Module 3: Proportional Reasoning

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Module 3 Proportional Reasoning [Videotape]

Introduction

In Piaget's theory, concrete operational thought is characterized by serial ordering, simple classification, and conservation logic applied directly to objects. A concrete thinker doing a Piagetian task must be able to observe objects and/or manipulate them. Formal operational thought involves proportional reasoning, separation of variables, elimination of contradictions, and class inclusion or exclusion operations. A formal thinker is able to work in situations where he does not deal with tangible objects. The formal thinker can apply the operations used by a concrete thinker, but goes beyond these operations when solving problems.

In the video-tape you are about to see, you will observe Francis P. Collea working with college science students who are responding to two Piagetian tasks. The tape clearly demonstrates that a college population includes students who approach certain tasks with concrete reasoning patterns, while others apply formal reasoning patterns. The students' responses indicate a wide range of variation even among science and mathematics majors, a highly selected group.

Objectives

To assist you in describing and/or identifying responses that indicate concrete or formal thought as applied to Piagetian tasks.

Procedure

Begin this module by reading the Overview of the tasks in the attached videotape notes. Then view the videotape. To help you understand the students' remarks in spite of their soft voices, a complete transcript of the dialogue is included at the end of the instructional materials. You may wish to glance at the transcript while you are watching the videotape.

Videotape Notes

Overview of the Tasks

In preparing the videotape, we selected responses of individual students so as to present a variety of approaches. We did not attempt to give an accurate impression of the frequency distribution that might be obtained from college students.
Module 3 Instructional Materials

Equal Arm Balance Task

The Balance Beam Task requires students to apply proportional reasoning and other elements of formal thought to a somewhat difficult problem. The interviewer poses the following four questions (in order) as each student observes the balance beam and attempts to predict the balance conditions.

1. Use a 7-weight and a 3-weight to balance a 10-weight placed 10 units from the fulcrum.
2. Use a 5-weight to balance a 10-weight placed at 10 units from the fulcrum.
3. Use a 7-weight to balance a 10-weight placed at 9 units from the fulcrum.

Dave

Our first student is shown responding to Problems 1, 2, and 3. He used proportional reasoning on #2, which involved a 2:1 ratio, but applied an additive process to #3: he placed the 7-weight at 12 units, three further than the 10-weight. He finally balanced the beam by trial and error with the 7-weight at 13 units, not clearly recognizing the relationship between location and weight. Dave appears to have begun the transition from concrete to formal thinking in relation to the balance beam.

Gary

The next student answered Problems 1, 2, and 3 quickly, using direct and inverse proportion with ease.

Celia

Had no difficulty with #4; she placed the 7-weight at 13 units and the 3-weight at 3 units.

Rosa

The fourth student on the tape, she succeeded on #1 (not shown) but did not handle any of the other problems successfully.

Jeff

The last student performing the balance beam task approached Problem 2 in a concrete way (direct correspondence of distance and weights), but quickly changed his mind when he observed the tipping of the beam. This is an example of self-regulation, where Jeff re-examined his strategy in the light of new data. Still, Jeff was not able to solve the more difficult Problem 3. Like Dave, he has begun the transition from concrete to formal thought.

Ratio Puzzle

The second task being used here is an extension of the Ratio Puzzle introduced in Module 1. There the student was told that two figures, Mr. Short and Mr. Tall, had heights of four and six buttons, respectively. After measuring Mr. Short with paper clips, the student had to predict the height of Mr. Tall in paper clips. We now include a second question dealing with Mr. Tall's fourteen-paper-clip-wide car; how wide is it in buttons?
Harley

The first student displayed his command of the proportional reasoning operation by determining a ratio of two measurements and then using it to calculate the dimension of Mr. Tall, an object he cannot observe.

Jackie

The next student on the tape quickly set up similar ratios and solved the problem quite easily.

Tracy

The third student working on the ratio puzzle set up the same ratios as Jackie and easily solved the problem.

Eddie

The next student on the videotape did not solve the problem; he thought he could not proceed unless he knew the size of the buttons. His reasoning pattern is concrete or pre-concrete.

Martha

Our last student tried in a very complicated way to establish a ratio between buttons and paper clips. Eventually she arrived at a solution that could be classified as transitional because she did exhibit proportional reasoning in her thinking, but did not apply it simply and consistently. She appeared to have an intuitive notion about establishing ratios.

Transcript of Videotapes

Equal Arm Balance Task

Frank: Now here is what I would like you to do. Here is a 7 weight and a 3 weight, put them on the other side so the beam will balance.

Dave: Okay. Put them together?

Frank: Sure you can. Okay, Dave, why do you think they will balance?

Dave: Because they are both equal distances.

Frank: Because they are both equal distances. Okay, shall we try it? (Tests it.) Very good. Let's try another one, Dave. This time, Dave, I'm going to give you a 5 weight. Could you place the 5 weight on the other side so the beam will balance again?

Dave: Okay. It will work.

Frank: Okay, why do you think it will work out there?

Dave: Because it's twice the distance and only half the weight.

Frank: Okay. Shall we try it?

Dave: Yes.
Frank: Very good. Very good. Let's try one more, Dave. This time, Dave, I'm going to put the weight, I'm going to move the ten weight to the nine slot. Okay. Now what I would like you to do is to take the 7 weight and balance the beam. (PAUSE) Why did you put it there, Dave?

Dave: Because there is three less than that, and you moved that in one and I moved this one out two.

Frank: Okay, you think it will balance?

Dave: I hope so.

Frank: Want to try it again?

Dave: Okay. I'll use one more.

Frank: Okay, now explain why you did it - how you did it.

Dave: Well, this one just went down and there's just the heavier weight moved that over one.

Frank: So you couldn't figure it out. You just did it by observation.

Dave: Yes.

Frank: Okay, shall we try another one. Shall we try it. Good observation. That's good, thanks a lot Dave.

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Gary: Seven

Frank: A 7 and a 3 weight. You think it will work now?

Gary: I think so.

Frank: Why?

Gary: You put 10 and 10 an equal distance apart.

Frank: (Tests balance arm) Okay, it works. Let me change it just a little bit. Let me ask you to put a 5 weight, where do you think you will put the 5 weight on the beam balance?

Gary: Twenty notches away.

Frank: Why do you think that it will work?

Gary: Because it is twice as far away and half the weight.

Frank: One more, okay? Let me move this to -- let's move it to 9, okay?

Gary: Okay.
Frank: Now I'll give you a 7 weight. Where would you put the 7 weight so that the beam will balance?
Gary: Oh, you couldn't put it on any notch.
Frank: Put it on the closest one.
Gary: Oh, okay. Could I lay it down?
Frank: Sure, if you want to. But leave it on the closest notch.
Gary: Okay.
Frank: Put it on the nearest one. Let's see, did that work? Okay, how did you figure that out?
Gary: Ninety pulling down against 90 on the other side (gestures), and 13 times 7 is about ninety.

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Frank: Want to try one more?
Celia: Okay.
Frank: Let me take this one off. Let me take this now and let me put it back at 10. Let me give you the 7 and 3. Okay. Now can you put the 7 and 3 on the other side so that the beam will balance? But you can't put them both at the same place.
Celia: Can't put them at the same place. (LONG PAUSE)
Frank: What did you do now?
Celia: Put 7 times 13 and 3 times 3, 9.
Frank: Think it will work now?
Celia: I hope.
Frank: Do you think it's a ratio of some kind?
Celia: Nope.
Frank: How did you figure it out?
Celia: Well, there's going to be a 100, so I had to match it over here and a combination here.
Frank: Okay, want to try it?
Celia: Okay, it works.
Frank: Very good. Thank you very much.

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Frank: Want to try another one now.

Rosa: Okay.

Frank: Let me take these off and let me give you a 5 weight. Now where would you put the 5 weight so that the beam will balance. (PAUSE) Okay, can you explain why you put it there?

Celia: Um, I'm not sure if I'm supposed to put it there or over here. It's a lighter weight, so I'm going to put it here it will go up, but over here close to center it will balance.

Frank: So you think it will balance then. It didn't work. Want to try it again? Want to put it some other place? Why did you put it there, Rosa? What number is it?

Rosa: Fifteen.

Frank: Why did you put it there?

Rosa: Because it tops the weight off, the 10 weight. It has to be half the distance over here more.

Frank: Shall we try it?

Rosa: Yes.

Frank: Didn't work again. Let's try another one. Suppose I put this at 9, okay and I gave you a 7 weight, where would you put the 7 weight so that the beam would balance?

Rosa: At 11.

Frank: At 11, want to put it at 11, please. How did you figure 11, Rosa?

Rosa: (indistinct response)

Frank: Okay, shall we try it? Didn't work. Okay, thank you very much.

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Frank: Let's try another one, Jeff. This time I'm going to give you a 5 weight. Where would you put the 5 weight so that the beam will balance? Alright, can you explain why you put it there?

Jeff: It's half as heavy, so put it out half as much.

Frank: Okay, shall we try it? Didn't work. Want to try it again?

Jeff: Yes.

Frank: Where did you put it now?
Jeff: Twice as far.

Frank: Why did you do that?

Jeff: It's half as heavy.

Frank: Half as heavy, so what does that mean? You have to put it farther away. Do you think it will work now?

Jeff: It works.

Frank: Okay, one more. Jeff, I'm going to take the 10 weight and I'm going to move it at the 9 spot. Okay, now I'm going to give you a 7 weight and I would like you to put it on the side so that the beam will balance. (LONG PAUSE) Okay, can you figure it out?

Jeff: No.

Frank: Do you have any idea where it could go?

Jeff: It should be down towards the end.

Frank: Towards the end. Want to put it someplace. Can you figure why you put it there?

Jeff: The weight's a little bit heavier.

Frank: Okay. Shall we try it? Okay, you can't figure out where it should be.

Jeff: No.

Frank: Okay, Jeff, thanks a lot.

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RATIO PUZZLE

Frank: This morning I measured him in my office with some buttons and I found him to be 4 buttons tall. Okay.

Harley: Yes.

Frank: Now, I've got another friend, Mr. Tall, who I didn't bring here this morning but I measured him this morning in my office with the same buttons and he was 6 buttons tall. Okay?

Harley: Yes.

Frank: Now, Harley, what I would like you to do is I would like you to measure the height of Mr. Short with these paper clips.
Harley: Yes.
Frank: Kinda keep this close to you. Okay?
Harley: Okay. What height? From his feet to his head?
Frank: Yes.
Harley: He is 5-1/2.
Frank: He is 5-1/2 paper clips tall. Okay, Harley, could you figure out how tall Mr. Tall is in paper clips?
Harley: It's 5, it's proportional. 4, 5-1/2 is 6 to cross multiplying at 33 divided by 4. Which is 8 and 1/4.
Frank: So Mr. Tall is 8-1/4 paper clips tall. And how did you figure that out now.
Harley: By proportion. Four buttons to 5-1/2 should be 6 to 8-1/4.
Frank: Okay. Let me ask you another question, Harley. Mr. Tall has a car, and the car is 14 buttons wide, 14 paper clips wide. Could you tell me how wide that car is in buttons?
Harley: Okay, it's the same proportion. It's 14 buttons no it's 14 paper clips wide. Okay, it's 14 paper clips, x is to 14 paper clips as 6 is to 8-1/4, 6 buttons to 8-1/4 paper clips which is a proportion as 14 x 6 which is 80-1/4 divided by 8.25. Which is approximately 10. Little bit over 10.
Frank: So you are saying that Mr. Tall's car is 10 buttons wide.
Harley: Yes.
Frank: And you figured it out by what?
Harley: A proportion.
Frank: Thanks, Harley.

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Frank: How did you get that answer, Jackie? 8.25 paper clips.
Jackie: Well, because Mr. Short was about 5'5", I just set up a ratio.
Frank: You set up a ratio. Can you explain how you set up that ratio?
Jackie: Well, Mr. Short is 4 buttons and Mr. Tall is 6 buttons and Mr. Short is 5'5" in paper clips so I put that over x.
Frank: Okay. That's how you got the answer.
Jackie: Yes.

Frank: Okay. Let me ask you another question, Jackie. Mr. Tall has a car and the car is 14 paper clips wide. Can you tell me how wide that car is in buttons? (PAUSE)

Jackie: Is this okay to do? Well, I guess it would be.

Frank: What's that Jackie?

Jackie: Well, if one's tall and the other is width, can you do that?

Frank: Can you do what? Make a proportion?

Jackie: Yes.

Frank: Try it. How did you figure that out, Jackie?

Jackie: Setting up a proportion.

Frank: Could you explain that, how you set up that proportion?

Jackie: Okay. The car is 14 paper clips so you put that over x buttons and Mr. Tall was 8.25 paper clips and 6 buttons so I just figured it.

Frank: And you solved for the answer.

Jackie: Yes.

Frank: Okay, thanks Jackie. That was very good.

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Tracey: Nine paper clips tall.

Frank: How did you figure that out Tracey? Can you tell me?

Tracey: Proportion.

Frank: Could you explain it please.

Tracey: Alright, it is 4 buttons x 6 paper clips reduces 2 to 3 so you want to set up a proportion between 6 buttons and x paper clips that equals 2 to 3, 18 equals 2 x. x equals 18 over 2 equals 9.

Frank: Okay, Tracey. That's pretty good. Let me ask you another question, Tracey. Mr. Tall has a car and the car is 14 paper clips wide. Could you tell me how wide that car is in buttons.

Tracey: 14 paper clips.

Frank: Wide.

Tracey: Okay, that is 2 over 3 equals 14 over x equals 2, 21.
Frank: Okay, can you explain how you got that answer, Tracey?

Tracey: I did the same thing. 2 buttons for 3 paper clips is 14 buttons for x paper clips cross multiply and solve for x.

Frank: Okay, thank you very much.

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Eddie: 5-1/2 paper clips tall.

Frank: Okay, good, Eddie. Could you tell me how tall Mr. Tall is in paper clips.

Eddie: I don't think I will be able to figure it. Mr. Short is 4 buttons tall that would be about 2 more buttons taller -----------which comes out inaccurately as 5 1/2. What did I say? 5-1/2?

Frank: 5-1/2.

Eddie: 5-1/2 paper clips tall. I don't think I will be able to.

Frank: Okay.

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Martha: 5 paper clips and 1/2.

Frank: 5-1/2 paper clips. Now could you figure out for me how tall Mr. Tall is in paper clips?

Martha: Okay. 2, 4. Mr. Short is 4, right?

Frank: 4 buttons.

Martha: And Mr. Tall is 6 buttons.

Frank: You want some paper or something, Martha? Here's some paper and a pencil if you want.

Martha: 1, 2, 3, 4, 5, that's 5-1/2.

Frank: Paper clips.

Martha: Paper clips equals 4 buttons and 6 buttons so half of 5-1/2 is 2-1/2 also half of half is a fourth which would be 2 and 3/4 so it would be 6-3/4 equals.

Frank: So you're saying Mr. Tall is 6 and.

Martha: No, that's Mr. Short. No, Mr. plus two, ---------
Frank: Explain to me how you got that?

Martha: Okay, let's see. Mr. Short is 5-1/2 paper clips.

Frank: Yes.

Martha: Which is 4 buttons. Okay? And a half and Mr. Tall is 6 buttons.

Frank: Right.

Martha: Which would be 1/3 of 4 buttons. And 1/3 of 6 buttons is 2. Okay, so I added 2 buttons to 4 which would make Mr. Tall and I figured half of 5-1/2 is 2-1/2 plus the 1/4 which is 2 and 3/4.

Frank: Okay. You say he is 8-3/4 buttons tall.

Martha: I think so.

Frank: Okay, 8-3/4 paper clips.

Martha: Paper clips, right.

Frank: Okay, good.