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**Problems With Problem Solving: Assessing Written Solutions of
Mathematical Habits of Mind Problems**

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Professional development Institute Partnership
Action Research Project Report

in partial fulfillment of the MA Degree
Department of Teaching, Learning, and Teacher Education
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Problems With Problem Solving: Assessing Written Solutions of Mathematical Habits of Mind Problems

Abstract

This paper explores a study of sixth grade mathematics students in a middle school setting. I investigated the effects of assigning Habits of Mind problems related to the curriculum, district standards and objectives to increase problem-solving opportunities and skills. I discovered integrating problem solving into the given curriculum allowed students to make connections to math in an opportune way that evoked communication on paper, interactions with partners or small groups, and presentations of solutions to the large group. Using the challenging Habits of Mind problems deliberately chosen to match specific content objectives within my district at the sixth grade level provided an opportunity for students to get more practice and improve upon their problem-solving skills. Planning and thinking about mathematics in ways that were unique in comparison to the components of instruction I typically used helped me incorporate weekly Habits of Mind problems. Analysis of classroom interactions and students' solutions as well as the students' reflection on their own experiences provided evidence that students experienced some difficulty in making their emergent thinking known to peers in a way that would allow for productivity for all involved. As a result of this research, I plan to continue to explore the notion of challenging sixth graders with featured Habits of Mind problems relative to the curriculum based on the positive feedback I gained from this short-term action research project.

Introduction

Through my journey in professional development, I learned that becoming a good problem solver requires taking risks, developing good communication skills, understanding basic facts and number sense, acquiring knowledge of mathematics vocabulary, using common sense, making connections to previous experiences, organizing my thinking and written work, and working constructively with others. For some people, this is easier than others. In fact, I grew by leaps and bounds in my ability to problem solve as well as my ability to teach problem solving. While I enjoyed the challenging and creative problems presented to me over the past two years, I sometimes struggled to know how and where to get started, how to connect what I already knew to the problem at hand or how to implement new teachings into solving the problem given. It was a painstaking journey for me that provided huge dividends by way of my own increased knowledge, self-confidence, and ability to work cohesively with others in a group on challenging tasks. Since this type of problem solving can be such a challenge for people, and yet so rewarding, I chose this as my research focus in the classroom.

The aforementioned research was conducted in a Midwestern city with a population of more than 200,000 people, consisting of two universities and two community colleges where earning an education and continuing education was part of the culture for many people. The sixth grade classroom studied was in a public middle school, grades 6 through 8, of approximately 800 students located in a middle- to upper-income neighborhood where parent and community involvement was prevalent. The school consisted of predominantly Caucasian students where 29 of approximately 240 sixth graders participated in the research in an on-grade level mathematics classroom.

None of the students had Individual Education Plans (IEP's) for learning disabilities or other identified special needs. All students scored in the middle to upper range on their Metropolitan Achievement Tests (MAT) and were on grade level in reading.

The students in my class did not come from the same elementary schools and thus did not have the same math teachers in previous years. The majority of my students were not excelling in problem-solving or homework story problems I had assigned, based on how many students skipped the problems altogether, had questions and could not complete the problems independently, or lacked confidence, ability or drive to even get started on problems. Successful problem solving in mathematics seemed to depend on the type of learner one was, such as being a mathematical and spatial thinker, a visual learner, a kinesthetic learner or auditory learner. I also believed student success depended on one's training and background knowledge, willingness to take risks and persevere, plus one's ability to read and sort information. Therefore, I saw it as my job to help my students all learn to become better problem solvers in mathematics and offer many opportunities to hone their problem-solving abilities. After all, this was a life skill and one that would help a student in his or her future ambitions.

In the Holt Course 1 mathematics textbook used by the public school system at the sixth grade, I was impressed by George Polya's problem-solving steps (Understand, Plan, Solve, and Look Back) used as the introduction of problem solving in this text. His approach was simple, clear and attainable by most sixth grade students. First, he acknowledged that understanding the problem was critical to solving it correctly. Second, Polya suggested that one should make a plan on how to solve a problem. This might include making a table, a drawing, creating a flow chart or one of many others he suggested and I, in turn, taught from the Holt Course 1 text. Next, it was time to solve

and find a solution to the underlying problem. Finally, one had to look back at the answer attained with confidence it made sense by checking one's work. These simple steps made the task of problem solving less daunting for students and for myself.

It was my goal to develop a worksheet that had these four steps described so that I could eventually assign problems and have students communicate their understanding in this way. I visualized solving problems all together and modeling how to complete this worksheet using a variety of problems and strategies for solving. My purpose was not to create robotic problem solvers, rather to model what good communication and understanding looked like. I expected students to use this format independently and comfortably to solve future problems and tackle tasks assigned. In sixth grade, student confidence seemed to be a factor that halted solving problems and often students stopped to ask me, "Is this right? Does this look good? What do I do next?" Independent thinking and problem solving by using Polya's four steps should be possible with this worksheet I designed and the numerous opportunities to solve Habits of Mind problems, which could increase their confidence in their problem-solving abilities.

I used the NCTM Principles and Standards book more and more to locate problems appropriate for my grade level, for clarification of standards and mathematics expectations, and to look ahead to mathematical expectations of upcoming grade levels and courses for my students. I found it important to understand how what I was teaching related to future student coursework so I could communicate this through instruction and prepare students as best I could. I believed all math teachers should be privy to a copy of the NCTM Principles and Standards to use as an additional educational resource available to guide teaching and instructional practices. I was more aware of

the principles and standards presented by the NCTM than ever before due to my affiliation to Math in the Middle, an intensive program for middle level educators, and saw the need for my action research to be based upon sound advice published in the NCTM Principles and Standards. Because I chose to research problem solving, I taught, reinforced, and observed students' written and oral communication. Problem solving, connections and communication were all critical NCTM Standards that drove my research.

I hoped increased experiences with problem solving and Polya's four simple steps would help students become more confident in their abilities to reason and solve problems, communicate processes and solutions, as well as make deeper connections between percents, decimals and fractions. Developing the habits that made a person a better problem solver occurred with more practice. The habits I was interested in improving were similar to those observed in mathematicians and articulated by Math in the Middle (reference the Math in the Middle Web site). Someone who possessed a rich set of mathematical habits of mind did some, if not all, of these things:

1. Understands which tools are appropriate when solving a problem
2. Is flexible in their thinking
3. Uses precise mathematical definitions
4. Understands there exist (therefore encourages) multiple paths to a solution
5. Is able to make connections between what they know and the problem
6. Knows what information in the problem is crucial to its being solved
7. Is able to develop strategies to solve a problem
8. Is able to explain solutions to others
9. Knows the effectiveness of algorithms within the context of the problem
10. Is persistent in their pursuit of a solution
11. Displays self-efficacy while doing problems
12. Engages in a meta-cognition by monitoring and reflecting on the processes of conjecturing, reasoning, proving, and problem solving

By utilizing others' published research, I hoped to assist my educational peers in the area of problem solving, offer input based on my classroom experiences and be a

resource for other interested educators while ultimately benefiting the students in my classroom.

Problem Statement

Habits of successful humans had been of interest to authors, researchers, and motivated individuals who sought better solutions to most anything such as building a business, saving money and building wealth, utilizing “best practices” in an educational setting to achieve better test scores, or parenting young children in a successful way. Mathematics was no different. Habits of great mathematical problem solvers were studied and recorded so that others could improve upon their skills. Students had experienced word problems throughout their mathematics education. Had they had frequent chances to solve multi-layered open-ended problems like those labeled Habits of Mind problems? It was my intent to increase the number of opportunities to solve problems that were more complex and layered than word problems that likely evoked one quick answer.

I wanted to offer opportunities to my students in which how to solve the problem was not immediately apparent. As I planned my research, I created the hypothesis that students would engage in learning and add more meaning to their mathematical understanding when it was presented in a different format. That is, when instruction was not so teacher-led, as I had done for years prior to the Math in the Middle Experience, and included peer presentations of solutions. I also hypothesized that small groups working on weekly Habits of Mind problems improved individual problem-solving skills when they witnessed peer work and had more opportunities to communicate during math class. Finally, I hypothesized that students’ solutions would be more meaningful since writing about one’s own thinking may aid in synthesizing

understanding of a given task. Developing a solid supply of habits of a mathematical thinker would benefit the students. It was my goal during this action research project to model good habits of mathematical thinking as well as point out what I saw students doing that appeared to mimic these habits. When students developed strategies to solve problems, were persistent while pursuing their solutions, were flexible in their thinking and made connections to what they already knew, I noted this to the class or directly to the individual, praising these skills, knowing students built schema for future problem solving.

Developing good habits and problem-solving skills as a mathematical thinker had become more important to me as a teacher after participating in professional development offered by my school district and local university resulting in a middle level mathematics master's degree program that further impacted my beliefs. Based on my professional development, where good mathematical habits and classroom management were modeled, I found myself wanting to convey its importance to my students. During one university course Professor Ken Gross of the Vermont Mathematics Institute stated, "After you solve a math problem - that's when your thinking begins. What have you learned from solving a problem?" He pushed us to think about the importance of thinking about our problem solving and solutions. Achievement and assessments were also of high importance, as I witnessed more testing being put into place in the educational field. In order to encourage growth that positively affected student performance on assessments, I strived to align my teaching practices with the NCTM Principles and Standards and suggested instructional practices. In my role as a teacher and a learner, it was my duty to take the current best

practices into consideration and implement what I could to assist students as they became better problems solvers and mathematicians.

Literature Review

Problem solving was a means to increase mathematical connections to life outside of school as well as other subjects in education. Educators, be they in a kindergarten classroom or a university lecture hall, could find some part of their mathematics curriculum that lends itself well to problem solving and offer the opportunity for connections and deeper understanding beyond temporary learning. While rote memorization had its place in mathematics, so did fostering a learning environment of problem solving with student-peer and student-teacher communication. Educators often expected speedy retrieval from students' memories with rote practice and I planned to slow the process down enough to offer opportunities for immense growth in the area of problems solving. The National Council of Teachers of Mathematics (NCTM) (2000) noted, "Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas" (p. 256). My intent was to bring problems into the curriculum that directly matched objectives required for the sixth graders. I researched online and in educational resources to locate Habits of Mind problems that best matched the skills I was teaching during second semester.

With problem solving as the basis of my research, I also discovered overlying themes emerged while reviewing literature. These themes supported the importance of mathematical problem solving: distinguishing between problem solving and exercises, examining mathematical thinking and the power of problem solving, and understanding the role of teachers and learners.

Distinguishing Between Problem Solving and Exercise

Educators used terms that could be interchangeable: computation, problem solving, story problems and word problems. However, I was interested in how these terms were different and how they affected student understanding. The major distinction between computation and problem solving was the addition of linguistic information that required children to construct a problem model. Whereas a computation problem was already set up for finding a solution, a word problem required students to use the text to identify missing information, construct the number sentence, and derive the calculation problem for finding the missing information (Fuchs, Fuchs, Stuebing, & Fletcher, 2008). The National Council of Supervisors of Mathematics (NCSM) offered a representative and generally accepted definition of problem solving:

Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations ...problem solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. (NCSM, 1989, p. 471)

Distinguishing between problem solving and exercises, as the definition above clarified, was important to my research, since I wanted to increase and improve problem solving in my classroom rather than utilize guided practice and independent exercises 100% of the time. In the past, I had relied on guided practice followed by independent practice heavily. When planning lessons prior to my research I included one “Problem Solving” worksheet with each lesson taught to give students the practice of reading mathematics and solving a problem. However, these problems I assigned were quick to solve and did not consist of many layers to solve. These problems were helpful, however, they were not as challenging as I desired for students who were very bright. Rickard (1996) stated that, through his experiences, practicing a procedure or algorithm helped us obtain a correct answer quickly. However, using an algorithm to explore and express

the relationship of a given problem could be much more challenging for students.

Mathematics was not a spectator sport, in other words. Practice of math skills spiraled throughout the grade levels in such a way that students were building on knowledge in hopes that connections increased achievement and overall understanding. I chose problems that were new to students but required them to rely on previous knowledge to help them make decisions on where to begin solving and how to proceed.

Examining Mathematical Thinking and the Power of Problem Solving

It was critical to teach students to be problem solvers as well as engage students with problems that connected to the curriculum and the interests of students. One needed to know what assisted and hindered the application of problem-solving skills. Andersson (2006) conducted research exploring the contribution of working memory to mathematical word-problem solving in 69 children in grades 2, 3, and 4. Students were given measures of mathematical problem solving, reading, arithmetical calculation, fluid IQ and working memory. Andersson's findings suggested that central executive functions and the phonological loop contributed to mathematical problem solving in children. He noted that chronological age was related to nearly all tasks, especially mathematical problem solving, arithmetical calculation, verbal-fluency tasks and reading. Likewise, age appropriate problems and expectations were of importance and were a challenge for me to generate. With that said, Andersson believed that older children did not have to rely so much on their ability to simultaneously process and store information during mathematical word-problem solving.

Instead they're more efficient word problem solving skill seems to rely on the ability to shift between operations and to instantly activate and retrieve information from long-term memory. The younger children who are less developed in these specific central executive functions (i.e. shifting, accessing information from long-term memory) and reading have to rely more on their ability

to simultaneously process and store information during mathematical problem solving” (p. 1212).

With this information from Andersson, I inferred that sixth graders had acquired adequate opportunities and a lengthy mathematical foundation so that I could comfortably encourage them to advance their problem-solving skills without overwhelming concern.

Language was part of the process of solving problems as was mathematical symbols that needed to be stored with meaning and then retrieved upon command. The development of word identification skill is facilitated by vocabulary knowledge (cf. Perfetti, 1992), the link between word identification and problem-solving skill suggests that language may play a role in math problem solving. Consistent with previous research, reading skill was a unique predictor of children’s mathematical word-problem performance even when arithmetical calculation was included (Andersson, 2006, p.1212). Reading is a complex cognitive activity including processes such as phonological processing and retrieval of long-term information and it might contribute to mathematical problem solving in a number of ways (Hecht, Torgesen, Wagner, & Rashotte, 2001; Just & Carpenter, 1992; Lee et al., 2004). In the class, I researched and found that all students were reading at or above the sixth grade level and students were able to understand exactly which parts of a problem involving reading they could not comprehend so they could ask specific questions. My role as a teacher was to facilitate learning and understanding being mindful of problem adaptability and students’ reading skills.

As an educator, it was important for me to know the best methods to help students access information and make decisions about which tools to access for which situations. According to Schoenfeld’s (1992) research, the tools of mathematics were

abstraction, symbolic representation, and symbolic manipulation, and being trained to use these tools was similar to being able to manipulate shop tools. Knowing how to use these tools did not make a person a craftsman and learning to think mathematically meant; (a) developing a mathematical point of view - valuing the processes of mathematics and abstraction and having the tendency to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure - mathematical sense-making” (Schoenfeld, 1992, p. 3). Fostering this mathematical point of view and students’ confidence in math boosted their problem-solving skills. Using student inventories and interviews, I gathered feedback for the purpose of research to guide my instruction and study the effects of my teaching.

As with any subject taught, there were challenges that must be addressed in my research. For example, Burns (2000) shared that students often had erroneous understandings of algorithms and might not be able to address the concepts that justify the algorithm so that it made sense. Memorizing algorithms could give students confidence to solve problems, but being unable to read and understand a problem was yet another challenge. Just as Kail, Hall, Lee and Swanson (2006) concluded that reading skill correlated to word-problem performance, Fuchs, Fuchs, Stuebing, and Fletcher (2008) explained there were further roadblocks to be cautious of when addressing problem solving. “Results also suggest that poverty and language play critical roles in the development of problem-solving difficulty and that inattentive behavior and poor processing speed may inhibit the development of computational skill” (Fuchs, Fuchs, Stuebing, & Fletcher, 2008, p. 45). This might affect the nine cognitive dimensions: language, semantic retrieval fluency, concept formation, matrix reasoning,

verbal working memory, numerical working memory, word identification, attentive behavior and processing speed. Problem solving needed to engage all learners while language and economic background must not be a limitation.

Role of Teachers and Learners

Student engagement and skill predicted the success of problem-solving practice. Educators preached in the past that students must take charge of their own education. The factors that played a role in the outcome of my research were; student interest in subject matter, the attitudes of the community of learners, and the educator's willingness to encourage risk-taking while supporting students. Each factor was not easy to address separately when observing student successes. Teachers needed to expect students to use the language of mathematics orally and in writing. The following information from *Everybody Counts* (Schoenfeld, 1992, p. 4) typified this view, and echoed themes in the *NCTM Standards* (NCTM, 1989) and *Reshaping School Mathematics* (National Research Council, 1990a).

Mathematics is a living subject which seeks to understand patterns that permeate both the world around us and the mind within us. Although the language of mathematics is based on rules that must be learned, it is important for motivation that students move beyond rules to be able to express things in the language of mathematics. This transformation suggests changes both in curricular content and instructional style. It involves renewed effort to focus on:

- Seeking solutions, not just memorizing procedures;
- Exploring patterns, not just memorizing formula;
- Formulating conjectures; not just doing exercises.

As teaching begins to reflect these emphases, students will have opportunities to study mathematics as an exploratory, dynamic, evolving discipline rather than as a rigid, absolute, closed body of laws to be memorized. They will be encouraged to see mathematics as a science, not as a canon, and to recognize that mathematics is really about patterns and not merely about numbers. (National Research Council, 1989, p. 84)

Mathematical competence and solid skills in the problem-solving realm opened many doors for students' futures as well. Being capable of analyzing situations and

examining their work and thinking gave students an edge. Schoenfeld believed the following to be true:

From this perspective, learning mathematics is empowering. Mathematically powerful students are quantitatively literate. They are capable of interpreting the vast amounts of quantitative data they encounter on a daily basis, and of making balanced judgments on the basis of those interpretations. They use mathematics in practical ways, from simple applications such as using proportional reasoning for recipes or scale models, to complex budget projections, statistical analyses, and computer modeling. They are flexible thinkers with a broad repertoire of techniques and perspectives for dealing with novel problems and situations. They are analytical, both in thinking issues through themselves and in examining the arguments put forth by others. (Schoenfeld, 1992, p. 4)

The issue became not how meaningful the learning was to an individual, but whether learning that was construed as generally meaningful could be committed to memory and reproduced at the right time (Boaler, 1999). My role as an educator in a mathematics classroom was to recognize that environment and how learning styles affected learning. Ultimately, the goal, as teachers, was to implement the curriculum in a meaningful way that made connections so that information was stored in students' long-term memories.

While converting fractions to decimals and percents had been a difficult part of the sixth grade curriculum for students in the past, I hoped to help students commit these skills to memory after assigning a Habits of Mind problem on that topic. I matched Habits of Mind research problems directly to the objectives required enhanced students' learning. For example, the problem "Show Me The Money" was solved by using fractions, decimals, and percents in a way that was interesting and challenging to students. Will Ima Brainiac truly receive the full \$1,000,000 she was promised by the game show by earning a fraction of the million with that fraction decreasing each year? Students found out that Ima will never earn the million dollars in her lifetime and were

very surprised how little she earned after 20 years. One student, Kaylan, stated during her presentation of her solution,

“I didn’t think the cents were imperative so I cut off the cents and everything after the decimal. I just looked at the dollars,” when presenting Show Me the Money. “What I found was that there was a pattern where each amount was half of the previous year. After 20 years she only earned 50 cents for her payment. She is not getting her million so I would sue.”

The yearly payment got smaller and smaller as the denominator of the fraction got larger and larger. During student interviews, this problem was frequently reported as the problem students liked the most. It was challenging and interesting so students enjoyed this Habits of Mind problem. In a small group interview conducted, the following was reported by students when asked the question: What was the most challenging HOM assigned?

- All four students: Show Me the Money.
- Leighton: That was hard.
- Claire: I really liked that one.
- Leighton: The Bonus Box I think was harder; it was a real challenge. It actually got smaller ‘cuz the fractions would get into millionths and billionths.
- Leighton: I felt successful with Show Me the Money but I didn’t feel it was the easiest thing in the world. Like the problems I felt confident with like sharing my work would probably be the Snail Escape and the Three Little Fences and Super Chocolates.
- Claire: I liked Show Me the Money because after you found the pattern it was kind of easier and a lot of people didn’t find the pattern so then it was kind of satisfactory because I did.

Students did not seem as frustrated by converting fractions to decimals and percents this year as they have in past years. It was possible that students this year were more skilled in this area or were a bright bunch of students to begin with. It was also possible that the work in “Show Me The Money” was meaningful and gave them schema to address these problems in the future with little to no confusion.

Of course, achievement and success on assessments drove much of the educational arena and affected my preparations. I planned to encourage growth that

positively affected student performance on assessments by aligning my teaching with the NCTM Standards and suggestions.

Moreover, a growing body of research suggests that students who use mathematics curricula that is aligned with the NCTM Standards, including mathematics as problem solving, tend to outperform students using traditional textbooks (e.g., R. Reys, B. Reys, Lapan, Holliday, & Wasman, 2003; Rivette, Grant, Ludema, & Rickard, 2003).

Rickard also reported that how teachers shape curriculum to meet the needs of their students is informed by their own understanding of mathematics content, problem solving, classroom discourse, and other factors (Rickard, 2005, p.3). My involvement in the intense professional development encompassed in a master's program to benefit middle level mathematics educators (Math in the Middle) surely improved my teaching and learning as well as student learning. My role as a teacher was important and students needed the structure and guidance I provided as noted below by Glenda Lappan, past president of the NCTM in her President's Message (October 1998);

"Teachers are the key to improving mathematics education. ... Regardless of the curriculum or the assessment process in a school district, the person in charge of adapting materials for a particular classroom and student is the teacher. It is through teachers' efforts that students have opportunities to learn mathematics. ... [T]eachers need access to high-quality materials, the support of parents, and ongoing, focused professional development."

As Lappan provided her input on the role of the teacher, Rickard found through research that teacher relationships involving reflection and self-reflecting dialogue bred success. Rickard's observations of a case study involving a middle school math classroom and "Bob", the subject of his research included:

The sustained reflection and dialogue Bob engaged in was a form of long-term professional development that, together with enabling conditions such as access to supporting curriculum materials, in-service, being confident in his mathematics knowledge, and openness to revising his practice, resulted in more closely aligning his practice with reform goals. Moreover, while these enabling conditions are important, it is not certain that Bob would have been as successful in aligning

his practice with problem solving reform goals without sustained reflection and dialogue (2005, p. 14-15).

Active professional development encouraged educators to improve the learning environment and study their own teaching. Through Math in the Middle and the Professional Learning Community in my school I have had more of these reflective conversations and dialogue of mathematics teaching practices. However, this had not always been the case for me. In the past I had limited opportunities to meet with teachers formally to discuss mathematics teaching. Any discussion of mathematics often occurred during a planning session or in an informal lunchtime conversation. I experienced barricades that made collaboration with peers difficult. There were many barricades that led to weaknesses in preparing future and tenured teachers according to Lloyd and Frykholm (1999), authors of *On the development of "book smarts" in mathematics: Prospective elementary teachers' experiences with innovative curriculum materials*. They recognized important issues in the area of mathematics preparation below:

Many prospective teachers possess weak knowledge and narrow views of mathematics and mathematics pedagogy that include conceptions of mathematics as a closed set of procedures, teaching as telling, and learning as the accumulation of information (Ball, 1990, 1991; Brown, Cooney, & Jones, 1990; Frykholm, 1996; Thompson, 1992; Wilson, 1994).

The increased mathematical background and experience in my graduate coursework was what I expected to pass on to students in my classroom. I wanted to use what I had learned to teach students to be better problem solvers.

Conclusion

Being stagnant in mathematics teaching, using rules for algorithms, reinforcing there was one means to an end, and discouraging communication in a classroom does not support the previously described meaningful and lasting change for learning in a

mathematics classroom. Change that was both meaningful and lasting summarized my professional goal and what interested me most in how the increased focus on problem solving would benefit students. As NCTM (2000) noted, “Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas” (p. 256). My goal as a practitioner-researcher was to be able to observe my students while examining my own teaching practices and beliefs with a focus on problem solving in a sixth grade mathematics classroom. The unique relationship I had as the researcher and teacher of students revealed interesting insight and perspectives on learning and problem solving.

Purpose Statement

The purpose of my research was to investigate what happened when I implemented strategic problem-solving skills and increased communication opportunities for sixth grade students through the use Habits of Mind problems in anticipation of improved student confidence and skill retention. Students in my math classes over the 12 years I had been teaching had had one thing in common: they often skipped difficult problem-solving questions on homework or begged to only do homework problems that did not include words. I might even admit that I sided with them at times not wanting to stumble along at explaining the answers and how to arrive at a solution. I also believed that students with reading difficulties might struggle with these word problems so I often assigned rote practice problems.

Then, along came professional development that impacted my thinking and my teaching. Providing one explanation to solve a problem no longer sufficed and I began to feel validated as a learner of mathematics that the way I solved a problem was just as

legitimate as a professional peer who taught high school or college mathematics. My confidence grew and I knew that I could reach more students with my newfound knowledge that how one arrived at a solution was indeed meaningful. I was fortunate to be surrounded by a community of learners that celebrated the process just as much as the solution and I wished to mimic this in my classroom. A solution did not have to look like a polished write-up in a college-level mathematics book, but could be derived by simpler drawings, patterns and other attempts to find a solution. I thought my students would be able to relate to my beliefs and understand how I felt when I solved problems differently than my cohort peers and felt simple-minded in comparison. I wanted students to teach and assist one another so that everyone could be successful in problem solving.

It was my goal to increase the amount of instructional time spent on problem solving to offer more opportunities to develop those feelings of success. I aspired to teach problem solving strategies in such a way that empowered students in their mathematical thinking and increased confidence. In the Holt Course 1 mathematics textbook used by this public school, I had been impressed by George Polya's problem solving steps (Understand, Plan, Solve, and Look Back) and thought there was a valid reason to pay more attention to modeling problem-solving strategies. I wanted students to listen to their peers, parents, grandparents or siblings and thoughtfully solve Habits of Mind problems rather than seeking a quick and concise "correct answer." I wanted to focus ample classroom time modeling these strategies that embodied the "habits of the mind of mathematical thinkers," or skills that were helpful for problem solving.

Observing changes that occurred in students when they were allowed such engaging opportunities was what made teaching fulfilling. I wanted to provide students

the opportunities described in Principles and Standards for School Mathematics (NCTM, 2000) of an ideal classroom:

“Teachers help students make, refine and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it” (Principles and Standards, 2000).

By choosing to research problem solving, I was teaching, reinforcing, and observing students’ written and oral communication. Problem solving, connections and communication were all critical standards laid out by the National Council of Teachers of Mathematics (NCTM) that would drive my research. I wanted to be competent and confident enough to offer students what they deserved and prepare students, while developing the “habits of mind,” for more than just seventh grade.

Research Questions

The purpose of my project was to investigate what happened when I implemented a weekly Habits of Mind problem among sixth grade mathematics students and scored them using a rubric. I examined the students’ solutions, including their ability to explain concepts and their confidence in their mathematical problem-solving ability, while I sought to answer the following research questions:

- What happens to my mathematics teaching when I implement weekly Habits of Mind problems that connect to the curriculum?
- What will student learning look like when I emphasize George Polya’s four problem solving strategies?
- What happens to my mathematics teaching when students present solutions to problems for their peers?
- What happens to student understanding when I require written explanations solving weekly Habits of Mind problems?

Method

Each week students received a Habits of Mind problem (see Appendices A-G) that correlated to the upcoming or current content provided by the school system on the objective card (see Appendix H) for sixth graders during research conducted from February 1, 2009-April 15, 2009. My original plan was to assign the Habits of Mind problem on Friday. The following Friday we would present our solutions and then assign the next Habits of Mind problem. For the most part, this was the case, but, depending on various circumstances, we sometimes had to move the presentation and newly assigned Habits of Mind problem to Monday. If presentations of solutions took the majority of the 60-minute class time, I would wait and introduce the new Habits of Mind problem on Monday to set it up well and not rush a good thing. Stopping a rich discussion of Habits of Mind problems and solutions to move onto something new was not part of my overall goal and I tried to allow learning to occur as needed. I would budget my time later to stay on-track with my overall research and curriculum plans. Students had one week and one weekend, from one Friday to the next Friday, to create a solution for the many layers of the problems by working with peers, independently or with siblings or adults outside of school. Throughout the week, students were given about 30-45 minutes of work time at the end of class in order to work with peers, gauge whether they were on-target, ask questions if unable to progress, and create a written solution for the Habits of Mind problem.

Student solutions were presented the following week using an ELMO projector in order to emphasize the answers as well as the many ways to arrive at an answer. After volunteering to present their solutions, students turned in their Habits of Mind solutions to me and I graded them using a 4-point rubric (see Appendix I) making comments on

their work and giving them feedback. After grading the solutions using the rubric, I chose some additional students to present their work the next week if they had not volunteered to present. If I found an example of something the whole class needed to improve upon or something I wanted to make a special point about, I would ask the chosen student to present. I found this peer modeling to be a helpful way to encourage students and help them gain confidence in my expectations and their work.

The instruments used to collect data included a weekly teacher journal evolving from a pre-made journal prompt (see Appendix J) to guide my thinking and gather needed data from the week. I typed my journals on my laptop, then saved and printed my compiled journals with a copy on my zip drive for safekeeping as well. I kept the journal in a 3-ring binder along with original copies of all my research documents. I also collected selected student work, including a weekly Habits of Mind assignment, any relevant class work, homework, quizzes and chapter tests. These materials were photocopied and kept in files organized by assignment name, class work, quiz or chapter test. In addition, for the purpose of research, I randomly selected 10 students and interviewed them during the lunch period. I used the same questions for each group of students I interviewed and recorded student responses on a paper template (see Appendix K) I created to guide my interview. I also recorded the interviews on my audiocassette recorder to assist in compiling accurate field notes. Finally, I gathered student opinions by conducting a pre- and post-survey (see Appendix L) consisting of questions about mathematics, confidence and interest in mathematics. With these methods in place I was able to collect the data needed to paint a qualitative picture of the experiences during the action research timeframe.

During the timeframe I was conducting research our Professional Learning Community (PLC) consisting of the mathematics department in my middle school chose a SMART-goal of improving open-ended problem solving in grades 6, 7 and 8. These problems we chose were compiled from past National Council of Teachers of Mathematics (NCTM) magazines, problem-solving Web sites, and professional mathematics resource books. We met once each month with the math department consisting of 10 regular education math teachers from grades 6 through 8 to discuss and choose appropriate open-ended problems for each grade level, assess student work and interpret data collected to satisfy district and state School Improvement requirements. In our PLC there is a Math in the Middle graduate in grade 7 and 8 that have completed intensive coursework on mathematics instruction and pedagogy. Upon my graduation we will have one Math in the Middle graduate in each grade level at our middle school. With these experienced teachers and mathematics leaders coming together once a month with a common purpose of improving instruction of open-ended problem solving, students in grades 6-8 at my middle school benefited from this teacher collaboration including increased opportunities to practice and improve upon problem solving skills.

As a PLC we discussed areas in mathematics we would like to see our students improve upon and decided on a goal that spans across three years in the middle level experience to accumulate baseline data and a large quantity of data tracking students and trends as students move from 6th to 7th grade and 7th to 8th grade. I had already spent a great deal of time collecting Habits of Mind problems which are open-ended problems similar to what our PLC decided to use and I knew I wanted to incorporate problem solving in my action research. During the 2008-2009 school year each math

teacher gave four open-ended problems and scored them on a 4-point rubric. Our goal was to have all students achieve a 3 or 4 average on all four problems to be considered proficient. These problems took students about 20 minutes to complete in one class period. We did not spend this much time on one problem on a normal basis due to the curriculum we taught consisting of many objectives and we had a limited amount of time. Our PLC found open-ended problem solving similar to Habits of Mind problems an important aspect to study and improve upon as it was often being overlooked in our classrooms and I suspect this additional practice in problem solving affected my students.

Findings

Conducting research on problem solving allowed me to examine individual progress on mathematical objectives I was teaching. I was able to gather information on the whole group by anonymously gathering student beliefs about math in general and my classroom. I collected data by using 16 statements where students marked a Likert scale choosing among always, sometimes or never. It was my intention before starting action research to gather baseline data on whether students were open to working in groups, sharing their mathematical ideas with the whole class or working problems more independently from the teacher. Based on the results, my initial impression was that this group of 29 students felt they received good grades in math and liked math, were open to working in groups, and were open to working on math problems that really made them think.

Some of the data that I collected did not change from the pre- to the post-survey. The same results occurred from the pre-survey to the post-survey in problems 1, 3, 4, 6,

7, and 16. Students looked forward to coming to class, believed they got good grades in math, believed math was something they did needed to understand, and sometimes liked when math problems really made them think. I would consider the students in my class to be conscientious students who wanted to do well so I was not surprised by these results.

After looking at student responses I wanted to pay special attention to the questions that best matched my action research plans. I believed the most important statements I should focus on were; 9.) Working in groups helps me understand math better, 10.) I'm not sure of my answers, even on simple problems, 13.) I am comfortable sharing my mathematical ideas with the whole class, and 16.) I understand math better when we do activities rather than the teacher just explaining it from the book. With these four statements the number of students agreeing with the statement increased except for number 16 which showed identical responses from the pre- to the post-survey. In my action research I used small group work and wanted students to experience that working with peers can improve learning and communication of mathematics, as indicated in statement number 9. I was interested in students' confidence in statement number 10 and was pleased that the number of students stating they felt comfortable with a simple answer increased. Another important aspect of the action research was for students to present their solutions to the whole class and learn from one another. I suspected that some students would be leary of presenting in front of their peers and others would be volunteering to present solutions of the assigned Habits of Mind problems. The number of students indicating they were comfortable presenting mathematics in front of their peers increased which I feel has a direct correlation to the opportunity given to present formally to peers using the ELMO. These statements

assisted me in formulating conclusions about whether my action research had a positive effect on students in my mathematics classroom.

When looking at the pre-survey data, it showed that 21 out of 29 students or 72.4% of the students believed activities improved their learning over teacher lecture. With that evidence, I believed I had the right group with the right attitude for implementing Habits of Mind problems. I believed students enjoyed and engaged in learning math objectives with challenging problems compared to when I used more lecture and step-by-step methodical teaching styles. When reflecting upon my research question, "What happens to my mathematics teaching when I implement weekly Habits of Mind problems that connect to the curriculum?" I made the assertion that students learned to solve mathematical problems and use many different methods to solve the same problem as well. Madeline gave helpful feedback when asked, "What is the most interesting idea you have learned this year?" on the final Math in the Middle student survey given with the alternate assessment by stating, "That you can figure out math problems more than one way. So you can see the problems in different ways." Students were actively engaged in learning math objectives with these unique, weekly Habits of Mind problems and shared their favorite Habits of Mind problems in the student interviews I conducted. I did not have anyone outwardly objecting to the assigned Habits of Mind problems and I found they were open to a change from the monotony of school in the second semester.

There were many factors that influenced students' beliefs, but I believed the climate of my classroom and changes in teaching had an effect on their attitudes about mathematics. On April 19, 2009, I wrote,

I've seen students take on the Habits of Mind problems without complaint, with confidence and with comfort to work in groups and ask questions. I've seen risk-

taking and growth. I am proud of the changes I've made and the work my students have done.

During the student interviews conducted Leighton mentioned, "It (Show Me The Money) was a challenge but it was harder to get it on my own because I rounded too much so I had to go back and fix it". As Leighton worked with Kayla each week along with a couple of other students, I noticed how much they relied on one another to interpret the Habits of Mind problems given and achieve a solution so I knew that working in groups helped them to better understand mathematics. Leighton added, "I liked working in groups because you get other peoples' opinions and it's kind of nicer that way." Hallie said, "It's good to get started as a group and then go home and figure it out. I like to choose my own groups because some people like to mess around and they take all the answers". Savanna added, "Some problems that I don't really get, I like to see what other people do to see if I know it [the answer]". When referring back to my research question, "What happens to student understanding when I require written explanations solving weekly Habits of Mind problems?" I made the assertion that students improved their mathematical communication by writing explanations and labeling their work thoroughly. Students were able to explain their thinking as well as verbalize their work with very little prompting from their teacher. Peers benefited from witnessing each other's work and were able to stretch their thinking and communication with collaboration from peers.

I interpreted other improvements to be apparent in numbers 2, 5, 8, 9, 10, 12, and 13. Number 13, I am comfortable sharing my mathematical ideas with the whole class, was a big "ah-ha" for me as I found that students stated they were more comfortable sharing solutions in class. Twelve out of 29 (41.4%) students marked "always" on the pre-survey and 15 (51.7%) marked "always" after the action research concluded. I attribute this to more opportunities to share, the presence of an ELMO

projector in my classroom offering a way to share with the whole group easily, and the positive climate in the classroom where students feel more comfortable sharing with the large group and willing to take risks. The question, “What happens to my mathematics teaching when students present solutions to problems for their peers?” is one that I believe can be answered by stating that the learning community changed instantly as several of the same students volunteered to present each week. Three students in the class presented their solutions after each Habits of Mind problem assigned. Four others presented during at least three problem presentations out of six assigned. In other words, there was a desire to communicate their mathematics with their peers when given the opportunity. By using the ELMO when presenting solutions I asserted that my mathematics teaching caused students to understand that there were many ways to interpret a problem eliciting different answers some of the time and that there were numerous ways to solve a problem. By viewing peer diagrams, models, patterns found, tables and lists in students’ presentations, students saw first-hand signs of growth and accomplishment and acknowledged one another’s work as correct even with the many differences in solutions.

Students also began to mimic solutions other peers had presented by drawing, making a table, finding patterns and more. By using the ELMO in my classroom, I was able to witness students understanding there were more ways than one to interpret problems. Students were able to explain their thinking as well as verbalize their mathematical processes while presenting with very little prompting from me. One student, Madeline, struggled mightily when the Habits of Mind problem, “Show Me The Money,” was assigned. She had a lot of questions and a look of despair on her face. Throughout the week, she kept asking questions to understand the problem assigned

and even received help from her older sister. On the day of the presentation she came in all smiles and said, "I got the H. O. M.!" She was so proud of herself and asked if she could be the first presenter showing the solution. This was the very first formal presentation we had, beyond sharing homework on the ELMO, so I was impressed by her confidence and it was so enjoyable to watch her present with pride using the ELMO with a written explanation she had even typed for the big day. Here is some of her work:

$\frac{1}{3}$ of 1 million
 333333.40

+

$\frac{1}{6}$ of 1 million
 166,666.67

+

$\frac{1}{12}$ of 1 million =
 83,333.34

+

$\frac{1}{24}$ of 1 million
 41,666.67

+

$\frac{1}{48}$ of 1 million =
 20,833.35 $\div 2$

645,833.44 after he 5th payment

60% of 1,000,000 is 600,000

She gets half of what she got the year before.

500,000.7 \star

583,333.41 \star

625,000.08 \star

$\frac{1}{24} \div 24$ of $\times 1,000,000$

Year 4
 5 (half of previous year payment)

did she get a...?

She was not the top student in my class when it came to assessments, but she certainly had great success with this problem in just one week's time.

After witnessing the excitement by several students over presenting solutions to Habits of Mind problems using the ELMO projector, I would have liked to spread the presentations across more students during my action research. I frequently had the same students wanting to present solutions and I allowed those who wanted to share the time and opportunity to do so. After all, their excitement would be contagious and I hoped students would be more positive about math and problem solving overall. It was my belief that forcing students to present to their peers might have a negative affect on their attitudes toward problem solving, Habits of Mind problems, and mathematics in general. After charting names of students who presented, I found there were seven students out of the 29 who presented the most, which gave many students the opportunity to sit back and watch math happen. When reflecting on my research question, "What happens to my mathematics teaching when students present solutions to problems for their peers?" I think that presentations impacted many, but I could reach more and do better by changing my expectations so that each student is required to present his or her solution to a Habits of Mind of choice at least one time per quarter. Making this part of the summative grade will raise their level of concern and interest and push them to write a complete solution they would be proud to share.

While taking graduate mathematics courses, I was able to experience technology that aids in the ability to have impromptu classroom presentations. Technology that allowed students to showcase their work and gain immediate feedback while increasing their level of concern through this opportunity to model problem solving skills for one another has been a positive factor in my classroom. An ELMO projector was used during the research and we all benefited by the opportunity to quickly show our work, explain our processes, ask questions, and add to our work all the while having the

whole class as an audience. Students were also more interested in coming to the projector to show work than I have experienced in the past. I have used an ELMO in my classroom for about 9 weeks during the 2007-2008 school year. Having the ELMO for second semester during the action research was critical to the success of student's learning and viewing models of great work done by peers.

With peers presenting mathematics solutions and group work becoming a bigger part of my classroom climate, students indicated they felt more comfortable asking questions in math class on statement number 12 of the attitude inventory. Confidence on even simple math problems improved on statement number 10 and students preferred to work on math more than other subjects on number 2. Other statements that were notable to me were numbers 14 and 15 where students felt working in small groups just made them more confused about math and students were more comfortable sharing their ideas in a small group than with the whole class. Shari wrote on the interest inventory,

I would not prefer to work in groups but if I had to I would pick: Braeden because when I don't understand a math problem I can go to him, Angel because she has very good ideas on challenging problems, or Claire because if I don't understand something she gives examples of my life or hers.

I found, based on interview responses, that students were sometimes more confused because of how each individual interpreted the problem given. Students indicated it was confusing when another student got a different answer and made them unsure of whether their own answer was still correct. Students are programmed and accustomed to having one correct answer and are now being stretched to understand there can be several interpretations and ways to get a solution that satisfies a habits of mind problem. In my teacher journal on April 19, 2009 I wrote,

During interviews I had students who admittedly get confused by peer solution presentations when they do not have the same answer as the student presenting. This is understandable since they were starting to question whether their answers were correct so their confidence was shaken. I don't believe that students are always completely comfortable with the answers peers give as some solutions are more clear (simple) than others. They find it frustrating and then want to resort back to the teacher just telling them the answer. This does show the tensions in the type of teaching and learning occurring in my classroom.

In addition, I believed that some of the habits of mind problems have been rigorous enough that the students were not as willing to share those answers with the whole class or were not interested in giving their interpretation orally. I believed, with more time for action research, I would have been able to have more students take risks and increase their confidence. In future years, I would be better able to anticipate the interpretations of the new problems I used this year and be able to boost the confidence of students with my own increased experience, confidence and abilities.

Student Mathematics Attitude Inventory

	Always		Sometimes		Never	
	Pre-Survey	Post-Survey	Pre-Survey	Post-Survey	Pre-Survey	Post-Survey
1. I look forward to math class.	16 55.2%	16 55.2%	13 44.8%	13 44.8%		
2. I would rather work on any other subject than math.			13 44.8%	14 48.3%	15 51.7%	15 51.7%
3. I get good grades in math.	24 82.8%	24 82.8%	5 17.2%	5 17.2%		
4. I think math is something I don't need to understand.			3 10.3%	3 10.3%	26 89.7%	26 89.7%
5. I admire other students who are good at math.	11 37.9%	13 44.8%	15 51.7%	13 44.8%		

6. I like when math problems really make me think.	7 24.1%	7 24.1%	16 55.2%	16 55.2%		
7. I don't think I am good at math.			7 24.1%	7 24.1%	21 72.4%	21 72.4%
8. I think some kids just know how to do math and some kids just don't understand it – and studying more won't help.			8 27.6%	5 17.2%	20 70.0%	23 79.3%
9. Working in groups helps me understand math better.	9 31.0%	9 31.0%	17 58.6%	18 62.1%		
10. I'm not sure of my math answers, even on simple problems.			13 44.8%	9 31.0%	14 48.3%	18 62.1%
11. It is important for me to do well in math.	28 98.6%	26 89.7%	2 6.9%	3 10.3%		
12. I feel comfortable asking questions in math class if I don't understand something.	19 65.5%	22 75.9%	8 27.6%	6 20.0%		
13. I am comfortable sharing my mathematical ideas with the whole class.	12 41.4%	15 51.7%	16 55.2%	12 41.4%		
14. Working in small groups just makes me more confused about math.			6 20.0%	12 41.4%	20	15 51.7%
15. I am more comfortable sharing my ideas in a small group than with the whole class.			16 55.2%	18 62.1%	6 20.0%	2 6.9%
16. I understand math better when we do activities rather than the teacher just explaining it from the book.	21 72.4%	21 72.4%	6 20.0%	6 20.0%		

Students learned to solve mathematical problems and used many different methods to solve the same problem with 29 students to compare and collaborate. The survey results showed students enjoyed and engaged in learning math objectives with challenging, weekly Habits of Mind problems compared to when I just explained math from the book. In relation to Polya's problem solving steps, students learned by using

the strategies of understanding the problem, making a plan, solving the problem and looking back to make sure the solution makes sense. While Kayla and Leighton worked together on “Show Me The Money” they used their calculators to find $\frac{1}{3}$ of 1,000,000 dollars. The answer was 333,333.3 repeating and they had to continue with the problem solving steps to make sure they understood whether they knew how to change this to dollars and cents and whether it was a logical answer. Once they created a table they began to find patterns that helped them move forward with confidence.

Years	Amount
3 years	1,000,000
2 years	500,000
1 year	250,000
0 years	0

Ima wants to know if the P&H and final payment is really \$1,000,000

After five years Ima

5	20,000	639,375
6	10,000	649,375
7	5,000	659,375
8	2,500	669,375
9	1,250	679,375
10	625	689,375
11	312.5	699,375
12	156.25	709,375
13	78.125	719,375
14	39.0625	729,375
15	19.53125	739,375
16	9.765625	749,375
17	4.8828125	759,375
18	2.44140625	769,375
19	1.220703125	779,375
20	0.6103515625	789,375

330,000
100,000
47,500 = About 5%

47,500
12,500
57,500 = About 6%

20,000 = 639,375

After 20 years Ima finally has 789,375

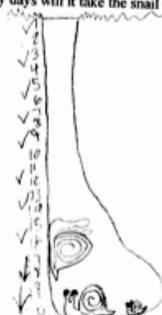
NO!

They learned from peer work and solutions shared on the ELMO and by doing partner and small group work. They were carefully solving problems by using the following strategies, generally, when attempting a Habits of Mind problem:

- *Draw a Diagram
- *Make a Model
- *Guess and Check
- *Work Backward
- *Find a Pattern
- *Make a Table
- *Solve a Simpler Problem
- *Use Logical Reasoning
- *Use a Venn Diagram
- *Make an Organized List

When compiling data to reinforce my assertions for the question, “What will student learning look like when I emphasize George Polya’s four problem solving strategies?” I made the assertion that students learned by using the strategies of understanding the problem, making a plan, solving the problem and looking back. They were learning from peer work and solutions shared on the ELMO and by doing partner and small group work. One student drew a hole and a snail climbing out of it in the Habits of Mind

1. Now that Spring is upon us the creatures that have been gone for the winter begin to reappear. One lonely snail was in a hole that was 20 feet deep. He is able to move 3 feet up the hole each day, but retreats down the hole 1 foot each night seeking warmth. How many days will it take the snail to climb out of the 20 feet deep hole?



one day makes 2 feet - 2 day so it would take 10 days to make it out.

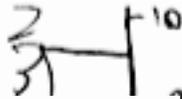
2. How many days will it take the snail to climb out of the 20 foot deep hole if he moves up 3 feet each day and down 2 feet (in a 24 hour period)?



It only makes +1 foot a day so it would take 20 days. +3 - 2 = 1

3. What if we have an extra...

problem “Snail Escape,” while another student drew a number line and used movements upward (positive integers) and movements downward (negative integers) in her explanation.



H.O.M.
Snail Escape

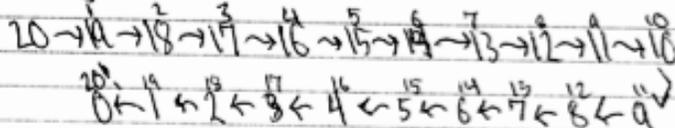
With the 1st snail he was moving up + out 2 feet a day out of a 20ft. hole he would start at 20 so really, instead of 20:2 of $\frac{20 \text{ feet}}{2 \text{ feet}} = 10$ days you would need to do:

The formula was: $\frac{19 \text{ feet}}{2 \text{ feet}} = 9.5$ days

$9.5 = 9$ because you could just count \rightarrow but if you would do it by climb + sleep you would get $X = \text{climb}$ $H = \text{sleep}$

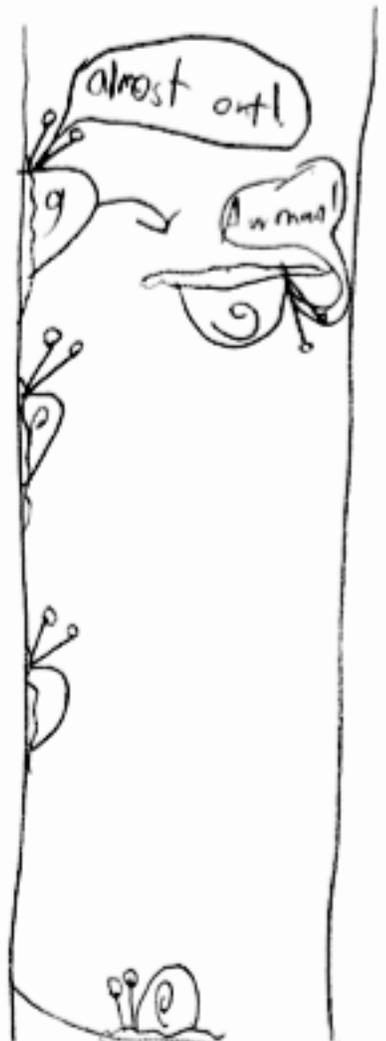
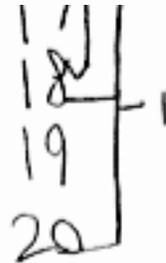
So you can get 9.5 + round up to 10 or you can also get 10 out of all the ways I did it I got 10 most often.

If the snail only went 1 foot a day starting at 20 to 0 it would take 20 days because of having to go to 0 after 1



If the snail traveled 4ft each day it would take him 4 days

19, 17, 15, 14, 13, 12 ~ 11, 10, 8, 7, 6, 5, 4 ~ 3, 2, 1, 0



Chances Are ...

On the top 1-4

1.) If you take the 10 min during 42 min. class you can make it: $10:42 = \frac{10}{42}$ and you have to change it into a percent so, you have to simplify it and you get $\frac{5}{21}$ so, then divide $5 \div 21$ to get a percent which is, 23.8%.

2.) If you take the amount of homework and the days you can make it $2:5 = \frac{2}{5}$ and you have to change it into a percent so, you have to simplify it and you get the same number because it is already simplified. So, then divide $2 \div 5$ to get a percent which is, 40%

3.) If you take the 9 periods of labs and the 6 weeks of school you can make it $9:30$ (because of the 6 weeks $5 \times 6 = 30$) $9:30 = \frac{9}{30}$ and you have to change it into a percent, but first simplify it to $\frac{3}{10}$ so, then divide $3 \div 10$ to get a percent which is, 30%.

4.) If you take the number of problems the teacher lets the students use it is $3:11 = \frac{3}{11}$ so, you have to simplify it but you can't because it already is so you have to get a percent you have to divide $3 \div 11$ which is, 27.2%

On the Bottom 1-3

1.) The probability for 1-4 is: 1.) $10:42$; 2.) $2:5$; 3.) $9:42$; and 4.) $3:11$.

Turn →

On the
HOM

“Chances Are” students made tables to show their daily routine and the percentages of the 24 hours each activity represented. Once one student found a strategy that worked, other peers nearby often followed suit or tried this strategy.

The chart on the right shows the percent, decimal, and fractional part of the day that each category or activity consumes for one student on the Chances Are Habits of Mind solution.

Event	Time	Time spent	Percent	decimal	Fraction
Sleeping	9:00-6:00	9 hours	37.5%	.375	$\frac{9}{24}$
Going to school	8:00-3:00	7 hours	29%	.29	$\frac{7}{24}$
Doing Homework	3:30-5:00	2 hours 30 min.	10.4%	.10	$\frac{2.5}{24}$
Eating	5:00-9:30	30 min.	2.1%	.02	$\frac{2}{24}$
Talking/Text	5:30-5:45	30 min.	2.1%	0.02	$\frac{2}{24}$
T.V.	7:30-9:00	1 hour 30 min.	6.3%	.06	$\frac{1.5}{24}$
Friends/Drinks	6:00-7:00	1 hour	4.2%	.04	$\frac{1}{24}$

On the M² Mathematics Beliefs Inventory completed in the spring of 2009 following the action research conducted, 88.9% of the students (24 out of 27 with two students absent) marked one of the following; Always, Usually, Half the time, or Sometimes, for the statement, “We write about our ideas in math class.” Only three students marked “Never” for this statement. Writing about our solutions and mathematical thinking has become part of the culture in our classroom. For the statement, “We talk about how different ideas in math are connected to each other,” 100% of the students marked Always, Usually, Half the time, or Sometimes for this statement. Again, communicating mathematics was an important part of math learning.

By using the ELMO while presenting solutions, my mathematics teaching caused students to understand that there were many ways to interpret a Habits of Mind problem, which could elicit different answers some of the time, and that there were numerous ways or paths to take to solve a problem. When students were required to have written explanations while solving weekly Habits of Mind problems, they improved

their mathematical communication by explaining their thinking and labeling their work thoroughly so peers could follow their work and understand their solutions. Below are examples of written or typed solutions one student created for Rectangle Area and Three Little Fences that show the ability to articulate the mathematics needed to solve the problem assigned.

I found the answers by using formulas like for the rectangle I (we) used



A = Area
L = Length
W = Width

And for the Area of a square we did $64 \div 4 = 16$ and took and used the same formula as rectangle and we did the area of circle as a class using pie.

The 3rd pig has the largest farm with the 2nd pig and the 1st. The largest rectangle the 1st have as a yard would be 176

Students were able to explain their thinking as well as verbalize their work with less prompting from me

as time went on during the action research. Two students in particular stand out that

H.O.M.
The Three Little Fences

1st little pig.....255 sq. ft.
2nd little pig.....256 sq. ft.
3rd little pig.....326.69 sq. ft.

In order the from largest to smallest:
3rd pig-326.69 sq. ft.
2nd pig-256 sq. ft.
1st pig-255 sq. ft.

The 1st little pig with a rectangle yard could have at the biggest size is a 255 sq. ft. yard.

The Cow Jumped Over the Moon



If the cow jumped over a full moon that had a circumference of 6,790 miles and it was 221,463 miles from the earth the moon. How many feet would the cow have to jump if it went up to the moon, over the top, and back to earth?

Answer: 1,205,175,840ft that the cow jumped

work
 $216,790 \div 2 = 3,315$
 $\times 14$
 $\times 10$
 $\times 10$
 ft. in a m.c.

were quite different as students, but showed fantastic work from start to finish during research conducted. Braeden is a student who possesses strong habits of a mathematical thinker, algorithms and recall of facts. He is confident in his mathematical abilities and was able to assist his peers when they struggled. I was concerned about how I might challenge Braeden all year and was confident he would be placed in pre-algebra for seventh grade due to his mathematical skills. Braeden always passed all of the test objectives and always got an A on his chapter tests. He worked hard and excelled on the Habits of Mind problems while exceeding my expectations. After I modeled a typed solution to students he presented his first solution to Show Me the Money below with accuracy, neatness and confidence.

		1,000,000.00
1/3	33.3333333%	333,333.33
1/6	16.6666667%	166,666.67
1/12	8.3333333%	83,333.33
1/24	4.1666667%	41,666.67
1/48	2.0833333%	20,833.33
		645,833.33
1/96	1.0416667%	
1/192	0.5208333%	
1/384	0.2604167%	
1/768	0.1302083%	
1/1536	0.0651042%	
1/3072	0.0325521%	
1/6144	0.0162760%	
1/12288	0.0081380%	
1/24576	0.0040690%	
1/49152	0.0020345%	
1/98304	0.0010173%	
1/196608	0.0005086%	
1/393216	0.0002543%	
1/786432	0.0001272%	
1/1572864	0.0000636%	
1/3145728	0.0000318%	
	66.6666349%	

We took the fraction and divided. Example $1/3 * 1,000,000$ equals 333,333.33. So we did that for every year. Then we made a total amount and added each year's profit. If you add all of the total profits up she will only have about 65% of her money by the fifth payment.

In the next 20 years she will not have her full million dollars. She will only have \$666,666.67. *NOPE 66 of 67%*

On the other spectrum was a student named Hannah who struggled on daily work, was timid and lacked confidence in her mathematical abilities. Hannah was diagnosed with Selective Mutism and did not often initiate conversations or questions with me or her peers. She struggled to keep up with homework. Her mother and I had been in contact about our concerns for Hannah and the need for her to come in for extra help. She frequently retested objectives on her chapter tests and needed extra time to finish tests. Her first Habits of

Mind solution, Show Me the Money, was more of a creative work of art than a complete answer to the multi-layered problem. While she did write ideas down during student presentations, she had minimal work to turn in. However, her explanations began to improve and soon I could see she communicated very well in written form. Her solutions were labeled, accurate and in-depth. I even used one of her PLC problem solutions as a peer sample of what a great explanation looks like.

(Show Me the Money - first solution)

(Extension to Three Little Fences - Improved write-up)

ANSWER-By Hannah
 Irma will NOT get her millions because after 5 years she will only get 20833.33 of her millions. I got this by dividing 3, 6, 12, 24, & 48 by 1 million to get my answers

bands box poor HUMPTY DUMPTY

Humpty Dumpty climbed up a wall. He fell down ~~24~~ ~~24~~ 24 ft. ~~With~~ Dr. Bob Couldn't figure out how many ~~E~~ bones he broke. He had to use math to figure it out. If every 2 ft. Humpty fell, he broke a bone, & ~~2~~ he hit 6 trees on the way down. ~~He broke~~ ~~1~~ bone & 2 bones per tree. when he landed on the ground, he broke many bones from the # of ~~words~~ 34-35. It is an even & ~~E~~ a Composite #. It is the # of how many teeth a polar bear have. If you divide it by ~~3~~ 3 you'll get the answer of 14. How many bones did humpty break in all?

Emergency Hospital

J L
 Poor Humpty Dumpty.

Answer

Dr. Bob

Hannah's willingness to try, and ability to write made all the difference in the world for this student who struggled on daily assignments. I was proud of her effort and improvement. The next table shows the culminating scores for each Habits of Mind problem solution handed in and graded using the rubric for scoring Strategy and Procedures, Neatness and Organization, Explanation, Mathematical Concepts, and Mathematical Terminology and Symbols. You can see that Braeden was proficient with 80% or above on each solution while Hannah was proficient with 80% or above on only

3 out of the 6 problems. However, her last three problems were the best scores and she improved as time passed.

Habits of Mind Problem	Show Me the Money	Chances Are	One-Wheeled Wonder	Snail Escape	Three Little Fences	Rectangle Area
Hannah's scores (out of 20 possible points)	10/20	12/20	11/20	20/20	16/20	16/20
Braeden's scores (out of 20 possible points)	20/20	16/20	16/20	19/20	19/20	20/20

It is also helpful to compare the number of students proficient in the entire class on each of the assigned Habits of Mind problems as you can see below;

Habits of Mind Problem	Show Me the Money	Chances Are	One-Wheeled Wonder	Snail Escape	Three Little Fences	Rectangle Area
Number of students scoring 80% & above	18 (62.1%)	13 (44.8%)	10 (34.5%)	22 (75.9%)	20 (69.0%)	20 (69.0%)
Number of students scoring 79% and below	11 (37.9%)	16 (55.2%)	19 (65.5%)	7 (24.1%)	9 (31.0%)	9 (31.0%)

Students proficient for the Habits of Mind problems during the action research varied from 34.5% to 75.9%. Why are the percentages so low? Many factors play into the assessment of student solutions. First, the difficulty level of the problem and amount of background knowledge needed to solve a problem has an effect. One-Wheeled Wonder became a difficult problem due to the multiple interpretations that

were occurring for what the problem was asking to solve. I soon called it my “One-Wheeled Disaster” after seeing student solutions all over the board and confusion on students’ faces. However, it was an opportunity to show students that interpreting and understanding a problem plays a role in how each of us attempt to solve the problem and that, essentially, each interpretation with a valid explanation is accurate. This was another opportunity for me to show why giving an explanation of your thinking is so valuable. Next, I had to assess whether a problem was a correct fit for the age/grade level, whether it matched the curriculum, was too wordy, lacked clarity, or, on the other end of the spectrum, was not challenging enough. Finally, I found my rubric needs tweaking as it was difficult to get 4’s. Some of the solutions would have been a 3 or 4 or above 80% proficient based on some of the work shown by students. In turn, my rubric does not fit my expectations now that I have had the opportunity to utilize the rubric.

In April, we compiled student data collected in the 2008-2009 school year and found that 51 out of 256 sixth graders had a 3 or 4 on their PLC problems using a different rubric than what I chose to implement during action research, but with similar attributes. This equates to 20% of the sixth graders scoring in the proficient range. In seventh grade 110 of 238, or 46% of the students were proficient, and in eighth grade, 84 of 257, or 33% of the students were proficient on PLC problems. Within my own classroom 9 out of 29 students received 3’s and 4’s on average, which was 31% of the class scoring in the proficient range. Even though these percentages seemed quite low, I considered this to be a success as this group of 29 students achieved an overall average higher than the 20% average for all sixth graders in the middle school. Although this percentage seemed low, the highest percentage for all three grades was in seventh grade at 33% proficient so 30% of my class scoring proficient was good.

These scores would likely improve as students have more opportunities to complete open-ended problems, view peers' work and more teacher modeling of problems occurs. The scores were likely so low at this time because of several factors. The lack of experience with these types of layered story problems played a role in student performance as well as the difficult task of choosing appropriate problems for the grade level matching the curriculum. Since reading and comprehending the problems were a factor, this could be challenging for students who struggle in reading or are below grade level. A problem with too many words can be difficult for students to find the critical information and solve correctly.

Conclusions

Completing action research brought exciting changes into my classroom and I was able to implement what I learned about problem solving during my Math in the Middle experience. Using small groups, leading large group discussion, offering opportunities to build upon their problem-solving skills and improving habits of mathematical thinking were taken from Math in the Middle and put into play as these were practices that were good for kids. The National Council of Teachers of Mathematics (NCTM) (2000) published the following; "Problem solving is central to inquiry and application and should be interwoven throughout the mathematics curriculum to provide a context for learning and applying mathematical ideas" (p. 256). This summed up my attempts and actions during the action research and was achieved by aligning expected curriculum objectives with the open-ended, challenging Habits of Mind problem solving I implemented weekly.

Along the way, I found that student solutions to open-ended problem solving became more detailed and students showed more work than they had in the past prior

to my action research project. Student beliefs and attitudes changed moderately in the group of students I had, but after conducting a pre- and post-inventory, I found that they were positive overall even though the results did not change greatly. These students came into my classroom with mostly positive experiences and attitudes toward mathematics.

My overall impression of my action research journey was that students were more willing to solve Habits of Mind problems after several positive experiences coupled with the ability to retrieve information and assistance from peers, the teacher or family members. Students quickly adapted to the challenge of writing solutions that showcased their learning. These students were age 11 or 12 and have a lot of background experience and schooling to pull from when given the assignment of completing a complex Habits of Mind problem. During my professional researching of problem solving I found that Andersson (2002) stated students' more efficient word problem solving skill seemed to rely on the ability to shift between operations and to instantly activate and retrieve information from long-term memory. I committed my role as a teacher to asking meaningful questions when assistance was needed. I monitored the students' ability to shift operations and activate information by giving just enough assistance to help them move forward and progress on the problem assigned.

Many students researched were skilled at solving open-ended problems early on in the school year and continued to improve upon their skills and habits of mind throughout the research timeframe. I believe some students have developed their habits of mind of a mathematical thinker in past experiences or are predisposed to think and solve math problems well. However, some students struggled to complete these open-ended problems and several turned nothing in when the time came to present Habits of

Mind problems. The same five students inconsistently turned in homework and HOM solutions during the research. I found that successful students appeared to have the disposition to be better problem solvers with minimal experiences other than the four PLC problems and monthly SIGMA problems produced by Dr. Steve Dunbar of the University of Nebraska-Lincoln. SIGMA is a mathematics competition that schools can choose to participate in each school year with challenging, multi-step problems. Some students had great responses that showed they were proficient in their problem solving. In some aspects, I thought the “treatments” I performed during action research might or might not have greatly improved their habits of a mathematical thinking. They did get opportunities to communicate in mathematics like they may not have in the past, however.

Just like some people were skilled in spelling and others were definitely not, I found that some students were skilled in problem solving and needed little coaching from me while others were putting effort into the process and still struggling to understand the problem or solution. This idea interested me and I wondered if, in time, the skills would develop given more and more opportunities to solve Habits of Mind problems and view peer work in small groups. I also knew that reading ability was directly related to success in problem solving. Consistent with previous research, reading skill was a unique predictor of children’s mathematical word problem performance even when arithmetical calculation was included (Andersson, 2006, p.1212). I could research students’ reading history and utilize this data when gaining understanding to why some students were skilled and some students struggled mightily with Habits of Mind problems.

In addition to problem solving being challenging for students overall, as a PLC and in my own preparation for research, I had difficulties in choosing appropriate open-ended problems. As a PLC, we knew how hard it was from one problem to the next to see if it was a good fit for age level or grade level, if it matched the curriculum or background experiences, if it was too wordy or lacked clarity, or was not challenging enough. For that reason, I found it difficult to make conclusions like, “Chapter test scores improved because of the learning during Habits of Mind problems,” or, “Students improved from the first Habits of Mind problem to the last based on data collected,” because each problem was entirely different. Making assertions about how deeply student learning was affected was difficult when these problems and student learning was layered and complex as in this case of studying student problem-solving techniques, thinking, processing, previous knowledge and the implications upon their learning.

While I made the assertion that the payoff of my efforts and changes in teaching were worthwhile for students, I did recognize that these changes came at a cost. I knew that assigning Habits of Mind problems took time in the classroom and outside of the classroom and that was one reason I made a concerted effort to find time for a weekly Habits of Mind problem to see how it would change my teaching and change students learning. Teaching step-by-step processes to solve problems became very comfortable to me and was not always easy to abandon. Notetaking and rule-following algorithms had their place in mathematics but I wanted students to move beyond this comfortable format. I knew students might not like the changes from their past classroom experiences, since the structure of my teaching may differ as there was a level of organized chaos involved in learning and working in a classroom that allowed group

work, collaboration and communication. Although the language of mathematics was based on rules that must be learned, I found, through researching numerous articles and readings that it was important for motivation that students move beyond rules to be able to express thinking in the language of mathematics. This transformation was found in changes in both curricular content and instructional style. This transformation was what my research was all about.

Implications

Conducting research in one's own classroom and making a concerted effort to reflect on the teaching was a valuable experience for me as an educator. The extensive reading of current research and practices impacted my learning and teaching as well. With this information and increased confidence came the ability for me to implement change in my classroom that was endorsed by the National Council of Teachers of Mathematics. I desired to be "someone who possesses mathematical knowledge for teaching (Ball, Thames, & Phelps, in press) and pedagogical mathematical habits of mind and was able to do the special work of mathematics teaching" and do these things:

- Presenting mathematical ideas
- Responding to students' "why" questions
- Finding an example to make a specific mathematical point
- Recognizing what is involved in using a particular representation
- Linking representations to underlying ideas to other representations
- Connecting a topic being taught to topics from prior or future years
- Explaining mathematical goals and purposes to parents
- Appraising and adapting the mathematical context of textbooks
- Modifying tasks to be either easier or harder
- Evaluating the plausibility of students' claims (often quickly)
- Giving or evaluating mathematical explanations
- Choosing and developing useable definitions
- Using mathematical notation and language and critiquing its use
- Asking productive mathematical questions
- Selecting representations for particular purpose

- Inspecting equivalencies

(Ball, Thames, & Phelps, p. 34)

So much of this was accomplished by including more large-group discussion and small-group work in my classroom and using the technology available to add to the rich discussions.

In the future, I would like to require students to present their solutions to their peers more frequently to challenge them to write better solutions and communicate their thinking. My future goal is to require each student to present a Habits of Mind problem once a quarter following the initial first quarter after I introduce Habits of Mind problems and use this presentation as a percentage of their overall grade rewarding them for their work. I believed presenting a solution could cause students to pay extra attention to a given problem and write a solution they would be proud of in front of their peers and they would eventually become more comfortable with presenting to a large group.

While presenting, students added to their own solution and mimicked the great work of their peers. With the positive experiences in my classroom in mind, I plan to increase the amount of Habits of Mind problems I assign that correspond to the curriculum in future years. I believe the path to better student problem-solving skills is by way of modeling, teacher feedback offered by way of a rubric, and more and more opportunities to practice. Creating or locating high-interest problems is helpful while being cognizant of the wordiness and level of mathematics necessary to solve a problem. All of this combined makes choosing problems a venture I will continue until I can find at least one habits of mind problem per chapter.

Students appreciated the kind of specific feedback they received from me as their classroom teacher on these weekly habits of mind problems, but they were glad to

have feedback and communication with classmates. By allowing and observing student-chosen small groups and expecting written communication, this assisted many students in their problem-solving journey and allowed me to assess student understanding. While some students preferred to work alone, many chose to work with at least one peer so they could bounce ideas off one another and bring their thoughts together to better solve assigned problems. The small groups and opportunities to present solutions were effective tools for enriching instruction during the action research and students experienced mathematics in a unique format that hopefully will impact them for years to come. After intensive reflection and revision of my teaching strategies during the action research, I look forward to adapting my teaching for the benefit of mathematics students in the coming school year.

Appendix A

(The following problems are from *POW! Problem of the Week; Grades 5-6*, The Education Center, 2002)

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Appendix B

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

(POW! Problem of the Week; Grades 5-6, The Education Center, 2002)

Appendix C

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Appendix D

The Three Little Fences
Holt - 4th quarter
Habits of Mind
Chapter 10

Name _____

Let's talk about the circular fence first!

circumference = pi x diameter

C = circumference d = diameter

$$C = 3.14 \times d$$

We know that the circumference is 64 so if we take 64 divided by 3.14 we get a diameter of 20.3821...

Let's round it to $d = 20.4$ to make it simpler.

If diameter (d) = 20.4 then the radius is half of this, $20.4 \div 2 = 10.2$ so
 $r = 10.2$

The area of the circular fence = pi times radius squared or 3.14×10.2 squared.

area (a) = pi x r squared

$$a = 3.14 \times 10.2$$

$$a = \underline{\hspace{2cm}}$$

Now let's talk about the possible areas of the square and rectangular yards!

We know that the perimeter of the yard must be 64 feet. The following equations represent the whole number possibilities to make a square or rectangular yard.

Remember; perimeter = width + length + width + length

$$16 \times 16 = 256$$

$$17 \times 15 = 255$$

$$18 \times 14 = 252$$

$$19 \times 13 = \underline{\hspace{2cm}}$$

$$20 \times 12 = \underline{\hspace{2cm}}$$

$$21 \times 11 = \underline{\hspace{2cm}}$$

$$22 \times 10 = \underline{\hspace{2cm}}$$

$$23 \times 9 = \underline{\hspace{2cm}}$$

$$24 \times 8 = \underline{\hspace{2cm}}$$

$$25 \times 7 = \underline{\hspace{2cm}}$$

$$26 \times 6 = \underline{\hspace{2cm}}$$

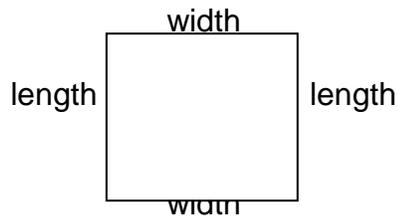
$$27 \times 5 = \underline{\hspace{2cm}}$$

$$28 \times 4 = \underline{\hspace{2cm}}$$

$$29 \times 3 = \underline{\hspace{2cm}}$$

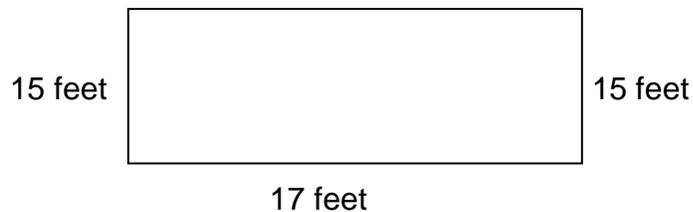
$$30 \times 2 = \underline{\hspace{2cm}}$$

$$31 \times 1 = \underline{\hspace{2cm}}$$



Example:

17 feet



$$\text{Perimeter} = 17 + 15 + 17 + 15$$

$$P = 64$$

$$\text{Area} = \text{length} \times \text{width} \quad A = 17 \times 15 \text{ or } 255 \text{ feet}^2$$

Appendix F

Habits of Mind: Rectangle Area

length



width

Name _____

Period _____

Date _____

1) A certain rectangle has length and width that are whole numbers of inches, and the ratio of its length to its width is 4 to 3. Its area is 300 square inches. What are its length and width?

2) How did you find your answer? What methods did you use to discover the length and width of the rectangle?

(continued on back side)

3.) The length of a rectangle is four times as long as its width. If the area is 100 meters squared, what is the length of the rectangle?

4.) The area of a rectangle is 360 meters squared. If its length is increased by 10 meters and its width is decreased by 6 meters, then its area does not change. Find the perimeter of the original rectangle.

Appendix G

HABITS OF MIND PROBLEM: SNAIL ESCAPE

Name _____ Date _____

1. Now that Spring is upon us the creatures that have been gone for the winter begin to reappear. One lonely snail was in a hole that was 20 feet deep. He is able to move 3 feet up the hole each day, but retreats down the hole 1 foot each night seeking warmth. How many days will it take the snail to climb out of the 20 feet deep hole? Show your work and thinking.



2. How many days will it take the snail to climb out of the 20 foot deep hole if he moves up 3 feet each day and down 2 feet (in a 24 hour period)?

3. What if we have an extra motivated snail and he moves up 5 feet each day and retreats 1 foot in the same day (up 5 down 1 foot)? How many days will it take for him to get to the top?

4. What pattern do you begin to notice? Do you see how integers are used with positive and negative numbers? Could you make a vertical number line showing how the snail moves on #3?

Appendix H

Objective Card For Sixth Grade

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Appendix D Math Problem-Solving Rubric

Student Name: _____

Habits of Mind Problem Title: _____

Date: _____

	4	3	2	1
Strategy/ Procedures	Uses an efficient and effective strategy to solve the problem	Uses an effective strategy to solve the problem.	Uses a strategy to solve the problem but it is not effective.	Does not use a strategy to solve the problem.
Explanation	Explanation is detailed and clear.	Explanation is clear.	Explanation is a little difficult to understand, but includes critical components.	Explanation is difficult to understand and is missing several components OR was not included.
Mathematical Concepts	Shows complete understanding of the mathematical concepts used to solve the problem.	Shows substantial understanding of the mathematical concepts used to solve the problem.	Shows some understanding of the mathematical concepts needed to solve the problem.	Shows very limited understanding of the underlying concepts needed to solve the problem OR is not written.
Mathematical Terminology and Symbols	Advanced, correct terminology and symbols used, making it very easy to understand what was done.	Correct terminology and symbols are used, making it easy to understand what was done.	Correct terminology and symbols are used, but it is sometimes not easy to understand what was done.	There is little use, or a lot of inappropriate use, of terminology and symbols.
Neatness and Organization	The work is presented in a neat, detailed, organized way that is easy to read.	The work is presented in a neat and organized way that is easy to read.	The work is presented in an organized way but may be hard to read.	The work appears sloppy and un-organized. It is hard to know what information

				goes together.
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Appendix J

Personal Weekly Journal Prompts

Week of _____

Ali Arndt

1. Write about 2 classroom situations this week that relate to problem-solving, formative/summative assessment, peer presentations, Polya's Steps or confidence levels in students.

2. Did the Habits of Mind match the student skill level? The curriculum and objectives well? What was notable about this week's problem and solutions? Would you use it again next year or change anything?

3. How are you doing with grading using the rubric? Are there notable changes in any student scores on the rubric?

4. Give examples of situations where you knew students were confident with the material. Was anything done to reinforce them?

5. What questions/prompts did you give to facilitate learning?

6. Give example(s) of situations where you knew students did not understand the material. What interventions were used? How was instruction changed?

7. What lesson plans/materials were used that fit well? Needed to be changed/adjusted?

8. Was there a quiz/test given this week? Summarize the results.

9. What are some tensions I felt this week between my role as a teacher and a researcher?

10. Comments:

Appendix K

Student Interview Questions

I will ask the following questions to check understanding on specific concepts. I will give the students a problem from a concept that has been taught in class and ask them to explain their reasoning and solution processes.

I would like you to work on this problem, saying aloud whatever it is you are thinking as you work through the problem. I especially want to hear you talk about how you decide what to do to solve the problem.

- How would you do this problem?
- Why did you do it in that manner?
- What are you doing here?
- How do you know that you are right?
- How would you know if you are wrong?

I will ask the following questions to understand their beliefs of assessment and learning styles.

- Do you feel like mathematics has gotten easier for you over the past few weeks? Why or why not?
- How much time do you spend on the Habits of Mind problems?

- Which Habits of Mind problems have you felt most successful solving? Why? Which Habits of Mind problems did you feel least successful solving? Why?
- Is there anything that you are doing differently when solving word problems? Is there anything that the teacher is doing differently?
- When you don't understand a concept or Habits of Mind problem, who do you go to or what do you do?
- Some students think they understand math better when it is explained by a student rather than by a teacher. How do you feel about that?
- What makes you most upset about the process of school? Is there anything you can do to fix it? Is there anything a teacher could do?
- Is there anything you want to know from me? Any questions?
- Is there anything else I should know about you to better understand your problem solving in math or your general math experience?

Appendix L

Student Attitude Survey:

Mathematics Attitude Inventory

Check one: A = Always
 S = Sometimes
 N = Never

Use a checkmark to match the best answer.

	Always	Sometimes	Never
1. I look forward to math class.			
2. I would rather work on any other subject than math.			
3. I get good grades in math.			
4. I think math is something I don't need to understand.			
5. I admire other students who are good at math.			
6. I like when math problems really make me think.			
7. I don't think I am good at math.			
8. I think some kids just know how to do math and some kids just don't understand it – and studying more will not help.			
9. Working in groups helps me understand math better.			
10. I am not sure of my math answers, even on simple problems.			
11. It is important for me to do well in math.			
12. I feel comfortable asking questions in math class if I don't understand something.			
13. I am comfortable sharing my mathematical ideas with the whole class.			
14. Working in small groups just makes me more confused about math.			
15. I am more comfortable sharing my ideas in a small group than with the whole class.			
16. I understand math better when we do activities rather than the teacher just explaining it from the book.			