7-2007

It Is Not Just the Solution Anymore

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Math in the Middle Institute Partnership
Action Research Project Report

in partial fulfillment of the MA Degree
Department of Teaching, Learning, and Teacher Education
University of Nebraska-Lincoln
July 2007

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Abstract

In this action research study of a fifth grade enrichment mathematics class, I investigate how problem solving strategies and group work opportunities influence the students' use of strategies and graphic representations. Though there are no large gains in the students' use of strategies, some students show minor gains in their use of representations. I also investigate the influence of group work upon usage of representations. At the beginning of the study most students indicate that working in groups is helpful in finding solutions. Toward the end, 52.9 percent of the students indicate that while group work is beneficial for "reasoning" problems, working in groups on story problems hinders their problem solving efforts. The research complicates our understanding of group work in teaching mathematics and specifically for the student use of graphic representations in problem solving contexts.

Introduction

This study describes my effort to increase attention to problem solving in a fifth grade mathematics curriculum. In this inquiry I focus on three interconnected things: increasing problem solving opportunities, the usage of graphic representations in problem solving, and doing these within the context small group work. A large number of representations exist. At times they may be needed to solve a problem. At other times representations might be used to organize information in order to better understand the problem and/or the processes to arrive at a solution. Representations may also be used to report the solution of a problem. Because of the number of representations, their varied uses, and their ability to make abstract information more concrete, emphasis was placed on the instruction and use of representations as a problem solving strategy, as well as small group collaboration to help promote the understanding and the use of a variety of problem solving strategies.

Many students' achievement test scores, related to mathematical problem solving, are low. This may indicate limited practice and/or limited knowledge of strategies. Along with the test scores, an overview of the curriculum shows an imbalance in the amount of time spent on computation and the time spent on problem solving within the classroom. In addition, daily work provides evidence of an over-reliance on computation, even within problem solving activities. Most students in my class use strictly computation when solving story problems. They look for what they think is the simplest and most direct route to a solution. However, often times their solution is incorrect. Important details tend to be overlooked. A complete understanding of the problem is not present. Students are not seeing the relationships that the problems entail. It appears that as a result they are unable to see connections and transfer advanced skills to other problems. Students are becoming stagnant in their mathematical abilities. This stagnation is reaching epidemic proportions. Each year the mathematical ability of most fifth grade students shows a decline from that of the class that preceded them. This is becoming a major concern of the teachers and the school. Increasing student performance in regard to problem solving is a school improvement goal. It seems appropriate to concentrate on problem solving as part of this action research study.

In doing so, I observe students who are high functioning in terms of computation. I want to see what effect a new curriculum with an increased emphasis on problem solving and a decreased emphasis on computation has on students' mathematical ability to solve problems and on their attitude toward problem solving. I do not want to hinder their established ability and confidence in computation. Instead, I want to increase their confidence, expanding their comfort zone to include a variety of problem solving strategies. I observe students while working in groups, noting their comments and discussions as well as their approaches to the solutions of the problems. I also observe students individually as they work on various problem solving activities.

This action research takes place in a small midwestern town in the United States. Because of the agricultural base of the area and the influences of transportation and industry, our community and school is a mixed population of those who are established members of the community and those who are more mobile. With this mobile society, many students attended three or more schools by the time they reach fifth grade. This mobility and switching of schools lends to the greater possibility of missing skills in one's educational experience. This may be one cause of the school population's recent decline in mathematical ability. Another factor may be the change in the socio-economic status of our community and school. During the 2006-2007 school year, 52 percent of the fifth grade students received free or reduced lunches. The overall school average is approximately 35 to 40 percent.

No matter what the reason for the decline in math skills, the fifth grade students need assistance. Such a large number of students struggle with basic computational skills that three sessions of the School Within A School math program are devoted to fifth graders. The School Within A School program concentrates on the basic grade level curricula. The instruction is often at a slower pace and modified to include more hands-on or concrete examples and practice. Because of the wide range of skills among the fifth grade students, I teach a fifth grade mathematics enrichment class. The purpose of the mathematics enrichment class is to provide students with more challenging work in computation, reasoning, problem solving, and geometry, without infringing on the sixth grade curriculum. It is an enhancement curriculum rather than a promotion program. Teachers administered a local pretest to the students in May of their fourth grade year to determine who would be eligible for the enrichment class. Teachers also studied outcomes of the April 2006 Terra Nova standardized test scores as part of the process to make the final selection of students for the fifth grade mathematics enrichment class. Test results and daily work make it evident that the students' strength in mathematics is their computation ability. Attention to detail and problem solving are areas of weakness.

I am conducting my action research in my fifth grade mathematics enrichment class. I have taught mathematics to fifth graders for 18 years, 14 years in this school district. However, this is the first year I am teaching a class in which all students show such a strong foundation in mathematics. In past years my math classes consisted of students with a wide range of skills. There have been years when I had students who did not even have the basics of number sense or place value, much less having the ability to add or subtract. I have also had English Language

Learners (ELL students) who could not communicate their questions or their solutions in English. Within the same classes I would have students who were slightly below grade level in math skills as well as those who were at grade level or above. At times I would also have one or two students who revealed high levels of math reasoning and ability. Often times in the past, because of the overwhelming number of students who still needed assistance in computation, I would spend too much time working with students on basic computational skills. This did not allow for the students to experience higher level thinking skills or to expect more of their own abilities, nor did it allow for those students with higher ability to be challenged. I also spent too much time presenting material and having students completing an assignment individually, rather than giving a short introduction to a concept and letting students collaborate and help each other find a solution. Quite often during second semester I would continue spending a lot of time on computation with some students while introducing other concepts that were part of the curriculum. I would have others move on to other mathematical skills and concepts at a faster pace, often giving them more challenging opportunities. Even though I was attempting to meet the needs of the students with varied abilities, I still had students work as individuals.

Within the past two years I have changed my philosophy and some of my practices in regard to math education in order to better serve the students. I am increasing my use of group sessions and discussions about math. I spend more time concentrating on reasoning skills and problem solving, reducing the amount of time spent on computation. In my enrichment class this year, I notice how much of a detriment over-emphasis on computation can be. Instead of producing students with a rich mathematical base, this limits the students' abilities. As a result, they may find it more difficult to adjust to the problem solving world around us. This year, especially during the action research, I am concentrating on problem solving and the benefits of group work and discussions. I am continuing my search for a balance between computation and problem solving and a balance between individualized and group work. Hopefully the information this action research can provide will give me insight as to the best course of action at this time for our students.

Problem of Practice

As a whole, the performance of fifth grade students the past few years shows low mathematical skills. Even though many students still have difficulty with computational skills, their skills in problem solving show even greater need for intervention. Achievement test scores confirm this. Students have limited capability of selecting appropriate strategies and graphics and of explaining their thoughts and approaches to solving problems. Therefore, they have difficulty transferring their reasoning and computational skills to new problem solving situations. They tend to look at each problem as a disconnected problem with totally new processes needed in order to solve.

The results of a fifth grade entrance test, given at the end of fourth grade, show that most students are unable to develop appropriate strategies and avoid the use of pictorial representations that would lead to a correct solution when working with computational problem solving. This year’s lessons provide evidence that the same inability to develop techniques appropriate for solving problems exists. This is true for both problems involving computation and those with greater emphasis on reasoning and patterns.

In the past I have not provided a good balance of computation and drill, story problems involving computations, and problems focused on reasoning skills. I have spent too much time presenting material rather that allowing students to discover patterns, strategies, and solutions. I have spent too much time focused on students performing as individuals instead of allowing students to work in groups, increasing their understanding of math by sharing their knowledge and ideas with each other. For student growth, I need to provide opportunities for thought as well as keep a balance with computation.

It is important to improve the problem solving skills of students, not only for the students themselves, but also for society as a whole. The world revolves around the connection of ideas and around problem solving. On a strictly educational basis, the school's math goal for North Central Accreditation is problem solving. An increase in the abilities to use graphics and select appropriate strategies should enhance the problem solving capabilities of the students. It should also allow teachers to devote more time to interact with students and enrich the curriculum. The annual yearly progress for No Child Left Behind is to mark improved problem solving skills. Also, two of the processing standards as listed by the National Council of Teachers of Mathematics (NCTM) are problem solving and connections. Working on these national standards is essential for a student in today's society.

Literature Review

In their action research study of third graders, Millard, Oaks, and Sanders (2002) note that in this high-speed world of technology and communications, where vast amounts of information are available in a microsecond, a person must be ready and able to constantly adjust to new problem solving situations. As stated in *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000, p. 21), "Learning with understanding is essential to enable students to solve the new kinds of problems they will inevitably face in the future." We do not know what the future brings. We cannot narrow our focus to preparing students for specific school endeavors and careers, as new careers and opportunities are developed with technological changes and needs for resources. Therefore, to best serve our students we must teach them in a way that will increase their understanding. We must teach them how to be flexible in their use of this understanding. We must prepare them for the unknown.

Stein, Grover, and Henningsen (1996) as well as Noss, Healy, and Hoyles (1997) studied middle school students and their understanding of mathematics. Their findings indicate that the understanding of mathematics is not just the knowledge of concepts and rules, but it consists of connections, linking new knowledge with that which was previously learned. It also embodies such abilities as recognition of patterns, problem solving, data analysis, and justification of reasoning and solutions. It is with the connection of these ideas that mathematics becomes meaningful and useful to individuals and society.

The importance of teaching thinking skills as a part of problem solving is a recent shift of emphasis. McKendree, Small, Stenning, and Conlon's (2002) study in the United Kingdom suggests that it is through the understanding of how people learn and transfer information that the role of representation comes to light. Representation refers to the process by which a concept or relationship is presented as well as the physical form of the presentation. The form in which ideas are presented determines how people can understand and use those ideas (McKendree, Small, Stenning & Conlon; NCTM, 2000).

Brenner, Herman, Ho, and Zimmer (1999) studied sixth grade students from America and compared them to students from the Asian countries of China, Japan, and Taiwan. The aim of their study was to see if the reason for high math achievement among Asian students was a better understanding of representations. Their study shows that a key component of mathematical competence, which involves reasoning and problem solving, is flexibility in the use of representations. This consists of using different forms of the same representation for different problem situations, as well as moving from one type of representation to another within a problem. Each type of representation relays information about a problem from a different perspective.

One finds representations in the integration and planning stages of problem solving. However, they are a result of the translation stage, which is the interpretation of the problem statement (Brenner, Herman, Ho, & Zimmer, 1999). McKendree, Small, Stenning, and Conlon (2002) suggest that within the framework of problem solving is the interpretation of the problem and the interpretation of the representation. A person must understand the problem well enough to apply representations in such a way that they are useful and will lead to greater understanding and effective reasoning. Each person brings to the problem his own experiences, beliefs, values, and assumptions. Often people read into the problem guidelines that are not present. These assumed guidelines add restrictions and can lead individuals down the wrong problem solving path. This became evident in Nunokawa's (2004) study of the use of representations by a graduate student in Japan. Nunokawa's data reveals that the interpretation of the problem is critical in the design of the pictorial representation. The noted subject of this study not only used the problem statement in developing his representation, but also used information from previous problems. It was in the transfer of information from one representation to another that restricted full understanding. Even with a good basic understanding of the problem and the mental manipulation of the drawing, the drawing became a detriment to finding a solution. With the subject limiting himself to assumed guidelines, the final solution was not evident. Constraints on the mental manipulation of the representation impeded the subject's ability in finding an appropriate solution (Nunokawa).

The understanding of representations affects students' use of appropriate drawings in mathematical problem solving. Their drawings, in turn, affect their understanding. In order for this cyclical operation to take place, representations must be learned. Part of this learning and practice includes the selection of appropriate representations for solving particular types of problems. Such a selection can aid in obtaining both understanding and the correct solution (McKendree, Small, Stenning & Conlon, 2002; Nunokawa, 2004).

Elia and Philippou (2004) studied the use of representations in problem solving by sixth grade high achieving students in Cyprus. They came to a similar conclusion as Neria and Amit (2004) did in their study of ninth grade students. Their assertion is that representation is an important strategy for problem solving in mathematics. Representation skills include the selection, construction, and use of representations such as pictures, tables, graphs, equations, and words. The skills also include communications and the thought processes of students in finding a solution. Even though pictorial representations can aid the communication process, communications itself is of equal fundamental value to the learning task and the thinking process. These two forms of representation compliment each other (Elia & Philippou; Neria & Amit; Nunokawa, 2004).

Some research indicates that pictorial representations act as bridges between the practical level of understanding and the theoretical level. Representations assist in the instruction and learning of mathematical concepts. They elicit and support reflection as well as the communication of ideas. To be an effective mathematical problem solver, one must be able to visualize relationships. In reference to problem solving in mathematics, to visualize is to be able to understand the problem and construct and/or use appropriate pictorial representations to help arrive at a solution (Elia & Philippou, 2004).

The research of Campbell, Collis, and Watson (1995) focused on visualization as it relates to visual processing. Their study, conducted with tenth graders, compared the students' abilities and uses of logic, visual imagery, and concrete images in solving problems of an abstract nature. Results indicate that logic contributed more to success in problem solving than visual imagery. However, students noted that visual imagery made problems more interesting and helped in the understanding of the problems' structures. During the research many students drew pictorial representations even when they had to mentally manipulate an object. In questioning students they responded that concrete images often gave better detail and understanding. Students also felt they were not limited by working memory when using concrete images. The findings of Neria and Amit (2004) indicate that when problems entailed high-level skills, most students tried to reduce the level of abstraction and reverted back to the core skills of numbers, words, and pictorial representations (Campbell, Collis & Watson, 1995; Neria & Amit).

These research studies suggest that students be taught to visualize a problem before attempting to solve it. This along with emphasis on abstract thinking and generalizations should help students clarify problem structures and check the reasonableness of answers. One must also remember to teach students the properties and uses of different pictorial representations as they provide support for and are preferred by many individuals during the problem solving process (Campbell, Collis & Watson, 1995; Neria & Amit, 2004).

The research conducted by Elia and Philippou (2004) proposes that there are four functions of pictorial representations in mathematical problem solving. Three of them, representational, organizational, and informational, are beneficial to both problem solving and the communication process. The fourth function, decorative, provides no assistance in understanding. In fact, decorative pictures can become distractions and hinder the problem solving process. In this study, when decorative pictures were present in a problem-solving situation, students arrived at a solution by using an analytical approach or by drawing their own picture or diagram. The original picture was of no use to them. Some research results suggest that for a pictorial representation to be truly meaningful, the problem solver should draw the representation rather than simply interpret a representation designed by an expert. By building one's own pictorial representation, the student continually analyzes his thinking processes (Elia & Philippou, 2004; Nunokawa, 2004).

Elia and Philippou (2004) imply that no matter what the function of the pictorial representation, its use in problem solving can cause internal conflict for the student. This conflict usually occurs when the text of the problem seems to approach the solution in a different manner than suggested by the representation. This internal struggle often becomes helpful in the solution effort. In striving for balance and continuity, students view the problem in greater depth. Sometimes, however, this is not the case. The pictorial representation can create an obstacle to learning. An overemphasis or concentration on irrelevant information or the inaccuracy of a drawing as demonstrated in Nunokawa's (2004) study is usually the cause. But the arrangement of the elements within the picture and the student's inability to mentally manipulate the picture to fit his needs, as was the case in the research by Elia and Philippou (2004) can also be the cause.

The view of Noss, Healy, and Hoyles (1997) is that too often students are encouraged to look for patterns and relationships and to construct pictorial representations to show these relationships without truly understanding the underlying nature of the relationship. Students may even build a symbolic generalization without being able to give a valid mathematical justification. The deep understanding is not there. For the students, the challenge of the activity is finding the solution, not finding understanding. The research showed that the students lacked skills to work through abstractions and apply generalizations. They did not have a solid, consistent support system that taught them understanding. The students were expected to automatically realize the why for processes and solutions without any formal guidance. Students were uncertain as to what to do because they did not have the knowledge base needed to deal with the problem.

Similar findings appeared when American students were compared to Asian students in relationship to mathematical competence and mathematical instruction. One factor that appeared to contribute to the difference in the performances of the students was the instructional material. Textbooks used in the Asian schools in the study linked multiple representations, such as verbal, pictorial, and symbolic, along with explanations and justifications. This was not found in America. Curriculum and textbooks in the United States generally focus on computation and other lower level skills (Brenner, Herman, Ho & Zimmer, 1999; Millard, Oaks & Sanders, 2002).

In line with this is the concept of an ideal problem solving environment in which there is a presentation of tasks that feature the possibility of more than one solution strategy, more than one form of representation, and written and/or oral forms of communication to justify processes and solutions. Seeing a mathematical problem in a number of ways helps with understanding and the ability to make connections to other problems. As English (1997) suggests after his one-year study with fifth graders, additional recommendations include those of assigning tasks that relate to a student’s knowledge base, providing sufficient time for the student to work through a problem without premature intervention, and providing opportunities for students to interact. Interaction helps students reflect on their thinking, increase their knowledge of concepts, and improve their justification ability. The hope is that through interaction, students will internalize their learning and become independent thinkers. A classroom environment, which is positive and conducive to problem solving, is needed for effective teaching and learning (English, 1997; Millard, Oakes & Sanders, 2002; Stein, Grover & Henningsen, 1996).

The research of Bottge, Heinrichs, Chan, and Serlin (2001), with eighth grade students in a remedial math class and those in a pre-algebra class, show that students lack the necessary strategies and skills needed for becoming effective problem solvers. Teaching strategies, along with allowing time for active participation in problem solving and having a good support system, allowed remedial math students to become successful problem solvers. Basic core skills are also deemed necessary, both for use with problem solving and for the prevention of a decline in computational skills. Research by Millard, Oaks, and Sanders (2002) shows evidence that students who are consistently exposed to the problem solving steps and strategies such as guess and check, representations, finding patterns, and working backwards, are able to demonstrate their abilities to use strategies and develop and justify solutions (Bottge, Heinrichs, Chan & Serlin, 2001; McKendree, Small, Stenning & Conlon, 2002; Millard, Oakes & Sanders, 2002).

Much research prior to the year 2000 does not focus on the connection between knowledge of core skills and problem solving, nor does it reflect the need for communications in terms of group work or justifications of thoughts and solutions However, recent studies take the approach that teachers have to provide opportunities for all students to learn core skills and a wide variety of strategies as they experience more types of problems involving computation and reasoning. As part of today's society, students need to learn how to share and justify their ideas. They need to become effective problem solvers. Even though problem solving is the basis of my research, this project differs from the published literature in a number of ways. This study involves fifth grade students in a mathematics enrichment class while the students in most of the literature were predominantly upper middle school or high school students. Of the studies cited, only English (1997) focused on fifth graders. Some studies involved remedial students or students from other countries. Also, while the published research focused on problem solving and some more specifically on representations, the purpose of this study differs from the research of others. Elia and Philippou (2004) studied the different functions of representations. Researchers, such as Noss, Healy, and Hoyles (1997) and Bottge, Heinrichs, Chan, and Serlin (2001) concentrated on the use of technological representations within a math program. Little focus has been placed on the combination of group work and representations in a math curriculum. The intention of this study is to see if teaching the properties and uses of a variety of strategies, with focus on pictorial representations, will assist students in becoming better problem solvers. With an increase of group problem solving opportunities focusing on strategies and representations, this study will hopefully provide insight as to the benefits of pictorial representations and group work on the ability of students to solve problems (Bottge, Heinrichs, Chan & Serlin, 2001; Elia & Philippou, 2004; English, 1997; Noss, Healy & Hoyles, 1997).

Purpose Statement

The purpose of this study is to evaluate how increased problem solving opportunities with the use of graphics and varied strategies affect the students' abilities to solve problems. I would like my students to develop a sense of comfort and confidence when approaching a problem and to become active participants in their learning. I would like my students to develop strategies for solving problems through use of already attained skills and to see connections among the skills and concepts when solving problems. I would like my students to make use of graphics to help them organize information, solve problems, and represent solutions. I would like my students to become better problem solvers.

My experience as a learner and as a teacher has shown me that the use of drawings is beneficial to the organization and understanding of a problem. This understanding can then help with the selection of an appropriate strategy that will lead to a solution. Drawings are also helpful as representations of solutions. My experience has also taught me that if an individual recognizes the relationships among skills and concepts, he/she is better able to transfer those skills to new problem solving situations. This, in turn, builds the comfort level and confidence. With the increased confidence students are more willing to become active participants in their learning. Through their sharing and their increased level of awareness, they begin to recognize more relationships among ideas. The cycle then continues, and new problem solving situations are willingly approached with purpose.

This study will attempt to answer these research questions:

- How do problem solving strategies influence students' capacities to apply graphic representations?
- Conversely, how do students' efforts at applying graphic representations have on their capabilities to solve problems?
- How do group problem solving opportunities shape students' capabilities to solve problems?

Methods

I increased the opportunities for problem solving by having short sessions once or twice a week plus a full class period of problem solving once a week. This was in addition to the story problems that are regularly part of an assignment. I taught problem solving strategies, such as looking for patterns, working problems backwards, guess and check, and using graphics. I stressed the use of drawings at all stages of problem solving and gave students the opportunity to work in small groups and be involved in the sharing of ideas. I also asked students to make class presentations, detailing the contributions they made to their group. I provided opportunities for students to journal about their experiences, thoughts, and questions regarding the problem solving sessions, by using free writes and journal prompts (see Appendix A).

My action research project began on February 9, 2007, with a colleague introducing the study and distributing the cover letter, the Parental Informed Consent Form, and the Child Assent Form to each student. She asked the students to return the forms to her by February 14, 2007.

After this, due to a number of unforeseen circumstances, my intended instruction and data collection did not coincide with the actual outcome. I had intended to have one class period each week devoted to the instruction of problem solving strategies with an additional two sessions for practice and evaluation of the strategies taught to that point in time. During the original nine weeks of the study, there were many two and three day weeks with a total of 18 days that I did not have math class. March and the first part of April posed the most problems with continuity and time for math instruction and evaluation. Thirteen of the missed math sessions were during the five and one-half weeks from March 1 through the first week in April. The many causes for missed days included a snowstorm, a state basketball tournament, spring break, parent/teacher conferences, staff development days, evacuation of the school, Terra Nova standardized tests, Easter, and a number of other scheduling conflicts.

Due to the number of missed classes, I decided I needed to extend the study beyond April 15. This was even difficult and somewhat ineffective because of additional scheduling conflicts. The first pieces of data were collected as planned, however, because of being responsible for other mathematical instruction to meet the requirements of the school's curriculum and because of working with the disjointed research plan, most data collection did not follow the original schedule.

I administered a survey to collect the initial data on the students' attitudes toward problem solving and working in groups, as well as their self-interpretation of their problem solving skills. This was a ten-question survey with a 5-point rating system, 1 being strongly agree and 5 being strongly disagree (See Appendix B). On February 12, 2007, I administered the survey to all 17 students. I had students complete this same survey again at the end of the study on May 1, 2007. This was approximately two weeks after the targeted date for the completion of the research.

The second source of data collection came from a problem solving worksheet I assigned on February 12, 2007. The three problems I chose to analyze could be solved using different forms of representations or using only computation. My intent with the analysis of these three problems was to see what approaches the students used to solve the problems. I used a rubric to score these problems on problem interpretation, use of strategies, and computation (See Appendix C). These problems gave me a baseline on which to compare student work following this initial assignment. It also gave me an insight as to the use of the rubric. After this worksheet I distributed a listing of the problem solving steps (See Appendix D) and began my instruction in the use of varied strategies in problem solving. The intended strategies included the use of a variety of representations (drawings, tables, graphs and diagrams), the use of patterns, working backwards, guess and check, and working a simpler problem. Because of the scheduling conflicts, I was unable to cover all the intended strategies. During the time of the research, students were also given the opportunity to work in groups.

A third data collection method was taking rubric scores on seven daily problem solving assignments. This included the initial assignment of February 12, 2007. I collected assignments approximately every one to two weeks. The number of days in which no math classes were held contributed to the inconsistency with the collection. I collected and analyzed the initial assignment in February. Students submitted three assignments in March (March 6, 13, and 26) and three in April (April 10, 18, and 25) for analysis. Differences in the scores seemed to relate to the strategy being emphasized and the difficulty of the problem. The amount of time between instruction and assessment also appeared to influence the scores.

Another form of data was the work prepared and presented by groups of students. Even though students engaged weekly in group work, I used only three separate group assignments as part of the collection. These three collections took place on February 22, March 29, and April 27. Students completed the majority of the work on the group assignments in class. I evaluated the group work using both the rubric and teacher observations. This data collection aimed to discover how group problem solving opportunities influence the students' capabilities to solve problems.

My study included data collection techniques such as teacher observations and journal entries aimed at answering a number of prompts (see Appendix E). These focused on all three research questions related to instruction in problem solving strategies, use of representations, and outcome of group work. Because of the number of days in which math classes were not held, I did not write detailed journal entries on student performance every week. Some entries centered on the lack of continuity for instruction and assessment rather than student performance. During the majority of the study, I made journal entries approximately once every two weeks. Students were also asked to write in their journals three times throughout the study, once in March, one time in April, and then again in May during the last week of the project. It was interesting to see how student attitudes changed from one entry date to another.

Findings

I started my research project by administering an attitudinal survey to all 17 students in my math enrichment class (see Appendix B). Even though the initial survey did reveal some interesting ideas of the students, the true value of the survey came with the comparison made to the survey given at the end of the research project. This comparison will be noted later in this paper. However, my first research question, which asked, “How do problem solving strategies influence students’ capabilities to apply graphic representations?” immediately became of particular interest with regard to the responses for one survey item.

Many students at the fifth grade level rely on computation for problem solving. This is what they are familiar with so it has become their comfort zone. It is their strategy of choice. Only with time and through formal instruction do students know and become capable of using a variety of strategies needed to solve problems. It is only through the practice of these strategies that students become comfortable and successful in their application. Without time and persistence, many students revert back to their old habits of relying on computation alone. The analysis of the students’ work throughout this study provided evidence of this reverting back to the use of computation. On the very first day, upon collecting the survey, I gave the students a worksheet they were to do on their own. I did not give any formal instruction in regard to strategies or to the concepts covered on the worksheet. Students had had similar problems in the past. Knowing how the students had approached such assignments previously throughout the year I anticipated similar work from the students, that is the reliance on computation and a lack of representations or the use of other strategies to solve problems. With 17 students completing the full sheet of 11 problems are a total of 187 problems. Findings reveal that of these, approximately 25.7 percent show evidence of the use of some form of representation. It should be noted that the person using the most representations to solve the problems also has the best grade.

This is where the one survey question becomes relevant. Number ten on the survey states, “I am able to use more than one strategy to solve most problems.” Results of the survey show that 11 of the 17 students indicated that they agreed or strongly agreed with this statement. Another four indicated a neutral rating, and two disagreed. Since the students rely so heavily on the computation for problem solving, I feel that perhaps they were interpreting the use of different strategies as the use of different forms of computation.

With the lack of representations on the previous day’s work I asked the students if they felt there was ever a need to use some form of representation (drawing, graph, or table) to solve a problem. The consensus of the group was that it wouldn’t be necessary. They expressed the thought that problems could always be solved using basic computational skills. I then began my instruction on the use of representations, specifically drawings, with the following problem, which I wrote on the board. “A fifth grade student left his home and walked 4 blocks east, 2 blocks south, 7 blocks west, 5 blocks north, and 3 blocks west. How far was the student from home after this walk?” Immediately the students arrived at their solutions. I asked each student individually for the answer and wrote each response on the board. Everyone responded that the student was 21 blocks from home. I then randomly chose five students and asked them how they arrived at the answer. They all remarked that all a person had to do was add the numbers to get the answer of 21 blocks. I then asked the students to raise their hands if they were confident they had the right answer. A few were hesitant, but most students’ hands shot up immediately. As noted in my journal, the first three students to raise their hands, showing confidence in their answers, are the three who generally rely most heavily on computation for problem solving. I then drew on the board a diagram resembling streets and city blocks. Together we counted the blocks as we figuratively walked the path. The students were surprised that the actual distance from home was only 9 blocks. Even though the students admitted that at times it was not only helpful but necessary to use a type of representation instead of computation to solve problems, convincing them to use such methods was a slow process.

After teaching the use of drawings and diagrams to assist in solving and reporting the solutions to problems, I then focused instruction on the use of other forms of representations. These included tables, frequency charts, line plots, and bar graphs. A state assessment on the use of tables followed group and individualized practice.

Even though students had some difficulty with the development of bar graphs, journal entries reveal mixed feelings about different strategies and preferred representations, including bar graphs. Three of the eight students who agreed to allow me to use their work as examples listed graphs as a favorite. Kyle[1], stated, “My favorite problem solving strategy is using graphs because . . . I understand them better than all of the other ones.” Jenny chose two preferred forms of representations with her comment, “I feel most comfortable drawing a picture or using a graph because it gives you a visual of what you are working with.” Some students simply preferred pictures as noted with Whitney’s journal entry. She wrote, “I like to use pictures . . . because if I lose my place I know automatically where I was, and it helps me organize my work.”

During the third week of my action research I reviewed previously taught strategies and I introduced the use of a Venn diagram as another representation that could assist a person in

[1] All names are pseudonyms.

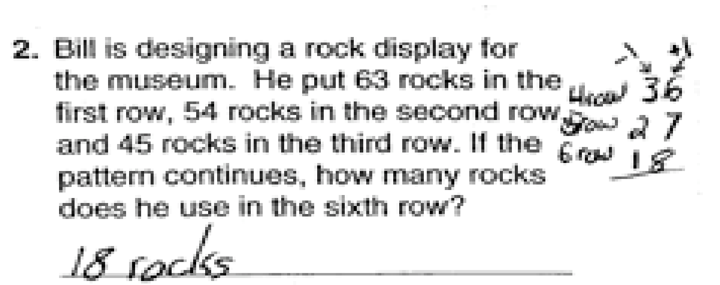
solving certain types of problems. I explained to the students that Venn diagrams are used to denote similarities and differences between items. I also showed the students there are times when the circles represent the characteristics and then items with one or both of the characteristics are listed in their appropriate sections. We went through a few simple, non-numerical examples and then transferred to problems with numerical values. Since the students had used Venn diagrams in the fourth grade, they were familiar with the basic form of two intersecting circles, and had little difficulty. When I added a third circle to the diagram the students quickly adjusted to the new form and easily worked examples, using the new Venn diagram with three intersecting circles. I informed the students that even though circles are most often used as the basic form of a Venn diagram, other shapes could also be used as long as they intersect.

I had the students practice for the remainder of the class period as well as on the following day. They were also assigned two story problems. Even though the one problem was ideal for the use of a Venn diagram and the second was perfect for the use of a table, only one student of seventeen used both of these representations. Three others used drawings (representing the items listed in the problem) on at least one of the problems. The remaining thirteen students either listed the answers without showing any work, or they solved the problems using computation. As we discussed the problems the following day, only one additional student mentioned she had thought of using a Venn diagram, and no one else had considered using a table to organize and solve the second problem. For most there was no application of skills related to the strategies that had been taught.

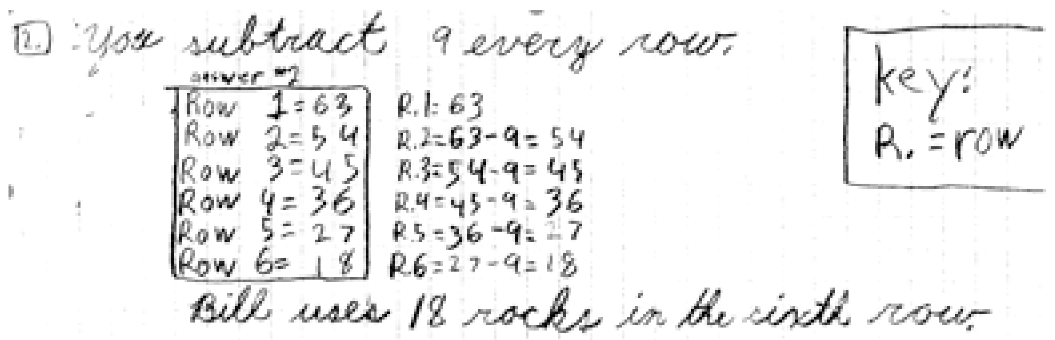
Once I had finished teaching the use of the various representations, I turned my attention back to the use of tables. I showed students that often, by keeping a running total while developing a table, the question within the problem is automatically answered. Showing students this process led right into the strategies of working backwards and finding and using patterns. Other than for solving problems related to elapsed time, most students did not care for working backwards. As Whitney stated, “I would say it is still difficult to work my problems backwards, mainly because it just confuses me.”

It was different with finding patterns. Students understood the basic idea behind finding patterns as we had worked with patterns throughout the year. However, most of the work previously done with patterns dealt with either visual/graphic patterns or patterns related more directly to reasoning skills. During the study we did spend some time on these types of patterns. However, at this point in the research, instruction centered on finding numerical patterns in order to write an equation or to decide the computational steps needed to solve the problem. I had the students use tables of information to compare numbers, seeing if there was a common difference, common factor, or some type of multiplication or division pattern. Once this was discovered I then showed them how to use this knowledge and the information given in the problem to find the required answer. By observing the students as they worked and by checking their assignments, it is quite obvious that most can easily determine the pattern or patterns for the problems and find the solutions to the problems. However, not all students find the same pattern within a problem. This fact helped me re-emphasize that often times there is not just one approach to a solution. Following are two such examples from students. The problem with the work from Kyle is shown first. Jon’s approach is shown second.

Kyle’s approach:



Jon’s approach:



I find the work of these two individuals quite interesting, as it is Kyle who at the beginning of the research project relied heavily on computation, and yet he is not one who used computation in this problem. He does, however, always display a sense of self-confidence. Jon is one who usually sees the more visual patterns easily; however, he does not show much confidence in his math ability. Perhaps in this problem the use of computation provided the confidence Jon needed and Kyle was confident enough to try a different approach.

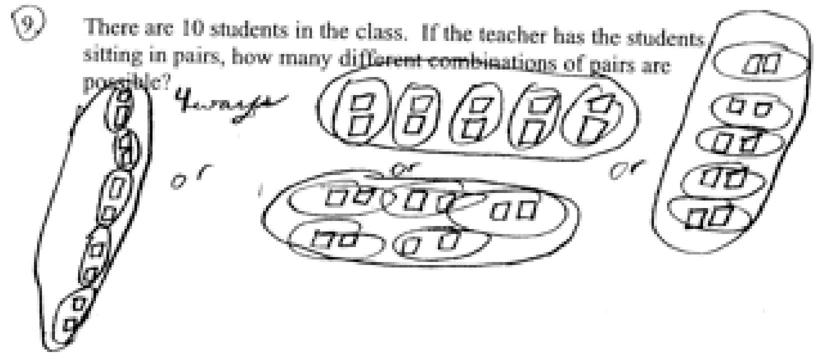
Because of the time constraints, we only spent part of two class periods with the concept of guess and check. Journal entries indicate that some students are fond of the guess and check method, while others see it as being too time consuming. Similar comments were made in the classroom. Time did not allow for instruction in the use of the strategy to work a simpler problem.

To complete my research I decided to spend two class periods giving the students a number of problems they had been assigned throughout the first part of this study. I wanted to see how their approaches to the problems had changed. I wanted to see if there would be a decrease in the use of computation and an increase in the use of other strategies. I reviewed problems the students had worked for practice and those worked as part of an assignment. I considered problems from each strategy and representation taught. I also looked for problems for which a variety of strategies could be used. One idea I hope I have left with the students is that there is more than one way to solve many problems. The selection of six problems previously used became the basis of the final assignment of my action research.

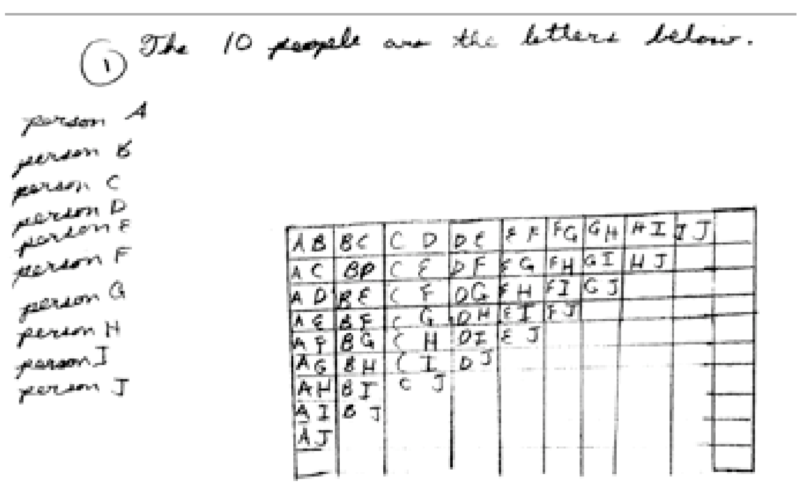
Most students had originally used computation when they solved the problems earlier in the study. This occurred even when a lesson on a certain strategy had just been covered in class. On this final assignment a variety of strategies were used. Some of the students who had relied so heavily on computation and mental math in the past finally broke through the barriers of their comfort zone. My data indicate that through constant review of problem solving strategies and representations previously taught, along with a minimum of two to three story problem sessions or assignments a week, students learn to use a variety of strategies to solve problems.

As many students also had difficulty with the interpretation of certain problems when first assigned, another curiosity arose during the study. Would a greater understanding of strategies assist in the interpretation of problems? For some students, such as Whitney, it appears it has. However, there could also be many other factors that contributed to a better understanding of the problem and the use of strategies in solving the problems. Below are Whitney's original and final solutions for the problem: There are 10 students in the class. If the teacher has the students sitting in pairs, how many different combinations of pairs are possible?

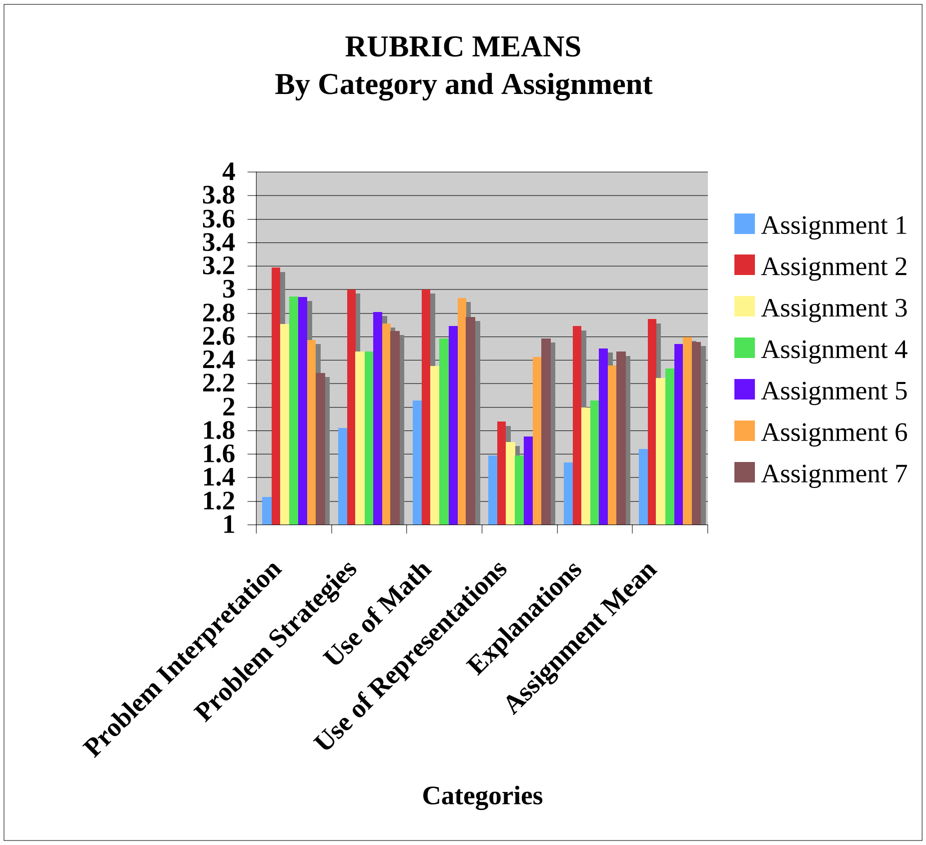
Whitney's solution on February 12, 2007:



Whitney's solution on April 25, 2007:



Journal entries, along with changes in rubric scores from the beginning of the action research to the end, indicate that the majority of students are now capable of and prefer using a strategy other than just computation in problem solving situations. Bethany said it simply by stating, "I only like story problems that you can draw pictures for." Some students find the use of representations inviting because of their discomfort with mental math. In her last entry Whitney writes, "I like to use representations to solve problems because I do not like mental math, also because I can find it quite difficult to take in so much information at once." Jon confirms that sentiment with the statement, "In this project I came to like problem solving way better than I did before. My favorite way to solve problems is drawing graphs and pictures because it helps me see it better. I cannot do mental math because it gets too confusing. I like to use representations." Keagan went from having computation as his top choice for problem solving to patterns and representations. In his first entry Keagan wrote, "Usually I like to use computation or patterns. I use guess and check too." By the end of the study he decided, "I like to use patterns in some problems. I also love to use pictures and graphs because I like to draw and it makes it easier." He continues by commenting, "At the beginning of the school year I hated to work in groups and do problem solving. And if I did do problem solving, I just tried to use adding and subtracting. Now I use patterns, graphs, pictures, or anything else I can think of." As noted with these comments and the rubric analysis of student work (see the following page and Appendices F and G), the teaching and continue review of strategies give students the tools needed to tackle, with confidence, a wide variety of problem solving situations. However, one must be cautious with the results because of the limited time of this study. Results of my study show a slight growth in the use of strategies and in the use of representations, but this growth could be due to the type of problem assigned or to the timing of the assignment. Also, one might ask what would happen if a longer period of time was devoted to the research. Would more growth be evident or would it level off and possibly even decline? After time, would students return to their previous methods of problem solving?



Because of a short 3-day school week for two consecutive weeks, Venn diagrams and other representations were reviewed before students received another assignment used as part of my research. This worksheet reflected my assertion regarding the need for time and persistence if students are going to use a variety of strategies instead of just returning to the use of computation. However, it also relates to my second research question, "…how do students' efforts at applying graphic representations have on their capabilities to solve problems?" The more familiar and comfortable the students become with the use of graphic representations, the more they see graphics as a tool to organize data and assist them with problem solving. As a result students have clearer pictures of the problems, better understanding of the problems, and more accurate solutions.

I gave the students the worksheet “Problem Solving Strategy: Draw a Picture/Diagram” on March 13, 2007, (Math in My World, 1998). I directed students to solve problems 1, 2, and 3 by using the strategy of drawing a picture/diagram (a representation). The instructions for problems 4 through 7 stated that any method could be used to solve each problem. For these problems, many students reverted back to using computation. The following table shows the results of the work of the eight students who agreed to be part of the research project. Even though the samples are limited in terms of the number of students and the number of assignments represented, the results of the worksheet suggest that the use of graphics (representations) can help students solve problems.

“Problem Solving Strategy: Draw a Picture/Diagram” Results

Student	Number Wrong Problems 1-3	Representation Used? Yes/No	Number Wrong Problems 4-7	*Representation Used? Yes/No
1	0	Yes	1	No
2	0	Yes	3	No
3	0	Yes	1.5	No
4	2	Yes	2.5	No
5	0	Yes	1	No
6	1	Yes	2.5	No
7	1	Yes	2.5	No
8	0	Yes	3	No
Totals	4 of 24		17 of 32	
% Incorrect	16.666%		53.125%	

*For problems 4-7, all eight students used strictly computation.

The next day I shared these statistics with the class. Students could not believe that they had a larger percentage of solutions incorrect when using computation than when using a type of representation. My observations of student work and student attitudes the remaining weeks of this project seem to indicate that this revelation became the turning point for many of the students. Even though students still used computation, there was an increase in the use of other strategies.

I included in my study the question as to how group problem solving opportunities shape students' capabilities to solve problems. I thought perhaps using the approach of group work would encourage students to use representations and a variety of other strategies when solving problems. Each time I introduced a new strategy I had the students work in groups to practice the new strategy and discuss its use as related to the problems. I had intended to assign a graded group problem every two to three weeks, after a number of strategies had been introduced. By doing this, the students could discuss within their groups the strategies they thought would best fit the problems. Because of the schedule conflicts and missed math classes, I was unable to follow through on this plan. I still gave students the opportunity to practice new skills in groups; however, I collected only three group projects.

The first group project came on February 22, 2007, after the introduction of pictures, tables, and graphs. I asked the groups to find the total number of each kind of vehicle the students counted in two days (see Appendix H). I had divided my class of 17 into 5 groups. There were a wide variety of results of the representations used. One group made a two-column counting chart in which each type of vehicle was represented with a different geometric shape. Another group designed a chart by dividing it into three main sections, a section for each type of vehicle. They drew circles for each vehicle counted on Monday, but then for Tuesday used computation. One group created a pictograph, and two groups made bar graphs. Each bar graph used a color-coded system, one using different colors to represent the days, the other using different colors to represent the type of vehicle. I stated in my journal, "The students within each group shared many ideas regarding the types of strategies that could be used to solve this problem" (Teacher Journal, February 23, 2007). All groups had mentioned computation, but steered away from that approach. Once a strategy had been selected, discussion then centered on how to present the information within their chosen form of representation. It was a very productive session. It appears that at the beginning of this study, students were more apt to use representations when working in groups than when working on their own. Students seldom relied on computation alone within group sessions.

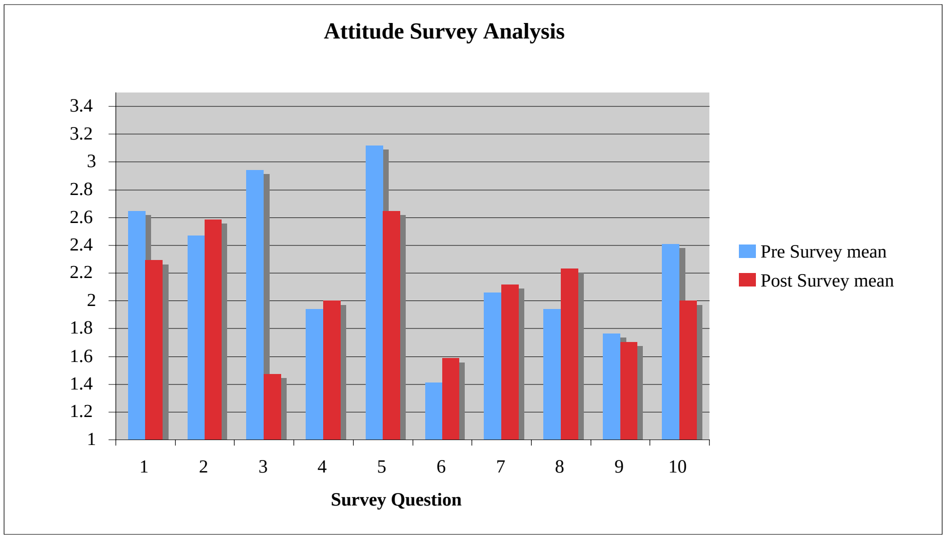
The second collected group project, which was given on March 29, 2007, consisted of finding what day of the week it would be related to a specified number of days in the future. Students could easily use computation to solve this problem. Two groups did use computation, three groups drew out calendars, and two groups made a listing of days.

I collected a third group assignment which related to the Fibonacci sequence. Some students recognized the pattern immediately, and their groups finished the project without much effort. Other groups struggled at finding the pattern, even though they had previously studied the Fibonacci sequence. Many of these students became frustrated with the situation and with working within a group.

My observations indicate that when first introducing the idea of using various strategies to solve story problems, group work is extremely important. The students have each other for support. Group work is also important immediately after the introduction of each strategy. However, it appears that as time passes, working in groups actually is a detriment to moving forward. The students' focus on the strategies and the problems decreases as does their relationship with each other. Observations and students' comments suggest that toward the end of the research students experienced more disagreement with each other, less cooperation, and a general dissatisfaction with the finished product. Students moved at a slower pace, each having different ideas and being less willing to come up with a unified approach. They also became less concerned with accuracy. Students began to take more care in the use of different strategies and different representations when working on their own than when working in a small group setting. The use of different strategies and an improvement in accuracy increased when students worked individually.

During the final stage of group work, students were verbalizing to me that they would prefer less time spent within the groups. They wanted to pursue the actual working of the problem on their own. Their journal entries, however, indicate that they liked working in groups for the sharing of ideas. One student remarked, "If I work in a group I can see other people's point of view and I can learn different observations and maybe find another way to solve a problem.' Another student wrote, "Working in groups … helps me learn what people think, and if I learn new strategies I may work better." The final entry written by Jon states, "Working in groups and learning different strategies has affected my ability to solve problems by helping me see more ways to solve the problems." After observing the students and their group work, reading their journal entries, and talking with them, I have come to the conclusion that group problem solving sessions are beneficial if they are used in moderation. Group work should be used, but closely monitored for optimal success.

A comparison study of the results of the initial survey and the second completion of the survey at the end of the study on May 1, 2007, shows a slight change in the attitude toward working in groups. However, responses to question 3 regarding the usefulness of drawing to solve math problems, question 5 about the use of patterns, and question 10 focusing on the use of more than one strategy, changed most significantly. It is only through concentrated instruction in problem solving strategies and an influx of related activities that students become comfortable and successful in using a variety of strategies to solve problems. Students come to recognize, as they did after Assignment 3, that these strategies are beneficial tools. The following graph indicates the changes in attitudes. Appendix I presents more detailed information on individual questions.



*A lower mean indicates a stronger agreement with the statement.

To better understand the chart and graph, the items on the survey are listed below. The complete survey can be found as Appendix B.

1. I find it easy to solve story-type problems.
2. I find it easy to solve problems that I have to use my reasoning skills with.
3. Drawing representations (pictures, tables, graphs) helps me solve problems.
4. I am able to show and explain the work required to solve math problems.
5. I use patterns to help me solve math problems.
6. I like to work in a group during problem solving sessions.
7. I ask questions of others when I work in a group.
8. Working in a group helps me better understand concepts.
9. Problem solving becomes easier with more practice.

10. I am able to use more than one strategy to solve most problems.

Conclusion

Through my observations and other forms of data collection, I find that if given the time and the opportunity, my students will show a willingness to try new approaches to problem solving and a willingness to improve their skills as problem solvers. My research suggests that problem solving strategies must be taught in order for students to show growth as problem solvers. If problem solving were left up to my students, without any formal instruction or practice, they would depend on the use of computation. Initially students were reluctant to move from their comfort zone of computation to the use of other strategies. Though some of my students might have been familiar with a number of the strategies, they did not have the structure or background needed to implement them. Even with the same instruction and practice, my students did not react to the use of the strategies in the same way. Each student has his or her own preference. For some students this preference is based on their ability and desire to draw. These students find the use of representations to be less difficult than some of the other strategies. I did find that those who relied most on computation seem to prefer the use of patterns along with tables and graphs as their most used form of representation. Although each of my students has a personal preference in regard to the problem solving strategy and the representation, the study and practice of all the strategies taught in this study are helpful to all the students. The students agree that the strategy they use not only depends on their preference, but also on the problem.

When I asked students to write a journal entry detailing which parts of the action research study had the most positive impact on their ability as problem solvers, they suggested that the two main influences were group work and learning how to use representations as a tool for problem solving. Researchers seem to agree that the use of representations is an important strategy for problem solving and one that needs to be practiced and learned through formal instruction. They also suggest that representations may assist in the understanding of problems and concepts as well as assist in communication skills. Other researchers, such as Elia and Philippou (2004), along with Nunokawa (2004) and Neria and Amit (2004), also suggest the helpfulness of representations and the idea of three major uses of pictorial representations. The three uses include the use of a representation that contains needed information to solve the problem, the use of a representation in the process or organizing and solving the problem, and the use of a representation to communicate to others the solution of the problem. Results from my study reveal all three of these uses to be present in student work. However, not all of my students use representation in the same manner, and the reason for their use seems to change over time. Some students, especially those who relied heavily on computation at the beginning of the study, often use their computational skills to solve the problem and then use representations as a way of communicating their results. Other students see representations as tools for organizing information and thoughts. Once the information is organized, they feel they can better understand the problem and proceed in finding the solution. As time progressed, more students were recognizing the usefulness of certain representations to help organize information to assist them in understanding the problem or to assist them in solving the problem.

By using a rubric to score student work I discovered that many of my students have difficulty with problem interpretation. They could often choose a strategy that was appropriate for solving the problem, but they missed important details in the problem, or they would give an erroneous answer. If I had not been using a rubric to focus on their process as well as their answers I would have possibly overlooked this difficulty. Being able to study their uses of strategies other than computation gives me insight to their thinking in terms of particular problems. I believe the use of the rubric and the student use of various strategies benefits me as a teacher in analyzing student performance. This in turn can benefit the students and improve their problem solving skills.

I see that the growth in problem solving skills is a slow process, and that time is a major factor in the development of such skills. The more time devoted to problem solving, the more competent the students become in the skill of solving problems. This action research was completed over a rather short period of time. Consistency and continuity of instruction was difficult due to scheduling conflicts. Even so, student scores show slight gains in problem solving capabilities. I would expect greater gains if more time could be devoted to the instruction and practice of each strategy and representation. This would require the strategies to be taught throughout the year. I suggest beginning the first part of the year with the instruction on using representations as these help students with the understanding of problems.

Other teachers who have students struggling with problem solving may find my research relevant and useful. For many students it shows both slight improvements in the use of strategies to solve problems as well as a shift of preferred methods from computation to representations. However, the short period of time and the low number of students involved in my action research does not provide enough information to make reliable assertions. More research must be completed on the teaching of problem solving strategies, the use of representations in problem solving, and the impact of group work on problem solving.

Implications

I will not be teaching an enrichment class this next year, however, I believe I can take my action research experience and adjust it where needed to be appropriate for my upcoming class. Even though I may not have been looking for it as part of my research, I did see growth in student desire to stick with a problem. This will be really important with some of the low achieving students I will have in class next year. Along with this idea I witnessed students involved in more discussion and planning prior to finding a solution. The students were thinking about the problem instead of just thinking about getting it finished so they could do something else. Group activities related to problem solving will be essential to help with understanding and to develop this productive form of communications. If these group sessions are truly going to be beneficial, I will need to relay my expectations for them and strictly monitor the sessions at the beginning of the school year.

Since there is evidence of slight growth in the use of problem solving strategies and a noted change in preference in terms of how to solve problems, I intend to continue with the instruction of problem solving strategies in my class. I will begin the school year with instruction in the use of representations as many students thought this strategy was the most helpful. As there are many forms of representations, I plan to spend the majority of the first quarter of school on this approach to problem solving. I will likely concentrate on the use of patterns toward the end of the first quarter of school and into the second quarter. This is because of how tables, a type of representation, are often used to help determine patterns. A third strategy I would like to introduce to the students is working a simpler problem. Even though this strategy was not used in my action research, I know it is an approach that is appropriate for fifth graders.

As I go through the different phases of problem solving instruction, I will need to help the students understand the problems and help them see that multiple approaches can be used to solve most story problems. Since my class will have students with varied abilities in problem solving, I may have to work more examples with the students to help with their understanding of the problems and the strategies.

I plan to spread the instruction in problem solving and the related activities through the entire school year. I believe it will make a difference if instruction and practice are not compressed into a short period of time as it was for the action research.

I will encourage my colleagues to include more instruction in problem solving strategies and related group work activities in their math classes. By using the results of this study and examples of student work, I will be able to demonstrate that students tend to show more growth in terms of understanding and interpreting problems, using appropriate strategies, and accuracy when given instruction in problem solving strategies along with being given fewer problems and more time to work with peers. I will stress that the time spent having students work more problems a day is better utilized by having them work in groups and discuss possible approaches to problems. Without a good understanding of problems and strategies, the number of problems assigned to students might actually be a detriment to their learning. The need for a balanced approach will be noted. I will also point out to colleagues that when having students use representations and other strategies rather than strictly computation to solve problems, a teacher receives insight as to the student's thinking. This helps narrow the focus of the difficulties students are having with the work. In turn, re-teaching can center more directly on student needs, and less time is spent re-teaching material students already understand. It would be my hope that once other teachers see the benefits of both group work and the instruction of strategies, that a ripple effect will occur and more emphasis will be placed on a balance curriculum, including more problem solving opportunities.

It is also my desire to start a problem solving club of fifth grade students that would meet after school once a month. Hopefully such a club would ignite curiosity and interest in mathematics and problem solving among both students and teachers. With time, such a club could grow to include students and teachers at other grade levels, as well as community personnel. However, the best result of such a club would be an increase in student interest in and understanding of mathematics.

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Appendix A

Prompts - Student Journals

1. Which problem solving strategy do you feel most comfortable using? Why?

2. Which problem solving strategy do you still find difficult to understand and use?

3. Which representation do you find most helpful when working story problems? Why? For what reason (to organize, to solve, to report) do you use it?

4. Do you prefer using representations to solve problems or solve problems without them? Explain your answer.

5. Explain how working in groups and learning about different strategies has affected your ability to solve problems.

Appendix B

Math and Cooperative Group Survey

Please give your honest response to each statement.

	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
1. I find it easy to solve story-type problems.	1	2	3	4	5
2. I find it easy to solve problems that I have to use my reasoning skills with.	1	2	3	4	5
3. Drawing representations (pictures, tables, graphs) helps me solve problems.	1	2	3	4	5
4. I am able to show and explain the work required to solve math problems.	1	2	3	4	5
5. I use patterns to help me solve math problems.	1	2	3	4	5
6. I like to work in a group during problem solving sessions.	1	2	3	4	5
7. I ask questions of others when I work in a group.	1	2	3	4	5
8. Working in a group helps me better understand concepts.	1	2	3	4	5
9. Problem solving becomes easier with more practice.	1	2	3	4	5
10. I am able to use more than one strategy to solve most problems.	1	2	3	4	5

Appendix C

Rubric-Mathematical Problem Solving & Representations

Problem Interpretation and Understanding Points ___ / 4	Misunderstanding of problem; little or no understanding 1	Limited but basic understanding of problem; some flaws in interpretation 2	Generally correct interpretation and explanation; appropriate response 3	Well analyzed and explained interpretation; clear understanding 4
Problem Solving Points ___ / 4	Inappropriate strategy, process, or solution 1	Incomplete or limited in application; not sure of process; logic is flawed 2	Appropriate process and application of strategies; shows analytical thinking 3	Well analyzed and explained interpretation; clear understanding 4
Use of Math Points ___ / 4	Inadequate use of math skills 1	Limited but adequate use of math skills; leaves out some important details 2	Competent and appropriate use of math skills 3	Complete competence in use of math; connections of mathematical ideas evident. 4
Use of Representations Points ___ / 4	No use or inappropriate use of representations; representation gives no relevant information 1	Use of representation that gives limited information about the problem 2	Use of representation that clearly organizes information, explains process, or portrays solution 3	Use of more than one representation, for multiple purposes; value and precision evident 4
Explanations and Justifications Points ___ / 4	Cannot explain why information is important or why certain strategies were used 1	Limited explanation; can explain why information <u>or</u> strategies were important, but not both 2	Appropriate process and application of strategies; shows analytical thinking 3	Well analyzed and explained interpretation; clear understanding 4
Total Grade Points **_____ / 20**	A: 20 pts.=100%; 19 pts.=96% (no 1) B: 18-16 pts.=90%; 15-14 =86% (no 1) C: 13-12 pts.=80%; 11-10=76% (no 1) D: 9-8 pts.=70% NC: 7-6 pts.=65%; 5=50%; 4-0=0%		Name ____________________ Pg./Ws._______ Date_______ By Carol Brown	

Appendix D

PROBLEM SOLVING

THINK

THINK ABOUT THE PROBLEM.
WHAT INFORMATION IS GIVEN?
WHAT IS THE QUESTION OR PROBLEM?

CHOOSE

STUDY THE PROBLEM.
THINK OF POSSIBLE OPERATIONS AND STRATEGIES.
PICK A STRATEGY.

CARRY OUT PLAN

CAREFULLY WORK THROUGH THE PROBLEM.

CHECK

DOES YOUR SOLUTION/ANSWER MAKE SENSE?
USE ANOTHER STRATEGY TO CHECK.
REMEMBER TO LABEL.

Appendix E

Questions/Prompts - Teacher's Journal

Prior to research study

Which strategies and representations will I choose to teach?

How much time will be spent on the teaching and practicing of each strategy and representation?

How will I select problems for practice and problems for evaluation purposes?

Question 1 - (teaching of strategies)

Which strategies and representations are most often used by students? What was indicated as the reason?

What changes have I seen this week? What changes have I seen since the beginning of the study?

What went really well this week during instruction? What did not go well?

Question 2 - (use of representations)

Does the use of certain representations correlate with the time of instruction?

Are students more willing to use representations on their own to solve problems? Has the use of representations increased?

Are representations being used mostly for organizing the information, solving the problem, or reporting the solution?

Question 3 - (group problem solving opportunities)

What went really well this week related to the extra problem solving opportunities?

What surprised me this week regarding the group approach to problem solving?

What changes have I seen in the ability of students to solve problems? How is this change related to the extra practice within a group?

Appendix F

Rubric Means
By Rubric Category and Assignment

	1 (2/12/07)	2 (3/6/07)	3 (3/13/07)	4 (3/26/07)	5 (4/10/07)	6 (4/18/07)	7 (4/25/07)
Problem Interpretation	1.235	3.188	2.706	2.941	2.938	2.571	2.294
Problem Solving-Strategies	1.824	3.000	2.471	2.471	2.813	2.714	2.647
Use of Math	2.059	3.000	2.353	2.588	2.688	2.929	2.765
Use of Representations	1.588	1.875	1.706	1.588	1.750	2.429	2.588
Explanations-Justifications	1.529	2.688	2.000	2.059	2.500	2.357	2.471
Overall Mean Per Assignment	1.647	2.750	2.247	2.329	2.538	2.600	2.553

Appendix G

Rubric Analysis of Assignments

Assignment 1

	1	2	3	4	Point Value	Mean
Problem Interpretation	14	2	1	0	21	1.235
Problem Solving - Strategies	4	12	1	0	31	1.824
Use of Math	1	15	0	1	35	2.059
Use of Representations	8	8	1	0	27	1.588
Explanations and Justifications	9	7	1	0	26	1.529
Number of Scores	36	44	4	1	85	**Overall Mean**
Total Point Value	36	88	12	4	140	**1.647**

Assignment 2

	1	2	3	4	Point Value	Mean
Problem Interpretation	0	2	9	5	51	3.188
Problem Solving - Strategies	0	6	4	6	48	3.000
Use of Math	0	7	2	7	48	3.000
Use of Representations	6	6	4	0	30	1.875
Explanations and Justifications	2	4	7	3	43	2.688
Number of Scores	8	25	26	21	*80	**Overall Mean**
Total Point Value	8	50	78	84	220	**2.750**

*One student absent.

Assignment 3

	1	2	3	4	Point Value	Mean
Problem Interpretation	0	5	12	0	46	2.706
Problem Solving - Strategies	0	9	8	0	42	2.471
Use of Math	0	11	6	0	40	2.353
Use of Representations	5	12	0	0	29	1.706
Explanations and Justifications	0	17	0	0	34	2.000
Number of Scores	5	54	26	0	85	**Overall Mean**
Total Point Value	5	108	78	0	191	**2.247**

Appendix G

Rubric Analysis of Assignments

Assignment 4

	1	2	3	4	Point Value	Mean
Problem Interpretation	0	2	14	1	50	2.941
Problem Solving - Strategies	2	7	6	2	42	2.471
Use of Math	2	6	6	3	44	2.588
Use of Representations	10	4	3	0	27	1.588
Explanations and Justifications	4	9	3	1	35	2.059
Number of Scores	18	28	32	7	85	**Overall Mean**
Total Point Value	18	56	96	28	198	**2.329**

Assignment 5

	1	2	3	4	Point Value	Mean
Problem Interpretation	0	4	9	3	47	2.938
Problem Solving - Strategies	0	6	7	3	45	2.813
Use of Math	0	9	3	4	43	2.688
Use of Representations	8	5	2	1	28	1.750
Explanations and Justifications	0	10	4	2	40	2.500
Number of Scores	8	34	25	13	*80	**Overall Mean**
Total Point Value	8	68	75	52	203	**2.538**

*One student absent.

Assignment 6

	1	2	3	4	Point Value	Mean
Problem Interpretation	1	5	7	1	36	2.571
Problem Solving - Strategies	1	2	11	0	38	2.714
Use of Math	1	2	8	3	41	2.929
Use of Representations	3	2	9	0	34	2.429
Explanations and Justifications	1	7	6	0	33	2.357
Number of Scores	7	18	41	4	*70	**Overall Mean**
Total Point Value	7	36	123	16	182	**2.600**

*Three students absent.

Appendix G

Rubric Analysis of Assignments

Assignment 7

	1	2	3	4	Point Value	Mean
Problem Interpretation	2	8	7	0	39	2.294
Problem Solving - Strategies	1	7	6	3	45	2.647
Use of Math	0	8	5	4	47	2.765
Use of Representations	2	6	6	3	44	2.588
Explanations and Justifications	3	6	5	3	42	2.471
Number of Scores	8	35	29	13	85	**Overall Mean**
Total Point Value	8	70	87	52	217	**2.553**

Appendix H

"Using Inference to Find Totals" (Nessel and Newbold, 2003)

Vehicles

Mr. Wong's class decided to tally the taxis, buses, and trucks they could see
from the classroom window between 9 A.M. and 9:30 A.M. on two days.
They formed 3 groups to do the counting.

On Monday, Group 1 counted 24 taxis, Group 2 counted 7 buses, and
Group 3 counted 31 trucks. On Tuesday, Group 1 counted 2 more taxis than they
counted on Monday. Group 2 counted 1 fewer bus than on Monday, and
Group 3 counted 3 more trucks than on Monday.

How many of each kind of vehicle did the students count all together?

Alternate Question: How many vehicles did they count all together?

Appendix I

Math and Cooperative Group Attitude Survey Analysis

Question	Survey (Beg.) (End)	Strongly Agree (1)	Agree (2)	Neutral (3)	Disagree (4)	Strongly Disagree (5)		*Mean	Mode
1	B	1	6	9	0	1		2.647	3
	E	3	6	8	0	0		2.294	3
2	B	3	5	7	2	0		2.471	3
	E	2	5	8	2	0		2.589	3
3	B	4	0	8	3	2		2.941	3
	E	11	4	2	0	0		1.471	1
4	B	5	9	2	1	0		1.941	2
	E	7	5	3	2	0		2.000	1
5	B	2	4	5	2	4		3.118	3
	E	3	5	5	3	1		2.647	2 & 3
6	B	13	2	1	1	0		1.412	1
	E	12	2	2	0	1		1.588	1
7	B	4	8	5	0	0		2.059	2
	E	5	7	4	0	1		2.118	2
8	B	8	3	5	1	0		1.941	1
	E	4	8	3	1	1		2.235	2
9	B	8	5	4	0	0		1.765	1
	E	7	8	2	0	0		1.706	2
10	B	1	10	4	2	0		2.412	2
	E	4	10	2	1	0		2.000	2

*A lower mean indicates a stronger agreement with the statement.