1975

CAPACITORS

Module _____

STUDY GUIDE

CAPACITORS

INTRODUCTION

Capacitors are important components of electronic circuits and of electrical machinery and power grids. You can find large oil-insulated capacitors on power-line poles or small ceramic-insulated capacitors in a radio. In each application the capacitor is used to store electrical charge and electrical energy - for example, sometimes for a short time in an alternating-current cycle, sometimes for a long time until the energy is needed, as in a strobe light for a camera. Your body can be a capacitor, storing up enough charge and energy to cause a painful spark when the capacitor discharges.

Practical capacitors are basically two conducting plates or sheets separated by an insulator (dielectric) such as air, oil, paper, plastic film, or even the oxide layer on one of the conducting surfaces. This module will treat the basic physics of capacitors.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Determine electric fields for charge distributions with planar, cylindrical, and spherical symmetries (needed for Objective 2 of this module)	Flux and Gauss' Law Module
*Determine electrostatic potentials for charge distributions with planar, cylindrical and spherical symmetries (needed for Objective 2 of this module)	Electric Potential Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. <u>Definitions</u> - Define the terms "capacitor" and "capacitance" and use these definitions to relate capacitance, voltage difference, and charge in a capacitor.

2. <u>Capacitance</u> - Derive and use expressions for the capacitance of capacitors that have planar, cylindrical, or spherical symmetry.

3. <u>Connected capacitors</u> - Determine the equivalent capacitance of a set of capacitors connected together, and determine the charge and voltage on each capacitor of the set.

4. Energy - Determine the energy stored in a capacitor or combination of capacitors, and compute the energy stored per unit volume in a region where an electric field exists.

5. Dielectrics - Describe the effect on a capacitor's capacitance, voltage, charge, and stored energy, as well as the electric field in the capacitor, if the space between the conductors of the capacitor contains dielectric material; describe qualitatively the distribution of polarization charges that accounts for these effects.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Sections 21.5, 21.6, 21.7, 21.13, and 21.14 in Chapter 21. Then read Chapter 26, Sections 26.1 and 26.2 for perspective before reading Sections 26.3 and 26.4. Study Illustration 21.6 and Problems A through H before working Problems I through N.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems (Chap. 26)
1	Sec. 21.5	A			
2	Secs. 21.6, 21.7	B, C		I	
3	Sec. 21.13	D, E, F	Illus.[a] 21.6	J	
4	Sec. 21.14	G		K, L	
5	Secs. 26.1, 26.2, 26.3, 26.4	H		M, N	1, 4, 7

[a]Illus. = Illustration

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 26. Section 26-5 is optional. Then study Problems A through H and Examples 1 through 5 before working Problems I through N and Problem 8 in Chapter 26.

Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems (Chap. 26)
1	Sec. 26-1	A				
2	Sec. 26-2	B, C	Ex.[a] 1, 2	I	Chap. 26, Prob. 8	21
3	Sec. 26-2	D, E, F	Ex. 3	J		12, 13, 18
4	Sec. 26-6	G	Ex. 4	K, L		34, 37
5	Secs. 26-3, 26-4	H	Ex. 5	M, N		30

[a]Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 27. Sections 27-7 and 27-8 (through the first column of p. 284) are optional. Study the Examples in 27-3 and 27-4 (pp. 378, 379) and Problems A through H before working Problems I through N.

Then take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	Sec. 27-1	A			
2	Sec. 27-2	B, C		I	
3	Sec. 27-3	D, E, F	Example (p. 378)	J	27-5, 27-11
4	Sec. 27-4	G	Example (p. 379)	K, L	27-4, 27-10
5	Secs. 27-5, 27-6, 27-8	H		M, N	27-15

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Read all of Chapter 26. Then study Examples 26-1 and 26-2 and Problems A through H before working Problems I through N. Take the Practice Test, and work some Additional Problems if necessary, before trying a Mastery Test.

WEINDER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 26-1	A			
2	Sec. 26-2	B, C		I	26-2, 26-4
3	Sec. 26-3	D, E, F	Ex.[a] 26-1	J	26-3, 26-10
4	Secs. 26-6, 26-7	G		K, L	
5	Secs. 26-4, 26-5	H	Ex. 26-2	M, N	26-8

[a]Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(1). An initially uncharged 5.0-μF capacitor is charged to a potential difference of 200 V by transferring charge from one plate of the capacitor to the other. How much charge was transferred?

Solution

Capacitance is defined as the ratio of the magnitude of the charge on one or the other of the conductors to the potential difference between them: $C = Q/V$ (the larger the capacitance, the more charge the capacitor will hold for a given potential difference.) The answer is therefore

$$Q = CV = (5.0 \times 10^{-6}\ \text{F})(200\ \text{V}) = 1.00 \times 10^{-3}\ \text{C}.$$

B(2). A spherical capacitor consists of two concentric spherical shells of radii a and b ($a < b$). Determine its capacitance.

Solution

Naturally, you can look up and memorize the formula, but you will clutter your mind with formulas if you do too much of this. So derive the capacitance: We know that if there is a charge Q on the inner conductor the electric potential in the space between the conductors depends on the distance from the center of the capacitor as $V = Q/4\pi\varepsilon_0 r$. Therefore the potential difference between the conductors is

$$V_a - V_b = \left(\frac{Q}{4\pi\varepsilon_0}\right)\left(\frac{1}{a} - \frac{1}{b}\right).$$

From the definition of capacitance $C = Q/(V_a - V_b)$ we obtain immediately

$$C_{sph} = \frac{Q}{(Q/4\pi\varepsilon_0)(1/a - 1/b)} = \frac{4\pi\varepsilon_0 ab}{b - a}.$$

C(2). The cylindrical capacitor in Figure 1, 2.00 cm long, consists of two conducting cylinders that are coaxial as shown. Determine the capacitance of this capacitor and compute the potential difference if the inner conductor is holding 2.00×10^{-9} C. ($R = 2.00$ mm; $r = 3.00$ mm.)

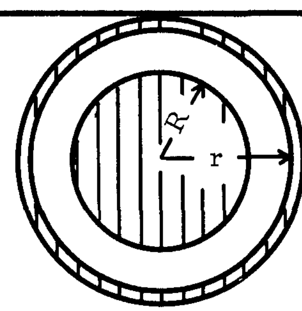


Figure 1

Solution

You should derive the expression for the capacitance: For a charge Q on the inner conductor, you can determine (do it!), using Gauss' law, that the electric field is given by

$$E = Q/2\pi\varepsilon_0 \ell r.$$

Then

$$V_a - V_b = \int_a^b E\, dr = \frac{Q}{2\pi\varepsilon_0 \ell} \ln\left(\frac{b}{a}\right).$$

Therefore

$$C_{cyl} = 2\pi\varepsilon_0\ell/[\ln(b/a)] = 2.74 \text{ pF} \quad \text{and} \quad \Delta V = \frac{Q}{C} = 730 \text{ V}.$$

D(3). Capacitors in parallel: Figure 2(a) shows three capacitors C_1, C_2 and C_3, connected in parallel (i.e., the plates of each capacitor are connected by wires to the same two terminals, a and b). Determine the equivalent capacitance of this arrangement [i.e., determine the size of the single capacitor C, Figure 2(b), which gives the same potential difference V between the terminals a and b if a given charge Q = CV is transferred from terminal b to terminal a and distributes itself among the three capacitors.]

Solution

Let the charges on C_1, C_2, and C_3 be Q_1, Q_2, and Q_3, respectively. The total charge Q is simply the sum of the charges on each capacitor: $Q = Q_1 + Q_2 + Q_3$. For this parallel connection, the potential difference V is the same for each capacitor (the upper plates are all at the same potential and the lower plates are all at some other potential), therefore

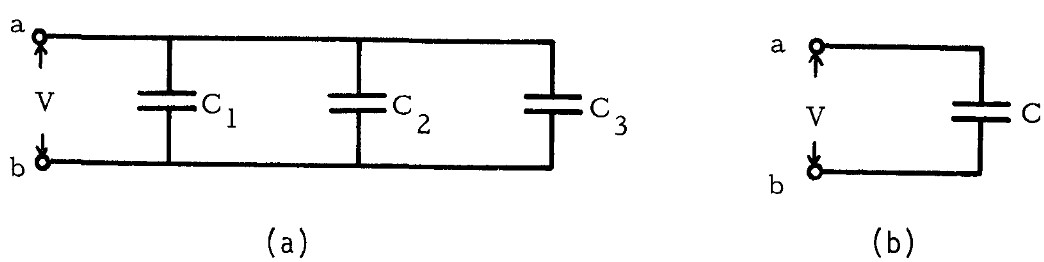


Figure 2

STUDY GUIDE: Capacitors

$$Q_1 = C_1V, \quad Q_2 = C_2V, \quad Q_3 = C_3V.$$

We can therefore immediately write

$$C = \frac{Q}{V} = \frac{Q_1 + Q_2 + Q_3}{V} = \frac{C_1V + C_2V + C_3V}{V} = C_1 + C_2 + C_3,$$

which says simply that "capacitors in parallel add." Your textbook treats the equivalent capacitor for capacitors connected in series.

E(3). Find the equivalent capacitance of the combination of capacitors shown in Figure 3 where $C_1 = 3.00\ \mu F$, $C_2 = 4.0\ \mu F$, and $C_3 = 5.0\ \mu F$.

Solution

In most practical cases, combinations of capacitors can be consolidated into groups of capacitors that are connected either in series or in parallel. The important thing is to be systematic as you work through a problem such as this. In this problem we first consolidate C_1 and C_2 into an equivalent capacitor $C_4 = C_1 + C_2$ to obtain the simplified Figure 4. Now we have C_3 and C_4 in series, which gives for the equivalent capacitance C the following relation:

$$\frac{1}{C} = \frac{1}{C_3} + \frac{1}{C_4}.$$

Inserting the values given for C_1, C_2 and C_3, we obtain $C = 2.92\ \mu F$.

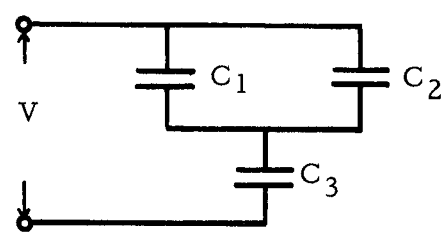


Figure 3

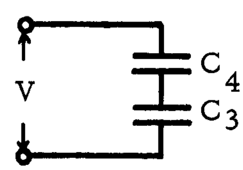


Figure 4

F(3). A potential difference of 300 V is applied to a 2.00-μF capacitor and an 8.0-μF capacitor connected in series.

(a) What are the charge and the potential difference for each capacitor?

(b) The charged capacitors are connected with their positive plates together and then with negative plates together, no external voltage being applied. What are the charge and the potential difference for each?

(c) The charged capacitors in part (a) are reconnected with plates of opposite sign together. What are the charge and the potential difference for each?

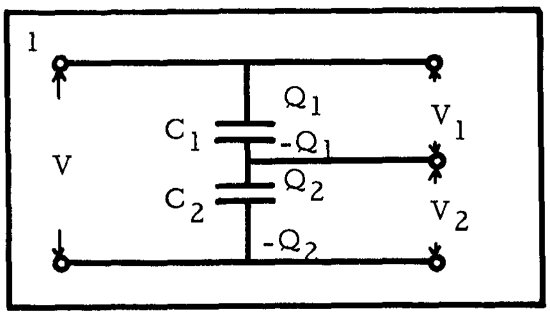
Solution

(a) You should be able to show that $Q_1 = 480$ C, $V_1 = 240$ V, $Q_2 = 480$ C, $V_2 = 60$ V.

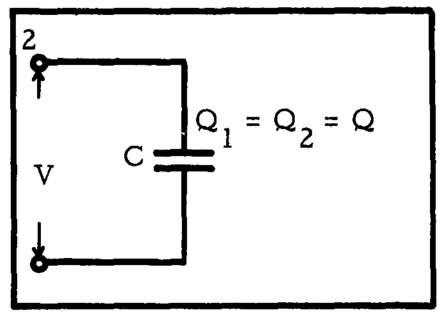
(b) The important part of this problem is to be sure that you can picture what is going on. For this purpose you may wish to illustrate each step of the problem in successive frames, as in Figure 5. Now, conservation of charge requires that the total charge on the upper plates in Step 4 of Figure 5 is

$$Q_1 + Q_2 = 2Q = Q_1' + Q_2',$$

and the negative of these on the lower plates. We can thus draw Step 5 showing

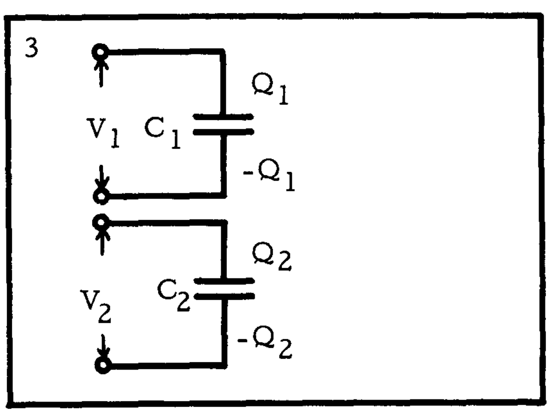


Step 1: Part (a) situation. Note that $Q_1 = Q_2 = Q$.

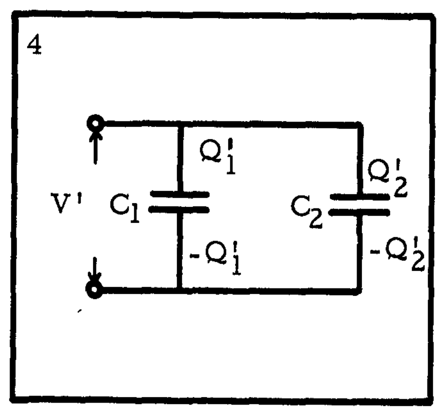


Step 2: Equivalent capacitor for Part (a).

Figure 5

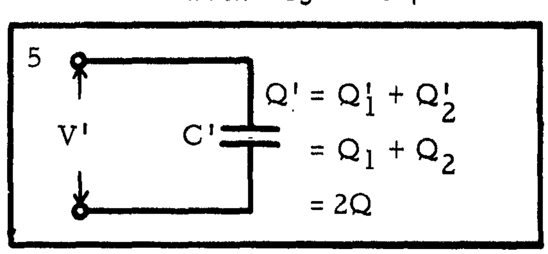


Step 3: Capacitors are disconnected from each other. $Q_1 = Q_2$.



Step 4: Capacitors are reconnected with positive plates together and with negative plates together.

Step 5: Equivalent capacitor for arrangement in Part (b).



an equivalent capacitance $C' = C_1 + C_2$, with charge $Q' = Q_1' + Q_2' = 2Q$ on it and potential difference $V' = Q'/C'$ across it. Having calculated V we can go back to Step 4 and determine the charges Q_1' and Q_2' on C_1 and C_2:

$$Q_1' = C_1V', \qquad Q_2' = C_2V'.$$

Sometimes you may find that writing out the answer in terms of the given quantities gives a rather complicated expression, and it may be easier to compute intermediate numbers. A good rule of thumb is to give writing out an algebraic expression a try before you give up and plug in numbers along the way. In this case,

$$Q_1' = C_1V = C_1Q'/C' = C_1[2Q/(C_1 + C_2)]$$

$$= \frac{2C_1V}{(C_1 + C_2)(1/C_1 + 1/C_2)} = \frac{2C_1VC_1C_2}{(C_1 + C_2)^2} = \frac{2C_1^2C_2V}{(C_1 + C_2)^2};$$

also

$$Q_2' = C_2V' = 2C_2^2C_1V/(C_1 + C_2)^2 ,$$

which equations are not too complicated to evaluate. The potential across both capacitors is

$$V' = \frac{Q'}{C'} = \frac{2Q}{C_1 + C_2} = \frac{2}{C_1 + C_2}\,\frac{V}{(1/C_1 + 1/C_2)} = \frac{2C_1C_2V}{(C_1 + C_2)^2}.$$

(c) In this case we have a similar sequence of steps. The difference is that now we have $Q' = Q_1 - Q_2 = 0$.

G(4). Two capacitors, one of 1.00 μF and the other of 2.00 μF, are each charged initially by being connected to a 10.0-V battery. Then the two capacitors are connected together. What is the total electric energy stored in each capacitor, if the capacitors are connected such that

(a) both positive plates are brought together, and
(b) plates of opposite charge are brought together?
(c) Account for the lost energy in part (a).

Solution

This problem is very similar to earlier problems except that now you must also compute the energy stored in each capacitor, $(1/2)CV^2$, and add up all this stored energy. The energy that "disappears" in this problem goes into heating the wires that make the connections or into electromagnetic radiation.

H(5). A parallel-plate capacitor is half filled with a dielectric of dielectric constant κ, as shown in Figure 6. What is the capacitance?

Solution

You can consider this capacitor to be two capacitors in parallel, one filled with dielectric and the other one not. Thus, if C is the original capacitance,

$$C = (1/2)C + (1/2)C\kappa = (1/2)C(1 + \kappa).$$

Problems

I(2). A capacitor is made of two flat conducting circular plates of radius 12.0 cm, separated by 0.50 mm. Determine the capacitance.

J(3). In the circuit shown in Figure 7, the battery provides a constant potential difference of 50 V and C_1 and C_2 are initially uncharged. Switch S_1 is closed, charging the 100-μF capacitor C_1. Then S_1 is opened, disconnecting the battery from the circuit. Following this, S_2 is closed. The value of the potential difference V across C_2 is then measured to be 35 V. Determine the capacitance of C_2.

K(4). An isolated metal sphere whose diameter is 10.0 cm has a potential of 8000 V. What is the energy density at the surface of the sphere?

L(4). Compute the energy stored in the system of Problem E if V = 50 V.

M(5). A Geiger counter is made of two long, concentric metal cylinders with a gas of dielectric constant κ between them. Neglecting end effects, use Gauss' law to calculate the capacitance of this configuration. The center rod has a radius a, the surrounding tube a radius b, and the length ℓ = b.

N(5). A parallel-plate capacitor of plate area A and separation d is charged by a battery to a potential B, and is then disconnected from the battery.
(a) Give expressions for the energy stored in and charge on the capacitor.
(b) A slab of dielectric with constant κ is then inserted into the capacitor, completely filling the space between the plates. Determine the capacitance, charge, potential difference, energy stored, and electric field in the capacitor. Describe qualitatively what happens to the "lost" energy, and the distribution of polarization charge in the dielectric.

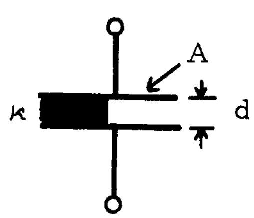


Figure 6

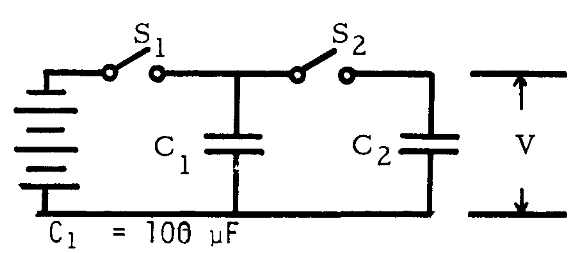


Figure 7

Solutions

I(2). Use the expression, which you should be able to derive, to determine that the capacitance is 8.0×10^{-10} F.

J(3). This problem is completely equivalent to a problem that reads: "A 100-μF capacitor is charged to 50 V, the battery then being disconnected. The capacitor is then connected across a second initially uncharged capacitor. If the potential difference drops to 35 V, what is the capacitance of the second capacitor?" Answer: 43 pF.

K(4). 0.100 J/m^3.

L(4). 3.7 mJ.

M(5). $2\pi\varepsilon_0\kappa\ell/\ln\frac{b}{a}$.

N(5). Q does not change. $E = B/d = Q/\kappa C_0 d$. $C = \kappa C_0$. Charge is fixed. $V = Q/\kappa C_0$.

PRACTICE TEST

1. Derive the expression for the capacitance of a capacitor that consists of concentric cylindrical conducting shells of radius a and b (a < b) and of length L.

2. A 5.0-μF capacitor is charged to 10.0 V and an 8.0-μF capacitor is charged to 5.0 V. They are then connected in parallel (positive plate of one to the negative plate of the other) with an initially uncharged 4.0-μF capacitor. Determine the energy stored in the final configuration.

3. A parallel-plate capacitor with plate separation d has a capacitance C.
(a) If a dielectric slab of thickness d/3 and dielectric constant 2.50 is inserted between the plates and parallel to them, determine the ratio of the capacitance with the dielectric in place to the capacitance without the dielectric.
(b) Make a sketch showing the location of free and polarization charges (with their signs) when the capacitor is charged with the dielectric in place.

Answers-- 1. $C = \dfrac{2\pi\varepsilon_0 L}{\ell n(b/a)}$; 2. 2.9×10^{-6} Joules ; 3. (a) 1.25.

CAPACITORS Date ____________________

Mastery Test Form A

pass recycle

1 2 3 4 5

Name ______________________________ Tutor ___________________________

1. Two oppositely charged ($\pm Q$) parallel plates have an area A and are separated by a distance d in vacuum. The electric field between the plates is perpendicular to the surface and has a magnitude $E = Q/\varepsilon_0 A$.

   (a) Write the defining equation for capacitance.

   (b) Determine the potential difference between plates.

   (c) Use the results from parts (a) and (b) to obtain an expression for the capacitance of two parallel plates.

   (d) Determine the energy per unit volume between the plates.

2. Given the capacitor scheme in Figure 1, with the indicated capacitances:

   (a) Reduce the various capacitors to a single equivalent capacitor.

   (b) Determine the charge on, voltage across, and energy stored in the 1.00-μF capacitor.

3. Two capacitors are connected in series with a battery as shown in Figure 2. Describe quantitatively what happens to the charge, potential difference, capacitance, and energy of each before and after a dielectric slab of constant κ is inserted into the bottom capacitor.

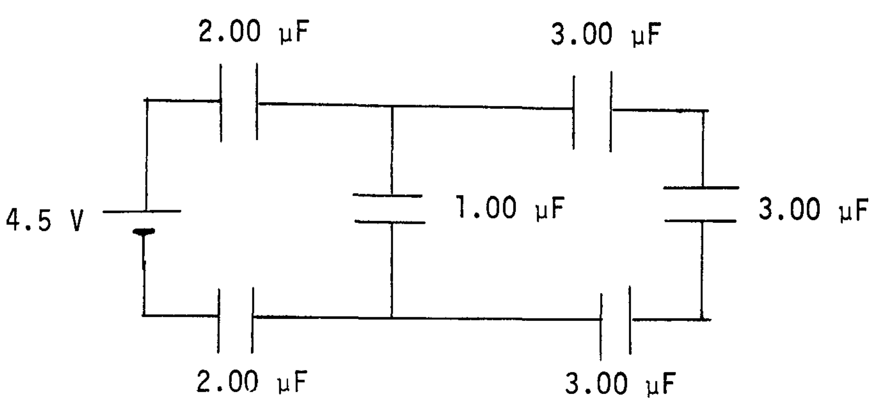


Figure 1

See Knote

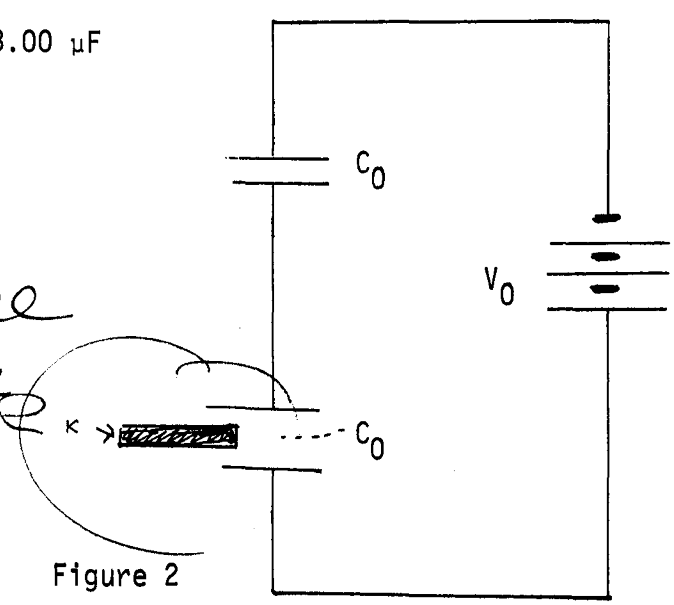


Figure 2

CAPACITORS

Date ______________

Mastery Test   Form B

pass   recycle

1 2 3 4 5

Name ______________   Tutor ______________

1. A spherical air capacitor consists of two concentric spherical shells. The inner sphere has a radius a and charge +Q. The outer sphere has a radius b and a charge -Q.

   (a) Write the defining equation for capacitance.
   (b) Determine the potential difference.
   (c) Use the results from parts (a) and (b) to obtain an expression for the capacitance of a spherical capacitor.
   (d) Determine the energy stored between the spheres.

2. Given the capacitor scheme shown in Figure 1:

   (a) Reduce the various capacitors to a single equivalent capacitor.
   (b) Determine the charge on, voltage across, and energy stored in the 3.00-μF capacitor.

3. The capacitor in Figure 2 is charged by using a battery, which is then disconnected. A dielectric slab that fills the volume between capacitor plates is then placed between them. Describe, quantitatively, what happens to the charge, capacitance, potential difference, electric field, and stored energy. Must you do work to insert the slab, or is work done on it by the field? Does the charge or the potential stay fixed as the slab goes in?

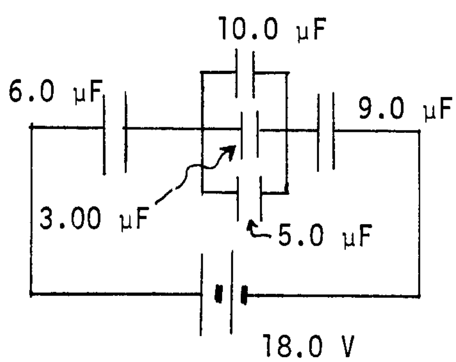


Figure 1

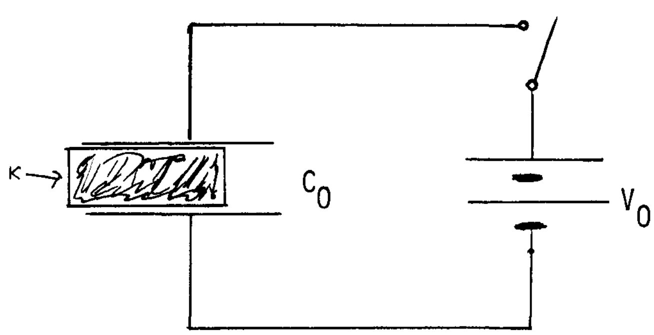


Figure 2

CAPACITORS

Date ____________________

Mastery Test   Form C

pass   recycle

1   2   3   4   5

Name ______________________   Tutor ______________________

1. A coaxial cable consists of an inner solid, cylindrical conductor of radius a supported by insulating disks on the axis of a thin-walled conducting tube of inner radius b. The two cylinders are oppositely charged with a charge per unit length of λ. The electric field between the cylinders is

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0}\left(\frac{1}{r}\right)\left(\frac{\vec{r}}{r}\right) ,$$

where $\frac{\vec{r}}{r}$ is a unit vector in the r direction.

(a) Write the defining equation for capacitance.
(b) Use the expression for the electric field to determine the potential difference.
(c) Use the results from parts (a) and (b) to obtain an expression for the capacitance of the coaxial cable.
(d) Determine the energy stored per unit volume in the region $a < r < b$.

2. Given the capacitor scheme shown in Figure 1, (a) reduce the various capacitors to a single equivalent capacitor; and (b) determine the charge on, voltage across, and energy stored in the 6.0-μF capacitor.

3. In Figure 2, two identical capacitors of capacitance C_0 are connected in parallel and charged to a voltage V_0. A slab of dielectric constant κ is then inserted in one of the capacitors. Describe, quantitatively, what happens to the charge, capacitance, potential difference, polarization charge, and stored energy of each. The capacitors are disconnected from the battery (switch open) before the dielectric is inserted. First decide if the charge or the potential difference is fixed as the slab goes in.

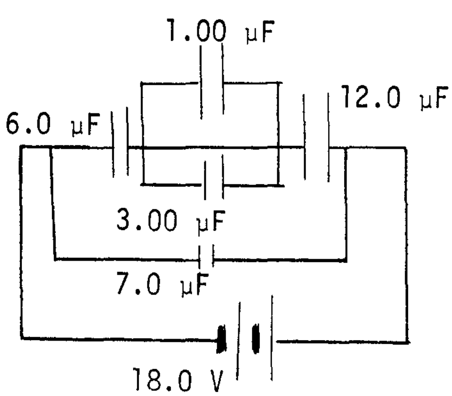


Figure 1

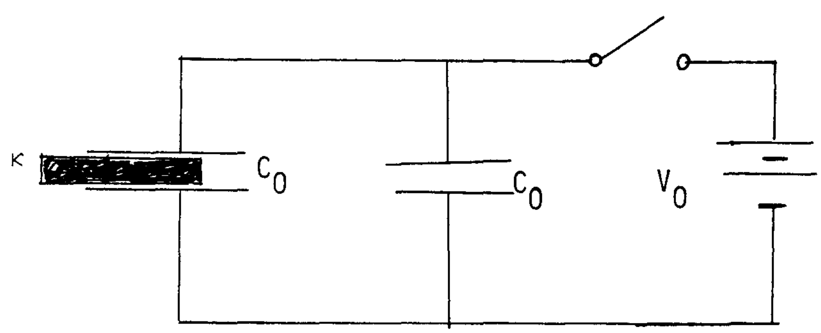


Figure 2

MASTERY TEST GRADING KEY - Form A

1. Solution: (a) $C = Q/V$. (b) $V = Ed = Qd/\varepsilon_0 A$. (c) $C = \varepsilon_0 A/d$.

   (d) $U = (1/2)\varepsilon_0 E^2 = (1/2)\varepsilon_0(Q^2/\varepsilon_0^2 A^2) = (1/2)(Q^2/\varepsilon_0 A^2)$.

2. Solution: (a) Find the equivalent capacitance:

   $\frac{1}{C_1} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$, $C_1 = 1.00\ \mu F$. $C_2 = 2.00\ \mu F$.

   $\frac{1}{C_T} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{3}{2}$, $C_T = 2/3\ \mu F$.

   (b) $V = 4.5$ V. $E = (1/2)CV^2 = (1/2)(10^{-6})(20) = 10^{-5}$ J.

   $Q = CV = (10^{-6})(4.5) = 4.5 \times 10^{-6}$ C.

recheck

$V = 1.5\ v$

$Q = 1.5\ mc$

$E = 1.125\ \mu j$

3. Before: $C_{eq} = C_0/2$. $Q = (C_0/2)V_0$ on each. $V = (C_0V_0/2C_0) = V_0/2$ on each.

   $C = C_0$ on each. $E = (1/2)C_0V^2 = (1/2)C_0(V_0^2/4) = (1/8)C_0V_0^2$ on each.

   After: $C_{top} = C_0$. $C_{bottom} = \kappa C_0$. $C_{eq} = \kappa C_0/(1 + \kappa)$. $Q = \kappa C_0 V/(1 + \kappa)$ on each. $V_{top} = \kappa V/(1 + \kappa)$. $E_{top} = (1/2)C_0[\kappa/(1 + \kappa)]^2 V^2$.

   $V_{bottom} = V/(1 + \kappa)$. $E_{bottom} = [\kappa C_0/(1 + \kappa)][1/(1 + \kappa)]^2 V^2$.

$$\frac{\frac{1}{2} K C_0 V^2}{\cancel{2}(1+K)^2}$$

MASTERY TEST GRADING KEY - Form B

1. Solution: (a) $C = Q/V$.

(b) $V = \frac{Q}{4\pi\varepsilon_0}(\frac{1}{a} - \frac{1}{b})$.

(c) $C = 4\pi\varepsilon_0 ab/(b - a)$.

(d) $E = \frac{1}{2}CV^2 = \frac{1}{2}(\frac{4\pi\varepsilon_0 ab}{b - a})(\frac{Q^2}{(4\pi\varepsilon_0)^2})(\frac{(b - a)^2}{a^2b^2}) = \frac{(b - a)Q^2}{ab4\pi\varepsilon_0}$.

8

2. Solution: (a) $\frac{1}{C} = \frac{1}{6.0} + \frac{1}{18.0} + \frac{1}{9.0} = \frac{6}{18} = \frac{1}{3}$. $C = 3.00\ \mu F$.

(b) $Q_3 = (3.00 \times 10^{-6})(3) = 9.0 \times 10^{-6}$ C. $V_3 = 3.00$ V.

$E = (1/2)CV^2 = (1/2)(3.00 \times 10^{-6})(9) = 13.5 \times 10^{-6}$ J.

$Q = C_{eq}V = 54 \times 10^{-6}$ C. $V_6 = 54/6 = 9.0$ V.

$V = 54/9 = 6.0$ V.

3. Solution: Q does not change. $E = V/d = Q/\kappa C_0 d$. $C = \kappa C_0$. Charge is fixed. $V = Q/\kappa C_0$. Work is done.

MASTERY TEST GRADING KEY - Form C

1. Solution: (a) $C = Q/V$.

(b) $V = \int \vec{E} \cdot d\vec{r} = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{b}{a}$.

(c) $C = \frac{2\pi\varepsilon_0\lambda\ell}{\lambda \ln(b/a)} = \frac{2\pi\varepsilon_0\ell}{\ln(b/a)}$.

(d) $E_{total} = \frac{1}{2}CV^2$,

$$U = \frac{E_{total}}{\pi(b^2 - a^2)\ell} = \frac{2\pi\varepsilon_0\ell\lambda^2 \ln^2(b/a)}{2\pi(b^2 - a^2)\ell \ln(b/a)\,(4\pi^2\varepsilon_0^2)} = \frac{\lambda^2 \ln(b/a)}{4\pi^2\varepsilon_0(b^2 - a^2)}$$

2. Solution: (a) $\frac{1}{C_1} = \frac{1}{6.0} + \frac{1}{4.0} + \frac{1}{12.0} = \frac{6.0}{12.0}$. $C_1 = 2.00\ \mu F$. $C_{eq} = 9.0\ \mu F$.

(b) $Q = CV = 9.0 \times 18.0 \times 10^{-6} = 162 \times 10^{-6}$ C.

$Q_7 = 7.0 \times 18.0 \times 10^{-6} = 126 \times 10^{-6}$ C.

$Q_6 = 36 \times 10^{-6}$ C. $V = Q/C = 36/6.0 = 6.0$ V.

$E = (1/2)CV^2 = (1/2)(6.0)(36) = 108 \times 10^{-6}$ J.

$\times 10^{-6}$

3. Solution:

	Initial	Final
Free charge	$2C_0V_0$	$2C_0V_0$
Capacitance	$2C_0$	$C_0(\kappa+1)$
Voltage	V_0	$2V_0/(\kappa + 1)$
Polarization charge	0	$2V_0C_0(\kappa - 1)/(\kappa + 1)$
Stored Energy	$C_0V_0^2$	$2C_0V_0^2/(\kappa + 1)$