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The Graphical Representation of Magnetic Theories

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this value down to 0.88. The preponderance of evidence in favor of the value 0.88 is very great. It is probably true that the very best way we have at the present time of determining candle power is to use the Hefner lamp and then to reduce by the use of this ratio.

The writer desires in conclusion to express his obligations to Mr. C. P. Matthews for timely assistance in making some of the above observations.

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The Graphical Representation of Magnetic Theories.

By Harold N. Allen.

THE induction theory of magnetism, introduced by Faraday, is now looked upon by all physicists as correct. The older theory which assumes the existence of magnetic fluids covering the ends of the magnet is in some cases mathematically simpler, and is for this reason often made use of. This, however, is apt to breed confusion as to the true nature of the induction or polarization in any given case. The difficulty Tyndall experienced in accepting Faraday's views as to diamagnetism, is accounted for by the fact that he was thinking in terms of the fluid theory, while Faraday was considering the magnetic polarization in the diamagnetic substance.

The object of this paper is to insist again upon the distinction between these two theories, and at the same time to consider some points in the induction theory itself. A number of diagrams will be described which illustrate the different aspects of the induction theory. They show how the molecules of the magnet are supposed to be polarized, and how this polarization is continued in the surrounding ether. The tubes of force, or "polarization tubes," and the equipotential surfaces are drawn in each case according to Maxwell's method. The figures must be revolved about the horizontal axis, so that the lines drawn will trace out, some the bounding surfaces between the tubes, and others the equipotential surfaces.

It has been found convenient to give the name polarization tube to a tube of force consisting of a bundle of 4π induction tubes. The polarization at any point is measured by the number of polarization tubes passing through a square centimeter of a surface, which cuts them at right angles, just as the induction is measured in the same way by means of the induction tubes.

If D_m is the magnetic polarization,
B_m the magnetic induction,
H_m the magnetic field intensity,

the following relations will hold:

$$D_m = \frac{B_m}{4\pi} = \frac{\mu H_m}{4\pi}.$$

A polarization cell or energy cell is a portion of the medium bounded on the sides by the walls of a polarization tube, and at the ends by two equipotential surfaces, differing from one another in value by unity. It is possible to regard each of these cells as containing half a unit (erg) of energy. This energy is to be regarded as in some way directed along the lines of force, as the energy of a bullet is directed along the line of its flight, or the energy of rotation of a fly-wheel is directed along its axis.

The length of the cell in a uniform field being

$$\frac{1}{H_m} = \frac{\mu}{4\pi D_m} = -\frac{dn}{dV_m},$$

and the area of its section being $\frac{1}{D_m}$,

the volume will be $\frac{\mu}{4\pi D_m^2}$.

Thus the density of the directed energy is $2\pi D_m^2$ in a medium of unit permeability.

According to one statement by Maxwell, the energy density in a medium of permeability μ is $\frac{2\pi D_m^2}{\mu}$. Ampère's theory of magnetism, however, implies, as will be shown, a much larger energy density than this.

In the first part of his treatise, Maxwell seems practically to assume that the polarization in a paramagnetic substance is made up of two parts, the polarization of the iron molecules or the intensity of magnetization I, and the polarization of the ether which occupies the same space $= D' = \frac{H'}{4\pi}$, where H' is the field intensity in the substance; that is, in a long uniform crevice, the direction of which coincides with that of the induction tubes.

Thus the total polarization is

$$D_m = \frac{H'}{4\pi} + I,$$

or

$$D_m = D' + I,$$

or in more familiar terms,

$$B_m = H' + 4\pi I.$$

The two polarizations are regarded as being superimposed, so that in some way they occupy the whole of the same space at the same time.

In Diagram II., Fig. 1, are shown the equipotential surfaces and polarization tubes produced, when an infinite cylinder of constant permeability, $\mu = 2$, is placed with its axis parallel to a uniform field of force $H = 1$.

Diagram I., Fig. 1, is a representation of the field before the introduction of the cylinder. The polarization tubes in these diagrams are bounded by the cylinders generated by the horizontal lines when the figure is rotated about AB. The line CD traces out the surface of the paramagnetic cylinder. The equipotential surfaces in the cylinder are simply continuations of those in the outside field. The polarization tubes are twice as numerous in the cylinder as outside.

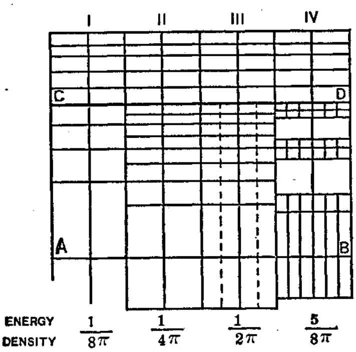


Fig. 1. Half size.

Half of the tubes are supposed to pass through the ether, just as if the iron were not present; the other half go through the iron molecules, which are supposed to fill out the whole volume of the cylinder.

The energy density in the air is $\frac{1}{8\pi}$, in the iron $\frac{1}{4\pi}$. The polarization is $\frac{1}{4\pi}$ in the air, and $\frac{1}{2\pi}$ in the cylinder.

Maxwell does not say anything about the energy density according to Ampère's theory of elementary current magnets. From the nature of the case any exact determination seems impossible, but some idea can be obtained by making certain assumptions with regard to the distribution of the elements, and supposing that the amount of conducting material is very small.

Suppose that the cylinder is saturated, and that all the elements are set at right angles to the field of force. In order to obtain a minimum value for the directed energy density we will assume that the elementary magnets consist of thin wires of infinite conductivity, bent into square or hexagonal circuits, which, for the sake of simplicity, are taken as arranged regularly in long lines side by side so as to form square or hexagonal magnetic solenoids. These latter are to fit together so as to leave only very narrow cracks between them. The permeability of the ether between the conductors and of the conductors themselves is unity.

Then the polarization inside each solenoid due to the currents in the elementary circuits is I, and to this must be added D', the polarization which would exist there if the iron were not present. Thus the total polarization is $D = D' + I$. In the very narrow cracks between the solenoids the polarization is D', the same as that in the space outside, and the distance between the equipotential surfaces is $\frac{1}{4\pi D'}$. The equipotential surfaces in the cracks are then simply continuations of the plane equipotential surfaces in the air around the iron. The energy density in the crack is $2\pi D'^2$. The distance between the equipotential surfaces inside the solenoids is $\frac{1}{4\pi D}$, and this is to the distance in the outside field in the ratio $D' : D$.

The density of the energy directed parallel with the field inside the solenoid is $2\pi D^2$.

Since the cracks are very narrow, this will be the density of the directed energy in the iron.

With an induction of 20,000 lines per square centimeter, this gives about 16,000,000 ergs per cubic centimeter, or about four-tenths of a calorie.

An attempt is made to represent the equipotential surfaces and polarization tubes according to this hypothesis in Diagram III., Fig. 1. The full vertical lines represent equipotential surfaces, both within and without the iron, while the dotted lines are additional equipotential surfaces inside the solenoids. $\mu = 2$, $B' = 1$, $D' = \frac{1}{4\pi}$, $D = \frac{1}{2\pi}$. Energy density in air $= \frac{1}{8\pi}$, in iron $= \frac{1}{2\pi}$, or twice as great as in Diagram II. Length of polarization cell in air and in cracks = one centimeter, length in solenoids = half a centimeter.

It is certain that the elementary circuits do not fill out the space in this way. If they did, the induction would soon reach a limit, beyond which it could not rise. For each of the elementary circuits is a perfect conductor, and on being turned round so that its axis is parallel to the field, will prevent any additional lines from passing through it, if there is an increase in the field.

The facts of saturation seem to indicate that the space guarded from the entrance of external lines by the elementary magnets is very small.

The molecules of magnetized iron are in all probability distributed very irregularly, but the problem can be simplified by assuming that they form long solenoids when the iron is saturated, and that these solenoids only occupy $\frac{1}{n}$th of the whole volume. The following results can be deduced.

The polarization D'' inside these solenoids remains constant from the moment they are formed, the current in the elementary magnets decreasing as the external field increases.

The average polarization of a cubic centimeter of the iron is

$$D = \frac{1}{n} D'' + \frac{n-1}{n} D'.$$

The quantity known as intensity of magnetization is defined by the equation

$$D = D' + I.$$

Thus
$$I = \frac{D'' - D'}{n},$$

or I diminishes after the constant value of D'' has been reached, the diminution being $\frac{1}{n}$th of the increase of D'. If n is large, this diminution will be small, and I will be fairly constant through a large range if $n = 100$.

The amount of energy in a cubic centimeter will be

$$\frac{2\pi}{n} D''^2 + \frac{2\pi(n-1)}{n} D'^2.$$

Taking certain results of Ewing on induction in iron in high magnetic fields and making $n = 100$, the amount of energy in one cubic centimeter is found to be 45 calories. This may perhaps be regarded as a maximum value.

In Diagram IV., Fig. 1, the condition of a portion of a very long cylinder in a uniform field $H = 1$ is indicated, on the assumption that $n = 2$, and μ the ratio between the average polarization of the paramagnetic substance and that in the intermolecular spaces is also two. The polarization outside the cylinder is $\frac{1}{4\pi}$, while the average polarization inside is $\frac{1}{2\pi}$, as in the two former cases.

One half of the space is occupied by the solenoidal molecular chains, which are represented in the figure, suitably divided by the polarization tubes and equipotential surfaces. These are of course much nearer together inside the solenoids than outside.

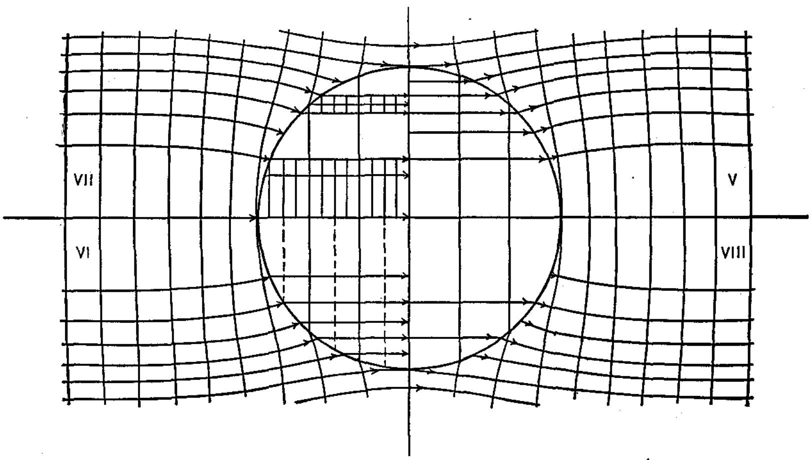
The polarization inside the solenoids is $\frac{3}{4\pi}$. Each of the cells shown, the volume of which is 4π c.c. outside the solenoids, and $\frac{4\pi}{9}$ c.c. inside, contains half an erg of energy; so that the energy density outside the solenoids is $\frac{1}{8\pi}$. Inside it is $\frac{9}{8\pi}$, and the average energy density in the cylinder is $\frac{5}{8\pi}$, as against $\frac{1}{8\pi}$ in the surrounding field, $\frac{1}{2\pi}$ inside the cylinder of Diagram III., and $\frac{1}{4\pi}$ in Diagram II.

Diagram V., Fig. 2, shows the result, according to the ordinary induction theory, of placing a sphere of radius 4 and permeability 2 in a unit field $B' = 1$. Inside the sphere are shown the equipotential surfaces $V' = -\frac{3B'x}{\mu + 2}$ which are $\frac{4}{3}$ cm. apart. At right angles to these run the tubes of polarization, which are bounded by the cylindrical surfaces traced out by the horizontal lines, when the figure is revolved about the horizontal diameter of the sphere. Of these there are

$$D = \frac{B}{4\pi} = \frac{3\mu B'}{4\pi(\mu + 2)} = \frac{3}{8\pi} \text{ per square centimeter.}$$

Hence through the 16π square centimeters of the equatorial plane there pass six of these polarization tubes, while in the original field, before the sphere was introduced, four tubes passed through the same area.

The polarization tubes outside the sphere are continuous on the surface with those inside, and at a distance from the sphere they become identical



Energy density in $\text{V} = \frac{9}{64\pi}$, $\text{VI} = \frac{9}{32\pi}$, $\text{VII} = \frac{27}{64\pi}$, $\text{VIII} = \frac{9}{128\pi}$.

Fig. 2. Half size.

with those due to the field $H' = B' = 1$. The intensity of magnetization is $\frac{3}{16\pi}$, and the polarization of the ether in the sphere is also $\frac{3}{16\pi}$, making a total polarization of $\frac{3}{8\pi}$ in the sphere as compared with $\frac{1}{4\pi}$ in the air at a distance. The energy density is only $\frac{9}{64\pi}$ in the sphere, while it is $\frac{1}{8\pi}$ in the original field.

Diagram VI., Fig. 2, shows the same problem treated so as to represent Ampère's theory, the molecular circuits being supposed to fill out the whole volume, as is also the case in Diagram III., Fig. 1. The dotted lines represent equipotential surfaces inside the solenoids, but not in the narrow cracks between them.

The energy density is twice as great as in the former case, being

$$2\pi D^2 = \frac{9}{32\pi}.$$

In Diagram VII., Fig. 2, the molecular chains or solenoids are only supposed to occupy one-third of the volume of the sphere.

In reality of course these solenoids must be supposed to be very narrow in comparison to their length, and the difficulty observed in the diagram, with regard to the connection between the tubes inside the sphere and those outside, does not occur.

If these solenoids could exist without the presence of the external field, they would send tubes back in the negative direction through the empty spaces between them, so that the polarization in these spaces when the external field is present is only $\frac{3}{16\pi}$, whereas in the distant parts of the external field it is $\frac{1}{4\pi}$. Inside the molecular solenoids the polarization is $\frac{3}{4\pi}$. The total flow of polarization through the sphere is the same in Diagrams V., VI., and VII. The energy density in the sphere is in this last case $\frac{27}{64\pi}$, or 50 per cent greater than in Diagram VI.

Diagram VIII., Fig. 2, shows a sphere of insulating material, the dielectric constant of which is unity. The electrical density on its surface is

$$\sigma = \frac{3}{16\pi} \cdot \cos\theta.$$

The sphere is suspended in a uniform electrostatic field $B'_e = 1$, so that the line $\theta = 0$ points along the direction of the field.

Outside the sphere the electrostatic equipotential surfaces and polarization tubes are geometrically identical with the magnetic lines in the previous cases. Within the sphere the equipotential surfaces are identical with those of Diagram V.

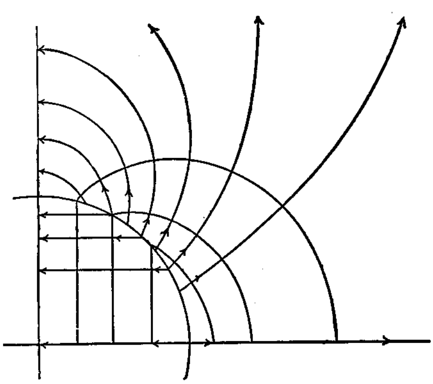


Fig. 3. Half size.

The electrostatic field intensity inside the sphere is only $\frac{3}{4}H'$, and the polarization is $\frac{3}{16\pi}$. The energy density is only $\frac{9}{128\pi}$. Three of the six tubes which come to the surface of the sphere on the negative side end in the three units of negative electricity distributed over that side, while three of the six tubes that start from the positive side, have their origin in the three units of positive electricity on that side. The remaining three tubes from the external field pass through the sphere.

The reason for the fact that the field in the sphere is smaller than that at a distance, is of course that the given electrical distribution would, if acting by itself, produce inside the sphere a field in the negative direction, as indicated in Fig. 3, the field intensity being $-\frac{1}{4}H'$. This is the "demagnetizing effect of the ends" of which we hear when instead of electric charges "free magnetisms" are supposed to exist on the surface of the sphere. It seems as though it would be almost worth while to treat all problems in which this mathematical method is simplest by means of the electrostatic analogy. That is, never to speak of free magnetism, but to work out the electrostatic problem, and to state that the required magnetic field is mathematically the same as the electrostatic field, wherever the permeability is unity.

While there may be nothing very new in what precedes, it is hoped that the diagrams with their explanations may prove of some use to those who, like the author, have experienced difficulty in keeping the different magnetic theories clearly separated.

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ON THE ALTERNATING CURRENT DYNAMO.

By W. E. GOLDSBOROUGH.

CONSIDER the case of a simple alternator having but one armature coil that rotates in a magnetic field of uniform intensity about an axis at right angles to the direction of the lines of force. If successive instants of time during one revolution of the coil are counted from the instant that the coil passes a line drawn through its axis of rotation and perpendicular to both the axis of rotation and the direction of the magnetic flux, the value of the induction piercing the coil at any instant during one cycle is expressed by the equation

$$N = N_{\max} \cos \omega t, \qquad (1)$$

in which $N_{\max}$ equals that portion of the flux that passes through the coil at the instant the plane of the coil is at right angles to the direction of the lines of force, and ω represents its angular velocity. The instantaneous value of the E.M.F. generated in the coil will be, by Faraday's law,

$$e = -\frac{dN}{dt} = \omega N_{\max} \sin \omega t = E \sin \omega t, \qquad (2)$$

since its maximum value $\qquad E = \omega N_{\max}. \qquad (3)$

If the coil is closed through a circuit of resistance R', inductance L' and capacity C', the resistance and inductance of the coil itself being R