Module 2: Concrete and Formal Thought

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Module 2 Concrete and Formal Thought

Introduction

You have just completed several activities in which you examined student responses to various problems involving observation and reasoning. Observations of many children and young people attempting to perform similar tasks have led Jean Piaget and other psychologists to formulate theories concerning the mental processes an individual uses to deal with problem situations. In this module, we shall introduce you briefly to stages of reasoning, a feature of Piaget's theory we consider important for physics teachers. Modules 3 and 4 will give you more details and examples to illustrate what we say here. Modules 5 through 11 will help you to apply Piaget's ideas to physics teaching materials and teaching approaches.

Objectives

To assist you in describing and identifying student behavior that indicates concrete thought and behavior that indicates formal thought.

Procedure

Begin by reading the article, "Piaget's Theory in a Nutshell" included in the attached instructional materials. An audiotape with comments coordinated with the article is available; you may wish to listen to the tape during your first reading or during a review. To follow the article, we have provided two more activities for you in this module -- analyzing the student answers to the puzzles in Module 1, and participating in a group discussion -- each at a designated station arranged by your workshop leader. The order of these two activities is optional.
Module 2 Instructional Materials

1. Piaget's Theory in a Nutshell

While you were reading the student responses to the four puzzles in Module 1, you undoubtedly recognized that Type A answers were more satisfactory, more adequate, than Type B answers. In fact, you may have been disturbed to learn that any college students gave Type B answers! We believe that each of the two types of answers is characteristic of a level of reasoning that corresponds to one stage in the intellectual development of children and adolescents as classified by the Swiss psychologist and epistemologist Jean Piaget. We shall therefore give you some background regarding Piaget's theory and then apply it to the problem-solving and reasoning strategies of the students who responded to the puzzles.

The principal concepts of the theory are stages of intellectual development and self-regulation; like concepts in any theory, they are idealizations helpful in analyzing and interpreting observations, and are no more or less real than a point particle or a frictionless plane. A stage of intellectual development is a period when a person's activities and reasoning are characterized by certain distinctive features. We shall give more details below. Self-regulation refers to the process whereby an individual's reasoning advances from one stage to the next. This very important idea is explained in Module 7.

Piaget has described human intellectual development in terms of four stages. The first two, called sensory-motor and preoperational, are usually completed before a child is ten years of age. The last two only are therefore of particular interest to us; they are called concrete thought and formal thought. To give you clues for distinguishing student behavior as falling into one or the other of these stages, we shall now enumerate some of their characteristic patterns of reasoning.

Clues to identify the stage of concrete thought* - affirmative answers to:

(C1) Does the individual make simple classifications and generalizations (e.g., all dogs are animals, only some animals are dogs)?

(C2) Does the individual apply conservation logic (e.g., if nothing is added or removed, the amount remains the same even though the appearance may differ)?

(C3) Does the individual arrange a set of objects or data in serial order and establish one-to-one correspondence between two sets (e.g., the youngest person at dinner gets the most dessert)?

In these respects the individual can reason and solve problems beyond his

*We have used parenthetical codes with the letters C and F to denote indicators of concrete and formal thought. Numbered items are principal clues, lettered items are illustrative examples.
ability in previous stages. Here are a few examples to illustrate these accomplishments. The individual now:

(Ca) understands concepts and simple theories that make direct reference to familiar actions and examples, and can be explained in terms of simple associations, orderings, or numerical equivalences/differences (e.g., objects that do something to each other are in interaction; the waves are high because there is a strong wind);

(Cb) follows step-by-step instructions as in a recipe, provided each step is correctly specified;

(Cc) relates his/her viewpoint to that of another in a simple situation (e.g., a girl is aware that she is her sister's sister).

Yet the advances in reasoning are limited as compared to those achieved at the stage of formal thought. These limitations may be detected as the individual now:

(Cd) searches for and identifies variables influencing a phenomenon, but does so unsystematically (e.g., investigates the effects of one variable but does not necessarily hold the others constant);

(Ce) relates observations and makes inferences from them, but does not consider all possibilities;

(Cf) responds to difficult problems by applying a related but not necessarily correct algorithm (i.e., relies on analogy or agreement more than on inconsistency or contradiction);

(Cg) processes information but is not spontaneously aware of his own reasoning (i.e., does not check his/her own conclusions against the given data or other experience).

**Clues to identify the stage of formal thought** - affirmative answers to:

(F1) Does the individual reason with propositions regardless of whether they are factual or hypothesized?

(F2) Does the individual consider all conceivable combinations of experimental or theoretical conditions, even though some may not be realizable in nature?

(F3) Does the individual recognize and interpret functional relationships in situations described by observable or abstract variables (e.g., field strength is inversely proportional to the square of the distance, the volume of a cube varies directly as the third power of the edge length)?

(F4) Is the individual aware and critical of his/her own reasoning (e.g., recognizes options in using various models or approximations, or tests a conclusion to see whether it is based on a fallacious step)?
Here are a few further examples to illustrate these achievements.

The individual now:

(Fa) engages in hypothetico-deductive reasoning (e.g. in the Islands puzzle, he/she would explain, "If there were a plane route between Islands A and C, then people could get by plane also from Island A to Island B.").

(Fb) plans experiments according to an overall design that investigates the effects of one variable while holding the others constant and also allows for unforeseen contingencies;

(Fc) uses theories and idealized models to interpret observations and draw conclusions;

(Fd) understands concepts defined in terms of other concepts or in terms of abstract relationships (e.g., ratios, mathematical limits);

(Fe) solves problems by introducing intermediate variables not given or asked for directly in the original statement.

In all these items it is the reasoning that counts; the answer or conclusions reached may or may not be correct, depending on whether relevant facts were remembered correctly.

The physics teacher who wishes to apply these ideas should know that many theoretical and experimental issues relating to the theory are currently being investigated. Piaget's original notion was that all persons progress through the stages in the same sequence, though not necessarily at the same rate. Yet recent studies suggest strongly that not everyone reaches the stage of formal reasoning. We have, therefore, earlier characterized the stages as idealizations; few advanced high school or beginning college students would fall clearly into the stage of concrete or of formal thought. Rather, we consider their overall behavior as transitional, partially consistent with each stage. Possibly the reasoning patterns of formal thought are only applied actively by individuals in areas in which they are interested and with which they are familiar.

This qualification leads to four additional points that must be kept in mind by the teacher. First, a person may use primarily formal reasoning patterns in relation to ideas with which he is familiar, while using concrete reasoning patterns in other areas. Second, the stage of formal thought is really open-ended, in that an individual may deepen his understandings, broaden the domains, and/or add new intellectual fields within which he can function formally with confidence. Third, one can enter the formal stage in any area only through self-regulation from the concrete stage, which must not be by-passed. Fourth, by applying memorized formulas to familiar problems, a student may appear to use formal thought though the reasoning pattern is actually concrete(Cf).

You may wonder whether Piaget's theory can be used reliably to improve physics teaching, in view of the fact that physics teaching has been taking place for many years without the theory's benefits. In fact, there are some ways in which Piaget's theory contradicts prominent theories of learning, according to
which individuals in the learner's environment shape his behavior through providing suitable stimuli (learning objectives, exercises) and selective reinforcement (grades, social esteem, academic failure). In our opinion, a sound teaching program reconciles these two approaches as follows: (1) all curriculum design and selection of achievement levels are carried out in accordance with Piaget's theory; (2) the interpersonal contacts between teacher and students rely on reinforcement in the sense that the teacher is the "stimulus" by serving primarily as role model for investigative and analytical attitudes and reinforcement is provided by the students' own sense of success, supported by social and verbal signals (smiles, admiration, encouragement) that acknowledge his success.

The theory's implications for physics teaching can be summarized as follows:

1. Be aware that some of your students approach topics in physics with concrete reasoning patterns, while others will approach the same topics using formal reasoning patterns.

2. Provide a teaching program that allows some success through the use of concrete reasoning patterns.

3. When introducing new topics, do so on the level of concrete thought, for two reasons -- (i) to allow students to gain at least a partial understanding through the use of concrete reasoning patterns, and (ii) to permit students to develop and apply formal reasoning patterns gradually through self-regulation.

4. Devote some effort to helping students establish formal reasoning patterns and thereby gradually raise their level of reasoning.

Modules 5 through 11 will expand on these items.

The thought of using Piaget's theory to improve educational programs systematically is relatively recent, having originated in connection with the elementary school science curriculum development projects during the nineteen sixties. In the last few years, researchers have begun to consider the implications for high school and college teaching, and have found in surveys that many students do not use the mental operations of formal thought when answering puzzles such as those included in Module 1. We shall therefore ask you to review these answers more carefully as another activity in this module.

(Note: if you have not yet used the audiotape commentary on "Piaget's Theory in a Nutshell," you may wish to do so, now or later, while reviewing the article.)
This module provides for two more activities:

(1) Analyzing the student answers to the Module 1 puzzles as revealing concrete or formal thought;

(2) Discussing "Piaget's Theory in a Nutshell" with other workshop participants and staff.

Follow your workshop leader's instructions with respect to these activities. At the conclusion, please answer the review questions on page 2-9.

2. Analysis of Student Responses in Module 1

Below is a chart on which we should like you to record your evaluation of the reasoning patterns used by the six students whose responses to the puzzles were given in Module 1. Please use the following more descriptive categories rather than the very superficial A/B designation that we employed:

- PC = Preconcrete
- C = Concrete
- Tr = Transitional from concrete to formal
- F = Formal
- ? = impossible to classify without more information

Category Tr is intended for responses that include several elements, some of which you would call C while others fit the description of F.

Choose first one student and examine his or her responses to each of the three puzzles. Record your evaluation of his/her reasoning patterns, thus making a "profile" of reasoning for this student. Please follow this procedure for at least three students -- more if you have time. Then read our general analysis and summary.

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<th>Puzzle Responses</th>
<th>Volume</th>
<th>Ratio</th>
<th>Islands</th>
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<tr>
<td>Deloris Johnson (19)</td>
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<td>Barbara Downing (21)</td>
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<td>David Kenting (19)</td>
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To give you specific illustrations of how the stages of reasoning in Piaget's theory can be applied to student work, we shall now give a general analysis of the responses to the puzzles in Module 1. The parenthetical codes refer to the items listed in Piaget's Theory in a Nutshell.

**Volume Puzzle**

**FORMAL THOUGHT (TYPE A).** Even though the weight is dynamically responsible for lifting the water, the combined volume of water plus marble limits the height to which the water can rise in the container. Since the combined volumes are equal for the two marbles, the water will rise to equal heights if the marbles are fully submerged (F1). Note the intermediate concept of the combined volume, or the alternate formulation that if equal marble volumes are added to equal water volumes, the final volumes will be equal (F1). The combined or final volume is not stressed in the statement of the puzzle, but must be introduced by the student (Fe).

**CONCRETE THOUGHT (TYPE B).** It is common sense that the weight of an immersed object is responsible for the force that lifts the displaced water (Ca). Hence the direct conclusion, given differing weights, is the greater the weight, the higher the water level (C3). Note that this reasoning leads to the correct conclusion for immersed bodies that float!

**Ratio Puzzle**

**FORMAL THOUGHT (TYPE A).** Each button corresponds to a certain number of paper clips, an intermediate quantity not stated in the puzzle nor asked for (Fe). Once this conversion ratio is known, the answer is found by simple calculation. Alternatively, the student might conceptualize the height ratio (Fd), another intermediate abstraction, and then reason that this ratio must be invariant with respect to the units of measurement (F1, F3).

**CONCRETE THOUGHT (TYPE B).** Since the height of Mr. Short measures more paper clips than buttons, simply add the extra amount to the height of Mr. Tall (C3). Even though the arithmetic difference in units is not stated or asked for; it is a much more direct measure of the qualitative difference than is the ratio, which comes from making a correspondence between each individual button and paper clip. Another concrete approach makes use of the height difference in buttons of the two figures, and associates that directly with the same difference in paper clips (C3). Note that extra buttons are equated to extra paper clips, in contradiction to the fact that the four buttons measuring Mr. Short are equal to six and not to four paper clips. This inconsistency is not noticed at the stage of concrete thought, but would be noticed at the formal stage and would lead the student who had originally made this mistake (self-regulation!) to re-examine his/her procedure (F4).

**Islands Puzzle**

**FORMAL THOUGHT (TYPE A).** On Question 2, the trip from Island B to Island C is conceptualized as possibly achieved by a change of planes or stopover
at Island D. In other words, the clues about plane routes are not only evaluated in terms of the direct information they provide, but also in terms of the inferences that are possible by using the general rules about connections that were stated in the introduction of the puzzle (F1, F2). On Question 3, the formal thinker imagines all possible routes from Island A to Island C in order to bring to bear the information available in the clues (F2). In particular, he must hypothesize that air travel is possible and evaluate this hypothesis for consistency with the data (F1, F4, Fa). Note that most of the Type A responses quoted in Module 1 did not make use of the formal approach to Question 3, but did on Question 2. This mixture of procedures is often observed in practice and indicates transitional reasoning, a reflection of the fact that the stages of Piaget's theory are idealizations which help one to classify observed behavior, but should not be used to classify people superficially.

CONCRETE THOUGHT (TYPE B). Since the clues do not give the answers to the questions directly, the concrete thinker either can't tell, selects certain details from the map (geographical placement, island separation) or postulates properties of each island to explain his ideas (C1). The properties of a single island (size, topography) used in this approach are conceptually simpler to manipulate than the plane routes, which represent relationships between islands. This approach also eliminates the need to make use of the rules for combining plane routes.

Summary

Below is a chart in which we have applied the above considerations to the responses of six students who attempted the three puzzles in Module 1. In looking at these responses you can see that only one subject gave all formal responses. This indicates that students are at varying levels in various subject areas. We would not expect college students to think formally in every content area. The transition from concrete to formal thinking depends a great deal on the kinds of experiences that any person has in a particular field of study. If a student is a formal rather than a concrete thinker in one area, however, he is more likely to make the transition to formal thought in another area when he is given suitable intellectual stimulation.

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Please discuss these results with a workshop staff member and other participants at a discussion table. Then complete the Module 2 Review Questions on the next page.
Module 2 Review Questions

Please answer these questions in writing. Then compare your ideas with those of other participants and with our answers below.

1. What are two characteristics of concrete thought?

2. What are two characteristics of formal thought?

3. How would you classify the answers to the following question? Explain in each case. "How many different license plates can be made with letters A, B, and C? Describe how you figured it out."

   Answer X: I made six ABC, CAB, BCA, CBA, BAC, ACB. I tried but can't make any more.

   Answer Y: It depends on whether you reuse the letter. If you use each one once, you have three choices for the first letter and two for the second and one for the third, three times two times one makes six. If you can have each letter more than once, like in ABB, then you have three choices for each of the three spots, that's three times three times three or twenty-seven. I'd hate to write them all down. There aren't any other possibilities because I took all into account.

   Your evaluation of X: 

   Your evaluation of Y: 

Formulations

1. Simple classification, serial ordering, one-to-one correspondence,
2. Functional relations, hypothesis-deductive reasoning, systemic
3. X - concrete; enumerates specific instances, concludes by trial
   and error.
4. Y - formal; uses general procedure, considers alternative problem

Answers: I, simple classification, serial ordering, one-to-one correspondence,
Hello! This tape offers comments and examples of the use of concrete and formal reasoning patterns in physics. It accompanies Module 2 of the Workshop on Physics Teaching and the Development of Reasoning produced by the American Association of Physics Teachers. I'm Bob Karplus.

And I'm Jane Bowyer. Have you read the article, "Piaget's Theory in a Nutshell" in Module 2? If so, you may find this tape instructive. If not, I'd suggest that you turn off the tape for now and read the article first, because it introduces the ideas on which this tape is based.

A transcript of the tape is included in your study guide beginning on page 2-10. If you'd like to follow the text, turn off the tape until you find the correct page and then turn it on again.

Piaget has described human intellectual development in terms of four stages during which individuals use certain patterns of reasoning.

Before continuing, I'd like to explain what I mean by a "pattern of reasoning." A pattern of reasoning is a mental process by which certain data, observations, or ideas are compared, organized, or transformed. For example, recognizing that a pendulum with mechanical energy of 20 joules and potential energy of 6 joules has kinetic energy of 14 joules, is a pattern of reasoning that involved comparing forms and amounts of energy. As another example, consider finding Mr. Ruthgren's telephone number between Rutherford and Ruthie; here one has to make use of the alphabetic order of letters and apply it successively to the first, second, third, fourth, and fifth letters in the names in the directory. A person who cannot conceptualize the alphabetic order of letters and apply it systematically is unlikely to find the listing.

Piaget uses the term OPERATION rather than pattern of reasoning, and describes it in his article reprinted in Module 11. We have avoided the term OPERATION because of its other meanings in physics.

Let's now go back to the four stages. The first two, called sensory motor and pre-operational, are usually completed before a child is ten years of age. Only the last two are therefore of interest to us; they are called concrete operational and formal operational. Bob and I will give examples of some characteristic patterns of reasoning associated with these two stages.
General clues to identify concrete thought were listed on pages 2-2 and 2-3:

(C1) Does the individual make simple classifications and generalizations?

Robert Karplus: An example is consistently sorting a collection of objects into electrical conductors and electrical insulators after testing them in a circuit.

Jane Bowyer: (C2) Does the individual apply conservation logic?

Robert Karplus: When a rocket of mass M ejects exhaust of mass ΔM, the student concludes that the rocket has remaining mass M-ΔM.

Jane Bowyer: (C3) Does the individual arrange a set of objects or data in serial order and establish one-to-one correspondence between the two sets?

Robert Karplus: Short organ pipes produce high pitched sound waves and long organ pipes produce low pitched sound waves.

Jane Bowyer: In these respects the individual can reason and solve problems beyond his/her ability in the preoperational stage. Items (C1), (C2), and (C3) are called concrete reasoning patterns, because they are applied to concrete objects and directly observable properties--electrical conductors, mass of a rocket, organ pipes, and audible pitch.

For comparison, we'll now describe a physics example that requires reasoning for which concrete patterns are not adequate. The example is an explanation of Archimedes's principle. Why is the buoyant force on body A when immersed in water equal to the weight of the displaced water?

Robert Karplus: First, imagine a hypothetical body B of exactly the same size and shape as A but composed of water. Since this water body is in equilibrium when immersed in water, the buoyant force it experiences is equal to its weight $W_B$. By the definition of body B, $W_B$ is also the weight of the displaced water. Furthermore, the buoyant force on body B is the net force exerted by the rest of the water across body B's bounding surface. The buoyant force on body A is the net force exerted by the rest of the water across its bounding surface, which is identical with the bounding surface of B. Hence the buoyant force on A equals the buoyant force on B, and this in turn is equal to the weight of the displaced water.

Jane Bowyer: The reasoning involved here was not limited to concrete patterns because the hypothetical water body B and the "displaced water" were never perceptually distinct. Furthermore, the reasoning
made use of certain propositions regarding the boundary surfaces and the equality of forces. The required reasoning comprised formal patterns.

Bob and I will now turn to formal reasoning patterns more broadly, with clues as listed on page 2-3:

(F1) Does the individual reason with propositions regardless of whether these are factual or hypothesized?

Robert Karplus  
The student who correctly finds the thermodynamic efficiency of an ideal heat engine with black body radiation as working medium uses propositions such as the first law of thermodynamics, the equation of state of the radiation, and hypothesized processes making up the carnot cycle. Similar reasoning was used in our explanation of Archimedes's principle. It is also used when Newtonian mechanics, electrostatics, group theory, or other subjects are derived from definitions and postulates rather than being inferred from concrete examples and observations.

Jane Bowyer  
(F2) Does the individual consider all conceivable combinations of experimental and theoretical conditions, even though some may not be realizable in nature?

Robert Karplus  
To solve the Islands puzzle, for instance, the individual had to be aware of all possible ways Island C could be reached from Island A. When inferring the construction of an electric network from measurements at its terminals, the student has to consider all possible ways in which resistors, capacitors, and other circuit elements could be assembled.

Jane Bowyer  
(F3) Does the individual recognize and interpret functional relationships in situations described by observable or abstract variables?

Robert Karplus  
Students who use inverse proportion of weight and distance when equalizing a balance arm apply this formal reasoning pattern. When graphing and interpreting experimental data, they smooth out small irregularities in the measurements and describe the relationship by a simple analytic formula.

Jane Bowyer  
(F4) Is the individual aware of and critical of his/her own reasoning?

Robert Karplus  
The formal operational student checks an answer by comparing the results of a calculation with other similar calculations. He/she verifies that the solution of a motion problem with friction falls between the solutions to the same problem without friction and with very large friction (no slipping at all).
On pages 2-3 and 2-4 there are additional examples of concrete and formal reasoning patterns. Unfortunately, we cannot give you a single, simple criterion for distinguishing between these two types of patterns.

You have to keep four additional points in mind, as described on page 2-4:

First, a person may use primarily formal reasoning patterns in relation to ideas with which he is familiar, while using concrete reasoning patterns in other areas with which he is unfamiliar.

Second, the stage of formal thought is really open-ended, in that an individual may deepen his understandings, broaden the domains, and/or add new intellectual fields within which he can function formally with confidence.

Third, one can enter the formal stage in any area only through self-regulation from the concrete stage, which must not be by-passed.

Fourth, by applying memorized formulas to familiar problems, a student may appear to use formal thought though the reasoning pattern is actually concrete.

You may wonder whether you should test your students to identify their developmental stage. In view of what we have just said, and the fact that the stages are idealizations, such a testing effort is likely to give unclear results. I would recommend that you observe your students' work on their physics problems for a period of a week or two and try to identify the reasoning patterns they use.

This is the end of our comments. We hope you are finding the workshop interesting. Do discuss these ideas with your fellow participants—they may have a very different point of view from yours. Before turning off the tape player, please rewind the tape so it can be used by other participants. Thank you for listening. Goodbye!