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Coordinating a Supply Chain With a Manufacturer-Owned Online Channel: A Dual Channel Model Under Price Competition

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Coordinating a Supply Chain With a Manufacturer-Owned Online Channel: A Dual Channel Model Under Price Competition

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***Abstract*—We consider a dual channel supply chain in which a manufacturer sells a single product to end-users through both a traditional retail channel and a manufacturer-owned direct online channel. We adopt a commonly used linear demand substitution model in which the mean demand in each channel is a function of the prices in each channel. We model each channel as a newsvendor problem, with price and order quantity as decision variables. In addition, the manufacturer must choose the wholesale price to charge to the independent retailer. We analyze the optimal decisions for each channel and prove the existence of a unique equilibrium for the system. We compare this equilibrium solution to the solution for an integrated system, in which the manufacturer owns both the online store and the retailer. To enable supply chain coordination, we propose two contract schemes: a modified revenue-sharing contract and gain/loss sharing contract. We show that, in cases where the retail channel has a larger market than the online channel, such contracts enable the manufacturer to maintain price discrimination, selling the products in different channels at different prices. Finally, we perform a comprehensive numerical study to consider the impact of the model parameters on the equilibrium and to demonstrate the performance of the proposed coordination contracts. We conclude that coordination is most critical for products which are highly price sensitive and for systems in which the online and traditional retail channels are not viewed as close substitutes.**

***Index Terms*—Competition, coordination, eBusiness, game theory, newsvendor problem with pricing, online direct channel.**

I. Introduction and Motivation

THIS paper considers competition and coordination in a retail supply chain with dual channels of distribution. A manufacturer distributes his products through both a traditional (independent) retail channel, e.g., a department store, as well as through a manufacturer-owned direct online channel. The past two decades have seen rapid growth in online channels, for a variety of reasons, including the development of the internet and information technology and the growth of third-party logistics providers [1], [2]. Adding a direct online channel has a number of advantages, such as increasing control over pricing and product selection, and enabling the manufacturer to reach a wider segment of customers and to engage in price discrimination. Of course, adding an online channel also has a number of potential disadvantages, including the introduction of competition into the system, which may lead to lower prices and potentially reduced profits for both channels [3]. Understandably, retailers often view manufacturer-owned direct online channels as a competitive threat [2], [4].

While there has been significant previous literature weighing these advantages and disadvantages to consider the question of whether or not it is beneficial for a manufacturer to add a direct online channel, as well as assessing the impact of the dual-channel approach on the retailer, little of this previous literature addresses the obvious next questions: given that the manufacturer has chosen to take a dual-channel approach to distribute his product, how can the system be designed in order to achieve supply-chain coordination? What kinds of mechanisms can be designed to reduce or eliminate channel conflict? Addressing these questions is the primary focus of this paper. Thus, we focus our analysis on systems in which there already exist dual channels of distribution, a traditional retail channel and a manufacturer-owned direct online channel, e.g., HP, Dell, Nike, and Apple. Because we consider a single period setting, our model is particularly appropriate for apparel, publishing, and electronics supply chains. As noted in [2] and [5], apparel manufacturers such as Polo Ralph Lauren, DKNY, and Liz Claiborne distribute their products through both manufacturer-owned channels (outlet stores and, more recently, online stores) as well as through major department store chains, such as Macy's and Kohl's.

In this paper, we model the horizontal price competition between the manufacturer-owned direct channel and the traditional retailer, and the vertical competition between the manufacturer and the retailer (e.g., [6]). We seek to understand how the equilibrium behavior of this supply chain will depend on a number of problem parameters. We study the impact of price competition on the profits of each firm and we investigate methods for achieving supply-chain coordination. As detailed in the next section, we believe we are the first to consider the problem of supply-chain coordination for a dual-channel supply chain in which the direct and retail channels compete on price. We do so in a setting in which demand is uncertain and in which both channels must make both price and quantity decisions.

Specifically, we consider a dual-channel supply chain in which demand in each channel is a random variable whose expected value is a function of the prices charged in each

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channel. Thus, the channels compete on price. In addition, demand in each channel will depend on customers' preferences regarding purchasing online versus in a traditional retail setting. For example, some customers may prefer the convenience and lower search cost associated with the online channel, while other customers may prefer the ability to immediately obtain the product through the retail channel. To reflect this, we adopt a commonly used linear demand substitution model (e.g., [7]–[9]). Given the uncertainty in demand, each channel's decision problem can be formulated as a newsvendor problem in which both price and order quantity are decision variables. In addition, the manufacturer sets the wholesale price.

Our equilibrium analysis is composed of multiple steps. First, for a given wholesale price, we characterize the Nash equilibrium decisions for the system, i.e., we determine the equilibrium price and quantity decisions for each channel and, under certain conditions, prove the existence of a unique equilibrium for the system. Next, we formulate a Stackelberg game to consider the manufacturer's choice of the wholesale price. We then study the equilibrium behavior of the system as a function of the problem parameters. We also compare this equilibrium solution to the solution for an integrated system, in which the manufacturer owns both the direct channel and the retailer. Given the solution to the integrated system, we propose a modified revenue-sharing contract, in which the manufacturer imposes a minimum retail price on the retailer, to enable supply-chain coordination.

In cases where the retail channel has a larger market than the online channel, such a contract enables the manufacturer to maintain a minimum price differential between the retailer and the online store, which helps to achieve price discrimination. In addition, we show that a gain/loss (GL) sharing contract can be used to coordinate the system. Finally, through a comprehensive numerical study, we consider how the equilibrium varies with the key model parameters and demonstrate that the proposed coordination contracts can improve the system's profit by 12.44%, on average, for 1080 problems tested.

The main contribution of this paper is the development of coordination schemes for a dual-channel system under price competition and demand uncertainty. As noted in the next section, while several authors have considered the problem of competition between online and traditional retail channels, to the best of our knowledge, none have considered coordination when the channels compete on price. In addition, we introduce a new modification to the classic revenue-sharing contract, which imposes a minimum retail price.

II. Literature Review

We next review the literature on competition in supply chains with multiple channels of distribution. While we focus on competition between a manufacturer-owned direct channel and an independent retailer, our research has some similarities to the literature on competition in a single supplier, n-retailer supply chain. While much of this literature considers retailers who compete only on inventory (e.g., [10]–[12]), the most relevant papers for our purpose are those in which demand is stochastic and the locations compete on price (e.g., [13] and [14]), or on both price and inventory (e.g., [9] and [15]).

A. Competition Between Direct and Retail Channels

We next review the literature on dual-channel supply chains in which the manufacturer sells directly to the end-users, either through online sales or through a manufacturer-owned store, as well as through an independent retailer. Unless specifically mentioned (e.g., [2] and [3]), none of these papers considers supply-chain coordination, the primary focus of this paper. The most relevant papers are those that consider dual channels that compete on price; most of these, however, do not consider production/order quantity decisions. For example, Dumrongsiri *et al.* [16] assume that the manufacturer fills direct sales demand on a make-to-order basis and must always fill the retailer's order. Thus, the manufacturer's production quantity is not a decision variable. Chiang *et al.* [1] assume the market share for each channel is a *deterministic* function of the prices and thus there is no quantity decision. Cattani *et al.* [4] also consider a dual-channel model in which the market share is deterministic. Like us, they use a Stackelberg game model with the manufacturer as leader and retailer as follower. Bell *et al.* [17] consider the impact of the introduction of a manufacturer-owned channel on the pricing strategies of the existing retailers under the assumption that demand is a deterministic function of prices and marketing effort.

The aforementioned papers, as well as this paper, all take retail price as a decision variable, and thus, the channels compete on price. Other research on manufacturer–retailer competition in dual-channel supply chains takes price as fixed. Here, the locations may compete on inventory or on other factors such as sales effort. The authors in [3] and [18] seek to coordinate a system in which the channels compete only on inventory. The authors in [2] and [19] consider a manufacturer who sells his product through one of three systems: retailer-only, direct sales-only, or a system with both retail and direct sales. Tsay and Agrawal [2] consider the impact of sales effort, while Hendershott and Zhang [19] consider the impact of search costs. Tsay and Agrawal [2] use their model to study channel coordination mechanisms, including a wholesale pricing scheme and two referral schemes.

B. Supply-Chain Coordination

In a single-channel supply chain, with one manufacturer and one retailer, operating under a wholesale price contract, it is known that the retailer will order less than the system optimal quantity. When demand is not dependent on retail price, several contract types have been shown to coordinate the supply chain and to arbitrarily divide its profit: buy back [20], revenue sharing [21], quantity flexibility [22], [23], sales rebate [24], [25], and quantity discount [26]. With price-dependent demand, a key question is whether contracts that coordinate the order quantity also coordinate the pricing decision. It is known that buy back, quantity flexibility, and sales-rebate contracts do not coordinate in this setting, while revenue sharing and quantity discount contracts achieve coordination under certain conditions [27].

A second segment of this literature considers coordination with multiple retailers. Research in this area includes a buyback contract to coordinate newsvendors who compete on both price and inventory [9], a wholesale price contract to coordinate retailers who compete only on inventory [28], a revenue-sharing contract to coordinate retailers who compete on quantity [21], and coordinating retailers who compete on price [14]. Cachon [29] provides a review of this literature.

One of the coordination contracts considered in this paper is the revenue-sharing contract. Numerous authors have considered this type of contract (e.g., [30]–[32]). We also consider a GL sharing contract, which is commonly observed in practice, although there are very few papers studying contracts of this form (e.g., [33]–[35]). In some industries, GL sharing contracts are referred to as profit-sharing contracts. Weinstein [36] examines profit-sharing contracts in the motion picture industry. In the supply-chain literature, Wang and Webster [5] investigate the GL sharing contract in a decentralized supply chain including one manufacturer and one loss-averse retailer. They show that such a contract can coordinate the supply chain under certain conditions.

A final segment of the literature considers a manufacturer who owns a direct sales channel, which competes with a traditional retail channel. Here, the supply chain experiences both vertical (between manufacturer and retailer) and horizontal (between direct channel and traditional retail channel) competition. Coordination in this setting has received little attention. Boyaci [3] shows that if the selling prices are fixed, the price-only, buy-back, rebate, and revenue-sharing contracts cannot coordinate the supply chain. Boyaci proposes a new type of contract, the compensation-commission contract, which can coordinate the system. Tsay and Agrawal [2] also consider this context, but assume deterministic demand (which is influenced by sales effort) and fixed prices. If the selling prices are decision variables and demands are price dependent, then coordination is challenging because it requires the price and inventory decisions in each channel to be system-optimal. Coordination in this environment, in which the supply chain experiences both horizontal and vertical competition, and in which the decision-makers must make both pricing and quantity decisions, is the focus of this paper. To the best of our knowledge, we are the first to study this problem, and the first to provide contracts which can coordinate the system and arbitrarily allocate profits between the manufacturer and retailer.

III. Model Formulation and Analysis: Deterministic Demand

We consider a supply chain consisting of a manufacturer (he), which we denote with the subscript M, who distributes his product through two channels: an independently owned and operated retailer (she), which we denote with the subscript R, and a manufacturer-owned online store, which we denote with the subscript O. Demand through each channel is dependent on the prices charged in both channels. Each channel must make both pricing and quantity decisions, while the manufacturer also sets the wholesale price charged to the retailer.

Since our goal is to investigate the impact of manufacturer-owned online stores in a competitive market environment, our demand model must adequately reflect the substitutability of the two channels. Thus, we adopt a typical linear demand substitution model. If we let $D_R(p_R, p_O)$ ($D_O(p_R, p_O)$) denote the total market demand seen by the retailer (online store) given the prices charged by the retailer p_R and the online store p_O, we can write

$$D_R(p_R, p_O) = \delta_R - \alpha_R \cdot p_R + \beta_R \cdot p_O \tag{1}$$

$$D_O(p_R, p_O) = \delta_O - \alpha_O \cdot p_O + \beta_O \cdot p_R. \tag{2}$$

The parameters α_i and β_i, $i \in \{R, O\}$, represent the own-price and cross-price sensitivities of demand. Generally, one would expect the retail demand to be more sensitive to the retail price than to the online price. Similarly, the online demand would be more sensitive to the online price than to the retail price. This would imply $\alpha_R > \beta_R$ and $\alpha_O > \beta_O$. Also, δ_i represents the base market size and captures the customers' base utility gained from purchasing through channel i. The parameter β_i captures the substitutability between the two channels, i.e., higher values of β_i imply that the channels are viewed as closer substitutes. Thus, δ_i and β_i will depend on the customers' preferences regarding purchasing online versus in a traditional retail setting. For example, some customers may prefer the convenience and lower search cost associated with the online channel, while other customers may prefer the ability to immediately obtain the product through the retail channel. The value of β_i, in particular, will reflect this tradeoff.

We now outline the sequence of events for our model and define some additional notation.

1) Prior to the start of the selling season, the manufacturer sets the wholesale price c_R charged to the retailer.
2) The retailer sets the retail price p_R and orders Q_R from the manufacturer at the wholesale price of c_R per unit.
3) At the same time, the manufacturer sets the online store price p_O and chooses a production quantity to be shipped to the online store Q_O.
4) The manufacturer produces $Q_M = Q_R + Q_O$, with unit production cost of c_M. We assume unlimited capacity at the manufacturer. Thus, the manufacturer is always able to produce the full quantity Q_M in time for the start of the selling season.
5) End-user demands occur simultaneously at the retailer and the online store.

We first derive the best response function for the retailer given the price for the manufacturer, i.e., we characterize the optimal p_R for a given value of p_O. Since, given the prices, demand is known with certainty, the retailer will choose to order a quantity Q_R just equal to demand. Thus, given p_O, the expected profit for the retailer Π_R is $\Pi_R(p_R|p_O) = (p_R - c_R)Q_R$, and the retailer will choose p_R to maximize $\Pi_R(p_R|p_O)$ subject to $Q_R = \delta_R - \alpha_R \cdot p_R + \beta_R \cdot p_O$. It is easy to show that $\Pi_R(p_R|p_O)$ is concave in p_R and that the optimal retail price is $p_R(p_O) = \frac{\delta_R + \alpha_R c_R + \beta_R p_O}{2\alpha_R}$.

We next characterize the manufacturer's best response function given the retailer's price, taking the wholesale price as fixed. Later, we will consider how the manufacturer chooses the optimal wholesale price. Since, given the prices, demand is known with certainty, the retailer will choose to order a quantity Q_R just equal to demand. Thus, given p_R, we can write the expected profit for the manufacturer Π_M as $\Pi_M(p_O|p_R) = (p_O - c_M)Q_O + (c_R - c_M)Q_R$, and the manufacturer will choose p_O to maximize $\Pi_M(p_O|p_R)$ subject to $Q_O = \delta_O - \alpha_O \cdot p_O + \beta_O \cdot p_R$. It is easy to show that $\Pi_M(p_O|p_R)$ is concave in p_O and that the optimal retail prices is $p_R^*(p_O) = \frac{\delta_O + \alpha_O c_M + \beta_O p_R + (c_R - c_M)\beta_R}{2\alpha_O}$. We next note that, since both profit functions are strictly concave in price, for a given wholesale price c_R, a unique equilibrium exists for the game between the retailer and manufacturer. Thus, the equilibrium prices can be found at the intersection of the best response functions, and can be written as follows:

$$p_R^* = \left(\frac{\beta_R}{4\alpha_O\alpha_R - \beta_O\beta_R}\right)[\delta_O + \alpha_O c_M + (c_R - c_M)\beta_R] + \left(\frac{2\alpha_O}{4\alpha_O\alpha_R - \beta_O\beta_R}\right)[\delta_R + \alpha_R c_R] \quad (3)$$

$$p_O^* = \left(\frac{\beta_O}{4\alpha_O\alpha_R - \beta_O\beta_R}\right)[\delta_R + \alpha_R c_R] + \left(\frac{2\alpha_R}{4\alpha_O\alpha_R - \beta_O\beta_R}\right)[\delta_O + \alpha_O c_M + (c_R - c_M)\beta_R]. \quad (4)$$

Notice that in order to have $p_R^* \geq 0$ and $p_O^* \geq 0$, we require $4\alpha_O\alpha_R - \beta_O\beta_R > 0$. This condition will be satisfied if $\alpha_R > \beta_R$ and $\alpha_O > \beta_O$, i.e., if the retail demand is more sensitive to the retail price than to the online price and, similarly, if the online demand is more sensitive to the online price than to the retail price.

We next consider how the manufacturer will set the wholesale price for the retailer. To answer this question, we consider a Stackelberg game between the manufacturer and retailer. In this game, the manufacturer sets the wholesale price taking into consideration the equilibrium prices, i.e., we are interested in the value of c_R that maximizes the manufacturer's profits, given p_R^* and p_O^*. By checking the second-order conditions, it is easy to show that the manufacturer's profits will be concave in c_R, and thus, there will exist a unique optimal c_R, as long as $\alpha_i > \beta_i$ and $\alpha_j > \beta_i$ for all $i \in \{R, O\}$ and $i \in \{R, O\}$. Unfortunately, while we can obtain a closed-form expression for the optimal c_R (see the online Supplementary Material), the exact expression is quite complex. However, if we consider a symmetric setting with $\delta_R = \delta_O = \delta$, $\alpha_R = \alpha_O = \alpha$, and $\beta_R = \beta_O = \beta$, we can simplify the expression for the optimal c_R to the following:

$$c_R^* = \frac{(8\alpha^3 + \beta^3)\delta + (\alpha - \beta)(8\alpha^3 + 2\alpha\beta - \beta^3)c_M}{2\alpha(\alpha - \beta)(8\alpha^2 + \beta^2)}. \quad (5)$$

We next present sensitivity analysis for this symmetric model with deterministic demand.

Theorem 1: Suppose $\delta_R = \delta_O = \delta, \alpha_R = \alpha_O = \alpha$, and $\beta_R = \beta_O = \beta$. Then, if $\alpha > \beta$:

1) Q_R is decreasing in p_R and increasing in p_O. Similarly, Q_O is decreasing in p_O and increasing in p_R. 2) The equilibriums Q_R and Q_O are increasing in β and decreasing in α. 3) The equilibrium p_R is increasing in β and decreasing in α. 4) The equilibrium c_R is increasing in c_M. 5) The equilibrium Π_R increases in β and decreases in α.

All proofs are provided in the online Supplement Material. The proof of Theorem 1 contains expressions for the equilibrium prices, order quantities, and profits. The condition $\alpha > \beta$ implies that each channels' demand is more sensitive to its own price than to the other channel's price.

The results of Theorem 1 are intuitive. Point 1 says that the optimal order quantity in each channel is decreasing in its own price, but increasing in the other channel's price. Point 2 says that the equilibrium order quantity in each channel is decreasing in its own-price sensitivity and increasing in the competitor-price sensitivity. Point 3 says that the equilibrium retail price is decreasing in its own-price sensitivity and increasing in the competitor-price sensitivity. Point 4 says that the equilibrium wholesale price charged to the retailer is increasing in the manufacturer's unit production costs. Point 5 says that the retailer's equilibrium profit is decreasing in its own-price sensitivity and increasing in the competitor-price sensitivity.

Finally, we consider how this system can be coordinated, i.e., we consider the type of contract that can make the decentralized system perform as well as an integrated system, in which the manufacturer owns both the retailer and the online channel. For this deterministic model, it is straightforward to see that a profit-sharing contract can coordinate the system. In such a contract, the manufacturer sets the wholesale price at $c_R = c_M$. Then, at the end of the selling season, the retailer keeps ϕ percent of the total system profit, while the manufacturer receives $1 - \phi$ percent of the total system profit. Given that coordination can be achieved for our deterministic model, we have the following result.

Theorem 2: Suppose $\delta_R = \delta_O = \delta, \alpha_R = \alpha_O = \alpha$, and $\beta_R = \beta_O = \beta$. Then, the retail and online prices in the coordinated (integrated) system are lower than the equilibrium retail and online prices in the decentralized system.

Theorem 2 indicates that coordination not only improves the overall system profit, but also makes the consumer better off by lowering the prices on both channels.

Finally, we note that a profit-sharing contract will require both parties to honestly share cost and profit information, which may create difficulties in implementation. In addition, when demand is stochastic, as in the next section, coordination is more difficult. However, we propose two contracts that coordinate the system and that are relatively easy to implement in practice.

IV. Model Formulation and Analysis: Stochastic Demand

We next extend the model considered in Section III to the case of uncertain demand so that we may use the model to capture the critical tradeoff between overstocking and understocking. To do so, we will use an additive model of uncertainty (see, e.g., [37]) and thus our demand functions can be rewritten as follows:

$$D_R(p_R, p_O) = \delta_R - \alpha_R \cdot p_R + \beta_R \cdot p_O + \epsilon_R \quad (6)$$

$$D_O(p_R, p_O) = \delta_O - \alpha_O \cdot p_O + \beta_O \cdot p_R + \epsilon_O \quad (7)$$

where $\epsilon_i, i \in \{R, O\}$, is a random variable defined on the range $[A_i, B_i]$. We let $f_i(\cdot)$ and $F_i(\cdot)$ denote the probability density function and cumulative distribution function of ϵ_i, with mean μ_i and standard deviation σ_i. To ensure that, for all values of p_j, there exists some range of p_i for which $D_i(p_i, p_j) > 0$, we require $A_i > -\delta_i$, for all $i \in \{R, O\}$.

The sequence of events in this model is identical to that in Section III, with the additional assumption that, at the end of the selling season, any excess inventory is salvaged with values v_R and v_O per unit at the retailer and online store, respectively. In addition, as noted previously, like much of the literature on supply-chain coordination, we assume a single-period newsvendor setting, and thus, our model will be most applicable for products with a short life cycle. Finally, to simplify the analysis, it is useful to write $Q_R = \delta_R - \alpha_R p_R + \beta_R p_O + z_R$ and $Q_O = \delta_O - \alpha_O p_O + \beta_O p_R + z_O$, where z_i represents the amount of safety stock held at location i, $i \in \{R, O\}$. With this reformulation, the decision variables for each player are (z_i, p_i), $i \in \{R, O\}$.

A. Retailer's Best Response Function

We first derive the best response function for the retailer, i.e., we characterize the optimal p_R and z_R for given values of p_O and z_O. We can write the expected profit for the retailer Π_R as

$$\Pi_R(p_R, Q_R | p_O, z_O) = E\left(-c_R Q_R + p_R \min[D_R, Q_R] + v_R \max[Q_R - D_R, 0]\right).$$

We can now apply the results obtained by Petruzzi and Dada [37], henceforth denoted as P&D. Specifically, our problem is the same as P&D, Section 1.1, but with the following notation: $h = -v_R, s = 0, \mu = \mu_R, a = \delta_R + \beta_R p_O, b = \alpha_R, c = c_R, A = A_R,$ and $B = B_R$. Thus, Lemma 1 and Theorem 1 in P&D apply and we can write the optimal p_R given z_R and p_O as follows:

$$p_R^*(z_R | p_O) = p_R^0(p_O) - \frac{\Theta_R(z_R)}{2\alpha_R} \quad (8)$$

where $\Theta_R(z_R) = \int_{z_R}^{B_R} (\epsilon_R - z_R) f_R(\epsilon_R) d\epsilon_R$ and $p_R^0(p_O) = \frac{\delta_R + \alpha_R c_R + \beta_R p_O + \mu_R}{2\alpha_R}$ is the optimal riskless price, which depends on the price charged by the online channel. Notice that this optimal price for the retailer depends only on the price at the online store, not on the order quantity.

We can plug $p_R^*(z_R | p_O)$ back into the profit function to get the profit as a function of just z_R, i.e., we can convert the retailer's 2-D strategy space into a single dimension, which will simplify our analysis. We apply Theorem 1 in P&D, which provides conditions under which there is a unique solution to the first-order condition for z_R.

1) $F_R(\cdot)$ should satisfy the following: $2r_R(z_R)^2 + \frac{dr_R(z_R)}{dz_R} > 0$ for all $A_R \le z_R < B_R$, where $r_R(z_R) = \frac{f_R(z_R)}{1 - F_R(z_R)}$ is the hazard rate.
2) The demand and cost parameters must satisfy: $\delta_R + \beta_R p_O - \alpha_R c_R + A_R > 0$.

The first condition will be satisfied for a variety of distributions, including the normal and log-normal. For the second condition, notice that, for any reasonable p_R and p_O, we should require $D_R \ge 0$. This implies that $\delta_R + \beta_R p_O - \alpha_R p_R + A_R > 0$. We also require $p_R > c_R$. Thus, we can safely assume that $\delta_R - \alpha_R c_R + \beta_R p_O + A_R > \delta_R - \alpha_R p_R + \beta_R p_O + A_R \ge 0$, and under reasonable conditions, the second condition should hold.

Finally, from Theorem 1 in P&D, we have the first-order condition for z_R

$$\frac{\partial \Pi_R}{\partial z_R} = -(c_R - v_R) + \left(p_R^0(p_O) - v_R - \frac{\Theta_R(z_R)}{2\alpha_R}\right) \times (1 - F_R(z_R)) = 0. \quad (9)$$

Let $z_R^*(p_O)$ denote the value of z_R that satisfies this condition for a given p_O. Given $z_R^*(p_O)$, the retailer's order quantity can be written as $Q_R^* = \delta_R - \alpha_R p_R^*(z_O) + \beta_R p_O + z_R^*(p_O)$.

B. Manufacturer's Best Response Function

We will first analyze the manufacturer's problem taking the wholesale price as fixed and deriving the best response function for the manufacturer given the price and order quantity for the retailer, i.e., we characterize the optimal p_O and z_O for given values of p_R and z_R. To do so, we can write the expected profits for the manufacturer Π_M, given p_R and z_R, as follows:

$$\Pi_M(p_O, Q_O | p_R, z_R) = E\left(c_R Q_R - c_M Q_M + p_O \min[D_O, Q_O] + v_O \max[Q_O - D_O, 0]\right)$$

where $Q_M = Q_O + Q_R$. Although the manufacturer's profit has additional terms to capture manufacturing costs and revenue from the retailer, its form is the same as the retailer's profit. Thus, we have the following optimal online price as a function of z_O and p_R:

$$p_O^*(z_O | p_R) = p_O^0(p_R) - \frac{\Theta_O(z_O)}{2\alpha_O} \quad (10)$$

where $\Theta_O(z_O) = \int_{z_O}^{B_O} (\epsilon_O - z_O) f_O(\epsilon_O) d\epsilon_O$ and $p_O^0(p_R) = \frac{\delta_O + \alpha_O c_M + \beta_O p_R + (c_R - c_M)\beta_R + \mu_O}{2\alpha_O}$ is the optimal riskless price. Finally, the first-order conditions for z_O are

$$\frac{\partial \Pi_M}{\partial z_O} = -(c_M - v_O) + \left(p_O^0(p_R) - v_O - \frac{\Theta_O(z_O)}{2\alpha_O}\right) \times (1 - F_O(z_O)) = 0. \quad (11)$$

Let $z_O^*(p_R)$ denote the value of z_O that satisfies this condition for a given p_R. Given $z_O^*(p_R)$, the retailer's order quantity can be written as $Q_O^* = \delta_O - \alpha_O p_O^*(p_R) + \beta_O p_R + z_O^*(p_R)$. The conditions needed to guarantee that there is a unique solution to the first-order condition for z_O are analogous to those for the retailer and are not repeated here.

C. Equilibrium Analysis

We next show that, for a given wholesale price c_R, the game between the retailer and manufacturer is supermodular and thus an equilibrium exists. In addition, under certain conditions, this equilibrium is unique. We start by noting that any equilibrium must satisfy the first-order conditions, which define the players' best response functions, specified in (8)–(11). Next, using (8) and (10), we can jointly solve for the prices for any given values of z_R and z_O

$$\begin{aligned} p_R^*(z_O, z_R) = &\left(\frac{\beta_R}{4\alpha_O\alpha_R - \beta_O\beta_R}\right)[\delta_O + \alpha_O c_M + \mu_O \\ &+ (c_R - c_M)\beta_R - \Theta_O(z_O)] \\ &+ \left(\frac{2\alpha_O}{4\alpha_O\alpha_R - \beta_O\beta_R}\right) \\ &\times [\delta_R + \alpha_R c_R + \mu_R - \Theta_R(z_R)] \end{aligned} \tag{12}$$

$$\begin{aligned} p_O^*(z_O, z_R) = &\left(\frac{\beta_O}{4\alpha_O\alpha_R - \beta_O\beta_R}\right) \\ &\times [\delta_R + \alpha_R c_R + \mu_R - \Theta_R(z_R)] \\ &+ \left(\frac{2\alpha_R}{4\alpha_O\alpha_R - \beta_O\beta_R}\right)[\delta_O + \alpha_O c_M + \mu_O \\ &+ (c_R - c_M)\beta_R - \Theta_O(z_O)]. \end{aligned} \tag{13}$$

Given the game defined by (9) and (11)–(13), we can now prove the following result.

Theorem 3: If $4\alpha_O\alpha_R - \beta_O\beta_R > 0$, then the game between the retailer and manufacturer is a supermodular game. Thus, there exists at least one equilibrium.

The proof of Theorem 3 shows that the profit functions have increasing differences in z_R and z_O, i.e., satisfy $\frac{\partial^2 \Pi_i}{\partial z_R \partial z_O} \geq 0$, for $i \in \{R, O\}$ [38]. This result implies that the decision variables are complements, i.e., an increase in z_O will cause the retailer to want to increase (or not decrease) z_R. The condition $4\alpha_O\alpha_R - \beta_O\beta_R > 0$ will be satisfied if $\alpha_R > \beta_R$ and $\alpha_O > \beta_O$, which, as discussed in Section III, is likely to hold. We next demonstrate that, under certain conditions, the equilibrium will be unique.

Theorem 4: A unique Nash equilibrium exists under the following conditions:

1) $4\alpha_O\alpha_R - \beta_O\beta_R > 0$, $c_R > v_R$, and $c_M > v_O$;
2) $2r_i(z_i)^2 + \frac{dr_i(z_i)}{dz_i} > 0$, $i \in \{R, O\}$;
3) $\delta_i - \alpha_i c_i + A_i > 0$, $i \in \{R, O\}$.

While the condition in Theorem 3 guarantees the existence of an equilibrium, the additional conditions in Theorem 4 guarantee that the equilibrium will be unique. If these conditions are not satisfied, the profit functions Π_R and Π_M may not be unimodal, and thus there could potentially be multiple equilibria. In that case, it becomes difficult to make specific predictions regarding the behavior of the system equilibrium in practice.

Finally, we can show the following properties of the equilibrium solution.

Theorem 5: Under the conditions specified in Theorem 4: 1) z_R^* is nonincreasing in c_R, while z_O^* is nondecreasing in c_R; 2) z_R^* is nondecreasing in v_R; and 3) z_O^* is nonincreasing in c_M.

The first result indicates that safety stock at the retailer (online store) decreases (increases) as the wholesale price increases. The second result says that, as expected, as the salvage value increases, the retailer's safety stock will increase. The third result also confirms our intuition that the online store's safety stock decreases as the production cost increases.

D. Setting the Wholesale Price

We next consider a Stackelberg game between the manufacturer and retailer to characterize the equilibrium wholesale price. Thus, we are interested in the value of c_R that maximizes the manufacturers profits, given the equilibrium price and order quantities, $p_R^*, Q_R^*, p_O^*, Q_O^*$. We start by proving the existence of a Stackelberg equilibrium:

Theorem 6: There exists a Stackelberg equilibrium for the game in which the manufacturer chooses the wholesale price c_R to maximize his own profits given $(p_R^*, z_R^*, p_O^*, z_O^*)$.

The proof of Theorem 6 follows [39], i.e., existence of an equilibrium c_R is guaranteed due to the fact that 1) the profit functions are real valued and continuous and 2) the strategy spaces are compact. This result only guarantees the existence of an equilibrium for the Stackelberg game, but says nothing about the uniqueness of such an equilibrium. While we are unable to prove uniqueness, our numerical results indicate that the equilibrium c_R appears to be unique. Specifically, to find the equilibrium c_R, we must solve a system of equations. The value of c_R that satisfies this system of equations appears to be unique.

To find the c_R chosen by the manufacturer to maximize his own profits given $(p_R^*, z_R^*, p_O^*, z_O^*)$, we write the first-order condition, $\frac{\partial \Pi_M}{\partial c_R} = 0$

$$\begin{aligned} \frac{\partial \Pi_M}{\partial c_R} = &\frac{\partial p_O}{\partial c_R}(\delta_O - \alpha_O p_O + \beta_O p_R + z_O) + (p_O - c_O) \\ &\times \left(-\alpha_O \frac{\partial p_O}{\partial c_R} + \beta_O \frac{\partial p_R}{\partial c_R} + \frac{\partial z_O}{\partial c_R}\right) \\ &- \frac{\partial p_O}{\partial c_R}\int_{A_O}^{z_O}(z_O - \epsilon_O)f(\epsilon_O)d\epsilon_O \\ &- (p_O - v_O)\frac{\partial z_O}{\partial c_R}F_O(z_O) \\ &+ (\delta_R - \alpha_R p_R + \beta_R p_O + z_R) + (c_R - c_M) \\ &\times \left(-\alpha_R \frac{\partial p_R}{\partial c_R} + \beta_R \frac{\partial p_O}{\partial c_R} + \frac{\partial z_R}{\partial c_R}\right) = 0. \end{aligned} \tag{14}$$

Since we do not have closed-form solutions for the equilibrium prices and quantities at the retailer and manufacturer, we use the implicit function theorem to write the partial derivatives of p_R, p_O, z_R, and z_O with respect to c_R. A more detailed discussion of our solution approach for finding the optimal wholesale price c_R^* can be found in the online Supplement Material.

V. Integrated System and Supply-Chain Coordination

We are interested in understanding how this dual-channel supply chain can be coordinated. We start by analyzing an integrated system in which the manufacturer owns both the retailer and the online channel. We then propose two contracting schemes to achieve coordination.

A. Integrated System

We consider an integrated system under the assumption that the manufacturer owns both the retailer and the online store. Prior to observing demand, the manufacturer chooses the production quantities and prices for each location. Demand is then observed and any excess demand at each location is salvaged. The total system profit, $\Pi_S^I(p_O, p_R, Q_O, Q_R)$, for the integrated system can be written as

$$\begin{aligned}\Pi_S^I &= E(-c_M Q_M + p_O \min[D_O, Q_O] + p_R \min[D_R, Q_R] \\ &\quad + v_R \max[Q_R - D_R, 0] + v_O \max[Q_O - D_O, 0]) \\ &= (p_R - c_M)(\delta_R - \alpha_R p_R + \beta_R p_O + z_R) - (p_R - v_R) \\ &\quad \times \int_{A_R}^{z_R} (z_R - \epsilon_R) f_R(\epsilon_R) d\epsilon_R \\ &\quad + (p_O - c_M)(\delta_O - \alpha_O p_O + \beta_O p_R + z_O) - (p_O - v_O) \\ &\quad \times \int_{A_O}^{z_O} (z_O - \epsilon_O) f_O(\epsilon_O) d\epsilon_O. \end{aligned} \tag{15}$$

In the following, we characterize the optimal solution of the integrated system.

Theorem 7: If the following conditions are met, the expected profit for the integrated system is unimodal (quasi-concave) and the optimal prices (p_R^I and p_O^I) and order quantities (Q_R^I and Q_O^I) can be found as the unique solution to the first-order conditions:

1) $4\alpha_R\alpha_O - (\beta_R + \beta_O)^2 > 0$;
2) $c_M > v_i, i = R, O$;
3) $2r_i(z_i)^2 + \frac{dr_i(z_i)}{z_i} > 0, i = R, O$;
4) $\delta_i - \alpha_i c_M + A_i > 0, i = R, O$.

To understand the first condition, consider the case in which $\beta_R = \beta_O = \beta$. In this case, the first condition in the theorem will hold if $\alpha_R > \beta$ and $\alpha_O > \beta$, i.e., if each channel's demand is more sensitive to its own price than to the other channel's price. Conditions 3 and 4 of the theorem are analogous to the conditions for unimodality required in Sections IV-A and IV-B.

Theorem 7 indicates that the integrated systems has a unique optimal solution. As shown in the proof of the theorem, the optimal prices for the integrated system, given z_R and z_O, are

$$\begin{aligned} p_R^I(z_R, z_O) &= \left(\frac{\beta_R + \beta_O}{4\alpha_R\alpha_O - (\beta_R + \beta_O)^2}\right) \\ &\quad \times \{\delta_O + (\alpha_O - \beta_R)c_M + \mu_O - \Theta_O(z_O)\} \\ &\quad + \left(\frac{2\alpha_O}{4\alpha_R\alpha_O - (\beta_R + \beta_O)^2}\right) \\ &\quad \times \{\delta_R + (\alpha_R - \beta_O)c_M + \mu_r - \Theta_R(z_R)\} \\ p_O^I(z_R, z_O) &= \left(\frac{2\alpha_R}{4\alpha_R\alpha_O - (\beta_R + \beta_O)^2}\right) \\ &\quad \times \{\delta_O + (\alpha_O - \beta_R)c_M + \mu_O - \Theta_O(z_O)\} \\ &\quad + \left(\frac{\beta_R + \beta_O}{4\alpha_R\alpha_O - (\beta_R + \beta_O)^2}\right) \\ &\quad \times \{\delta_R + (\alpha_R - \beta_O)c_M + \mu_r - \Theta_R(z_R)\}. \end{aligned}$$

It is difficult to analytically compare the p_R^I and p_O^I. However, our numerical study indicates that whether or not $p_R^I > p_O^I$ will depend on the parameters of the model.

Finally, we compare the optimal solution for the integrated system with the Nash equilibrium solution for the decentralized system. As discussed in detail in Section VI-B, our numerical results indicate that $p_R^* > p_R^I$, $p_O^* > p_O^I$, $z_R^* < z_R^I$, and $z_O^* > z_O^I$. The results for the prices match the deterministic model results, as given in Theorem 2. These differences can be attributed to the inefficiencies caused by both horizontal price competition and vertical double marginalization. Therefore, we next investigate two methods for achieving supply-chain coordination.

B. Coordination Using a Modified Revenue-Sharing Contract

Having analyzed the equilibrium behavior for the dual-channel model and having characterized the optimal system behavior for the integrated system, we now investigate a contracting scheme that will enable supply-chain coordination. Specifically, we consider a modified revenue-sharing contract in which the retailer shares a fixed portion of her revenue with the manufacturer. Under this contract, defined by the parameters (p_R^I, p_O^I, ϕ), the manufacturer imposes a minimum retail price p_R^I on the retailer, i.e., the retailer may choose to price at any level $p_R \geq p_R^I$, and the manufacturer sets his own online price at p_O^I. Then, at the start of the selling season, the retailer chooses her order quantity Q_R for which the manufacturer charges the unit wholesale price $c_R = \phi c_M$. Then, at the end of the selling season, the retailer keeps a fraction ϕ of her revenue and gives a fraction $(1 - \phi)$ of her revenue to the manufacturer. We call this contract a *minimum retail-price-constrained revenue-sharing contract*.

The minimum retail price constraint is commonly observed in practice. Lynette [40] studied 203 resale price maintenance cases and found that approximately 80% of the cases involved a minimum resale price requirement specified by the manufacturers to reduce price competition among the retailers. In the bicycle industry, some suppliers have been known to enforce minimum pricing, although not overtly [41]. In the automotive industry, car manufacturers have a long history of specifying a "manufacturer suggested retail price." They are now considering efforts to enforce the minimum retail price [42]. In the video game industry, Nintendo has been known to occasionally institute minimum price requirements [43]. Similar minimum price requirements are common for electronic goods, software, books, and pharmaceuticals.[1]

Minimum retail price constraint makes intuitive sense in a dual-channel supply chain in which the manufacturer distributes products through a traditional retailer and a manufacturer-owned online store. In such a system, the manufacturer will likely be

[1] "Century-Old Ban Lifted on Minimum Retail Pricing," by S. Labaton, *New York Times*, 06/29/2007 and "Justices Hear Arguments About Pacts on Pricing," by L. Greenhouse, *New York Times*, 06/27/2007.

interested in ensuring that the retailer does not undercut his price or, going even further, he may be interested in maintaining price discrimination, i.e., ensuring that the retail price is strictly greater than the online price. Our numerical study results indicate that the minimum price-constrained revenue-sharing contract enables this sort of price discrimination for cases in which the retail channel has a larger market than the online channel, i.e., when $\delta_R \geq \delta_O$.

To analyze this contract, we first show that, when the manufacturer imposes a minimum retail price, the retailer will always choose to price at exactly this minimum level.

Theorem 8: Under the revenue-sharing contract defined by (p_R^I, p_O^I, ϕ), where p_R^I is the minimum retail price imposed by the manufacturer, the retailer will always choose to price at p_R^I.

To understand this result, note that if the retailer operated under a revenue-sharing contract with no constraint on her price, given that the online channel prices at p_O^I, her optimal price would be less than p_R^I. Since her profit is unimodal in price, under the minimum price contract, the retailer would choose to set her price equal to the minimum level, p_R^I. We also note that this result does not require any assumptions regarding the size of p_R^I relative to p_O^I, i.e., we do not require $p_R^I > p_O^I$. Finally, this result coincides with what is generally observed in practice, i.e., when manufacturers impose a minimum retail price on their retailers, the retailers generally choose to set their price at exactly that minimum level [44].

We can now demonstrate that a revenue-sharing contract of the form described previously can coordinate the supply chain. We first define the following notation: let Π_R^* (Π_M^*) denote the retailer's (manufacturer's) optimal expected profit without the revenue-sharing contract, when $c_R = c_R^*$, the optimal wholesale price for the decentralized dual-channel model, and let Π_R^I (Π_M^I) denote the retailer's (manufacturer's) expected profit if both parties use the optimal solution for the integrated system, when $c_R = c_M$.

Theorem 9: When $c_R = \phi c_M$, the revenue-sharing contract defined by (p_R^I, p_O^I, ϕ) coordinates the dual-channel model. If $\phi \in \left[\frac{\Pi_R^*}{\Pi_R^I}, \frac{\Pi_R^I + \Pi_M^I - \Pi_M^*}{\Pi_R^I}\right]$, the contract will be Pareto-improving.

As shown in the proof of Theorem 9, the Pareto-improving region for ϕ is always nonempty, i.e., there always exists some value of ϕ that makes both parties better-off.

Next, we note that, depending on the relationships among Π_R^*, Π_R^I, Π_M^*, and Π_M^I, the parameter ϕ may be larger than 1, as discussed in the following.

1) When $\Pi_R^I \leq \Pi_R^*$, a feasible ϕ will always be greater than or equal to 1. In this case, the integrated system provides no benefit to the retailer over the decentralized system. Therefore, in order to motivate the retailer to participate, the manufacturer must share some of the benefits he obtains from the integrated system with the retailer. Thus, the revenue-sharing contract is similar in concept to a rebate contract, in which the retailer pays a wholesale price larger than c_M and then claims some refund for each item sold.
2) When $\Pi_R^I > \Pi_R^*$ and $\Pi_M^I > \Pi_M^*$, a feasible ϕ may be greater than 1, depending on the negotiating power of each party, e.g., when the retailer has more negotiating power, we would expect her to extract more of the system profits and thus ϕ may exceed 1.
3) When $\Pi_R^I > \Pi_R^*$ and $\Pi_M^I \leq \Pi_M^*$, a feasible ϕ will always be less than or equal to 1, as in a traditional revenue-sharing contract. In this case, the integrated system provides no benefit to the manufacturer over the decentralized system. Therefore, in order to motivate the manufacturer to participate, the retailer must share some of the benefits she obtains from the integrated system with the manufacturer.

While the value of ϕ may theoretically be greater than 1, in our numerical results, as described in Section VI, the Pareto-improving region for ϕ ranged from 15% to 51%.

1) Implementation and Legal Considerations: The minimum price-constrained revenue-sharing contract proposed here is different from what is known as a "minimum resale price maintenance contract." In *Dr. Miles Medical Co. v. John D. Park and Sons* (1911), the Supreme Court found that minimum resale price maintenance contracts violate the Sherman Antitrust Act [45]. However, that ruling has been a source of debate (see [46] and [47]). In a June 2007 ruling, the Supreme Court overturned *Dr. Miles*, concluding that vertical price restraints are not illegal across the board, but must be judged on an individual basis. The ruling came in the case *Leegin Creative Leather Products*,[2] a manufacturer of fashion accessories, who wanted to target customers willing to pay a higher price for extra service. The company instituted a policy that required retailers to adhere to a minimum price and refused to sell to the retailer who filed suit because that retailer sold goods at prices below the manufacturer's minimum price.

Thus, in a minimum resale price maintenance contract, the manufacturer refuses to sell products to a retailer if the retailer does not accept and execute the minimum retail price. Our proposed contract is different because the retailer is given two options: 1) she could buy from the manufacturer under the existing wholesale price contract or 2) she may participate in the price-constrained revenue-sharing contract. In order to increase his profits, the manufacturer seeks to motivate the retailer to choose option 2). Therefore, the manufacturer designs the revenue-sharing contract in a manner that is beneficial to the retailer, i.e., so that the profits earned by the retailer under 2) are at least as high as under 1).

Finally, since the prices charged by both channels under the minimum price-constrained revenue-sharing contract are equal to the optimal prices for the integrated system, these prices will likely be less than those charged without the contract, i.e., $p_R^I < p_R^*$ and $p_O^I < p_O^*$. Thus, consumers are likely to be better off under the proposed contract. This fact may be a compelling argument for the courts to consider when evaluating the legality of price maintenance contracts.

[2]See http://www.supremecourtus.gov/opinions/06pdf/06-480.pdf.

C. GL Sharing Contract

In this section, we propose a GL sharing contract to coordinate the dual-channel model. In a GL sharing contract, the upstream player (manufacturer) shares the gain or loss incurred by the downstream player (retailer). Such GL contracts are commonly observed in the insurance, banking, and third-party logistics industries [5], [33], [48], [49].

We assume that the manufacturer offers a GL sharing contract (W_L, W_G, T) to the retailer. The parameter T is the retailer's expected optimal profit without the contract, which can be thought of as the retailer's target profit. At the beginning of the selling season, the manufacturer sends the quantity Q_R^I to the retailer at the wholesale price c_R^*, as specified in Section IV-D, and asks the retailer to sell the product at the price p_R^I. The online store stocks the quantity Q_O^I and sells at price p_O^I. At the end of the selling season, if the retailer's realized profit is less than T, then the manufacturer pays the retailer W_L times the difference between the retailer's realized profit and T. Otherwise, the retailer pays the manufacturer W_G times the difference between the retailer's realized profit and T. Notice that, as with the minimum price-constrained revenue-sharing contract, the issue arises of whether such a contract is legal, given that the manufacturer attempts to dictate the selling price to the retailer. To avoid this problem, we take an approach similar to that of [51] to implement the price constraint without "forcing" the retailers to participate. Specifically, we assume that the manufacturer offers a menu, with two options, to the retailer upfront. In the first option, the retailer orders Q_R^I at a wholesale price c_R^*, as specified in Section IV-D, and sets retail price equal to p_R^I. At the end of the selling season, the manufacturer shares the retailer's gain or loss according to the GL sharing contract. In the second option, the retailer can order any order quantity at the wholesale price c_R^* and set any retail price. No GL sharing is offered in this option. As we will demonstrate in the following, the retailer will earn more profits under the GL contract than without it. Thus, the retailer will voluntarily choose to participate in the GL sharing contract.

We next specify some additional notation. Π_R^* is the retailer's optimal expected profit without the contract, Π_R is the retailer's expected profit under the contract, and Π_R^I is the retailer's expected profit if both parties use the optimal solutions for the integrated system. The manufacturer's profits, Π_M^*, Π_M, and Π_M^I, are defined analogously. Also, note that $T = \Pi_R^*$.

Theorem 10: A GL sharing contract (W_L, W_G, T) coordinates the decentralized dual-channel model. If $\Pi_R^I > T$, then $W_G > W_L$. Otherwise, $W_L > W_G$.

Thus, a GL sharing contract will always coordinate the system. In the proof of the theorem, we indicate how to determine values for the contract parameters W_L and W_G that will ensure that the contract is Pareto-improving, i.e., that both the retailer and manufacturer will benefit from participating in the contract. The relative size of these gain and loss parameters W_G and W_L will depend on the specific problem parameters. If, in expectation, the retailer does better under the integrated system than in the dual-channel system with no contract, i.e., if $\Pi_R^I > T = \Pi_R^*$, then the percentage of the retailer's gain that will be shared with the manufacturer W_G, in those instances in which the retailer does better under the contract, will be larger than the percentage of the retailer's loss that would be paid to the retailer by the manufacturer W_L in those instances in which the retailer does worse under the contract.

D. Comparison of Coordinating Contracts

We have presented two contracts, the minimum price-constrained revenue-sharing contract and the GL sharing contract, that can coordinate the dual-channel model. In addition, both contracts are completely flexible in dividing system profits between the players. Depending on the negotiation power of each party, under either contract, the manufacturer's profit can range from Π_M^* (i.e., the manufacturer gets no gain from coordination) to $\Pi_M^I + \Pi_R^I - \Pi_R^*$ (i.e., the manufacturer takes all of the gain from coordination). Similarly, the retailer's profit can range from Π_R^* to $\Pi_M^I + \Pi_R^I - \Pi_M^*$. One difference between the two contracts is the wholesale price. Under the revenue-sharing contract, the wholesale price is linearly increasing with the fraction ϕ of revenue that is kept by the retailer. Under the GL sharing contract, the wholesale price is fixed at c_R^*, the optimal wholesale price in the dual-channel model. The wholesale price can have a significant impact on a retailer if his cash is limited prior to the start of the selling season. Therefore, the revenue-sharing contract could be more attractive to the retailer than the GL sharing contract if the wholesale price is lower than c_R^*.

VI. Numerical Study and Observations

In this section, we conduct a series of numerical analyses to investigate the sensitivity of the equilibrium solution for the dual-channel model to various key parameters, as well as to compare the performance of the dual-channel model to that of the integrated system. Since we do not have closed-form solutions, we find the equilibrium solutions by solving the problems numerically. After reviewing the existing literature on channel coordination and demand substitution with a linear demand model (e.g., [8], [9], and [50]), we created a set of test problems by varying the key parameters, i.e., the firm's own-price sensitivity ($\alpha_i \in \{30, 40, 50, 60, 70, 80\}$), the firm's competitor-price sensitivity ($\beta_i \in \{0, 3, 6, 9, 12, 15\}$), and the unit salvage value ($v_i \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$). For the random component of demand ϵ_i, we tested uniform distributions on [0, B_i] for $B_i \in \{50, 100, 150, 200, 250, 300\}$. The production cost ($c_M$) is normalized at 1, while the base market demand (δ_i) is fixed at 2000.

A. Analysis of Dual-Channel Model Equilibrium

We first discuss the behavior of the equilibrium prices, safety stocks, and profits. We note that the behavior of the equilibrium is similar across all uniform distributions tested, although the magnitude of the profits will differ [see Fig. 1(a)]. While we discuss our general results, we have chosen to graphically display only selected results.

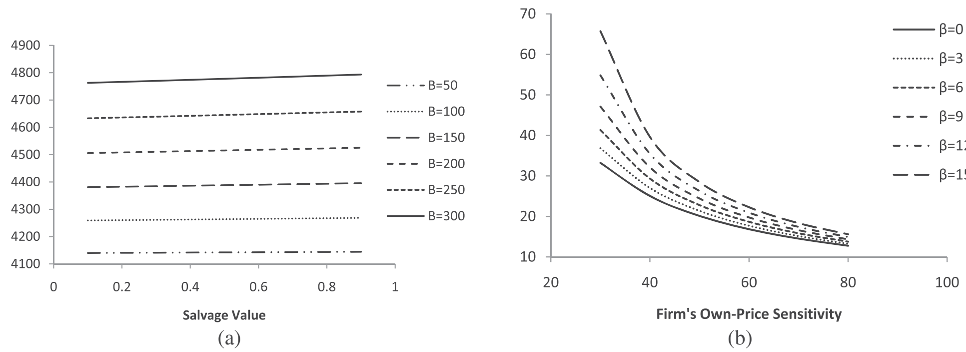


Fig. 1. Graphical illustration of equilibrium behavior. (a) Retailer's profits. (b) Manufacturer's wholesale price.

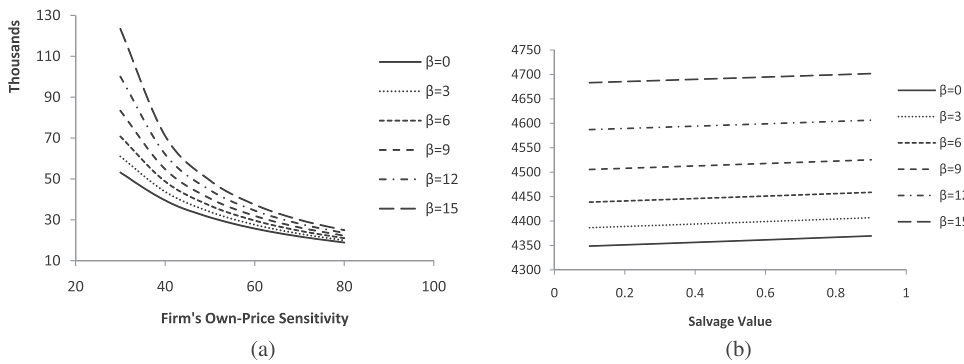


Fig. 2. Sensitivity analysis on dual-channel model. (a) Manufacturer's profit. (b) Retailer's profit.

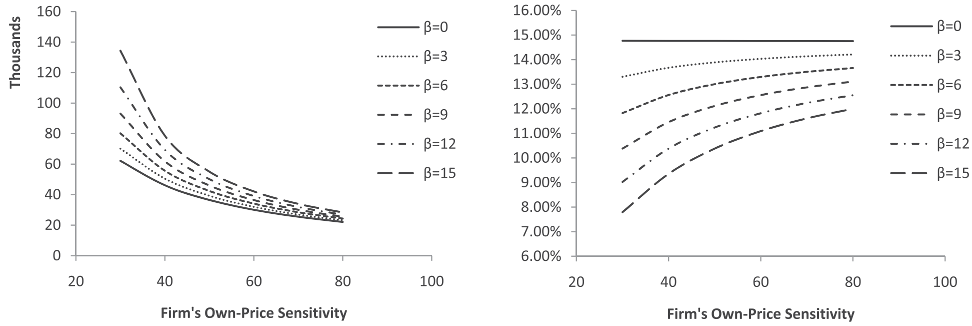
1) *Wholesale price:* As shown in Fig. 1(b), the manufacturer's optimal wholesale price decreases as the firm's own-price sensitivity increases. However, when demand is very sensitive to the competitor's price, the manufacturer will set a higher wholesale price. Interestingly, the manufacturer lowers the wholesale price as the salvage value increases.
2) *Safety stock:* As the savage value increases, the safety stock levels increase, as expected. Higher competitor-price sensitivity results in higher safety stock for the online channel but lower safety stock for the retailer. Interestingly, while higher own-price sensitivity reduces the safety stock for the online channel, it increases the safety stock for the retailer.
3) *Profits:* As depicted in Fig. 2, both channels' optimal profits behave as expected, and as predicted by the analysis of the deterministic model in Theorem 1. Specifically, as one firm's own-price sensitivity increases, its profits decrease. However, the effect of a firm's competitor-price sensitivity is exactly the opposite. Finally, as in a typical newsvendor model, a higher salvage value results in higher profits [refer to Fig. 2(b)].
4) *Prices:* The impact of these parameters on the optimal prices is identical to that for the optimal profits, and is as predicted by the analysis of the deterministic model in Theorem 1, i.e., firm i's price is decreasing in its own-price sensitivity and increasing in the competitor-price sensitivity. Also, a higher salvage value leads to higher prices. In addition, as noted previously, we found that the online price was always less than the retail price. More specifically, for our parameter values, we found that $p_O^*/p_R^* \in [70\%, 90\%]$.

Our numerical results provide managerial insights into recent developments in manufacturer-owned online channels. Since the manufacturer is the leader of the Stackelberg game, he optimally chooses the wholesale price to extract the retailer's profits. As a result, although we consider only symmetric channels, we observe that the manufacturer's profits are significantly higher, i.e., about seven times (on average) larger, than the retailer's profits.

B. Performance of Supply-Chain Coordination

One key question to be addressed in this research is the degree to which the proposed coordination contracts can improve the system's profits. To answer this question, we compare the dual-channel model with the integrated system for the 1080 problem settings tested. As expected, the system profits for the integrated system are higher than for the dual-channel model for every problem instance. The average percentage increase in profits under the integrated system is 12.44%, ranging from 6.29% to 14.95%. Note that these increases can be interpreted as the value of supply-chain coordination.

Further investigation reveals that the value of coordination increases with the own-price sensitivity and decreases with the competitor-price sensitivity [see Fig. 3(b)]. These results can be understood if we consider the behavior of total system profits in the dual-channel model, as illustrated in Fig. 3(a). Specifically,



(a) (b)

Fig. 3. Performance of coordination contract. (a) Total profits in dual-channel model. (b) Percentage increase in total profits with coordination.

as consumers become more sensitive to a firm's own price, the total system profits in the dual-channel model decrease. Therefore, there will be more room to improve performance through supply-chain coordination. In contrast, as the competitor-price sensitivity increases, and thus the two channels are viewed as closer *substitutes*, the total system profits in the dual-channel model increases and there is less room for improvement through coordination. In summary, we conclude that coordination is most critical for products which are highly price sensitive and for systems in which the online and traditional retail channels are not viewed as close substitutes, i.e., when consumers have strong preferences for one type of channel versus the other.

In addition, as predicted by the analysis of the deterministic model in Theorem 2, both the manufacturer and retailer set lower prices in the integrated system. On average, p_R (p_O) decreases by 26.52% (1.94%), ranging from 15.68% to 32.04% (0% to 5.05%), respectively. In the integrated system, the retailer can reduce her price significantly because the manufacturer does not exercise the Stackelberg leader's power, i.e., he does not set a high wholesale price to extract the retailer's profits. The manufacturer can then also reduce the price for the online channel. As a result of the decreased prices, the total system demand increases. The average percentage increase in expected demand in the integrated system is 28.20%, ranging from 14.10% to 33.18%. Finally, although the safety stock at the retailer z_R is always higher for the integrated system than for the decentralized model, the safety stock at the online store z_O is always slightly lower in the integrated system. However, the total order quantity ($Q_M = Q_R + Q_O$) in the integrated system increases by 31.21% on average, ranging from 19.73% to 35.88%.

The key to these results is the inefficiency caused by horizontal price competition and vertical double marginalization. While the online channel only suffers from horizontal price competition, the retailer incurs inefficiency from both horizontal price competition and vertical double marginalization. While horizontal competition will by itself tend to lower the retail price, implying $p_R^* < p_R^I$, vertical double marginalization will tend to increase the price, implying $p_R^* > p_R^I$. Thus, whether p_R^* is greater than or less than p_R^I depends on the balance between the vertical and horizontal competition effects. We find that the vertical double marginalization effect dominates, i.e., $p_R^* > p_R^I$. This higher price at the retailer enables the online channel to also charge a higher price, i.e., $p_O^* > p_O^I$. Next, we consider the safety stocks. Since the price for the online store is higher than the system optimal price, i.e., $p_O^* > p_O^I$, the newsvendor critical ratio is higher, and the safety stock held at the online store will be higher than the system optimal level, i.e., $z_O^* > z_O^I$. For the retailer, $p_R^* > p_R^I$ will not necessarily imply that $z_R^* > z_R^I$ because the effect of vertical double marginalization leads to lower safety stock at the retailer. From our numerical results, we find that the latter effect dominates, i.e., the retailer holds less safety stock than system optimal level, i.e., $z_R^* < z_R^I$.

VII. Managerial Implications

We conclude by discussing the key managerial implications of this research. We have considered a dual-channel supply chain in which a manufacturer sells a product to end-users through both a traditional retail channel and a manufacturer-owned direct online channel. We consider this problem in a newsvendor setting, in which products are sold for a single selling season. Thus, our research is particularly relevant in the apparel, publishing, and electronics industries. The issue of competition in a dual-channel setting is of increasing relevance in these industries. Adding a direct online channel has become quite popular in these industries due to the low barriers to entry, i.e., it is relatively easy for manufacturers to add a direct online channel due to recent development in information technology, and the low setup costs for online stores. The authors in [2] and [5] provide several examples of manufacturers from the apparel and electronics industries who participate in such dual-channel systems.

Given the increased popularity of manufacturer-owned online channels of distribution, it is important to understand the impact of this additional channel on the competitive environment. Our research considers this impact from the perspective of the manufacturer, the retailer, and the consumer. Specifically, we have demonstrated that adding a manufacturer-owned online channel can create supply chain conflicts and that ignoring the impact of the direct channel can reduce the retailer's profit significantly. Specifically, since the manufacturer is the leader of the Stackelberg game, he will attempt to set the wholesale price to extract all of the retailer's profits. As a result, in a supply chain consisting of a traditional retail channel and a manufacturer-owned online channel, we would expect the manufacturer's profits to be significantly higher than the retailer's profits. This is precisely what we observe through our numerical analysis (the manufacturer's profits are seven times higher, on average, than the retailer's).

In order to address the supply chain conflict created by the addition of a manufacturer-owned online channel, we have proposed two coordinating contracts. As discussed in [5] and [6], the use of supply contracts to improve supply-chain performance is relatively common in the apparel and electronics industries. By implementing the coordinating contracts proposed in this paper, the supply chain can restore its efficiency. In addition, our proposed contracts can arbitrarily divide the optimal system profits, allowing the profit allocation between the manufacturer and retailer to depend on the negotiating power of each party. Our numerical results demonstrate the value of the proposed contracts, indicating that coordination can lead to a 6–15% increase in total system profits. Our analysis also provides insights into when coordination has the most value. We find that coordination is most beneficial when 1) the products are highly price sensitive and 2) the two channels of distribution are not viewed as close substitutes, i.e., where consumers have clear preferences for one channel versus the other.

Our research also allows us to consider how the channel conflict created by the entrance of a manufacturer-owned online channel, in addition to a traditional retail channel, impacts the consumer. The results presented in this paper indicate that a decentralized dual-channel supply chain will have higher prices on both channels, and a lower total stocking quantity, than will the coordinated system. Thus, coordination of the dual-channel system not only improves the profits for the retailer and manufacturer, but also makes the consumer better off.

VIII. Conclusion

Finally, we highlight some of our theoretical contributions to the extensive literature on supply chain management. We have presented a model formulation for a dual-channel supply chain that has a mixture of simultaneous (Nash) and Stackelberg game features, in a stochastic model setting with both price and order quantity as decision variables for each channel. We believe that we are the first to study the problem of coordination in a dual-channel supply chain in which an independent retailer and a manufacturer-owned direct online channel engage in price competition. Our model setting allows us to reflect 1) the horizontal competition between the retailer and the manufacturer-owned online store; and 2) the manufacturer's power as the Stackelberg leader, setting the wholesale price. For this complex model setting, we have demonstrated the existence of a unique equilibrium and provided insights into the behavior of that equilibrium. To investigate the inefficiency of the dual-channel system, we have considered the performance of an integrated system, in which the supply chain is controlled by a single decision-maker (the manufacturer). Finally, we have proposed two supply chain contracts (modified revenue-sharing and GL sharing contracts) that enable supply-chain coordination. We have shown that such contracts can coordinate the supply chain and that they can significantly improve the system profits.

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