1975

COLLISIONS

COLLISIONS

INTRODUCTION

If you have ever watched or played pool, football, baseball, soccer, hockey, or been involved in an automobile accident you have some idea about the results of a collision. We are interested in studying collisions for a variety of reasons. For example, you can determine the speed of a bullet by making use of the physics of the collision process. You can also estimate the speed of an automobile before the accident by knowing the physics of the collision process and a few other physical principles. Physicists use collisions to determine the properties of atomic and subatomic particles. Essentially, a particle accelerator is a device that provides a controlled collision process between subatomic particles so that, among other things, some of the properties of the target particle can be studied.

In addition the study of collisions is an example of the use of a fundamental physical tool, i.e., a conservation law. A conservation law implies that something remains the same, i.e., is conserved, as you have seen in a previous module, Conservation of Energy.

Conservation laws play an important role in physics. In the study of collisions in this module we are interested in one of the fundamental conservation laws, conservation of linear momentum. If the sum of the external forces is zero, then the linear momentum is conserved in the collision. This is fortunate since it provides a way around the analysis of the forces of interaction between two bodies as they collide, an otherwise formidable task. Thus the conservation-of-linear-momentum law allows one to analyze the effects of a collision without a detailed knowledge of the forces of interaction. One can deduce the converse also, as does the particle physicist in accelerator experiments, for example - some of the properties of the target particles may be deduced from the law of conservation of linear momentum and other laws of physics.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Solve mechanical problems involving conservative and nonconservative forces, by applying the conservation-of-total-energy concept (needed for Objective 2 of this module)	Conservation of Energy Module
*Use the concepts of impulse and linear momentum to solve mechanical problems (needed for Objective 2 of this module)	Impulse and Momentum Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Conservation of linear momentum - Define or state: (a) elastic collision, (b) inelastic collision, (c) perfectly or completely inelastic collision, and (d) the law of conservation of linear momentum.

2. Collisions - Solve problems involving collisions between two or more bodies and/or the splitting up of a body into two or more fragments.

GENERAL COMMENTS

The important concepts presented in this module are

Elastic collision: a collision in which kinetic energy is conserved.

Inelastic collision: a collision in which kinetic energy is not conserved. Note: Kinetic energy may be either gained or lost during a collision.

Perfectly inelastic collision: a collision in which the colliding objects stick together after the collision.

Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant. Or, during a collision, if the interaction impulsive force is very large in comparison to the sum of all external forces such as gravity, then it is a good approximation to say that linear momentum is conserved.

Remember: Momentum is a VECTOR quantity and must be treated as such.

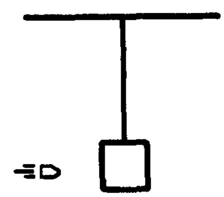
TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Sections 7.4, 7.5, and 9.3 through 9.6; work Problems 16 in Chapter 7, 15 in Chapter 9, and Problems A and B plus any two of the problems listed in the table below; answer Question 7 in Chapter 9. Also work the following problem:

A block of balsa wood whose mass is 0.60 kg is hung from a string of negligible weight. A bullet with a mass of 2.00 g and a muzzle velocity of 160 m/s is fired into this block at close range (horizontally) and becomes embedded in the block.

(a) Find the velocity of the block plus the bullet just after the collision.
(b) Calculate how high the block will rise.



When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

BUECHE

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	(work any two)
1(a)	General Comments, Sec. 9.3				
1(b)	General Comments, Sec. 9.3				
1(c)	General Comments				
1(d)	General Comments				
2	Secs. 7.4, 7.5, 9.3-9.6	A, B	A, B	Chap. 7, Prob. 16; Chap. 9, Prob. 15	Chap. 7, Quest.* 5, 7, 13, Probs. 4, 9-18; Chap. 9, Quest. 4, 7, 10, Probs. 10-17, 26, 28

*Quest. = Question(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 9, Sections 9-1 and 9-3 through 9-5; answer Question 6; work Problems 18, 22, 30, 40, plus Problems A and B.

Note: Definitions of elastic, inelastic, and completely inelastic collisions given in Section 9-4 apply to all collisions, not just to one-dimensional collisions.

When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text* (Chap. 9)	(Chap. 9)
1(a)	General Comments, Sec. 9-4				
1(b)	General Comments, Sec. 9-4				
1(c)	General Comments, Sec. 9-4				
1(d)	General Comments, Sec. 9-3				
2	Secs. 9-1, 9-3 to 9-5	A, B	A, B	Quest. 6, Probs. 18, 22, 30, 40	Quest. 1-8, Probs. 14-17, 21, 24, 28, 34, 37, 44

*Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 8, Sections 8-2 through 8-6; work Problems 8-6, 8-10, 8-20, 8-25, 8-37 plus Problems A and B. Note: Conservation of linear momentum can be used to a good approximation when the external forces are small compared to the interaction forces during the collision. For example: when a bat hits a ball, the interaction forces are large (generally) compared to gravity and the force exerted by the batter; therefore, in this case gravity and the force exerted by the batter can be neglected during the interaction.

When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems		Additional Problems
		Study Guide	Study Guide	Text	
1(a)	General Comments, Sec. 8-3				
1(b)	General Comments, Sec. 8-4				
1(c)	General Comments, Sec. 8-3				
1(d)	General Comments				
2	Secs. 8-2 to 8-6	A, B	A, B	8-6, 8-10, 8-20, 8-25, 8-37	8-5, 8-11, 8-12, 8-16, 8-23, 8-28, 8-29, 8-30, 8-35

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Sections 5-5 through 5-7 in Chapter 5 and Section 10-6 in Chapter 10. Work Problems 5-1, 5-11, 5-12, and 10-32 in the text plus Problems A and B.

Note: Even though the statement of the law of conservation of linear momentum was deduced for a particular two-body collision, it is valid in general. Conservation of linear momentum can be used to a good approximation when the external forces are small compared to the interaction forces during the interaction. For example: when a bat hits a ball, the interaction forces are large (generally) compared to gravity and the force exerted by the batter; therefore in this case gravity and the force exerted by the batter can be neglected during the interaction.

Your text makes a distinction between types of inelastic collisions that is not generally made, i.e., $\Delta K < 0$ inelastic and $\Delta K > 0$ explosive, where ΔK is the change in kinetic energy during the collision. Generally $\Delta K \neq 0$ is classified as an inelastic collision, as is done in the General Comments.

When you think that you know the material well enough to satisfy the objectives, take the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions: Study Guide	Assigned Problems: Study Guide	Assigned Problems: Text	Additional Problems
1(a)	General Comments, Sec. 10-6				
1(b)	General Comments, Sec. 10-6				
1(c)	General Comments, Sec. 10-6				
1(d)	General Comments				
2	Secs. 5-5 to 5-7, 10-6	A, B	A, B	5-1, 5-11, 5-12, 10-32	5-1, 5-2, 5-5 to 5-12, 5-15 to 5-21, 10-1, 10-6, 10-8, 10-27 to 10-35

PROBLEM SET WITH SOLUTIONS

A(2). In the absence of any external forces a particle with mass m and speed v is incident on a particle of mass M initially at rest (see Fig. 1). After collision, particle m is observed to go off at an angle θ_2 with respect to the initial direction with speed v_f (see Fig. 2). M is observed to go off at an angle θ_1 with respect to the initial direction with speed V.

(a) Find V in terms of all the other parameters except θ_1.

(b) Let each parameter in turn approach zero and comment on the reasonableness of the answer.

(c) What is the maximum value for V? Is this reasonable?

(d) What happens as the magnitude of $M \to \infty$?

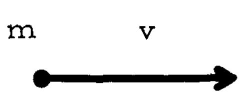


Figure 1

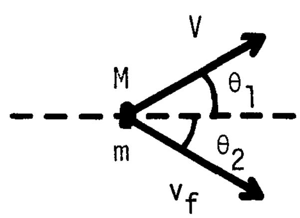


Figure 2

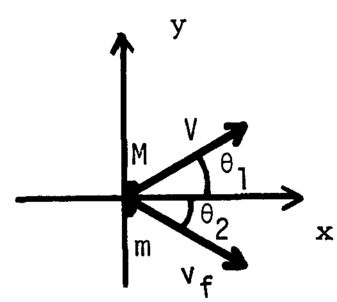


Figure 3

Solution

(a) Given m, M, v, v_f, and θ_2. Find V. Use momentum conservation (see Fig. 3). The x component of the linear-momentum-conservation equation is

$$mv = MV \cos \theta_1 + mv_f \cos \theta_2. \qquad (1)$$

The y component is

$$MV \sin \theta_1 = mv_f \sin \theta_2. \qquad (2)$$

Rearranging Eq. (1) we have

$$\begin{aligned} MV \cos \theta_1 &= m(v - v_f \cos \theta_2), \\ MV \sin \theta_1 &= mv_f \sin \theta_2. \end{aligned} \qquad (3)$$

Squaring the above equations and adding we have

$$M^2V^2(\cos^2\theta_1 + \sin^2\theta_1) = m^2(v^2 - 2vv_f\cos\theta_2 + v_f^2\cos^2\theta_2) + m^2v_f^2\sin^2\theta_2.$$

Using the fact that $\sin^2\theta + \cos^2\theta = 1$ and doing some rearranging we have

$$V = (m/M)\sqrt{(v_f^2 - 2vv_f\cos\theta_2 + v^2)}.$$

(b) Now let

$m \to 0$ and $V \to 0$:

Reasonable - consider a ping-pong ball colliding with a bowling ball.

$M \to 0$, V becomes large:

Reasonable - consider a bowling ball colliding with a ping-pong ball.

$v_f \to 0$, $V \to mv/M$:

Reasonable - all linear momentum transferred from m to M.

$v \to 0$, $V \to mv_f/M$:

Reasonable - explosion, total linear momentum zero.

$\theta_2 \to 0$, $V \to (m/M)(v_f - v)$:

Reasonable - linear momentum lost by m given to M.

(c) V_{max} at $\theta_2 = \pi$:

Reasonable - since m has maximum change in momentum, i.e., it transfers maximum momentum to M.

$$V_{max} = (m/M)(v + v_f).$$

(d) As $M \to \infty$, $V \to 0$:

Reasonable - since as M goes to ∞, V has to become smaller in order to conserve linear momentum.

B(2). In the absence of external forces a particle of mass m collides elastically with another particle of the same mass initially at rest. Show that if the collision is not head-on the two particles go off so that the angle between their directions is $\pi/2$.

(a) State what is given and what you are to find symbolically.

(b) Draw a diagram.

(c) Write down the relevant equation or equations. In this case use the laws of ________________ and ________________.

(d) Solve the equations for the relevant unknown or unknowns.

Solution

(a) Given $m_1 = m_2 = m$, show that $\theta_1 + \theta_2 = \pi/2$.

(b)

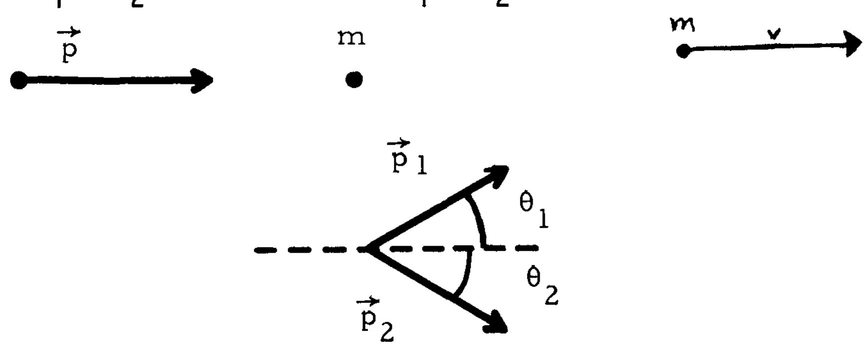


Figure 4

(c) Conservation of linear momentum:

$$\vec{p} = \vec{p}_1 + \vec{p}_2. \tag{4}$$

Conservation of kinetic energy:

$$K = K_1 + K_2, \tag{5}$$

where K stands for the kinetic energy of the incident particle before collision, etc.

(d) Now

$$K = p^2/2m. \tag{6}$$

Squaring Eq. (4) we have

$$p^2 = p_1^2 + p_2^2 + 2\vec{p}_1 \cdot \vec{p}_2.$$

Combining the above eqaation with Eq. (6) we have

$$2mK = 2mK_1 + 2mK_2 + 2\vec{p}_1 \cdot \vec{p}_2.$$

Combining the above equation with Eq. (5) we have

$$(\vec{p}_1 \cdot \vec{p}_2)/m = 0.$$

Assuming $p_1 \neq 0$; $p_2 \neq 0$ and $m \neq \infty$, all not very interesting cases, then $\vec{p}_1 \perp \vec{p}_2$, which was to be shown.

PRACTICE TEST

1. Define or state: (a) elastic collision; (b) inelastic collision; (c) perfectly inelastic collision; (d) the law of conservation of linear momentum.

2. A hockey puck B rests on a smooth ice surface and is struck by an identical puck A that was originally traveling at 60 m/s and that is deflected 30° from its original direction. Puck B acquires a velocity at an angle of 45° to the original velocity of A.
   (a) Compute the speed of each puck after collision.
   (b) Is the collision perfectly elastic? If not, what fraction of the original kinetic energy of puck A is "lost"?

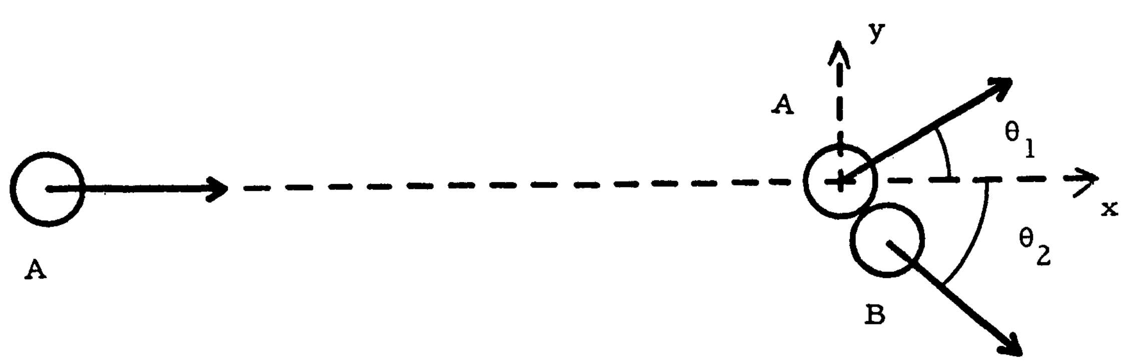


Figure 5

Practice Test Answers

1. (a) Elastic collision: a collision in which kinetic energy is conserved.
(b) Inelastic collision: a collision in which kinetic energy is not conserved. Note: kinetic energy may be either gained or lost during a collision.
(c) Perfectly inelastic collision: a collision in which the colliding objects stick together after the collision.
(d) Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant. Or, during a collision, if the interaction impulsive force is very large in comparison to the sum of all external forces such as gravity, then it is a good approximation to say that linear momentum is conserved.

Note: If you missed any of these definitions, MEMORIZE the ones that you missed.

2. (a) $V_{BF} = V_{Ai}/(\sin\theta_2 \cot\theta_1 + \cos\theta_2)$.

Check this answer for dimensions and reasonableness [see parts (b), (c), and (d) in the solution of Problem A for reasonableness check].

$V_{BF} = 31$ m/s.

$V_{AF} = V_{Ai} \sin\theta_2/(\sin\theta_2 \cos\theta_1 + \sin\theta_1 \cos\theta_2)$

Check this answer for dimensions and reasonableness.

$V_{AF} = 44$ m/s.

(b) $\Delta K/K_i = 0.20$, or the collision is inelastic.

Note: If you missed this problem, work some more of the optional problems in the text until you feel that you understand the material. When you understand the material, then ask for a Mastery Test. If you answered this Practice Test correctly, ask for a Mastery Test now.

COLLISIONS

Mastery Test   Form A

Date ________________

pass   recycle

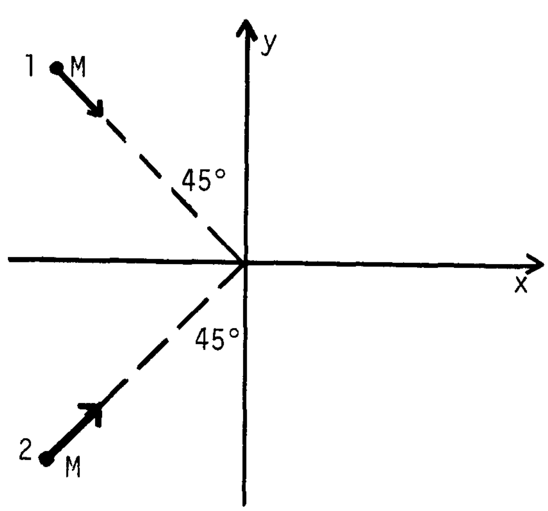
1   2

Name ________________________   Tutor ________________________

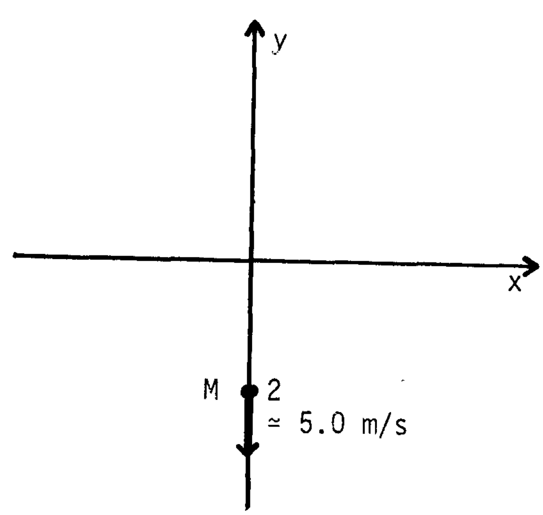
1. Define or state:
   (a) elastic collision;
   (b) inelastic collision;
   (c) perfectly inelastic collision;
   (d) the law of conservation of linear momentum.

2. Consider the collision as shown in the figure below. The colliding particles are identical and initially have a speed of 10.0 m/s. After the collision particle 2 moves as shown.
   (a) Find the velocity of particle 1 after collision.
   (b) Is this an elastic collision?

Before



After



COLLISIONS

Mastery Test Form B

Date ____________________

pass recycle

1 2

Name ______________________________ Tutor ______________________________

1. Define or state:
   (a) elastic collision;

   (b) inelastic collision;

   (c) perfectly inelastic collision;

   (d) the law of conservation of linear momentum.

2. A cannon mounted on a stationary railroad car fires a 100-kg projectile so the latter moves horizontally with a speed of 600 m/s at a sideways angle of 30.0° to the track. The car plus the cannon have a mass of 10 000 kg.

   (a) Make a sketch and describe in what way momentum conservation can be used to solve this problem, or explain why this is not the case.

   (b) At what speed will the railroad car recoil along the track? (Neglect friction with the track.)

COLLISIONS

Mastery Test Form C

Date ____________

pass 1 2 recycle

Name ____________ Tutor ____________

1. Define or state:
   (a) elastic collision;
   (b) inelastic collision;
   (c) perfectly inelastic collision;
   (d) the law of conservation of linear momentum.

2. As you stand at a lightly traveled street intersection, you are startled to observe the collision of a fire engine (mass = 6000 kg), a house trailer (mass = 25 000 kg), a steam calliope (mass = 4000 kg), and a dump truck (mass = 8000 kg). The four vehicles are, respectively, traveling northeast at 30.0 m/s, west at 10.0 m/s, south at 20.0 m/s, and east at 25.0 m/s.
   (a) Make a diagram of the vehicles immediately before the collision, and indicate their masses and vector velocities.
   (b) If the entire junk pile sticks together after the collision, what is its velocity before it has been slowed down by friction?
   (c) Is this collision elastic?

COLLISIONS

Mastery Test   Form D

Date ____________

pass   recycle
1   2

Name ____________   Tutor ____________

1. Define or state:
   (a) elastic collision;
   (b) inelastic collision;
   (c) perfectly inelastic collision;
   (d) the law of conservation of linear momentum.

2. A radioactive nucleus, initially at rest, decays by emitting an electron and an electron antineutrino at right angles to one another. The momentum of the electron is 1.20×10^{-22} kg m/s and that of the electron antineutrino is 6.4×10^{-23} kg m/s.
   (a) Find the momentum of the recoiling nucleus.
   (b) If the mass of the recoiling residual nucleus is 5.8×10^{-26} kg, what is its kinetic energy of recoil?

COLLISIONS

Date ____________________

Mastery Test   Form E

pass   recycle
1   2

Name ______________________________   Tutor ______________________________

1. Define or state:
   (a) elastic collision;

   (b) inelastic collision;

   (c) perfectly inelastic collision;

   (d) the law of conservation of linear momentum.

2. A body of mass $m_1 = 10.0$ kg moves to the right along a frictionless table top at a speed of 50 m/s and makes a head-on collision with another body whose mass m_2 is unknown, but which is originally moving to the left at a speed of 30.0 m/s. If the bodies stick together after the collision and move to the right at a speed of 20.0 m/s, what is the value of m_2? Is the collision elastic?

COLLISIONS

Mastery Test Form F

Date ____________

pass recycle

1 2

Name ____________ Tutor ____________

1. Define or state:
   (a) elastic collision;

   (b) inelastic collision;

   (c) perfectly inelastic collision;

   (d) the law of conservation of linear momentum.

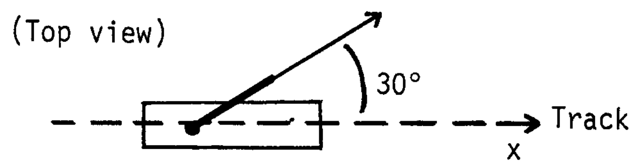
2. A ball with speed 3.00 m/s and mass 1.00 kg strikes off-center a second ball of mass 3.00 kg initially at rest. The incident ball is deflected 90° from its incident direction, and the collision is completely elastic. In what direction, relative to that of the incident ball before the collision, does the second ball leave the collision?

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1.(a) $\Delta K = 0$.	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$.	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision.	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant.
2. $\Delta\vec{P} = 0$.	2.(a) $\vec{p} = 2mv\hat{i}$: total momentum of particles 1 and 2 before collision.
Vector nature of $\vec{p}$	$\vec{p}_2 = -mv/2\hat{j}$: momentum of particle 2 after collision.
Is answer dimensionally correct?	$\vec{p}_1 = \vec{p} - \vec{p}_2$ $= 2mv\hat{i} + (mv/2)\hat{j}$.
Is the answer reasonable?	$v_1 = 2v\hat{i} + (v/2)\hat{j} = (14.0\hat{i} + 5.0\hat{j})$ m/s.
	(b) $K_i^2 = mv^2$, $K_f = (\frac{m}{2})(\frac{v^2}{4}) + (\frac{m}{2})v^2(2 + \frac{1}{4}) = mv^2(\frac{1}{8} + \frac{9}{8})$ $= mv^2(\frac{1}{8} + \frac{9}{8}) = mv^2(\frac{10}{8})$, $K_i \neq K_f$. Thus the collision is not elastic. Also, since v_1 has a j component it cannot be $\perp$ to v_2; therefore collision is inelastic.

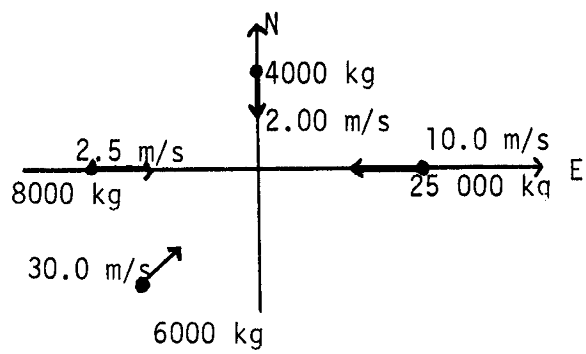
MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1.(a) $\Delta K = 0$;	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$;	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision.	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant.
2.(a) $\vec{p}$ conserved only in direction defined by track.	2.(a) Momentum not conserved in direction ⊥ to track.



What To Look For	Solutions
(b) $\vec{p}$ used is total $\vec{p}$ times cos 30°. Answer dimensionally correct? Units correct? Answer reasonable?	$P_x = 0 = (P_x)_{proj} + (P_x)_{car}$ $= [(100)(600) \cos 30.0°]$ m/s $+ (10\ 000)v$, $v = -[(100)(600) \cos 30.0°/10\ 000]$ m/s $= -(6 \cos 30.0°)$ m/s $= -5.2$ m/s.

MASTERY TEST GRADING KEY - Form C

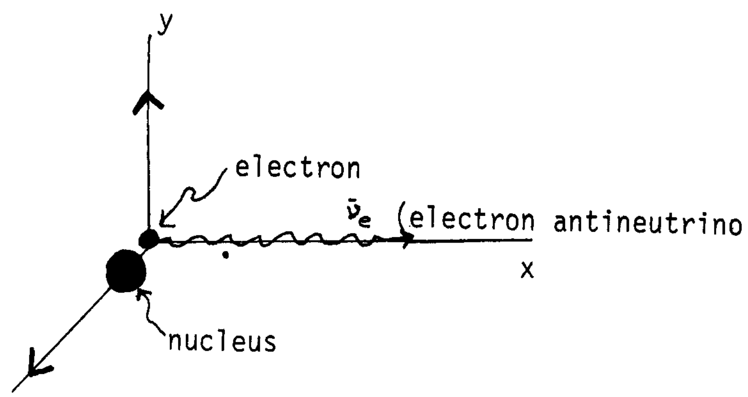
What To Look For	Solutions
1.(a) $\Delta K = 0$.	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$.	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: if the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant.
2.(a) Make sure diagram is clear!	2.(a) 
(b) $\Delta\vec{P} = 0$. Vector nature of $\vec{p}$. Answer dimensionally correct? Units correct? Answer reasonable?	(b) Momentum is conserved, junk pile moves with $\vec{v}_{c.m.}$
(c) recognize perfectly inelastic collision	(c) No - perfectly inelastic.

$$\vec{v}_{c.m.} = \frac{\vec{P}}{M} = \left(\frac{(6000)(15.0v_2\hat{i} + 15.0v_2\hat{j}) + (25\,000)(-10\hat{i})}{6000 + 25000 + 4000 + 8000} + \frac{(4000)(-20\hat{j}) + (8000)(25\hat{i})}{6000 + 25000 + 4000 + 8000}\right) \text{ m/s}$$

$$= (77\hat{i} + 47\hat{j})/(43) \text{ m/s} = (1.8\hat{i} + 1.1\hat{j}) \text{ m/s}.$$

MASTERY TEST GRADING KEY - Form D

What To Look For	Solutions
1.(a) $\Delta K = 0$.	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$.	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision.	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant.
2. $\Delta\vec{P} = 0$. Vector nature of $\vec{p}$. Answer dimensionally correct? Units correct? Answer reasonable?	2.



(a) $\vec{p}_f = 0$,

$\vec{p}_{nuc} = -(0.64\hat{i} + 1.20\hat{j}) \times 10^{-22}$ kg m/s.

(b) $K = \frac{p^2}{2m} = \frac{1.81 \times 10^{-44}}{2(5.8 \times 10^{-26})}$ kg m^2/s^2,

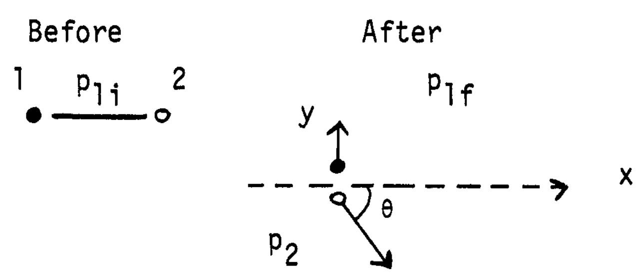
$K = 1.6 \times 10^{-19}$ J.

MASTERY TEST GRADING KEY - Form E

What To Look For	Solutions
1.(a) $\Delta K = 0$.	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$.	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision.	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: If the sum of the <u>external</u> forces acting on a system is zero, then the total linear momentum of the system remains constant.
2. $\Delta\vec{P} = 0$. Vector nature of $\vec{p}$? Answer dimensionally correct? Units correct? Answer reasonable?	2.  $\Delta\vec{p} = 0. \quad p_i = m_1v_{1i} - m_2v_{2i} = p_f = (m_i + m_2)v_f$. $m_2 = \frac{m_i(v_{1i} - v_f)}{(v_{2i} + v_f)} = 10.0\ \text{kg}\ \frac{(50 - 20.0)}{(30 + 20.0)} = 6.0\ \text{kg}$. Collision is inelastic since objects stick together; therefore it is not elastic!

MASTERY TEST GRADING KEY - Form F

What To Look For	Solutions
1.(a) $\Delta K = 0$.	1.(a) Elastic collision - a collision in which kinetic energy is conserved.
(b) $\Delta K \neq 0$.	(b) Inelastic collision - a collision in which kinetic energy is not conserved.
(c) Objects stick together after collision.	(c) Perfectly inelastic collision - a collision in which the colliding objects stick together after the collision.
(d) $\Delta\vec{P} = 0$ if $\Sigma\vec{F}_{ext} = 0$.	(d) Conservation of linear momentum: If the sum of the external forces acting on a system is zero, then the total linear momentum of the system remains constant.
2. $\Delta\vec{P} = 0$. Vector nature of $\vec{p}$? Answer dimensionally correct? Units correct? Answer reasonable?	2.



$\vec{p}_2 = \vec{p}_{1i} - \vec{p}_{1f}$,

$\vec{p}_{1i} = m_1 v_{1i}\hat{i}$, $\quad \dfrac{p_{1i}^2}{2m_1} = \dfrac{p_{1f}^2}{2m_1} + \dfrac{p_2^{\;2}}{2m_2}$,

$\vec{p}_{1f} = m_1 v_{1f}\hat{j}$, $\quad \dfrac{m_1^2 v_{1i}^2}{2m_1} = \dfrac{m_1^2 v_{1f}^2}{2m_1} + \dfrac{m_1^2}{2m_2}(v_{1i}^2 + v_{1f}^2)$,

$\vec{p}_2 = m_1(v_{1i}\hat{i} - v_{1f}\hat{j})$, $\quad v_{1i}^2 = v_{1f}^2 + (m_1/m_2)(v_{1i}^2 + v_{1f}^2)$,

$v_{1f}^2 = v_{1i}^2(m_2 - m_1)/(m_2 + m_1)$,

$v_{1f} = v_{1i}\sqrt{(m_2 - m_1)/(m_2 + m_1)}$,

$\tan\theta = \sqrt{(m_2 - m_1)/(m_2 + m_1)} = \sqrt{(2/4)} = \sqrt{2}/2$.

$\theta = \tan^{-1}(\sqrt{2}/2)$.

See diagram for definition of θ.