information? Did you exclude any information as irrelevant? Did you make any hypotheses and then test them? Please record your observations of your own procedure here.

3. Did you notice any similarity between the Islands Puzzle and four-terminal "mystery" circuit boxes sometimes used in general physics laboratories?

If you did not, turn back to the puzzle, try to construct an electric analogue, and compare the results obtained from this analogue with your answers to the puzzle. Did you find the comparison helpful? Please record your comments.

4. The next pages present written student responses to the Islands Puzzle. Please read the responses and compare them with yours.

**Student A₁ (John Blake - Age 16)**

1. Answer: Can't tell from the two clues.
   Explanation: There is not enough information given.

2. Answer: Yes
   Explanation: "They can go to Island B from Island C, then on to Island D."

3. Answer: No
   Explanation: There is no information on a direct flight from A to C, but if you could get to C from A then you could also get to B. Since you cannot get to B from A you cannot get to C from A.

**Student A₂ (Deloris Johnson - Age 19)**

1. Answer: Can't tell from the two clues.
   Explanation: "There was no information given concerning the two."

2. Answer: Yes
   Explanation: "They can fly from C to D, have a lay over and catch the plane from D to B."

3. Answer: No
   Explanation: "It was said you cannot go from A to B. There is no information about a flight direct from C to A - only C to D."
Student A₃ (College Student - Age 17)

1. Answer: Can't tell from the clues given.
   Explanation: "The two clues don't relate the upper islands to the lower ones.

2. Answer: Yes
   Explanation: They can go from B to D and then to C, even if there are no direct flights.

3. Answer: No
   Explanation: If they could go from C to A, then the people on B could go first to D, then to C, and then on to A. But this contradicts the second clue, that they can't go by plane between B and A.

Student A₄ (David Kenting - Age 19)

1. Answer: Can't tell from the two clues.
   Explanation: By information given they could if appropriate landing facilities were on Island B.

2. Answer: Yes
   Explanation: "Yes because planes go from C to D or vice versa and B to D and vice versa. Therefore all have facilities."

3. Answer: No
   Explanation: "No, because Island A has no landing facilities mentioned."

Student A₅ (Norma Kuhn - Age 21)

1. Answer: Yes
   Explanation: "If the trip from C to D includes a stop on B. The clues only state that one cannot go by plane between A and B. The introduction states that the flights need not be direct."

2. Answer: Yes
   Explanation: "Via D"

3. Answer: Can't tell from the three clues.
   Explanation: "The clues do not give any connections to A except via boat."

Student A₆ (Barbara Downing - Age 22)

1. Answer: Can't tell from the two clues.
   Explanation: "The clues tell nothing of the relation of B and D."

2. Answer: Yes
   Explanation: "If you can go from C to D and D to B, C to B should also be possible."

3. Answer: No
   Explanation: "If you can go from C to B, but not B to A, you should not be able to go from C to A."
Student B₁ (College Student - Age 30)

1. Answer: Yes
   Explanation: "You can't go from B to A but you can go from D to B, or go from D to C then to Island B."

2. Answer: Yes
   Explanation: "It doesn't say that you can't go. It says you can't go from A to B islands, you can cut across or go through D."

3. Answer: Yes
   Explanation: "You can as long as you don't go on to Island B."

Student B₂ (Harold O'Keefe - Age 20)

1. Answer: Yes
   Explanation: "Because B is bigger than D and listing is the same sequence."

2. Answer: No
   Explanation: "The sequence is broken."

3. Answer: Yes
   Explanation: "They are listing the same as in Question One."

Student B₃ (High School Student - Age 18)

1. Answer: No
   Explanation: "B and D are not far enough apart."

2. Answer: Yes
   Explanation: "They are a long distance apart."

3. Answer: No
   Explanation: "Not far enough apart."

Student B₄ (College Student - Age 17)

1. Answer: Yes
   Explanation: "Because the people can go north from Island D because in the clue it could be made in both directions."

2. Answer: No
   Explanation: "I am presuming both directions doesn't include a 45° angle from B to C."

3. Answer: Yes
   Explanation: "Because Island C is right below Island A."

5. Now please identify some features of the thinking used by Students A and B that set them apart from each other and from your thinking.

   Students A:

   Students B:

Please proceed to another puzzle or to the Review Questions on page 1-14.
Module 1 Review Questions

After you have completed two or three puzzle activities in Module 1, read the following responses and classify them as Type A or Type B. Compare your answers with those of other participants sitting near you and with ours (bottom of page).

Student X₁ (College junior) Volume Puzzle

Prediction: Rise above 8 to approximately 10
Explanation: The weight of the object placed in water displaces an amount of water. Thus if the steel marble was heavier, it displaced more water, causing it to rise above the level of water in which the glass marble was in.

Student X₂ (age 15) Ratio Puzzle

Prediction for Mr. Tall: 15 paper clips high
Explanation: "Guess. I'm really not sure how to do this."

Student X₃ (age 20) Islands Puzzle

1. Answer: Can't tell from the two clues.
   Explanation: No information about flights between B and D

2. Answer: Yes
   Explanation: Go from C to D and then to B.

3. Answer: Can't tell from the three clues
   Explanation: Not possible to fly from B to A, and there is no mention of a direct flight between C and A.

Type ________