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On the Transparency of the Ether

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UNIVERSITY STUDIES.

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I. — *On the Transparency of the Ether.*

BY DEWITT B. BRACE.

WHETHER light coming from the remotest members of the visible universe has not been enfeebled to a greater extent than the variation of distance would require, is still an open question. If there be absorption at all, it must be exceedingly small through spaces comparable with the dimensions of the solar system, in order that the light of these distant bodies may be perceived.

It is proposed in the present paper to investigate the phenomena which would occur if the energy were absorbed by the ether itself through frictional forces or imperfect elasticity. If absorption does take place, there must be a differential effect for varying wave-lengths, if the ether satisfies the equations of motion of elastic bodies. Several arguments have been advanced as proving that such an absorption takes place, of which those of Cheseaux, Olbers, and Struve are the most celebrated. Considerations on other grounds would seem to suggest such a conclusion. Cheseaux and Olbers, arguing from insufficient data as to stellar distribution, have shown that if the number of stars is infinite and distributed with anything like uniformity in space there must be absorption of light, as otherwise the sky would appear all over of a brightness approaching that of the sun, since the brightness at any point depends on the depth of the luminous layer and the solid angle which it subtends at that point.

The researches of both Herschel and Struve prove a non-uniformity of distribution in all directions, but a concentration of stars toward the medial plane of the Galaxy, more marked the smaller the magnitude of the star. Struve, from conclusions based on the supposition of an average uniformity of stellar distribution in layers parallel to this central plane, and on the assumption that the brightness is a measure of the relative distance, attempted to prove that absorption must take place. In fact, for a uniform distribution the number of calculated stars of different magnitude should vary inversely as their brightness. Now the number of calculated stars of any magnitude exceeds slightly the number observed, this excess being greater with the diminution of the magnitude. Hence it is concluded that absorption must take place to explain this increasing discrepancy, and that there must be a limit to the space-penetrating power of a telescope much lower than the enfeeblement of light with the distance would require. Later investigations regarding the constitution of the visible universe show that Struve's assumptions were false, and that no law of uniformity in distribution or in intrinsic brightness can be accepted. While it is at present impossible to ascertain with much accuracy the real magnitudes of the stars, there are sufficient data to show that both their volume and their intrinsic brightness, per unit surface, vary between wide limits. The annual parallax of several stars, as measured by different observers, gives approximately consistent results. A comparison of the brightness and distances of these stars with the intrinsic brightness and distance of the sun, as made by Zöllner, shows that the sun's volume is but a small fraction of that of these stars, supposing equal intrinsic brightness per unit of surface. This method of comparison, when applied to Sirius, gives a much greater volume than other methods would warrant, though far exceeding that of the sun in any case. It must hence be concluded that the intrinsic brightness of a unit of its surface is much greater than that of the sun. The results of spectrum analysis point to a wide variation in the age and temperature of different stars, some being very much farther advanced in the process of cooling than others. It seems certain from these considerations that not only the absolute size but the intrinsic brilliancy vary within very wide limits, some stars emitting several thousand times as much light as others.

The observations on stellar distribution indicate a much more complicated law than the earlier observers supposed. The more or less marked crowding together of stars in certain regions, with the existing intermediate voids, and the only partial resolvability of these aggregations, show a tendency to some system of clustering in which the various orders of magnitudes are actually intermingled. In certain regions the more minute stars are much more sparsely scattered than in others, while the distribution should approach more marked uniformity with diminishing magnitude. The absence of vast numbers of stars, with excessive crowding of the smallest magnitudes which such a distribution would require, shows that the telescope can penetrate to the bounds of the system in these regions.

The observations on the immense extent of the orbits of certain binary stars furnish evidence that there exists a connection between certain stars which have not heretofore been suspected as being members of the same system. The fact that the stars are gathered together in clusters principally in or near the Galactic zone indicates that they must form a part of the Galaxy, since there is no reason why, if they were outside our stellar system, they should not be more uniformly distributed toward the poles of this zone. These evidences of the complexity of the laws of distribution in magnitude and distance furnish strong proof that the present stellar system is finite, and that it does not appear so from the ultimate absorption of the light of the remoter members of an infinite system. Nothing but a cosmical veil of varying tenuity existing in interstellar spaces and closing our view more or less effectually from the infinite expanses beyond, could possibly explain these appearances, — a supposition which is wholly unallowable. If there be absorption in space, it must be determined by other methods than the one by which Struve attempted to prove it.

If the law of the Dissipation of Energy is absolutely universal, then it must be allowed that no distortion of the ether can take place without a certain loss of energy however small, so that the luminiferous vibrations would be gradually frittered down, and after an almost infinite number of such distortions be dissipated away so as to escape perception. On this hypothesis, from analogy with all known phenomena connected with ponderable bodies under similar conditions, a differential effect should be produced for different periods of vibrations, which would give a perceptible coloration in distant stars.

If an excessively diffused material substance be supposed scattered through space in a gaseous state, such a body could only absorb selectively through its atoms, its molecules being too widely scattered to allow of any transformation of energy into molecular friction. Hence the only loss, other than by selective absorption, would be in the ether itself. The absorption would then take place according to the same laws which determine it when such a substance is not present.

In a medium in which there were dissipative forces proportional to the rate of distortion, there would be a relative change in the velocity of propagation of transverse vibrations of different periods which, for sufficiently great distances, might be detected in the coloration produced by any sudden outburst or extinction of starlight. If the absorption were small, such a difference in the velocities of different rays would be exceedingly small, even for distances comparable with the greatest dimensions of the stellar system, so that the coloration could only last for a very short time.

The luminiferous medium bears a close analogy to the ponderable substances of nature in respect to its rigidity for high rates of distortion and its apparently perfect fluidity for motions of distortion of low rates. The existence of such apparently incompatible qualities does not seem so difficult to understand, when a material substance subjected to rates of distortion of far less range than the extreme limits at which these two qualities are observed to exist in the case of the ether appears in the one case like a rigid solid, and in the other like a very mobile fluid.

Maxwell found that, by rotating a cylinder rapidly in a liquid and passing a ray of polarized light close to its surface, the plane of polarization was altered, proving clearly a state of strain for ordinary liquids when the rate of distortion is sufficently high. Sir William Thomson has also shown how wax or pitch may, in the one case, vibrate like ordinary solids, and, in the other, allow bodies to pass very slowly through them without appreciable resistance. The luminiferous medium presents similar phenomena. For periods of vibration comparable with those of light, it acts like a very elastic solid. For low rates of distortion like those which the motions of the planets and comets as well as those which the molecules of a gas produce, there is no sensible resistance, and the medium seems to act like a perfect fluid. That this resistance is exceedingly small is shown by the fact that the comets, which are in general of extreme tenuity, give no definite indications of a resisting medium in space.

While the properties of the ethereal medium manifestly transcend those of ordinary matter, yet it seems to fulfil, in the qualities of elasticity and fluidity, the conditions of natural bodies. Very strong analogy to the ether is furnished by viscous substances, and these substances always dissipate more or less rapidly any vibrations to which they are subjected, proportionally to the rate of distortion, — at least for small rates. If the ether has a corpuscular structure, — and it is difficult to conceive of absorption otherwise, — and the analogy in respect to viscosity is extended to it, as well as the analogy in respect to its elasticity and fluidity, there should be a loss or transformation of radiant energy.

Loss of energy may also take place in other ways depending on imperfect elasticity alone, or the loss may arise both from viscous forces and from imperfect elasticity. In the one case we have the stress varying with the rate of distortion, and in the other, with the duration and magnitude of the strain. The existence of either will give a differential effect for the absorption of different rays.

Suppose that absorption does take place, the amplitude of a periodic motion would be some function of the distance, wave-length or period, and of the viscosity and imperfect elasticity. Let it be required to find the form of the function for parallel rays of light, propagated in the direction of the y-axis and with a displacement ξ parallel to the x-axis.

Let

$$\xi = A\epsilon^{-ipt} F(y, \lambda, \mu), \tag{1}$$

where $A\epsilon^{-ipt}$ represents a periodic motion at the origin, of amplitude A, and μ the coefficient of viscosity. When dissipative forces proportional to the relative velocities are present, the form of the function F is readily obtained. If the ether is perfectly elastic, the equation of motion for parallel rays is

$$\rho\frac{\partial^2\xi}{\partial t^2} = n\frac{\partial^2\xi}{\partial y^2}, \tag{2}$$

where ρ is the density and n the rigidity of the ether. Now Stokes has shown in his celebrated paper "On the Friction of Fluids in Motion"[1] that the expressions for the stresses in an isotropic solid may be obtained directly from those found for the case of a viscous fluid in motion by merely substituting the displacements ξ, η, ζ for the velocities u, v, w, and the rigidity n for the coefficient of viscosity μ. In the case under consideration, we have not only the rigidity n, but a viscous coefficient μ, each of which produces a shearing stress independently, so that the resulting stress will be the sum of the two, and the equation of motion becomes

[1] Collected Papers, Vol. I.

$$\rho \frac{\partial^2 \xi}{\partial t^2} = n \frac{\partial^2 \xi}{\partial y^2} + \mu \frac{\partial^2 u}{\partial y^2};$$

or, since

$$u = \frac{\partial \xi}{\partial t}, \tag{3}$$

$$\rho \frac{\partial^2 \xi}{\partial t^2} = n \frac{\partial^2 \xi}{\partial y^2} + \mu \frac{\partial^3 \xi}{\partial t \partial y^2}.$$

The left-hand side represents the force of acceleration per unit of volume; the first term of the right-hand side expresses the force arising from the distortion of the surrounding ether, and the second term the dissipative force arising from the rate of distortion. A particular solution of this equation is

$$\xi = A\epsilon^{\beta y - ipt}. \tag{4}$$

Substituting in equation (3), we have

$$-\rho p^2 = n\beta^2 - i\mu p \beta^2. \tag{5}$$

Since we are dealing with simple periodic motions, p must be real, and β must therefore be complex.

Let

$$\beta = -\kappa + i\gamma, \tag{6}$$

where $\gamma = \frac{p}{V}$. Substituting in (5), and separating the real and the imaginary terms, we have

$$\left.\begin{aligned} 0 &= \rho p^2 + n\kappa^2 - n\gamma^2 - 2p\mu\kappa\gamma, \\ 0 &= 2n\kappa\gamma + p\mu\kappa^2 - p\mu\gamma^2; \end{aligned}\right\} \tag{7}$$

whence

$$\left.\begin{aligned} \gamma^2 - \kappa^2 &= \frac{p^2 a^2}{a^4 + p^2 \nu^2}, \\ \kappa\gamma &= \frac{p^3 \nu}{a^4 + p^2 \nu^2}, \end{aligned}\right\} \tag{8}$$

where $\nu = \frac{\mu}{\rho}$, is the kinematic coefficient of viscosity. Now κy cannot be a large quantity, since the light of stars at a very great distance y reaches us; hence κ, and consequently

ν, must be very small. Neglecting squares of small quantities, we have

$$\gamma = \frac{p}{a},$$

$$\kappa = \frac{p^2 \nu}{2a^3} = \frac{2\pi^2 \nu}{a\lambda^2}, \qquad (9)$$

putting for p its value

$$\frac{2\pi}{\tau} = \frac{2\pi a}{\lambda}.$$

A very small change in the velocity may produce an appreciable retardation of different rays for very great distances. Expressing γ to the next order of approximation, we have on expanding

$$\gamma^2 = \frac{p^2}{a^2} - 3\kappa^2,$$

neglecting small quantities of higher orders than κ^2. Since κ is a very small quantity, we may put

$$\kappa = \frac{w}{y},$$

where y is a large quantity, and w has different but not large values. Putting

$$y = Vt' = at,$$

and substituting, we have

$$p^2\left(\frac{1}{a^2} - \frac{1}{V^2}\right) = \frac{3w^2}{y^2},$$

or

$$t^2 - t'^2 = \frac{3w^2}{p^2};$$

$$\therefore t - t' = \frac{3w^2}{8\pi^2}\frac{\tau^2}{t}, \text{ nearly.} \qquad (10)$$

This shows that the relative retardation of the different rays due to viscosity is too small to be observed, even when the time t, necessary for light from the most distant visible object to reach us, is very great.

Substituting now in (4),

$$\xi = A\epsilon^{-\frac{2\pi^2\nu}{a\lambda^2}y + i\frac{p}{a}y - ipt}$$

or, omitting the imaginary part,

$$\xi = A\epsilon^{-\frac{2\pi^2\nu}{a\lambda^2}y}\cos\left(\frac{py}{a} - pt\right) = A\epsilon^{-\frac{2\pi^2\nu}{a\lambda^2}}\cos 2\pi\left(\frac{y}{\lambda} - \frac{t}{\tau}\right). \quad (11)$$

Thus $$F = \epsilon^{-\frac{2\pi^2\nu}{a\lambda^2}y + i\frac{p}{a}y} \text{ approximately.} \quad (12)$$

Stokes[1] has advanced the view that a fluid may be an extremely plastic body which admits of "a finite, but exceedingly small amount of constraint before it is relieved from its state of tension by its molecules assuming new positions of equilibrium." He suggests that the ether may be like a very plastic substance, which allows of the free motions of solids through it, but which also admits of small amounts of constraint without permanent distortion. He instances[2] as an illustration a mixture of jelly and water in varying proportions, which will admit of a given amount of constraint without dislocation, this constraint being less and less as the mixture is made thinner. Now all solid bodies in nature seem to possess to a degree more or less marked the quality of "Elastic After-effect," as observed by Weber, Kohlrausch, and others, where the elastic recovery, as well as the stress produced by any strain, depends on the time. Extending this quality to the ether on the supposition that it is an extremely plastic body and should possess the same kind of qualities as other plastic substances, it would seem that a loss of energy could take place from this cause. In this case F will have a somewhat different form from (12).

Suppose that the stress at any time is independent of the rate of shear, but depends on the duration and magnitude of the strain, as it would if the "Elastic After-effect" were present.

[1] Collected Papers, Vol. I., page 125.

[2] Collected Papers, Vol. II., page 12.

The exact expression for this effect would be complicated, depending as it does on previous strains. But for a succession of waves of given period, the change in the stress would evidently be, on the whole, proportional to the distortion and the time, when the distortion did not vary greatly and the time was very small.

Since the duration of the shortest waves in the visible spectrum is very small and about half that of the longest waves, and since the relative distortion does not vary greatly for the vibrations of different rays in a normal spectrum, the mean relative diminution in the stress, and hence the relative diminution in the amplitude, may be taken proportional to the relative distortion, and to the duration of a wave period, directly. The distortion at any time is proportional to the displacement directly, and to the wave-length inversely. Hence the change in the displacement which takes place during any short time Δt, or in passing over a space Δy, is

$$\alpha \frac{\xi}{\lambda} \Delta t = \alpha \frac{\xi}{a\lambda} \Delta y,$$

where a is the velocity of propagation and α is approximately constant over wide ranges in the distortion. After passing over a distance $y = n \Delta y$, the displacement would be

$$\xi \left(1 - \frac{\alpha}{a\lambda} \Delta y \right)^n = \xi \epsilon^{-\frac{\alpha}{a\lambda} y} \tag{13}$$

since $\frac{\alpha}{a\lambda}$ is a very small quantity, and its square and higher powers may be neglected. Hence, since the motion is a periodic function of the time and distance,

$$\xi = A\epsilon^{-\frac{\alpha}{a\lambda} y + i\frac{p}{a} y - ipt} = A\epsilon^{-\frac{\alpha}{a\lambda} y} \cos 2\pi \left(\frac{y}{\lambda} - \frac{t}{\tau} \right). \tag{14}$$

Hence, in this case,

$$F = \epsilon^{-\frac{\alpha}{a\lambda} y + i\frac{p}{a} y} \quad \text{nearly.} \tag{15}$$

In the actual case of solid bodies, the decay of a vibration does not seem to follow either equation (14) or equation (12). Sir William Thompson found that the relative diminution in the amplitude of vibrating wires of different periods was less than it would be if it were due to viscosity alone, and greater than it would be if due to imperfect elasticity or the Elastic After-effect alone. However, the diminution was more rapid for short periods than for long ones, indicating a dependence on the rate of shear, as well as on imperfect elasticity. The law of decay, if due to both these causes, would be expressed by the equation

$$\xi = A\epsilon^{-\left(\frac{\theta_1}{a\lambda}+\frac{\theta_2}{a\lambda^2}\right)y} \cos 2\pi\left(\frac{y}{\lambda}-\frac{t}{\tau}\right), \qquad (16)$$

where θ_1 and θ_2 are unknown, but may be determined by experiment for different substances subjected to different rates of distortion. The three formulæ

$$\text{I.} \qquad \xi = A\epsilon^{-\frac{2\pi^2 \nu}{a\lambda^2}y} \cos 2\pi\left(\frac{y}{\lambda}-\frac{t}{\tau}\right),$$

$$\text{II.} \qquad \xi = A\epsilon^{-\frac{a}{a\lambda}y} \cos 2\pi\left(\frac{y}{\lambda}-\frac{t}{\tau}\right),$$

$$\text{III.} \qquad \xi = A\epsilon^{-\left(\frac{\theta_1}{a\lambda}+\frac{\theta_2}{a\lambda^2}\right)y} \cos 2\pi\left(\frac{y}{\lambda}-\frac{t}{\tau}\right),$$

indicate the absorption as depending on the wave-length. If absorption takes place in the ether in a way analogous to that in ponderable substances, it must follow one of these laws, which include all modes of absorption for ordinary bodies, and hence coloration should occur in varying amounts with the distance. The equation II. represents the case of minimum coloration.

From what is known of the decay of vibrations in material bodies, it seems most probable that the conditions of the problem are most nearly satisfied by I. When the rate of distortion in solid bodies is considerable, the viscous resistance seems to increase less and less rapidly with the rate of shear. When this rate is diminished, the law of viscous resistance seems to become more and more nearly proportional to the rate. When the rate of distortion is very small, it is directly proportional to it. In III., then, the smaller the rate, the larger θ_2 becomes relatively to θ_1, until finally the term containing θ_1 may be neglected. Somewhat similar considerations show that, if absorption depends on the rate, as in natural bodies, the proportional law for viscous resistance must hold in the solution of our present problem. In solids, the viscous resistance to finite rates of shearing is finite, and hence, for very small rates, the viscous resistance must also be very small. For luminous vibrations, the rate of distortion must be very great, in any case, to be perceptible. Further, the range over which this rate extends must be excessively wide, since, applying the law of the inverse distance for the amplitude to the remotest visible stars whose light occupies several thousand years in reaching us, it is evident that the amplitude must be diminished many million times. If A is the original amplitude,

$$\xi = \frac{A}{y} \epsilon^{-\kappa y} \tag{17}$$

is the amplitude of a spherical wave at a distance y from the origin, if absorption is present; $\frac{A}{y}$ is the amplitude if it is not present. As y is always large, even for the nearest stars, κ must evidently be small, in order that their light may be sensible. Since, for the greater amplitudes or higher rates of distortion, the viscous resistance of ether must be small; for the lower rates of distortion, the viscous resistance must be very small stresses proportional to the rate of shear and subject to the principle of superposition, which has been assumed in deriving I. As a ray of light from such a star is diminished to a small fraction of its original amplitude before passing over a considerable portion of its path, it may be considered as following this law approximately. If this law were not followed until a further diminution in amplitude, the relative coloration between the nearer and remoter stars, depending on the distance, would only be the more marked, since the differential effect would be less for the nearer than for the remoter stars.

As the light must pass through our own atmosphere, a further absorption must take place, which also varies with the wave-length. It will be necessary to include this effect in the relative coloration to determine what the resultant appearance would be.

Let

$$\xi = A\epsilon^{-[\psi(\lambda)y+\phi(\lambda)y']} \tag{18}$$

represent the law of absorption, where $\psi(\lambda)y$ corresponds to the exponents in I., II., III., and $\phi(\lambda)y'$ is the corresponding exponent for atmospheric absorption through any thickness y'. As both ψ and ϕ are approximately independent of the amplitude, they are interchangeable as regards sequence in absorption, and we may suppose the atmospheric absorption to have taken place first. Hence in every case, we can leave out of consideration this effect and simply apply I., II., and III. to spectra as they are seen, to determine the relative coloration produced by absorption in space alone.

We have now to apply I., II., and III. to a normal spectrum to determine the amount of energy absorbed when coloration is perceptible. In plate I., the curve A^2 represents approximately the distribution of energy in the visible portion of the normal solar spectrum for different wave-lengths at high sun, according to Langley.[1] The effect of space-absorption on the solar spectrum would be inappreciable. Let now such a spectrum be carried to a very great distance; suppose the rays parallel, and absorption present. The loss of energy can be represented graphically by plotting curves with values obtained from I., II., and III. The intensity is proportional to the square of the amplitude or in I. and II. to

[1] Researches on Solar Heat, Plate I. Prof. papers of U. S. S. S., No. XV.

I *a.* $$\xi_1^2 = A^2 \epsilon^{-\frac{4\pi^2 \nu}{a\lambda^2}y}$$

II *a.* $$\xi_2^2 = A^2 \epsilon^{-\frac{2a}{a\lambda}y}.$$

Curve I_1 represents the distribution of energy according to I*a.* after the amplitude of a wave corresponding to .80 in the diagram has been diminished .01 of its original value. Curve I_2 and II_2 represent this distribution according to I*a.* and II*a.* respectively after a diminution in amplitude of .10 of its original value for the same wave. The law representing absorption according to III. would be a curve between these two. The curve A_1^2 represents what the normal distribution would be if no energy had been absorbed and the amplitude had been uniformly diminished by .10. From these curves we are able to determine the proportion of the rays lacking in the different parts of the spectrum which would when added give the original spectrum. Thus from I_1 of the red rays about .006 are lacking; of the orange and yellow, about .020; of the green, nearly .030; and of the violet, about .050. In the same way for curve I_2, about .06 of the red rays would be lacking, .15 of the yellow, .20 of the green, and about .50 of the violet. If the law of absorption is according to curve II_2, about .03 of the red would have been absorbed to .06 of the yellow, nearly .10 of the green, and nearly .40 of the extreme violet. It is thus evident that the greater the absorption, the redder the spectrum will appear.

Aubert has shown that less than one per cent of red mixed with white is perceptible. For sufficient intensity, curve I_1 would be within this limit, so that a hue near the orange-red would be perceptible. Either I_2 or II_2 would evidently give a very perceptible reddish tinge. Thus from the hue it is possible, for a given intensity, to determine the total loss in intensity and the diminution in amplitude. In the curve I_1 the amplitude of the yellow rays has been diminished from two to three per cent. In curves I_2 and II_2 the diminution has been about twenty per cent and fifteen per cent respectively.

In the case of the heavenly bodies there should then be a coloration, becoming more marked with the distance, this coloration also depending in part on the intensity. No regular gradation in hue is perceptible, and hence it may be concluded that the loss of energy is small, if any. From what is known of the spectra of incandescent bodies, the effect of increase of temperature is to displace slightly the position of maximum energy up the spectrum. Any irregular distribution of stars as regards temperature would not cause the average light of a certain number in one part of the heavens to differ materially from that of another number taken anywhere else in the heavens. To carry the test for absorption to the utmost limit possible, we have only to consider those milky patches of light visible to the eye in the Galaxy; or, better, those star clusters which are barely resolvable with the best telescopes; or, going still further, to consider those nebulæ whose spectra resemble the stellar spectra, and which consequently are probably resolvable into stars. From the vast number of stars which must constitute such a stellar mass, it may be concluded that if there were no absorption, the light with which such a mass would shine would be white. In traversing such vast distances, the absorption must be infinitesimal, not to produce a perceptible coloration. The general absence of gradation in color, even in the remotest visible bodies, shows that but a small per cent of their light can have been lost in space. This shows that $\epsilon^{-2\kappa y}$ cannot differ from unity by more than a small quantity. Hence κy will in general be less than unity, and κ will not be greater than $\frac{1}{y}$, which for the distances we have been considering, is excessively small. Referring to equation (10), we see that w is nearly unity, and hence the difference in time of propagation is a very small quantity, even for the remotest visible bodies.

Taking now our complete equation as it would be for plane polarized light propagated in spherical waves, we have for the intensity at any point

$$\xi^2 = A^2 \frac{\epsilon^{-2\kappa y}}{y^2} \qquad (19)$$

Thus the variation in intensity with distance becomes

$$\frac{\epsilon^{-2\kappa y}}{y^2} = \frac{1}{y^2} \text{ approximately,} \tag{20}$$

when y is taken as the dimension of the visible universe. In order that the effect of absorption might equal that due to the variation in distance, we should have to take a distance ny, such that

$$(\epsilon^{-2\kappa y})^n = \frac{1}{(ny)^2},$$

or (21)

$$(.9)^{2n} = \frac{1}{(ny)^2},$$

if the diminution in amplitude from absorption were ten per cent for a distance y. Thus n would have to be very great, and the system would be of dimensions n times as great as those of our own stellar system. To a close approximation, the system should have the same appearance whether absorption were present or not. The apparent finiteness of the stellar universe cannot thus be due to absorption, as Struve supposed, his assumption of uniform distribution requiring a loss of as much as one-third the light of stars of the ninth magnitude.

Either, then, the universe must be finite, or, if infinite in extent, the average density of distribution of self-luminous bodies outside our own system must be exceedingly small, as otherwise the sky would appear of a uniform brightness, approximating that of the sun.

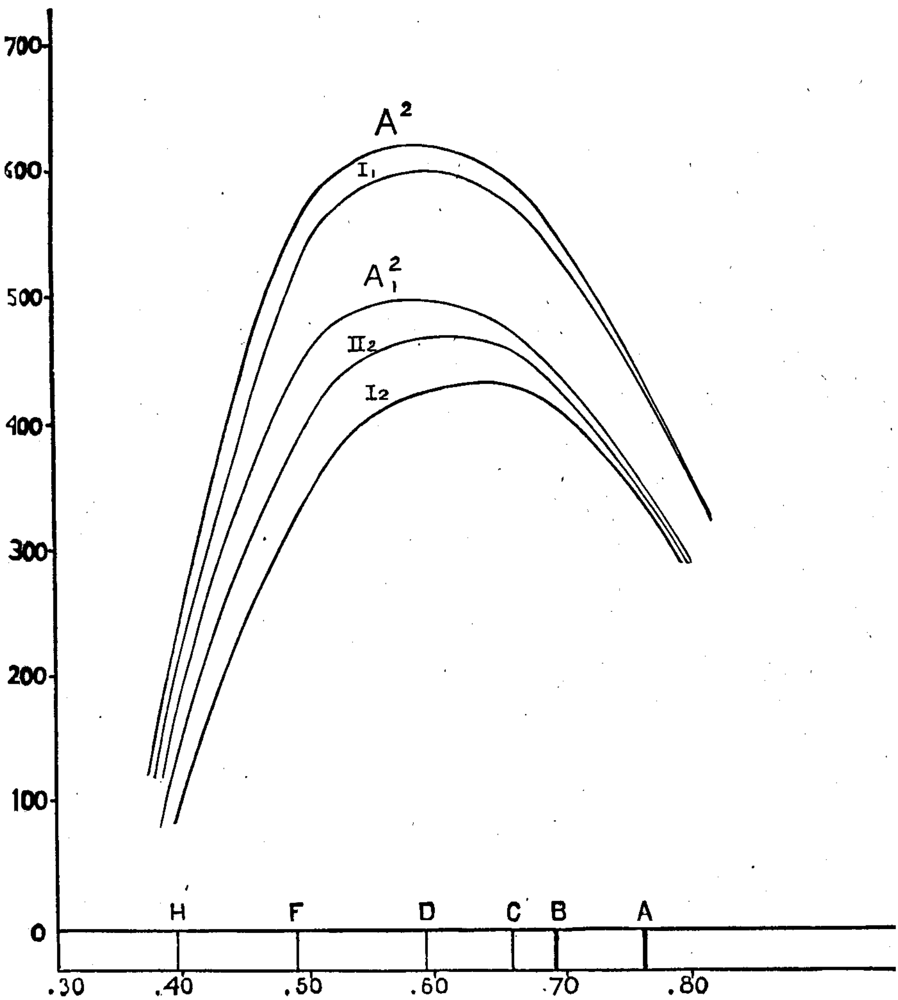
700
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Plate I.