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BUFFER LAW AND TRANSITIONAL ROUGHNESS EFFECT IN TURBULENT OPEN-CHANNEL FLOWS

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Abstract. A novel and complete velocity profile law is presented for turbulent open-channel flows. The law embeds the linear law in the viscous sublayer; the quartic law due to turbulent bursting that must satisfy instantaneous mass conservation, the modified log-wake law in the outer region and the effects of wall roughness. Specifically, an arctangent law smoothly connects the linear law with the log law of the wall. Combining the arctangent law with the modified log-wake law, a continuous velocity profile is proposed for smooth boundary conditions. A complete velocity profile is obtained after subtracting a new roughness function. Finally, the proposed laws have been validated with data for hydraulically smooth, transitional and rough turbulent regimes.

1. Introduction

Velocity profile laws describe turbulent boundary layer flows and have been extensively studied since the 1920s. Recent developments in open-channel flows are reviewed by Nezu (2005), who stated that "an analytical solution for the buffer layer between the viscous sublayer and the log-law layer is not available … the buffer layer plays a critical role in the turbulent bursting phenomenon." Similarly, an analytical expression for the transitional roughness effect is also an unsolved problem. These two gaps retard our overall progress in the study of bedload initiation and transport.

The objectives of this study are threefold: (i) to propose an arctangent law for the buffer layer in order to fill the gap in the law of the wall; (ii) to extend the modified log-wake law (Guo and Julien 2003, Guo et al. 2005) from the outer region to the inner region; and (iii) to propose a new transitional roughness function in order to create a complete velocity profile law.

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2. Arctangent Law in the Buffer Layer

Inspecting boundary layer data, one can hypothesize a buffer layer law that connects the laminar sub-layer to the log law through the log law additive constant. That is, the buffer layer model must asymptotically tend to a constant when $y^+ > 30$. As an approximation, it is assumed that the buffer layer law can be represented by a series of arctangent functions

$$u^+ = \sum_{i=1}^{\infty} k_i \arctan^i \frac{y^+}{A} \tag{1}$$

where u^+ and y^+ are mean velocity u and distance y, scaled by the shear velocity u_* and kinematic viscosity ν; the value of A and the first four coefficients k_i are determined with experimental data (Guo 2006) to match the following asymptote:

$$u^+ = y^+ - \frac{1}{4(1150)} y^{+4} + \cdots \tag{2}$$

One can expand Eq. (1) to the fourth power

$$u^+ = \frac{k_1}{A} y^+ + \frac{k_2}{A^2} y^{+2} + \left(\frac{k_3}{A^3} - \frac{k_1}{3A^3} \right) y^{+3} + \left(\frac{k_4}{A^4} - \frac{2k_2}{3A^4} \right) y^{+4} + \cdots \tag{3}$$

Comparing Eq. (3) with (2) gives

$$k_1 = A,\ k_2 = 0,\ k_3 = \frac{A}{3},\ k_4 = -\frac{A^4}{4(1150)} \tag{4}$$

Using the least-squares method, the value of $A \approx 7$ can be determined by fitting Nezu and Rodi's (1986) data in the buffer layer as shown in Fig. 1. The data of Osterlund and McKeon et al have also been examined in Guo (2006).

3. Extension of the Modified Log-Wake Law

This section aims at extending the modified log-wake law (Guo and Julien 2003, Guo et al. 2005) from the outer region to the inner region. One can take two steps to reach the end. First, one can merge the arctangent law, Eq. (3), and the log law by the following

$$u^+ = 7 \arctan \frac{y^+}{7} + \frac{7}{3} \arctan^3 \frac{y^+}{7} - 0.52 \arctan^4 \frac{y^+}{7} + \ln\left[1 + \left(\frac{y^+}{C} \right)^{1/\kappa} \right] \tag{5}$$

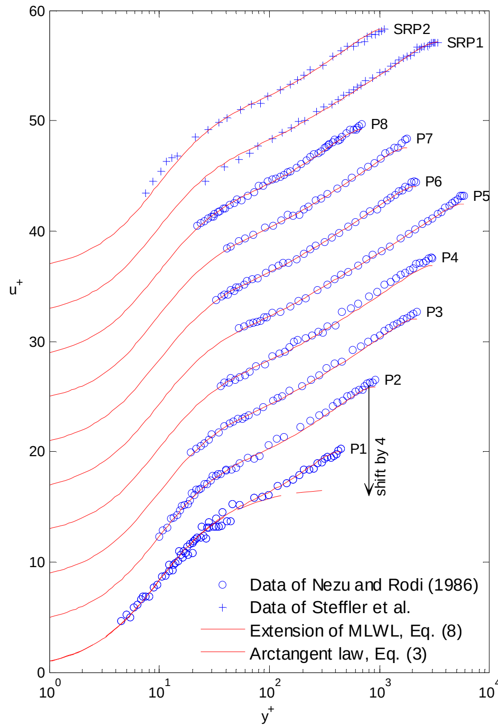

Figure 1. Test of arctangent law and extension of modified log-wake law in flumes

where κ = von Karman constant, and the value of C is found at $y^+ \to \infty$. That is,

$$u^+ = \underbrace{\frac{7\pi}{2} + \frac{7}{3}\left(\frac{\pi}{2}\right)^3 - 0.52\left(\frac{\pi}{2}\right)^4 - \frac{1}{\kappa}\ln C}_{B} + \frac{1}{\kappa}\ln y^+ \tag{6}$$

Obviously, the above equation reduces to the classic log law and B is the log law additive constant. Thus, the value of C can be written as

$$C = \exp[\kappa(16.873 - B)] \tag{7}$$

which depends on κ and B that slightly vary in literature.

To extend the modified log-wake law from the outer region to the inner region, replacing the log law in the modified log-wake law (Guo and Julien 2003, Guo et al. 2005) with Eq. (6) yields

$$u^+ = \underbrace{7\arctan\frac{y^+}{7} + \frac{7}{3}\arctan^3\frac{y^+}{7} - 0.52\arctan^4\frac{y^+}{7} + \ln\left[1+\left(\frac{y^+}{C}\right)^{1/\kappa}\right]}_{\text{the proposed law of the wall}} + \underbrace{\frac{2\Pi}{\kappa}\sin^2\frac{\pi\xi}{2} - \frac{\xi^3}{3\kappa}}_{\text{the modified wake law}} \tag{8}$$

where Π is the Coles wake strength, and ξ is the distance y scaled by the boundary layer thickness δ. Equation (8) is a new velocity profile law for hydraulically smooth boundaries. Note that although Eq. (8) is a data-driven equation or empirical equation, it satisfies all boundary conditions and asymptotes. For example, in the inner region where $\xi \to 0$, Eq. (8) reduces to the proposed law of the wall. In the outer region where $y^+ \to \infty$, one can show that Eq. (8) reduces to the modified log-wake law (Guo and Julien 2003, Guo et al 2005).

Equation (8) is also plotted in Figure 1, where for Nezu and Rodi's (1986) including Steffler's flume data, $(\kappa, B, \Pi) = (0.412, 5.29, 0.2)$ and C = 115. The solid lines in Figure 1 validate the extension of the modified log-wake law, Eq. (8), which well describes the entire velocity profiles for open-channels.

4. Roughness Function

The log law for rough turbulent flows is often written as

$$u^+ = \frac{1}{\kappa}\ln\frac{y}{y_0} \tag{9}$$

where y_0 = zero-velocity position from the theoretical wall that varies with roughness k_s and the roughness number $k_s^+ = k_s u_* / \nu$, as shown in Figure 2. By analogy with the Shields equation (Guo 1997), Figure 2 can be modeled by the following empirical expression

$$\frac{y_0}{k_s} = \frac{1}{9k_s^+} + \frac{1}{30}\left[1 - \exp\left(-\frac{k_s^+}{26}\right)\right] \tag{10}$$

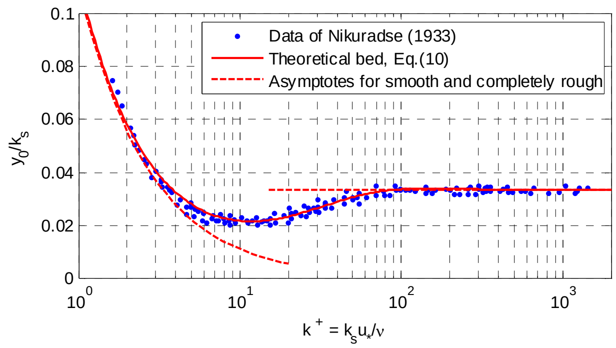

Figure 2. Effect of roughness on the theoretical bed position

which reduces to $y_0 / k_s = 1/9k_s^+$ for the hydraulically smooth boundary where k_s^+ is very small, and $y_0 / k_s = 1/30$ for the hydraulically rough boundary where k_s^+ is very large.

To find the roughness function, one can rearrange Eq. (9) as

$$u^+ = \frac{1}{\kappa}\ln y^+ - \frac{1}{\kappa}\ln\left(\frac{y_0}{k_s}k_s^+\right) = \underbrace{\frac{1}{\kappa}\ln y^+ + B}_{\text{the log law}} - \underbrace{\left[B + \frac{1}{\kappa}\ln\left(\frac{y_0}{k_s}k_s^+\right)\right]}_{\text{roughness function } \Delta B\left(k_s^+\right)} \tag{11}$$

The term in the brackets is called the roughness function, which can be written as

$$\Delta B = \frac{1}{\kappa}\ln\left\{1 + 0.3k_s^+\left[1 - \exp\left(-\frac{k_s^+}{26}\right)\right]\right\} \tag{12}$$

where Eq. (10) and $B = 1/\kappa \ln 9 \approx 5.5$ have been used. Subtracting the roughness function (12) from Eq. (8) gives

$$\begin{aligned} u^+ = &\underbrace{7\arctan\frac{y^+}{7} + \frac{7}{3}\arctan^3\frac{y^+}{7} - 0.52\arctan^4\frac{y^+}{7} + \ln\left[1 + \left(\frac{y^+}{C}\right)^{1/\kappa}\right]}_{\text{the law of the wall}} \\ &+ \underbrace{\frac{2\Pi}{\kappa}\sin^2\frac{\pi\xi}{2} - \frac{\xi^3}{3\kappa}}_{\text{the modified wake law}} \underbrace{- \frac{1}{\kappa}\ln\left\{1 + 0.3k_s^+\left[1 - \exp\left(-\frac{k_s^+}{26}\right)\right]\right\}}_{\text{the roughness function}} \end{aligned} \tag{13}$$

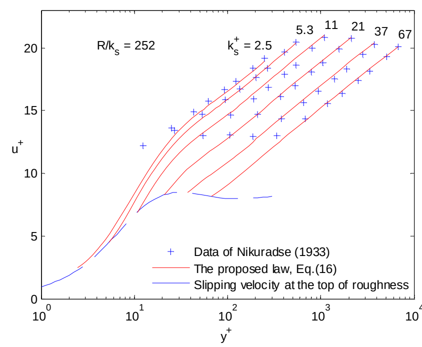

Figure 3. Effect of roughness on mean velocity profiles.

which is the major result of this article. It is a complete velocity profile law for turbulent boundary layers. It embeds all existing asymptotes; it can describe the buffer layer velocity profiles; and it includes the transitional roughness effect.

According to Eq. (13), Nikuradse's (1933) pipe data for transitional and rough flow regimes can well be replicated in Figure 3, where the dashed line represents the velocity at the top of roughness.

5. Conclusions

1) The buffer layer velocity profile can be described by the arctangent law, Eq. (3), which embeds the linear law in the viscous sublayer.
2) The combination of the arctangent law with the modified log-wake law, Eq. (8), can describe the velocity profile for smooth boundaries.
3) The complete velocity profile includes roughness effects Eq. (13), and is valid for hydraulically smooth, transitional and rough boundary conditions.
4) The arctangent law, the modified log-wake law, and roughness function have been tested with data in fumes and pipes.

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