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Reality and Theory in a Collision

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NOTE

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Billiard-ball collisions are frequently cited in introductory physics textbooks, usually as examples of elastic collisions. Many articles describing such collisions have appeared in this journal and elsewhere,[1–6] but comparisons between theoretical results and actual collisions are rare, and most of the theoretical analyses have simply assumed friction to be negligible during the collision time. Students trying to simulate billiard-ball collisions on a popular simulation program such as *Interactive Physics*® may encounter interesting collisions, such as ones in which the cue ball is thrown up into the air when the frictional coefficients are set at even modest levels. We decided to use a video-analysis tool designed for the physics classroom (*VideoPoint*®) to examine a slow-motion film of a head-on collision made some years ago and currently available on *Physics: Cinema Classics*.[7] In this paper, we compare some of the experimental data we obtained with theoretical results and the simulation results.

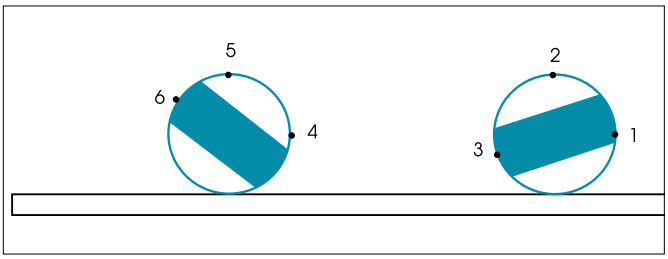

Fig. 1. Points used to determine position and rotation angle of colliding billiard balls in *VideoPoint*.

The Film and the Theory

A. Description of the Video Clip

The *Physics: Cinema Classics* disc provides a *Project Physics* film of a head-on collision between two billiard balls shot nominally at 3000 frames per second by a fixed camera. Each ball has a wide stripe around a diameter making rotation visible, as shown in Fig. 1. The film shows a cue ball rolling without skidding across a level surface. It then strikes a stationary target ball head on. What happens next can really startle a class of complacent physics students who have just finished studying collisions between particles and have seen demonstrations of Newton's cradle. As they watch, the target ball starts to skid across the surface without any initial spin, but it gradually slows down as its spin increases. Meanwhile, the cue ball slows down sharply at first while continuing to spin, and then it begins to accelerate in pursuit of the target ball. Friction between the balls and the surface is, of course, the gremlin responsible for such behavior. During the collision, the frictional torque on each ball was relatively small, so there was little change in a ball's rotation during the extremely brief collision time, but eventually the effect of such torques became substantial.

B. Simplified Model

A simple model frequently adopted in textbooks dealing with such collisions is (1) to assume it to be perfectly elastic (i.e., kinetic energy is conserved), (2) to ignore any friction during the short collision time, and (3) to assume the surface friction subsequent to the collision to be the same for both balls.

The solution of this model is straightforward. Immediately after the collision, the cue ball has lost all its linear momentum but its angular momentum has not changed; meanwhile the target ball has acquired all the linear momentum but has no angular momentum yet. As time passes, the frictional forces between each ball and the surface change the linear and angular momenta of each ball until its linear and angular velocities satisfy the no-skidding condition, $\omega = v/r$. Letting τ_i be the duration of the skidding for each ball and f the surface frictional force, we obtain two pairs of equations for τ_i and the final linear velocities v_{if}:

Linear Momentum:

Cue Ball $\quad f\tau_c = mv_{cf}$

Target Ball $\quad f\tau_T = mv_0 - mv_{Tf}$

Angular Momentum:

Cue Ball

$$fr\tau_c = \frac{2}{5}(mv_0 r - mv_{cf} r)$$

Target Ball

$$fr\tau_T = \frac{2}{5}(mv_{Tf} r)$$

These are readily solved yielding

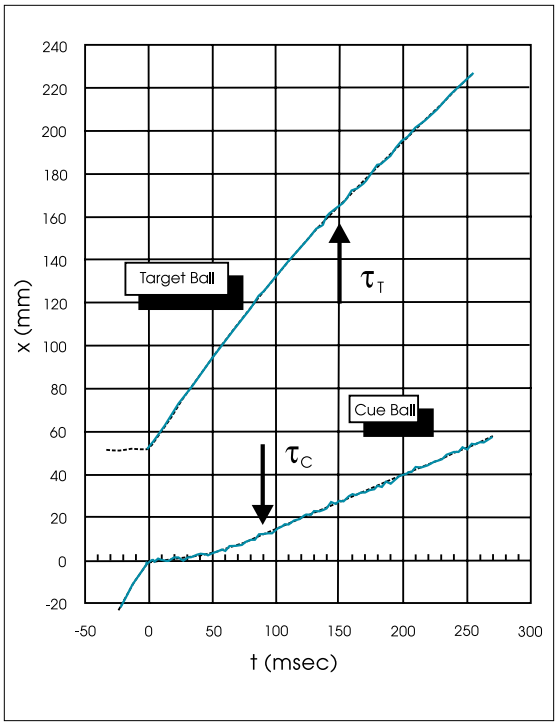

Fig. 2. Displacements of colliding billiard balls. Collision occurs at $t = 0$. Raw data and least-squares fitted curves are shown. Skidding of cue ball stops at time τ_c; skidding of target ball stops at τ_T.

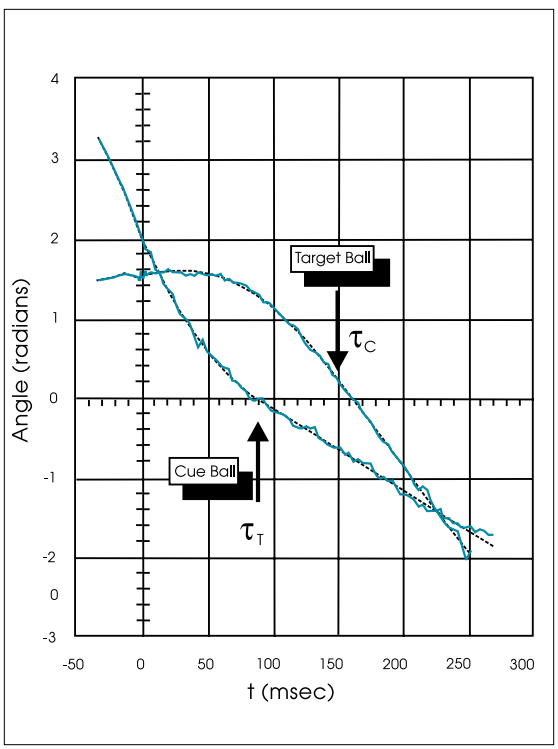

Fig. 3. Rotation angles of colliding billiard balls in Fig. 2. Raw data and least-squares fitted curves are shown. Skidding of cue ball stops at time τ_c; skidding of target ball stops at τ_T.

$v_{cf} = \frac{2}{7}v_o$ and $v_{Tf} = \frac{5}{7}v_o$ and $\frac{f\tau_c}{m} = \frac{2}{7}v_o = \frac{f\tau_T}{m}$; that is, the skidding times are, interestingly enough, the same for both balls.

Data

A. Data collection

Thirty years ago this film was available on a film loop, and students in *Project Physics*[8] were asked to view it carefully, describe verbally what happened as thoroughly as possible and to make a simple measurement such as checking to see whether momentum was conserved between the two balls. This could have been done by projecting single, stopped frames and marking the ball positions on a screen. Since the camera frame rate and diameter of a billiard ball are given quantities, both linear and angular velocities can then be roughly computed. Later on, with the availability of video players, students viewing the clip could make marks on a transparent sheet covering a television screen and then take measurements off the sheet. In either of these situations, tracking the motion of the balls through many frames would have been tedious and error prone. These days, a video clip can be digitized, viewed on a computer, and studied in detail with software such as *VideoPoint*.[9] With such software, the user can view the video clip one frame at a time and use the computer mouse to mark the position of interesting points. The coordinates of the points are then stored in tables. This makes it easy to take data from many frames of the film and to save it in a format that is convenient for analysis.

We collected data much as students might by marking three points on each ball as shown in Fig. 1. In principle, two points would have sufficed, but the resolution of *QuickTime*® movies is limited, and the high-speed camera did not register frames very well, so an additional point was used as a cross check. We made measurements every 10 frames, which is considerably more measurements than students would ever have tried with pencil-and-ruler techniques.

We assumed that the stated frame rate (3000 fps) was accurate and that the film had been accurately transferred to videodisc. Sizes of billiard balls for different games may vary slightly. For calibration purposes, we used the following numbers supplied with the film:

mass = 170.3 g
diameter = 5.24 cm = $(2\frac{1}{16}$ in)

We also assumed each ball to be of uniform density so that its moment of inertia is $I = \frac{2}{5}mR^2$. We imported the raw data from *VideoPoint* into a spreadsheet for analysis. Because image resolution was limited and because we measured only a few points per frame, the position and angle measurements shown in Figs. 2 and 3 exhibit noticeable jitter.

B. Analysis

We determined that, just before the collision, the cue ball was moving with a velocity of 0.98 m/s and rotating with an angular velocity that is consistent with no skidding.

The general features of the motion are apparent in Fig. 2: the *cue ball* moving with constant velocity, then halting suddenly and undergoing a period of acceleration and finally

achieving a constant velocity again; the *target ball* being struck, starting to move but decelerating as it skids across the surface and begins to rotate, finally reaching a constant velocity. In the film the target ball visually seems to rotate backwards slightly after the collision, but the motion is so slight and happens so quickly that a viewer could easily dismiss it as an illusion. However, Fig. 3 confirms the visual impression. Such reverse rotation indicates that friction between real billiard balls during their collision is measurable.

The post-collision motions are divided into a skidding phase followed by a rolling phase. We fitted the data from each phase to simple curves using an iterative procedure. From the graphs, we visually estimated the time at which skidding stopped by using the time-honored tradition of viewing the graphs at a grazing angle. We then computed least-squares fits of the linear and rotational motions to quadratic functions of t during the skidding phase (constant acceleration) and to linear functions of t during the rolling phase (no acceleration). We then adjusted our estimate of the onset time of the rolling phase until the computed velocities at the end of the skidding phase matched those at the start of the rolling phase. The skidding times, τ, determined from linear motions or from rotational motions could differ by nearly 10%. This is one estimate of the precision of our collected data. The fitted curves have been superimposed on the data in Figs. 2 and 3.

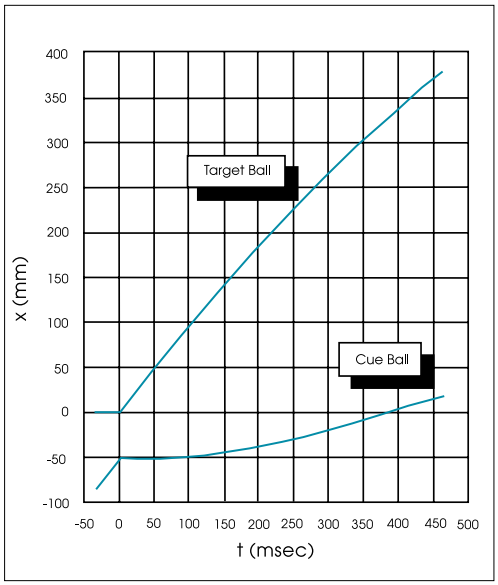

Fig. 4. Displacements of colliding billiard balls simulated by *Interactive Physics*. Initial velocity of cue ball = 1 m/s. Frictional coefficients = 0.08.

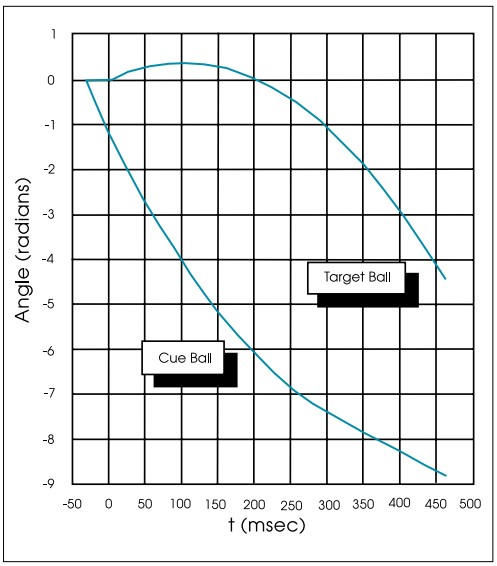

Fig. 5. Rotation angles of colliding billiard balls simulated by *Interactive Physics*. Initial velocity of cue ball = 1 m/s. Frictional coefficients = 0.08. See Fig. 4.

If we assume that all the measurements are normally distributed and have the same standard deviation and that the fit of the curve to the data is good (as it seems to be to the eye), then during the skidding phase of the motion, the acceleration of the target ball (using 50 data points) is about –1.83 m/s^2 with a standard error of about 5%, and the acceleration of the cue ball (using 35 data points) is about +3.13 m/s^2 with a standard error of about 10%.

Here is a list of the interesting measurements we obtained.

1. After the collision, 10% of the linear momentum had been lost. (0.167 kg m/s versus 0.151 kg m/s).
2. After the collision, 17% of the translational kinetic energy of the system had been lost.
3. After the collision, 15% of the rotational kinetic energy was lost.
4. Overall, 17% of the total kinetic energy of the system was lost during the collision.
5. During the skidding phase, presumably the only force acting on each ball is the friction between it and the surface. Consequently we expected to see accelerations that were identical in magnitude. Instead, the acceleration of the cue ball (+3.13 m/s^2) and the acceleration of target ball (–1.83 m/s^2) were *significantly* different. The skidding times are different: τ_c = 90 msec, τ_T= 150 msec.
6. The final velocities of the cue ball and the target ball were 0.251 m/s and 0.583 m/s respectively, so their ratio is 0.43. In the simple model presented above with an initial cue ball velocity of 0.98 m/s, the computed values would be 0.28 m/s and 0.70 m/s for a ratio of 0.40.

Simulations

Since the real data contain significant noise, students might be tempted to use a computer simulation program to try to understand what is happening. How well can a simple simulation program like *Interactive Physics*® reproduce the data observed here? It is easy to set up a head-on collision in *Interactive Physics*.[10] The calculations in *Interactive Physics* are done by numerical integration of the equations of motion. In simulating collisions, accuracy is achieved by limiting the amount of overlap of two colliding bodies (their inter-penetration) and choosing smaller time-steps for the integration. You can choose the masses and speeds of the balls colliding, the coefficient of restitution (elasticity parameter), and coefficients of friction. We chose the masses for the

simulation to be the same as the masses of actual billiard balls, set the initial velocity at 1 m/s, and set the elasticity parameter to 1, fully elastic collisions. We also specified that the objects not overlap by more than 0.1 mm. Choosing a smaller overlap made it too difficult to position the balls at the start of a simulation: they tended to bounce off the surface when released.

Some shortcuts programmed into *Interactive Physics* limit its usefulness in this situation. For instance, you assign *each* object its own coefficient of friction and whenever two bodies interact, the program actually uses the smaller of the two friction coefficients. In this situation, it means that you cannot have smaller frictional coefficients between the balls than between a ball and the surface—as would be needed in order to replicate the physical situation.

Consequently, in order to model the ball/ball interaction properly, we had to use small coefficients of friction everywhere so that the balls would not interact too strongly with each other, but this means that in the simulation the balls skid for much longer time than they did after the actual collision. Figures 4 and 5 show the result of a simulation using a coefficient of friction $\mu = 0.08$. The basic qualitative features are the same as the actual collision; however, quantitatively, the final speeds of the balls are 0.229 m/s and 0.657 m/s, that is, the cue ball is slower and the target ball faster than the actual experiment but both velocities are in reasonable agreement with the simple model presented earlier. Even though the program's elasticity parameter e was set equal to 1, in the simulation more than 10% of the kinetic energy of the cue ball was lost during the collision, so frictional effects during the collision time seem to be important in the simulation as they are in the actual experiment.

Conclusions

Once again we find upon careful examination that nature provides us with simple events that challenge our understanding. The qualitative features of a head-on billiard ball collision are well predicted by our simple theories. However, when we look in detail at the collision there are some features that puzzle us, e.g., the relatively large loss of energy for an "elastic" collision and the difference in post-collision accelerations. We have not tried to film other collisions between billiard balls to see if the data are reproducible. We offer it as a challenge to physics classes everywhere: what is really happening here?

We urge physics teachers to use the analytical tools made possible by computers to enable their students to carefully study everyday events. Let the students discover for themselves the stimulating richness of nature's mysteries. Let them have the satisfaction of having wonderful ideas of their own.

Acknowledgments

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9. *VideoPoint*® is published by Lenox Softworks (www.lsw.com/videopoint) and distributed by PASCO scientific, 800-772-8700.
10. *Interactive Physics*® is available from Knowledge Revolution, 800-766-6615.