2014

Two Stage Procurement Processes With Competitive Suppliers and Uncertain Supplier Quality

Yue Jin
Alcatel-Lucent, yue.jin@alcatel-lucent.com

Jennifer K. Ryan
Rensselaer Polytechnic Institute, ryanj6@rpi.edu

Walter Yund
GE Global Research, wyundiv@gmail.com

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Yue Jin, Jennifer K. Ryan, and Walter Yund

Abstract—**This paper considers a sourcing problem faced by a manufacturer who outsources the manufacturing of a product to one of several competing suppliers, whose cost and quality capabilities are unknown. We consider a two-stage sourcing process in which the first stage is the qualification stage, while the second stage is the supplier selection stage. In the first stage, the manufacturer exerts effort to learn about the quality level of each of the suppliers and then must determine the set of qualified suppliers, subject to some tolerance for error. In the second stage, the manufacturer runs a price-only procurement auction, in which the qualified suppliers compete for the manufacturer's business. We model this two-stage sourcing process with the goal of obtaining insights into manufacturer's optimal decisions. We seek to determine the optimal qualification standard, the optimal amount of effort to be exerted in the qualification process and the appropriate tolerance for error in the qualification process, and to understand the interactions between these decision variables. We are particularly interested in understanding how the manufacturer can design the process to 1) ensure the firm only sources from qualified suppliers and 2) encourage competition among the suppliers during supplier selection.**

Index Terms—**Competitive analysis, decisions under risk and uncertainty, economic modeling, manufacturing supply chain, supply chain integration.**

I. Introduction and Motivation

The dramatic increase in the use of outsourcing over the past several decades has been well documented. Manufacturers who make the decision to outsource often do so with the goals of reducing costs and obtaining operational efficiencies [21]. However, the decision to outsource also creates a new set of business problems for the manufacturer. In particular, the manufacturer must now make critical decisions regarding the design of the sourcing process, including determining the capabilities to be required of the selected suppliers, how those capabilities will be assessed, and how to select among a set of potential suppliers.

Manuscript received September 17, 2012; revised March 18, 2013; accepted May 20, 2013. Date of publication June 18, 2013; date of current version January 16, 2014. Review of this manuscript was arranged by Department Editor S. Talluri.

Y. Jin is with Bell Labs Ireland, Alcatel-Lucent, Dublin, Ireland (e-mail: yue.jin@alcatel-lucent.com).

J. K. Ryan is with Industrial and Systems Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180 USA (e-mail: ryanj6@rpi.edu).

W. Yund is with the Complex Systems Engineering, GE Global Research, Schenectady, NY 12345 USA (e-mail: wyundiv@gmail.com).

This paper has supplementary downloadable material available at http://ieeexplore.ieee.org.

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TEM.2013.2266276

In this paper, we consider the sourcing problem faced by a manufacturer who has made the decision to outsource the manufacturing of a critical product to one of several competing suppliers. These suppliers differ in their capabilities for producing high-quality products, and in their unit production costs. In addition, the manufacturer does not have perfect information regarding the suppliers' capabilities and costs. Therefore, the manufacturer must design the sourcing process to 1) enable learning about the suppliers' quality capabilities and 2) encourage competition between the potential suppliers.

When a manufacturer has imperfect information regarding the suppliers' cost and capabilities, it is common to use a two-stage procurement process in which the first stage is the qualification stage, while the second stage is the supplier selection stage. In the qualification stage, potential suppliers are screened for their quality capability, i.e., the manufacturer must determine whether a given supplier is capable of producing products with a given quality level. A key goal of this stage is to "mitigate the risk of supplier nonperformance" on quality [31]. In the supplier selection stage, those suppliers that have been qualified are invited to compete for the manufacturer's business by participating in a sealed-bid price-only procurement (or reverse) auction. A key goal of this stage is to induce competition between the qualified suppliers, so that the manufacturer can obtain the best pricing terms. Thus, the design of a two-stage process requires a fundamental tradeoff: A more stringent qualification stage implies that the manufacturer sources from only the most qualified suppliers. However, a more stringent qualification stage reduces the number of qualified suppliers and the level of competition in the supplier selection stage. Therefore, a key challenge when designing the two-stage process is achieving the right balance between qualification and competition.

Manufacturers who procure goods through reverse auctions often do so in combination with a qualification process, i.e., as part of such a two-stage process [12].[1] This is because, as Merson [22] notes, "[i]n a process that. . .results in an award to the lowest bidder, it is imperative to have established, responsible competitors who offer high-quality products." However, qualification can require substantial time and costly effort on the part of the manufacturer. In addition, while qualification can allow the manufacturer to refine her understanding of the suppliers' capabilities, it is unlikely to provide perfect information regarding those capabilities. Thus, when designing the qualification process, the manufacturer must determine not only the level of capability required for a supplier to be considered qualified, but also the level of costly effort to exert to learn about the potential

[1] A two-stage process is used in the military [4], as well as in the telecommunications [16], pharmaceutical [19], and automotive [25] industries.

suppliers, and the appropriate tolerance for error in the qualification decision. To address these issues, we develop a model of a two-stage sourcing process. We use this model to study how the manufacturer can design the sourcing process to ensure that she only sources from qualified suppliers, while still maintaining competition among the suppliers in the supplier selection stage, and to understand the complex interactions between the manufacturer's three decision variables, as well as the impact of key input parameters on the manufacturer's decisions.

A. Problem Statement

We consider a buyer who must select a single supplier from a set of potential suppliers by considering both price and quality. The buyer's profit consists of revenue from sales of the product, along with the cost of procuring the product and the costs associated with poor product quality. The suppliers' profits consist of the revenue earned from the sale of the product to the buyer and the production costs, which are increasing in the product quality. The suppliers differ in their cost structure, as well as in their capabilities for producing high-quality products. The buyer uses a two-stage sourcing process, in which the goal of the first stage is to determine the set of qualified suppliers, i.e., those capable of meeting a target quality level. We assume an inspection-based qualification process, in which the buyer selects a sample of each suppliers' product in order to assess its quality level. In the second stage, i.e., the supplier selection stage, the buyer invites the qualified suppliers to compete in a procurement auction and chooses among them on the basis of price. The buyer's problem is to design the sourcing process, i.e., to set the target quality level used in the qualification stage, as well as the amount of effort to exert in the qualification process, i.e., the number of units to sample from each supplier, and the tolerance for error in the qualification decision.

B. Literature Review

We consider the design of a two-stage sourcing process with imperfect information regarding the potential suppliers' costs and quality capabilities, where supplier selection consists of a price-only procurement auction. We divide the relevant literature into four areas: papers considering 1) timing of the qualification process; 2) design of the supplier evaluation process; 3) procurement through multidimensional auctions; and 4) quality-related procurement.

1) Timing of the Qualification Process: When considering the design of a two-stage procurement process, one important question is when to perform supplier qualification. Cripps and Ireland [10] show that prequalification (i.e., qualification performed prior to bidding) and postqualification (i.e., qualification performed after bidding) are revenue equivalent when price and quality are evaluated at the same time. Wan and Beil [31] note that prequalification is expensive when there is a large number of potential suppliers, while postqualification can lead to a higher winning bid price. Wan *et al.* [32] consider the question of when to qualify suppliers in a setting with competition between an "incumbent" (i.e., already qualified) supplier and an "entrant" (i.e., not yet qualified) supplier. Finally, bid evaluation can be thought of as qualification during the competition (rather than pre- or postcompetition). Carr [6] considers a system with costly bid evaluation and characterizes conditions under which it is optimal for the buyer to evaluate all bids. In this paper, we assume that the buyer employs prequalification, i.e., the buyer inspects and qualifies the suppliers prior to the bidding process.

2) Design of the Supplier Evaluation Process: A key aspect of the supplier qualification stage is the evaluation and assessment of the suppliers' capabilities. There is a vast literature on supplier evaluation, most of which considers the problem of ranking, scoring, or categorizing a set of potential suppliers when there are multiple attributes that matter to the buyer. Recent surveys of this literature include [7], [15], [1], and [5]. While the literature considers a wide variety of supplier attributes, the most commonly studied are price, quality, and delivery time. A wide variety of supplier evaluation approaches have been proposed. Among the most common are analytic hierarch process, analytic network process, data envelopment analysis (DEA), mathematical programming (MP) techniques, multiobjective programming (MOP), artificial intelligence approaches, and fuzzy set theory approaches. However, the vast majority of this literature assumes that the buyer has perfect information regarding the suppliers' characteristics. In contrast, in this paper we consider a buyer who has imperfect information on the suppliers' costs and capabilities.

The literature on supplier evaluation under imperfect information is more limited. As noted in [7], fuzzy approaches are the most common for handling imperfect information. However, some authors have considered chance-constrained DEA and MP approaches. For example, Talluri *et al.* [29] present a chance-constrained DEA approach in which the outputs, conditioned on the inputs, are assumed to follow a joint Normal distribution. Wu [33] and Wu and Olson [34] also take a stochastic DEA approach to supplier evaluation. Wu and Olson [35] compare three approaches (chance constrained programming, DEA, and MOP) to supplier evaluation when the suppliers' unit price, acceptance rate, and on-time rate are uncertain. Bilsel and Ravindran [2] use a chance-constrained programming approach when there is uncertainty regarding demand, supplier capacities, and supplier costs. Dogan and Aydin [11] present a Bayesian Network approach which allows the buyer to incorporate his domain specific knowledge into the evaluation process.

The research considered in this paper differs from this previous work in two key ways:

1) Our model is appropriate when the buyer uses a two-stage process in which the second stage is a reverse auction. In such a setting, the suppliers choose their bid prices under competition with other qualified suppliers. Hence, decisions regarding supplier evaluation and qualification, which influence the number of suppliers participating in the auction, have an impact on the suppliers' bid prices. To our knowledge, this interaction has not previously been considered in the supplier evaluation literature. In addition, because we model supplier selection using an auction, in our paper the prices charged by the suppliers are endogenous and functions of the buyer's decisions regarding the design of the qualification process. In

contrast, most of the supplier evaluation literature takes the suppliers' prices as exogenously specified.

2) Our model allows for imperfect information about the suppliers' costs and capabilities. In addition, our model captures the fact that, in practice, the buyer must decide how much costly effort to exert in order to learn more about the suppliers' capabilities. Finally, since the buyer cannot perfectly assess the suppliers' capabilities, we also allow the buyer to determine the optimal tolerance for error in the qualification decision. These issues have not been jointly considered in the existing supplier evaluation literature.

3) Multidimensional Procurement Auctions: An alternative to the two-stage procurement process and supplier evaluation processes described earlier is a multidimensional auction, in which the suppliers' bids consist of multiple attributes, such as price, quality, and logistics costs. Like some of the supplier evaluation approaches, most of the multidimensional auction approaches require the buyer to combine the various supplier attributes into a single score. Unlike the much supplier evaluation literature, the auction models incorporate the effects of imperfect information and competition on the suppliers' bidding strategies. Thiel [30] demonstrates that, under certain conditions, designing a multidimensional auction is equivalent to designing a single-dimensional auction. Che [8] and Branco [3] consider multidimensional auctions in which the buyer evaluates the suppliers using a score function that combines price and quality. The aforementioned papers assume that the nonprice attributes are determined endogenously. In contrast, Engelbrecht-Wiggans *et al.* [13], Shachat and Swarthout [26], and Kostamis *et al.* [18] consider various types of multidimensional auction mechanisms when the suppliers' nonprice attributes, such as quality or logistics costs, are exogenously specified. In our paper, the suppliers' nonprice attribute is modeled as quality, which is determined endogenously, subject to an exogenously specified limit on the suppliers' capability for achieving high quality.

4) Quality-Related Procurement: The literature on quality-related procurement, in which the buyer evaluates the suppliers based specifically on quality, is also relevant. See Zhu *et al.* [37] for a recent review of this literature. As noted by Yan *et al.* [36], this literature can be divided into two categories: i) papers that consider a single buyer and a single supplier and ii) papers that consider competition among multiple (potential) suppliers. Papers in category (i) include Reyniers and Tapiero [24], who present a model of the supplier's quality decision and the buyer's inspection decision, given a procurement contract that specifies penalties (price rebates and shared repair costs) to be incurred by the supplier in the event of a defective unit, and Lim [20], who study a similar model, but with information asymmetry. While we also consider a buyer with imperfect information on supplier quality, we take a different approach than Lim [20], who considers the design of a screening contract. In contrast, we assume the buyer uses the qualification stage to learn about supplier quality. Papers in category (ii) include Tagaras and Lee [28], who consider the problem of evaluating suppliers on both cost and quality. In addition, the literature on quality-related procurement includes numerous papers considering quality improvement decisions on the part of the supplier and/or the buyer. Of particular relevance is Starbird [27], who considers a buyer who uses acceptance sampling to monitor supplier quality and to encourage supplier quality improvement.

II. Problem Description

We consider a buyer (she) who sells a commodity-like item to consumers at a fixed price. Total consumer demand, denoted by V, is fixed and known. The buyer can purchase the item from any of n potential suppliers (denoted by $i = 1, \ldots, n$). The buyer has chosen to single source, and thus must select a single supplier from the set of potential suppliers. We take the number of potential suppliers, n, as exogenously specified. Relaxing this assumption would require the introduction of a third stage into our model, i.e., the supplier's decision regarding whether to participate in the qualification stage, which would complicate the analysis.

In the next section, we provide an overview of the two-stage sourcing process used by the buyer. Then, in the following two sections, we provide problem descriptions for the buyer and suppliers. In Section III, we characterize the optimal decisions for the buyer and suppliers at each stage of the process.

A. Two-Stage Sourcing Process

The buyer has imperfect information regarding the costs and quality capabilities of the suppliers. Thus, she has chosen to use a sourcing process consisting of two stages. The first stage is the qualification stage, in which the buyer exerts effort in order to learn about the quality capabilities of the potential suppliers, and then determines the set of qualified suppliers. When designing this qualification stage, the buyer has three key decisions:

1) The buyer determines the minimum acceptable quality level for the suppliers, denoted by Q. We will refer to Q as the qualification threshold. The buyer would like to qualify only those suppliers who are capable of achieving quality level Q.
2) The buyer chooses the level of effort, denoted by e, to exert in the qualification process. The suppliers' capabilities for producing high-quality products are initially unknown to the buyer, i.e., prior to qualification the buyer only knows the distribution of the suppliers' capabilities across the set of suppliers. However, the buyer can exert effort during the qualification process, e.g., the buyer can inspect a sample of size e of each supplier's product, in order to learn about that supplier's capability. This effort, although costly, enables the buyer to a) differentiate between the capabilities of the suppliers and b) reduce the uncertainty regarding a given supplier's capability.
3) The buyer sets the tolerance for error in the qualification decision, denoted by α. After exerting effort in the qualification process, the buyer has a different probability distribution for each supplier, representing her beliefs regarding the supplier's quality capability. Given these distributions, the buyer must determine whether a given supplier, whose true quality capability is still uncertain, should be labeled as qualified. The buyer does so by specifying a tolerance for error, where an error occurs when

the buyer qualifies a supplier whose true quality capability is less than the threshold Q. Thus, α represents the probability of qualifying a supplier who cannot achieve quality level Q.

Given the design of the qualification stage, i.e., given (Q, e, α), the buyer will exert effort e to inspect each of the potential suppliers and then will determine the appropriate set of qualified suppliers based on the qualification standards, Q and α.

The second stage of the sourcing process is the supplier selection stage, in which the buyer selects a single supplier from the set of qualified suppliers. When the suppliers' costs are uncertain, a common approach is to use a procurement auction, in which the qualified suppliers submit sealed price-only bids, and the buyer chooses among them based only on price. The bid prices are determined by the suppliers based on their own costs and capabilities, as well as their beliefs regarding the costs and capabilities of the other suppliers. The bid prices will also depend on the number of competing suppliers, i.e., a larger number of qualified suppliers competing in the supplier selection stage will lead to lower bid prices.

B. Buyer's Problem

We assume that the buyer designs the qualification stage, i.e., chooses (Q, e, α), in order to maximize her expected profit, which is composed of several components.

1) The buyer's revenue depends on the total consumer demand V and the selling price to the consumer s. Because we take both s and V to be exogenously specified, the buyer's total revenue, sV, is fixed.
2) The buyer incurs a procurement cost per unit, which is the price offered by the winning supplier in the supplier selection stage. We use c_p to denote the buyer's expected unit procurement cost. As will be seen, c_p depends on the buyer's decision variables.
3) The buyer incurs costs associated with poor quality products provided by the winning supplier. For simplicity, we model these costs as warranty costs. In this case, the measure of quality for supplier i, denoted q_i, will be the probability that a randomly selected unit of supplier i's product will not require warranty work. Therefore, q_i takes values between 0 and 1, with 1 representing the highest possible quality level. The cost to the buyer of a single unit of product requiring warranty work will be denoted by w. Then, if we let $\bar{q}$ denote the expected quality level provided by the winning supplier, the buyer's expected warranty cost per unit is $w(1-\bar{q})$. As will be seen, $\bar{q}$ depends on the buyer's decision variables and the distribution of quality levels across the suppliers.
4) The buyer incurs a cost associated with effort exerted in the qualification stage. We let e denote the level of effort exerted on each supplier, which is assumed to be the same for all suppliers, and we let $c_E(e)$ denote the cost to exert effort level e. Thus, if there are n potential suppliers, the buyer's total cost of effort is $n \times c_E(e)$.

The buyer's total expected profit is $\pi_B = V\{s - [c_p + w(1-\bar{q})]\} - nc_E(e)$. Since the revenue is fixed, we will minimize the buyer's expected cost. It will be convenient to normalize by the volume V and to consider the buyer's expected unit cost, which we will denote by c_B:

$$c_B = c_p + w(1-\bar{q}) + \frac{nc_E(e)}{V}. \tag{1}$$

The buyer's problem is to choose (Q, e, α) to minimize this total expected unit cost.

C. Supplier's Problem

Supplier i's profit is a function of his bid price p_i, his chosen quality level q_i, and the cost to manufacture an item with quality level q_i. We will assume that the unit production cost for supplier i, who produces a product with quality level q_i, is $c_i q_i^z$, where z is a constant greater than or equal to 1, common to all suppliers in the industry. Here c_i represents the cost incurred by supplier i to produce a unit of product with perfect quality ($q_i = 1$). These costs are assumed to vary across the suppliers. To enable closed form results, we assume that the c_i are uniformly distributed between lower and upper bounds, denoted by c_L and c_H, respectively. For simplicity, we will refer to c_i as the unit cost for supplier i. Notice that this production cost is an increasing and convex function of quality, i.e., the marginal cost of quality is increasing, as would be typical in many industries. A cost function of this form is commonly used in the literature. See, for example, [9] .

Supplier i has a maximum achievable quality level, which we denote by $q_i^{\max}$. Thus, the supplier may choose to produce at any quality level q_i, subject to $q_i \le q_i^{\max}$. We refer to $q_i^{\max}$ as supplier i's quality capability. Each individual supplier's capability is a function that supplier's experience, access to skilled labor, and production technology. Thus, the $q_i^{\max}$ will vary across the suppliers. To enable closed form results, we assume that the $q_i^{\max}$ are uniformly distributed between lower and upper bounds, denoted by q_L and q_H, respectively. This assumption might apply in industries where many varying processes yield varying qualities to meet the needs of many different buyers.

The profit for supplier i, given that he wins the buyer's business with bid price p_i and quality level q_i, can now be written as $\pi_i = V[p_i - c_i q_i^z]$. Supplier i's problem is to select his quality level q_i and selling price p_i, to maximize his expected profit, subject to $q_i \le q_i^{\max}$, taking into consideration the buyer's mechanism for choosing among the potential suppliers, and the costs and capabilities of the other suppliers. In doing so, we assume each supplier knows three things: 1) his own cost structure and capability, 2) the underlying distribution of costs c_i within the industry, and 3) the underlying distribution of maximum quality levels $q_i^{\max}$ within the industry.

III. Designing the Optimal Two-Stage Sourcing Process

We can now determine the optimal decisions for each party, i.e., the optimal bid prices and quality levels for the suppliers, and the buyer's choice of Q, e, and α. We will work backward, i.e., we will first consider the supplier selection stage, assuming a fixed number of suppliers have passed the qualification stage. We will then consider the qualification stage.

A. Supplier Selection Stage

We first analyze the competition between the qualified suppliers, who submit price-only bids to the buyer. We assume that some number m of the original n suppliers have been qualified, where $m \leq n$. The analysis presented here is similar to that in [16] .

To determine how the m qualified suppliers will bid in the supplier selection stage, we must first consider how these suppliers will set their quality levels. Recall that the buyer would like to qualify only those suppliers who are capable of achieving the qualification threshold, i.e., those suppliers with $q_i^{\max} \geq Q$. However, the buyer cannot observe the exact values of $q_i^{\max}$ and thus the buyer may make an error in the qualification process, i.e., may qualify a supplier who has $q_i^{\max} \geq Q$. Thus, we must consider two cases. First, if $q_i^{\max} \geq Q$, then supplier i is capable of achieving quality level Q. Since the buyer's supplier selection decision is made only on the basis of price, to keep his costs as low as possible, supplier i will choose to produce a product with the minimum possible quality level, i.e., will choose to set his quality level equal to the threshold, even if he is capable of achieving higher levels of quality. Thus, if $q_i^{\max} \geq Q$, supplier i will set $q_i = Q$. Second, if $q_i^{\max} < Q$, we assume that supplier i will choose to set $q_i = q_i^{\max}$ because doing so gives him the best chance of maintaining the buyer's business. Thus, overall, we have $q_i = \min\{Q, q_i^{\max}\}$.

Supplier i must also determine p_i, i.e., the price at which he will offer the product to the buyer. Recall that the buyer requires the qualified suppliers to participate in a sealed-bid procurement auction. Therefore, when modeling the bidding process for the suppliers, we draw on the results of auction theory (e.g., [17]). In a conventional first-price sealed-bid ascending auction (where the highest price wins and is paid by the highest bidder), the bid price for bidder i, given there are m bidders, is

$$b(u_i) = u_i - \frac{\int_{\underline{u}}^{u_i} [F_u(x)]^{m-1} dx}{[F_u(u_i)]^{m-1}} \tag{2}$$

where u_i is the value of the item to bidder i, which has distribution $F_u(\cdot)$, defined on $[\underline{u}, \overline{u}]$. In our model, however, the winning bidder will be the one with the lowest price. Also, the economic relationship in a standard auction is reversed, i.e., it is the bidder (supplier) who will provide the item and be paid, not the seller. Thus, we modify the aforementioned to obtain the price offered by the winning supplier, say supplier i, assuming he wins, as follows:

$$p_i = p(c_i) = Q^z \left(c_i + \frac{\int_{c_i}^{c_H} [1 - F_c(x)]^{m-1} dx}{[1 - F_c(c_i)]^{m-1}} \right) \tag{3}$$

where $F_c(\cdot)$ denotes the cumulative distribution function (CDF) for the c_i, which is defined on $[c_L, c_H]$. If we compare the two expressions in (2) and (3), we see that in a traditional auction, bidder i's bid price is less that his valuation u_i for the production. On the other hand, in a reverse auction, supplier i's bid price is greater than his unit cost c_i. Also, notice that in (3), we use the complement of the CDF, $1 - F_c(\cdot)$, which is the probability that an individual supplier's cost exceeds some value, while in (2), we use the CDF, $F_u(\cdot)$, which is the probability that an individual bidder's valuation for the good is less than some value. Intuitively, this is due to the fact that in the procurement auction, supplier i wins only if all other suppliers' unit costs exceed c_i, while in a traditional auction, bidder i wins only if all other bidders' valuations are less than u_i.

Also notice the multiplier Q^z in the expression for p_i. When bidding, each supplier must consider his beliefs regarding the unit costs of the other suppliers. Recall that these unit costs are $c_i q_i^z$, where $q_i = \min\{Q, q_i^{\max}\}$. However, we assume that the suppliers are unaware of the potential inaccuracy of the qualification process, i.e., we assume that supplier i believes that all of the other suppliers competing in the supplier selection stage have $q_i^{\max} \geq Q$, where the qualification threshold Q was announced in the qualification stage. Thus, supplier i believes that $q_i = Q$ for all the other suppliers competing in the supplier selection stage, i.e., supplier i believes that the qualified suppliers' unit costs are of the form $c_i Q^z$. Therefore, each supplier's bid price depends only on his beliefs about the distribution of the c_i across the suppliers, and the announced qualification threshold Q.

To obtain closed form results, we assume the supplier costs c_i are independent random draws from a uniform(c_L, c_H) distribution. This assumption is common in auction theory when the goal is to obtain closed-form results. See, for example, [14], [17], and [23]. In this case, supplier i's bid price is

$$p_i = p(c_i) = Q^z \left[c_H - \left(\frac{m-1}{m} \right) (c_H - c_i) \right]. \tag{4}$$

Given the bids from the suppliers, the buyer will choose to source from the qualified supplier with the minimum price. From (4), it is clear that the qualified supplier with the minimum cost, denoted by $c_{(1)}$, will submit the lowest bid. Thus, the buyer will procure the product at price $p(c_{(1)})$. Since $c_{(1)}$ is the minimum value of m draws from a uniform(c_L, c_H) distribution, $E[c_{(1)}] = \frac{1}{m+1} c_H + \frac{m}{m+1} c_L$. Therefore, the buyer's expected unit procurement cost c_p is just the expected price at which the buyer procures the good, which can be written as

$$c_p = E[p(c_{(1)})] = Q^z \left[c_H \times \frac{2}{m+1} + c_L \times \frac{m-1}{m+1} \right]. \tag{5}$$

Finally, from (5), we can see that the expected winning bid price is decreasing in m. In other words, when there are more qualified suppliers competing in the supplier selection stage, the buyer's expected unit procurement cost decreases.

B. Qualification Stage

We next consider the design of the qualification stage, which determines m, the number of qualified suppliers who compete in the supplier selection stage. We assume an inspection-based qualification process in which the buyer inspects a random sample of size e of each supplier's product in order to measure its quality level. We assume the sample size is the same for all suppliers. The sample size e represents the measure of effort exerted in the qualification process, at cost $c_E(e)$ per supplier.

During the qualification process, supplier i produces products with his maximum quality level $q_i^{\max}$ with the goal of maximizing the likelihood that the buyer will find supplier i to be qualified. As noted earlier, the inspection process is not perfect. If the supplier's true quality level is $q_i^{\max}$, for a given sampled unit of product, $j = 1, \ldots, e$, the buyer will observe the quality level $q_{ij} = q_i^{\max} + \epsilon_{ij}$, where ϵ_{ij} represents the independent and identically distributed (i.i.d.) measurement error, which is assumed to follow a Normal distribution with mean 0 and known variance σ^2, for $i = 1, \ldots, n$ and $j = 1, \ldots, e$. Given a sample of size e, we define $\hat{q}_i = \frac{\sum_{j=1}^{e} q_{ij}}{e}$, i.e., $\hat{q}_i$ is the sample mean for supplier i, which has a normal distribution with mean $q_i^{\max}$ and standard deviation $\sigma/\sqrt{e}$.

Given $\hat{q}_i$, the buyer must decide whether supplier i is qualified. She does so using a hypothesis test with $H_O = q_i^{\max} < Q$ and $H_A = q_i^{\max} \geq Q$. The null hypothesis H_O is assumed to be true and is rejected in favor of the alternative H_A only if the data are highly improbable given the null hypothesis. Thus, with this formulation, the buyer assumes the supplier cannot achieve the qualification threshold Q, and only rejects that assumption if the data are sufficiently convincing.

The hypothesis test has a confidence level α, which represents the probability of type I error, i.e., the probability that the buyer rejects the null hypothesis when it is true. Thus, α is the probability the buyer declares a supplier to be qualified when they do not meet the qualification threshold, i.e., have $q_i^{\max} < Q$. Therefore, α represents the buyer's tolerance for error in the qualification process. Based on this hypothesis test, there is a threshold, $G(Q, \alpha, e) = Q + z(1-\alpha)\sigma/\sqrt{e}$, on the observed value of $\hat{q}_i$, where $z(1-\alpha)$ is the value of z that satisfies $P(Z \leq z) = 1 - \alpha$, and Z is a standard normal random variable. If $\hat{q}_i$ is greater than or equal to $G(Q, \alpha, e)$, the supplier is said to be qualified; otherwise, the supplier is said to be not qualified. Thus, $G(Q, \alpha, e)$ depends on the qualification threshold Q the tolerance for error α and the effort exerted in the qualification process e. In summary, the qualification rule is to qualify supplier i if and only if $\hat{q}_i \geq G(Q, \alpha, e)$.

Given this qualification process, the probability supplier i, whose true maximum quality level is $q_i^{\max}$, is considered qualified can be written as

$$P(\hat{q}_i \geq Q + z(1-\alpha)\sigma/\sqrt{e}) = 1 - \Phi\left(\frac{Q - q_i^{\max}}{\sigma/\sqrt{e}} + z(1-\alpha)\right) \tag{6}$$

where $\Phi(\cdot)$ the CDF for a standard normal distribution.

The level of competition in the supplier selection stage is determined by the number of qualified suppliers m. It is now clear that m will be a random variable and will depend on the design of the qualification stage, i.e., on the buyer's choice of Q, α, and e. We can write the expected number of qualified suppliers, which we denote by $m(Q, \alpha, e)$, as follows:

$$m(Q, \alpha, e) = nE\left[1 - \Phi\left(\frac{Q - q_i^{\max}}{\sigma/\sqrt{e}} + z(1-\alpha)\right)\right]. \tag{7}$$

Not every supplier that passes the qualification process will actually have $q_i^{\max} \geq Q$. Thus, it is useful to compute the expected quality of the qualified suppliers, which we denote by $\bar{q}(Q, \alpha, e)$ and refer to as the expected delivered quality, assuming that those suppliers with $q_i^{\max} \geq Q$ will choose to set their quality levels just equal to Q (8), as shown at the bottom of the page.

Notice that (8) reduces to $\bar{q}(Q, \alpha, e) = Q$ if $Q \leq q_L$, i.e., if the buyer sets the qualification threshold to be less than the lower bound on the $q_i^{\max}$, then all suppliers will be able to achieve the qualification threshold, i.e., $q_i^{\max} \geq Q$ for $i = 1, \ldots, n$.

We have assumed that those suppliers with $q_i^{\max} \leq Q$ will choose to set their quality levels just equal to $q_i^{\max}$. In reality, it is difficult to predict what quality level these suppliers will provide. We consider the best case, i.e., these suppliers will set their quality levels as high as possible, under the assumption that, if selected, the supplier will make every possible effort to meet the requirements of the buyer in order to maintain the buyer's business.

We can now state the following theorem, which characterizes the behavior of the expected number of qualified suppliers $m(Q, \alpha, e)$ and the expected delivered quality level $\bar{q}(Q, \alpha, e)$ under the assumption that the $q_i^{\max}$ are uniformly distributed between q_L and q_H.

Theorem 1: For the inspection-based qualification process with qualification threshold Q, level of effort e, and tolerance for error α, we have the following results when $q_i^{\max}$ follows a Uniform(q_L, q_H) distribution.

1) The expected delivered quality level $\bar{q}(Q, \alpha, e)$ is increasing in Q and decreasing in α.
2) The expected number of qualified suppliers $m(Q, \alpha, e)$ is decreasing in Q and increasing in α. If the buyer sets $\alpha \leq 0.5$, then the expected number of qualified suppliers $m(Q, \alpha, e)$ is decreasing and convex in Q for $Q \leq \frac{q_L + q_H}{2} + z(1-\alpha)\frac{\sigma}{\sqrt{e}}$, but decreasing and concave in Q for $Q > \frac{q_L + q_H}{2} + z(1-\alpha)\frac{\sigma}{\sqrt{e}}$.

The proof of Theorem 1 can be found in the online supplementary material. The theorem indicates that the expected

$$\begin{aligned}
\bar{q}(Q, \alpha, e) &= \frac{E\left\{\min[q_i^{\max}, Q] \times \left[1 - \Phi\left(\frac{Q - q_i^{\max}}{\sigma/\sqrt{e}} + z(1-\alpha)\right)\right]\right\}}{E\left[1 - \Phi\left(\frac{Q - q_i^{\max}}{\sigma/\sqrt{e}} + z(1-\alpha)\right)\right]} \\
&= Q - \frac{\int_{q_L}^{Q} (Q - q_i^{\max}) \left[1 - \Phi\left(\frac{Q - q_i^{\max}}{\sigma/\sqrt{e}} + z(1-\alpha)\right)\right] \left(\frac{1}{q_H - q_L}\right) dq_i^{\max}}{\int_{q_L}^{q_H} \left[1 - \Phi\left(\frac{Q - q_i^{\max}}{\sigma/\sqrt{e}} + z(1-\alpha)\right)\right] \left(\frac{1}{q_H - q_L}\right) dq_i^{\max}}.
\end{aligned} \tag{8}$$

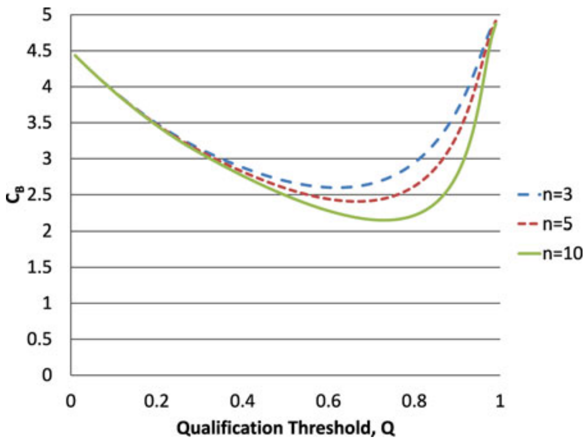

Fig. 1. Buyer's cost as a function of quality threshold, Q, for various values of n, for $z = 2, q_L = 0.3, q_H = 1, c_L = 1, c_H = 3, s_w = 5, s_e = 0.5, \sigma = 0.1, e = 50, \alpha = 0.05$.

delivered quality is increasing in the qualification threshold and decreasing in the tolerance for error, while the number of qualified suppliers is decreasing in the qualification threshold and increasing in the tolerance for error.

Given $m(Q, \alpha, e)$ and $\bar{q}(Q, \alpha, e)$, we can write the buyer's expected cost as a function of Q, α, and e, using (1), as follows:

$$c_B(Q, \alpha, e) = c_p(Q, \alpha, e) + w(1 - \bar{q}(Q, \alpha, e)) + \frac{nc_E(e)}{V} \tag{9}$$

where from (5) the buyer's expected unit procurement cost is computed as

$$c_p(Q, \alpha, e) = Q^z \left[c_H \times \frac{2}{m(Q, \alpha, e) + 1} + c_L \times \frac{m(Q, \alpha, e) - 1}{m(Q, \alpha, e) + 1} \right] \tag{10}$$

where $m(Q, \alpha, e)$ is as given in (7). Note that, rather than taking the expectation of (5) over the random number of qualified suppliers, we have instead replaced the random variable with its expected value. We make this approximation for analytical convenience.

The buyer's problem is to find the optimal Q, α, and e to minimize $c_B(Q, \alpha, e)$. Unfortunately, it is difficult to prove anything about the behavior of $c_B(Q, \alpha, e)$. For example, Fig. 1, which shows the buyer's cost as a function of the threshold Q for various values of the number of potential suppliers n demonstrates that the buyer's cost is not strictly convex in Q. Instead, we find that, for large Q, the cost function switches from convex to concave. This is due to the behavior of $m(Q, \alpha, e)$ for $\alpha < 0.5$, as described in Theorem 1. This behavior results in the procurement cost c_p becoming concave for large values of Q. Thus, the buyer's total cost c_B also becomes concave for large Q. However, the figure does demonstrate a key insights, i.e., the buyer's costs are decreasing in the number of potential suppliers, while the optimal quality threshold is increasing in the number of suppliers. Thus, with more potential suppliers, the buyer can be more rigorous in the qualification stage, while still maintaining competition in the supplier selection stage, resulting in lower costs.

IV. Numerical Study

To gain insight into the buyer's optimal decisions, we conducted a full factorial experiment in which the key parameters were allowed to take three values, as specified in the following.

1) The warranty cost is set equal to a multiple of the average cost to produce a unit of average quality, i.e., we set $w = s_w \times \left(\frac{c_L + c_H}{2}\right) \times \left(\frac{q_L + q_H}{2}\right)^z$, where $s_w \in \{2, 5, 10\}$. We have chosen to model the unit warranty cost in this way in order to reflect the fact that the warranty cost is likely to be correlated with the cost to produce a unit of product. In addition, we allow the multiplier s_w to be large in order to capture the fact that the unit warranty may also incorporate a goodwill cost, i.e., the customer dissatisfaction resulting from poor quality and inconvenience.
2) Bounds on the distribution of costs in the industry: the upper bound is fixed at $c_H = 3$, while the lower bound can take values of $c_L \in \{1, 2, 2.9\}$.
3) Bounds of the distribution of quality in the industry: the upper bound is fixed at $q_H = 1$ (perfect quality), while the lower bound can take values of $q_L \in \{0.3, 0.5, 0.7\}$.
4) Number of potential suppliers: $n \in \{5, 10, 20\}$.
5) Cost of quality parameter: $z \in \{1, 2, 4\}$.
6) The standard deviation of the measurement error of testing: $\sigma \in \{0.08, 0.1, 0.12\}$.
7) The cost of effort for each potential supplier is linear in effort, where the unit cost of effort was set equal to a multiple of the average cost to produce a unit of average quality, i.e., we set $c_E(e) = c_e \times e$, where $c_e = (1 + s_e) \times \left(\frac{c_L + c_H}{2}\right) \times \left(\frac{q_L + q_H}{2}\right)^z$ and $s_e \in \{0.5, 2.5, 5\}$.
8) The buyer's total production volume V is taken to be fixed at 1 million units.

For each of the seven key input parameters, we consider three values (high, medium, and low), which results in a total of 2187 experiments. To find the optimal Q, α, and e, we conducted a 3-D grid search for the combination of values that minimized the buyer's expected total cost, as given in (9), where numerical integration is used to evaluate $\bar{q}(Q, \alpha, e)$, as given in (8). For the number of samples e, we searched over integer values between 1 and 50. For the tolerance α, we discretized the search area, considering values of α ranging from 0.01 to 0.96, in increments of 0.05, as well as $\alpha = 0.99$. For the quality threshold, we also discretized the search area, considering values of $Q \in [0.01, 0.03, 0.05, \ldots, 0.99]$.

A. Overview of Key Results and Insights

Before discussing our detailed results, we will summarize the key insights and tradeoffs. The results reported in the Sections IV-B–IV-D will support the discussion in this section. Our problem has three decision variables, which interact in complex ways. The behavior of the optimal qualification threshold Q is easiest to understand. If we are more stringent in the qualification process, we qualify fewer suppliers, which leads to lower warranty costs but higher procurement costs, due to less competition in the supplier selection stage. Thus, when the problem parameters are such that warranty costs dominate (e.g., the unit warranty cost is high and/or the number of potential suppliers is large), the optimal Q will be high. However, when the problem parameters are such that procurement costs dominate (e.g., the number of potential suppliers is low and/or the warranty cost is low), the optimal Q will be low. These results are consistent with those in [16].

To understand the behavior of the remaining two decision variables, it is useful to think in terms of type I and type II errors. For our model, type I error occurs when we qualify a supplier who does not meet the qualification threshold. Type II error occurs when we do not qualify a supplier who does meet the qualification threshold. Type I error reduces the expected delivered quality and leads to higher warranty costs, while type II error reduces the number of qualified suppliers and leads to a higher procurement price. Thus, when the model parameters are such that the warranty costs dominate, the buyer tries to reduce the probability of type I error. When the model parameters are such that the procurement costs dominate, the buyer tries to reduce the probability of type II error. Finally, note that reducing type I error also leads to fewer suppliers being qualified.

Next, we consider how the probabilities of type I and type II errors are affected by α and e. Recall that α is the probability of type I error. Thus, increasing α will increase the probability of type I error, but will decrease the probability of type II error. On the other hand, increasing e, the sample size, will decrease both types of errors. As noted earlier, when the model parameters are such that the warranty costs dominate, the buyer tries to reduce the chance of type I error. To achieve this, she has two alternatives: a low tolerance for error, α or a high level of effort, e. Since a high e reduces the chance of both types of error, it is preferred to a low α, as long as effort is not too costly. In addition, when the effort level is high enough, and thus the variance of the sample mean is very small, the qualification decision is fairly accurate even when the tolerance is high. Thus, when the model parameters are such that the warranty costs dominate, we tend to see cases in which the optimal effort level is high, while the optimal tolerance is less predictable.

When the model parameters are such that the procurement costs dominate, the buyer will try to reduce the probability of type II error in order to qualify more suppliers. This can be achieved by setting a high tolerance for error α. The optimal level of effort, however, is less clear. Increasing effort reduces both type I and type II errors. Reducing type I error leads to fewer qualified suppliers, while reducing type II error leads to more qualified suppliers. Which effect dominates depends on the value of α. If α is low, a higher level of effort increases the number of suppliers qualified. However, if α is high, the opposite is true. Thus, when the procurement costs dominate, we tend to see cases in which the optimal α is high. Then, depending on the magnitude of the optimal α, i.e., depending on exactly how high α becomes, the optimal e may be either high or low.

B. Behavior of the Optimal Solution: Examples

We next discuss a series of examples, as shown in Table I. Example 1 has a low warranty cost and a large number of potential suppliers. We find the following optimal solution: $Q^* = 0.49, \alpha^* = 0.56, e^* = 12$. Thus, the buyer

TABLE I
EXAMPLES OF OPTIMAL SOLUTION, FOR
$z = 2, \sigma = 0.1, c_L = 2, q_L = 0.3, s_e = 2.5$

Example	s_w	n	Q^*	α^*	e^*	c_B^*	$m(Q^*, \alpha^*, e^*)/n$	$\bar{q}(Q^*, \alpha^*, e^*)$
1	2	20	0.49	0.56	12	1.590	73.5%	0.499
2	2	5	0.43	0.91	28	1.647	85.0%	0.429
3	10	20	0.95	0.99	36	2.952	12.7%	0.940
4	10	5	0.93	0.99	30	3.529	16.1%	0.921

should calculate $G(Q^*, \alpha^*, e^*) = Q^* + z(1 - \alpha^*)\sigma/\sqrt{e^*} = 0.49 + z(0.44) \times 0.08/\sqrt{12} = 0.486$. Then, the buyer should sample and inspect 12 units from each potential supplier and qualify only those suppliers whose average quality level is at least 0.486. This will result in, on average, 73.5% of suppliers being qualified, i.e., $m(Q^*, \alpha^*, e^*) = 14.7$, and an expected delivered quality of $\bar{q}(Q^*, \alpha^*, e^*) = 0.489$.

Example 2 considers an identical set of input values, except that the number of potential suppliers is reduced from 20 to 5. The buyer should sample 28 units from each potential supplier and qualify those suppliers whose average quality is at least $G(Q^*, \alpha^*, e^*) = 0.405$. This will result in, on average, 85.0% of suppliers being qualified, i.e., $m(Q^*, \alpha^*, e^*) = 4.3$, and an expected delivered quality of $\bar{q}(Q^*, \alpha^*, e^*) = 0.429$. Examples 1 and 2 demonstrate that when the number of potential suppliers decreases, qualifying a larger percentage of potential suppliers is critical in order to maintain sufficient competition in the supplier selection stage. In this example, this is achieved by reducing the qualification threshold, as well as increasing both the tolerance for error and the sample size, both of which reduce the chance of type II error, i.e., the chance of not qualifying a supplier who does meet the qualification threshold.

Examples 3 and 4 are identical to examples 1 and 2, but with a higher unit warranty cost. This results in a higher qualification threshold, a higher expected delivered quality, and a smaller percentage of suppliers qualified. The impact of the higher warranty cost on the optimal tolerance for error and effort is as predicted in Section IV-A. The optimal effort level is higher in examples 3 and 4 (high warranty cost) than in examples 1 and 2 (low warranty cost). Thus, when the unit warranty cost is high, the buyer will increase the level of effort in order to reduce type I error, to ensure that only suppliers who meet the qualification threshold are allowed to compete in the supplier selection stage. This higher level of effort results in a very low variance for the sample mean, enabling the buyer to increase the tolerance for error, α, without significantly reducing the accuracy of the qualification process.

In all four examples in Table I, the expected delivered quality $\bar{q}(Q^*, \alpha^*, e^*)$ is very close to the optimal quality threshold Q^*. From our computational study, we find that this result holds in general. Across the 2187 experiments, the maximum value of $Q^* - \bar{q}(Q^*, \alpha^*, e^*)$ was 0.048, while the median value was 0.003. Thus, even when the optimal tolerance for error is high, the buyer is still able to achieve close to the target quality level Q^*.

We conclude this section by summarizing our general observations regarding the behavior of the optimal solution as a function of the key problem parameters.

1) In all experiments, the optimal qualification threshold Q, optimal expected delivered quality $\bar{q}$, and optimal expected unit cost, c_B are nondecreasing in the unit warranty cost, w while the percentage of suppliers qualified is nonincreasing in the unit warranty cost. The optimal effort level, e is increasing in the warranty cost in most (greater than 86%) of the experiments.
2) In most (greater than 95%) of the experiments, the optimal qualification threshold, Q is nondecreasing in the number of potential suppliers, n while the optimal expected unit cost, c_B is nonincreasing in the number of potential suppliers.
3) In all experiments, the optimal expected unit cost, c_B is decreasing in the parameter, z due to the fact that the cost to produce a unit with given quality is decreasing in z.
4) In most (greater than 90%) of the experiments, the optimal expected unit cost, c_B is increasing in the lower bound on the unit costs c_L. In most (greater than 99%) of the experiments, the optimal tolerance for error α is nonincreasing in c_L.
5) In most (greater than 99%) experiments, the optimal threshold Q is nondecreasing in the lower bound on the distribution of quality q_L. In most (greater than 92%) of the experiments, the optimal expected delivered quality $\bar{q}$ is nondecreasing in q_L.
6) The optimal effort level is always nonincreasing in the unit testing cost c_e while the optimal tolerance for error α is nonincreasing in the cost of testing in more than 99% of the experiments. The optimal qualification threshold Q does not change with the testing cost in most (greater than 99%) experiments. The optimal expected unit cost c_B is increasing in the testing cost in all experiments.

C. Behavior of the Optimal Solution: Extreme Values

Extreme results such as $\alpha^* = 0.99, e^* = 1$ or $e^* = 50$ are fairly common in our numerical study. We summarize these results as follows.

1) $\alpha^* = 0.99$ in 1221 out of the 2187 experiments. For these cases with $\alpha^* = 0.99$, we have the following results:
   1) There are 173 cases in which $e^* = 1$. All of these cases occur when the warranty cost is low (i.e., $s_w = 2$).
   2) There are 78 cases in which $e^* = 50$. The bulk (67) of these cases occur when the warranty cost is high (i.e., $s_w = 10$).
2) $e^* = 50$ in 453 out of the 2187 experiments. This result is most likely to occur when the warranty cost is large (255 of these cases have $s_w = 10$) and the lower bound on the suppliers' unit production cost is high (299 of these cases have $c_L = 2.9$).
3) $e^* = 1$ in 175 out of the 2187 experiments. In all of these cases, the warranty cost is low ($s_w = 2$), while for most of these cases the optimal tolerance is very high ($\alpha^* \geq 96\%$).

These results are in line with the discussion in Section IV-A. The cases with $\alpha^* = 0.99$ tend to be those in which it is optimal for the buyer to qualify a large number of suppliers in order to maintain a high level of competition in the supplier selection stage. When the warranty cost is low, many of the cases with $\alpha^* = 0.99$ have a very small optimal sample size. This is due to the fact that when α^* is large, a low level of effort leads to a larger number of qualified suppliers. Thus, when warranty costs are low, and competition in supplier selection is the main concern, e^* should be low. However, when the warranty cost is high, $\alpha^* = 0.99$ tends to be associated with large e^*. In other words, when the warranty cost is high, the buyer will choose to increase the level of effort in order to reduce the chance of type I error, i.e., of qualifying a supplier who cannot meet the qualification threshold. As noted earlier, when e^* is quite large, the qualification process will still be fairly accurate, even when $\alpha^* = 0.99$.

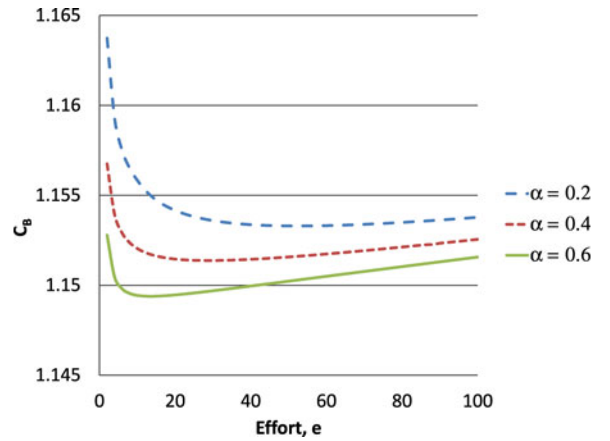

Fig. 2. Buyer's cost as a function of effort e for various values of α, for $z = 2, q_L = 0.3, q_H = 1, c_L = 1, c_H = 3, s_w = 2, s_e = 0.5, \sigma = 0.1, n = 20$.

Finally, we note that in 63 out of the 2187 cases, the optimal qualification threshold Q^* was less than the lower bound on the quality levels for the industry q_L. These 63 cases all had a low warranty cost ($s_w = 2$) and high lower bound on quality ($q_L = 0.7$). In addition, almost half of these cases had a small number of potential suppliers ($n = 5$) and a large unit production cost ($c_L = 2.9$). We can explain these results by noting that the buyer's total cost is composed of two factors: the expected unit procurement cost and the expected warranty cost. The procurement cost is affected by both the level of competition, i.e., the number of qualified suppliers $m(Q, \alpha, e)$ and the suppliers' unit production costs, which are proportional to Q^z. When the number of potential suppliers n and the unit warranty cost s_w are both small, the procurement costs dominate the warranty costs. To reduce the total cost, the buyer can increase the number of qualified suppliers $m(Q, \alpha, e)$ which creates more competition in the supplier selection stage. This can be achieved by setting $Q = q_L$. In addition, it may be beneficial for the buyer to reduce Q further, i.e., to set $Q < q_L$, in order to reduce the suppliers' unit production costs, which are proportional to Q^z. Thus, if the warranty costs and the number of potential suppliers are sufficiently low, while the unit production cost is relatively high, the buyer may set Q to be less than q_L, resulting in full competition in the supplier selection stage and reduced production costs. Finally, when $Q^* < q_L$, all suppliers will be qualified, regardless of the values of e and α. Thus, it is optimal to exert no effort and the tolerance for error has no effect on the buyer's cost.

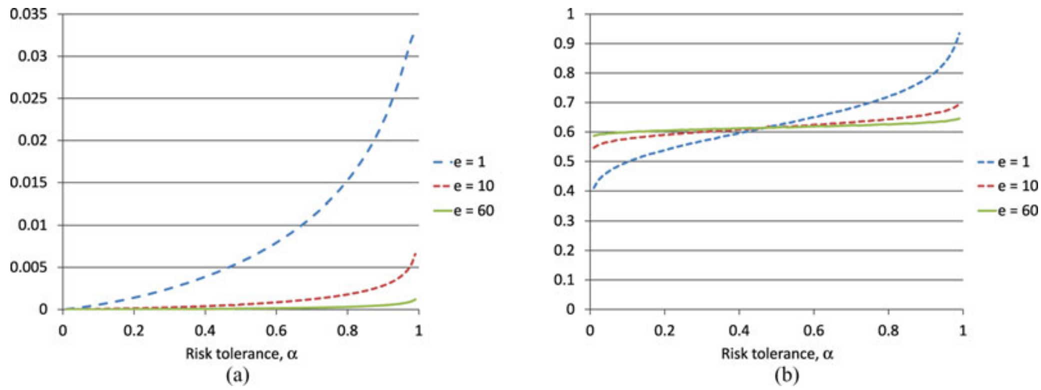

Fig. 3. Expected quality shortfall, percentage of suppliers qualified as a function of α for various e, for $z = 2, q_L = 0.3, q_H = 1, c_L = 1, c_H = 3, s_w = 2, s_e = 0.5, \sigma = 0.1, n = 20$. (a) Expected quality shortfall. (b) Percentage of suppliers qualified.

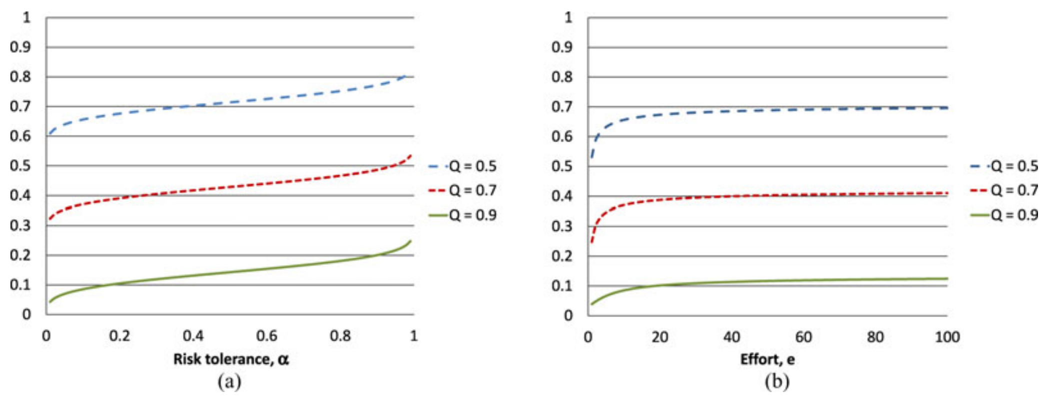

Fig. 4. Percentage of suppliers qualified as a function of α and e for various values of quality threshold, Q, for $z = 2, q_L = 0.3, q_H = 1, c_L = 1, c_H = 3, s_w = 2, s_e = 0.5, \sigma = 0.1, n = 20$. (a) Versus Tolerance. (b) Versus Effort.

D. Interactions Between Decision Variables

We next discuss the interactions and tradeoffs between the three decision variables, Q, α, and e. Fig. 2 shows the buyer's cost as a function of effort e for various values of the tolerance, α. We see that the optimal level of effort is decreasing in the tolerance for error. When the tolerance for error is low, more effort allows the buyer to qualify more suppliers, thus increasing competition in the supplier selection stage and reducing the expected unit procurement cost. More effort also allows the buyer to ensure that only suppliers who can meet the qualification threshold are qualified, which reduces the expected warranty costs.

Fig. 3(a) shows the expected quality shortfall, i.e., $Q - \bar{q}(Q, \alpha, e)$, as a function of the tolerance α for various levels of effort, while Fig. 3(b) shows the percentage of suppliers qualified, i.e., $m(Q, \alpha, e)/n$, as a function of the tolerance α for various levels of effort. We see that the expected quality shortfall is decreasing in effort and increasing in tolerance for error, as expected. Fig. 3(a) also demonstrates that effort has a more significant impact on the expected quality shortfall than the tolerance for error. Fig. 3(b) demonstrates that when the tolerance for error is small, the percentage of qualified suppliers is increasing in effort, while the opposite is true when the tolerance for error is large. In other words, if the buyer is more willing to accept an error in the qualification process, most suppliers will be considered qualified initially, and thus exerting effort will only serve to eliminate some suppliers. The opposite is true when the tolerance for error is low.

Fig. 4(a) and (b) shows the percentage of qualified suppliers as a function of tolerance α and effort, e for various values of the qualification threshold Q. As expected, this percentage is increasing in the tolerance for error and decreasing in the qualification threshold.

V. Conclusion and Managerial Insights

In this paper, we considered the design of a two-stage sourcing process for a buyer who outsources the manufacturing of a product to one of several competing suppliers. The buyer has imperfect information on the suppliers' capabilities. Thus, in the qualification stage, the buyer must exert costly effort to learn about the suppliers' capability for producing high-quality products. The buyer also has imperfect information regarding the suppliers' costs. Thus, in the supplier selection stage, the buyer uses a sealed-bid first-price procurement auction to select among the qualified suppliers. In designing this two-stage process, the buyer must determine the qualification standard, the level of effort to exert in the qualification process, and the appropriate tolerance for error, in order to maximize her expected profits. The buyer's goal is to ensure that the firm only sources from qualified suppliers, while maintaining sufficient competition among the suppliers in the supplier selection stage. Our results demonstrate the complex interactions between the buyer's decision variables, as well as the impact of key parameters on the buyer's cost. Below, we provide insights into the buyer's optimal decisions:

1) When the unit warranty cost is high and/or the number of potential suppliers is large, the optimal qualification threshold will be high. In these cases, it is more critical for the buyer to ensure a high level of quality than it is to ensure competition in the supplier selection stage. In contrast, when the unit warranty cost is low and/or the number of potential suppliers is small, the optimal qualification threshold will be low.
2) When effort is costly, the buyer's optimal effort level is generally limited. This is particularly true when the number of potential suppliers is small and the unit warranty cost is low. In this case, encouraging competition between the suppliers is more critical than ensuring that only high capability suppliers are qualified. This result provides useful insight for the supplier evaluation and the qualification literature, which generally does not consider the cost of effort. In particular, our findings imply that incorporating a large set of attributes in supplier evaluation, which is likely to make learning about supplier capabilities more costly, may not always be optimal, particularly for attributes that do not contribute substantially to the buyer's overall profit.
3) While intuition might suggest that the buyer should set a low tolerance for error in the qualification process, we find that the optimal tolerance for error is often quite large. This is particularly true when the number of potential suppliers is small and the unit warranty cost is low, i.e., when encouraging competition between the suppliers is critical.

We have also obtained several insights regarding the impact of key problem parameters.

1) The number of potential suppliers has a significant impact on the buyer's decisions and optimal cost. As the number of potential suppliers decreases, the buyer is motivated to reduce the qualification threshold and increase the tolerance for error in order to ensure sufficient competition in the supplier selection stage. This results in increased warranty costs, but reduced procurement costs.
2) The magnitude of the unit warranty cost also has a significant impact on the results.
   a) When the unit warranty cost is low, the need for competition in supplier selection dominates the need to maintain a high level of quality. Thus, the buyer seeks to qualify more suppliers. This can be achieved through a low qualification threshold, a high tolerance for error or a low level of effort.
   b) When the unit warranty cost is high, the buyer wants to maintain a high level of delivered quality. Thus, in addition to setting a high qualification threshold, she must reduce the chance of error in the qualification decision. To achieve the latter, she has two alternatives: a low tolerance for error or a high level of effort. While both alternatives can reduce the chance of qualifying suppliers who cannot meet the qualification threshold, a high effort level also increases the chance of qualifying suppliers who are indeed qualified. Thus, the buyer will generally choose to increase her effort, along with the qualification threshold, when the warranty cost is high.

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Yue Jin received the Doctorate degree in industrial engineering and operations research from the University of Massachusetts at Amherst, Amherst, MA, USA.

Since 2008, she has been a Researcher at Bell Labs Ireland, Dublin, Ireland, after spending one and half years there as a Postdoctoral Researcher. Her main research interests include optimization and coordination in supply chain and service operations management. Her works include remanufacturing, order allocation policies, worker migration in service systems, and charging infrastructure for electric vehicles. She is constantly involved in internal and external research collaborations. She has also made major contributions in a number of projects that address practical problems for Alcatel-Lucent business units, including the reconfiguration of regional supply chains, the automation of spot carrier selection, and the engineer scheduling in customer delivery units.

Jennifer K. Ryan received the B.A. degree in mathematics and the social sciences from Dartmouth College, NH, USA, in 1990, and the M.S. and the Ph.D. degrees both from the Department of Industrial Engineering and Management Sciences, Northwestern University, NH, in 1996, and 1997, respectively.

She is currently an Associate Professor of industrial and systems engineering at Rensselaer Polytechnic Institute, Troy, NY, USA. Her research has been published in management science, operations research, production and operations management, naval research logistics, and IIE Transactions. Her research interests include the area supply chain management, with a particular focus on coping with uncertainty and the role of information in managing supply chains.

Dr. Ryan has received several National Science Foundation grants, including an NSF CAREER grant. She currently serves as an Associate Editor for IIE Transaction, Naval Research Logistics, and OMEGA.

Walter Yund received the B.A. degree in economics from Union College, NY, USA and the Ph.D. degree in decision sciences and engineering systems from Rensselaer Polytechnic Institute, Troy, NY, USA, where his research focused on systems integration and supplier/customer management in decentralized supply chains.

He is a Research Scientist in the Complex Systems Engineering Lab at GE Global Research, NY, USA. He has collaborated with several GE businesses in developing supply chain systems to support key business processes including strategic sourcing, logistics, manufacturing throughput, supplier quality management, and inventory optimization.