1975

CONSERVATION OF ENERGY

CONSERVATION OF ENERGY

INTRODUCTION

Imagine a bicycle rider coasting without pedaling along a road that is very smooth but has a lot of small hills. As he coasts up a hill, the force of gravity will, of course, slow him down; but it speeds him up again as he goes down the other side. We say that gravity is a conservative force because it gives back as much kinetic energy (KE) to the cyclist when he returns to a lower level, as it took away when he ascended to the top. We therefore assign a gravitational potential energy (PE) U_g to the cyclist, which depends only on his elevation. The lost kinetic energy is converted into this U_g. We then find to our delight that the sum $E = K + U_g$ is (approximately) constant: U_g is larger at the top of each hill, and smaller at the bottom, in just such a way that its change compensates for the change in the kinetic energy K! This is an example of the conservation of mechanical energy.

However, if we watch the cyclist for some time, we are disappointed to find that $K + U_g$ is only approximately conserved: frictional forces gradually slow the cyclist down; and after awhile he starts pedaling again, thereby increasing $K + U_g$. But still, all is not lost. The energy-conservation law can be saved by defining other kinds of energy (for example, chemical, thermal, and nuclear) that are produced by the action of so-called nonconservative forces. If we call these nonmechanical energy forms E_{nc}, then $E = K + U + E_{nc}$ is exactly conserved. In fact, energy conservation is one of the great principles of physics, and one that holds even outside the domain where Newton's laws are valid.

Another example of energy transformation is provided by hydroelectric power production, beginning with the water stored behind a high dam. As the water rushes down the intake pipes it gains kinetic energy, then does work on the turbine blades to set them in motion; and, finally, the energy is transmitted electrically to appear as heat in the oven in your kitchen. Experience with energy transformations of this kind led to the formulation of the law of conservation of energy in the middle of the nineteenth century: ENERGY CAN BE TRANSFORMED, BUT NEITHER CREATED NOR DESTROYED. This law has survived many scientific and technological developments since that time, and our conception of the possible forms of energy has been enlarged. Whenever it seemed that energy was created or destroyed, physicists ultimately have been able to identify a new energy source (for example, thermonuclear energy in the sun) or a new energy receiver (such as neutrinos in beta decay).

In this module we shall be concerned only with mechanical energy and energy exchanged by doing work. We shall therefore be describing examples of mechanical energy conservation - the case of the ideal bicycle rider, and nonconservation of mechanical energy - the case of the real bicycle rider or the hydroelectric power plant. As a matter of fact, all practical, physically realizable phenomena involve friction, air resistance, and similar effects that result in some heating and a corresponding loss of mechanical energy. We shall therefore deal with idealized situations in which frictional forces are absent or are of a simple form. Since these forms of mechanical energy loss are often very small, our descriptions will be adequate approximations for many phenomena, and they will illustrate the law of conservation of energy as applied to mechanical processes.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate the work done by constant or variable force (needed for Objectives 3 and 4 of this module)	Work and Energy Module
*Apply the work-energy relationship to solve problems involving conservative and/or nonconservative forces (needed for Objectives 3 and 4 of this module)	Work and Energy Module

LEARNING OBJECTIVES

After you have mastered the contents of this module, you will be able to:

1. Forces - Define a conservative or a nonconservative force, or distinguish between them in problems.

2. Potential energy - Calculate the potential energy function U(x), given a conservative force F(x) depending on one coordinate; or conversely, given U(x), find F(x).

3. Conservation of mechanical energy - Use the law of conservation of mechanical energy for conservative forces to solve problems involving particle motion in one dimension.

4. Conservation of total energy - Apply the law of conservation of total energy, specifically including frictional forces, in the solution of problems of particle motion in one dimension.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 8, Sections 8.5 through 8.8 and Chapter 9, Sections 9.6 through 9.9. Bueche treats the objectives in this module in different order from most texts. We would suggest that you read Sections 8.5 and 8.6 in which the author shows that the ability of the gravitational field to do work on a man can be defined as gravitational potential energy, Eq. (8.6); then skip to Chapter 9 and read Section 9.6 up to Illustration 9.5, in which the potential energy of a spring is derived in Eq. (9.16). These are the two forms of potential energy we will use most often. Many others exist in nature, however, and the point of Objective 2 is the general relation between force and potential. At the end of Section 8.5 (p. 119), the work of gravity is shown as a sum of increments over a path in y. This would result, in the limit, in a general relation for an arbitrary conservative force:

$$U(x) = -\int_{x_1}^{x_2} F(x)\,dx.$$

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 8.5	A			
2	Sec. 9.9	B, C		M	Chap. 8, Prob. 2, Chap. 9, Probs. 30, 31
3	Secs. 8.5-8.7	D, E	Illus.* 8.6, 8.7, 9.5	N, O, P	Chap. 8, Quest.* 6, Probs. 17, 23, 25; Chap. 9, Probs. 21, 25, 27
4	Sec. 8.8	F	Illus. 8.8, 8.9, 9.6, 9.8	Q, R	Chap. 8, Quest.8, 14, Probs. 16, 18, 21, 22

*Illus. = Illustration(s). Quest. = Question(s).

Now read Section 9.9, which derives the differential relation between F(x) and U(x):

$$F = -\,dU/dx.$$

(You need not worry about more than one dimension.) With these equations the potential energy function can be found for an arbitrary force, and conversely F(x) can be found for a given U(x). Problems B and C in this module and Problems 30 and 31 in Chapter 9 of the text are keyed to Objective 2. You have already read about conservative and nonconservative forces in Section 8.5. Conservation of mechanical energy is treated in Section 8.7. Note that the author does not use the term "mechanical energy," preferring to use the symbolic form (U + K) instead. In Objective 3, "(U + K)" is equivalent to "mechanical energy."

The last logical step is to extend conservation ideas to nonconservative forces. This is done in Section 8.8. It should be pointed out that in Illustration 8.8, the "friction work" is the energy output equivalent of the actual work done by friction on the body (which is a negative quantity).

Read the General Comments; study the solutions to Problems A through F; and solve Problems M through R. If you need more practice, you may wish to work some of the optional problems listed above. Take the Practice Test before attempting a Mastery Test.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read all of Chapter 7 except Sections 7-6 and 7-9. You should have no difficulty with Objectives 1 and 2, which are well described in the text. The important principles of energy conservation are developed logically but rather briefly in Sections 7-7 and 7-8. You should note that Eq. (7-12b) is a statement of conservation of mechanical energy and is applicable to problems that do not involve friction. Section 7-8 discusses how to include friction or any other nonconservative force in the conservation principle. The last equation on p. 125 is a statement of conservation of total energy. The formulas given in General Comments may help you solve problems.

Read the General Comments; study the solutions to Problems A through F; and solve Problems M through R. If you need more practice, you may work some of the optional problems listed below. Take the Practice Test before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 7)
1	Secs. 7-2, 7-7	A			Quest.* 2, 3, 4
2	Secs. 7-3, 7-4	B, C	Ex.* 1, 2, 3	M	Probs. 2(a), 3(a), 15(a), 22
3	Secs. 7-3 to 7-5	D, E	Ex. 5	N, O, P	5, 7, 17, 18, 25, 28, 30
4	Secs. 7-7, 7-8	F		Q, R	23, 32, 33, 37, 39 [part (d) is difficult]

*Ex. = Example(s). Quest. = Question(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 10, Sections 10-1 through 10-5. Although the order of presentation is different from the order of the objectives, we suggest you read the text in order. You may omit Eq. (10-7), since we are limiting this module to motion in one variable. All of the material in Sections 10-1 through 10-4 is a development of the concept of potential energy, although Section 10-4, Property 5 will help you the most to develop problem-solving skills for Objective 3.

In Section 10-5, the conservation of total mechanical energy is defined, $E = K + U =$ constant. The total energy of an isolated system is always conserved, but mechanical energy is conserved only in the absence of friction and other nonconservative forces. Read the General Comments; study the solutions to Problems A through F; and solve Problems M through R. If you need more practice, you may work some of the optional problems listed below. Try the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Sec. 10-4	A				
2	Secs. 10-1 to 10-4	B, C		M		10-17, 10-18
3	Sec. 10-5	D, E	Ex.* 10-1 to 10-6	N, O, P		10-2, 10-3, 10-4, 10-9, 10-10, 10-11, 10-14
4	Sec. 10-5	F		Q, R		10-16, 10-19, 10-20

*Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 7, Sections 7-4 through 7-8. The authors develop the principle of conservation of mechanical energy but do not state it in a form directly applicable to Objective 3. Equation (7-14) is a statement of total energy conservation (Objective 4) in which W contains the nonconservative forces. As the authors state, when $W = 0$; $\Delta E_k + \Delta E_p = 0$ = conservation of mechanical energy. Some further discussion of this is given in the General Comments. Note that the text notation is different from the more common notation we have used in the General Comments; $E_p \equiv U_i$, $E_k \equiv K$, $W \equiv W_{nc}$. Note that Objective 1 is not covered in the text until Section 7-6. This change in order from other texts is unimportant and should cause you no difficulty.

Guidance for Objective 2: In the previous module Work and Energy, the work-energy theorem was introduced in terms of the kinetic energy;

$$\Delta E_k = \int_{s_1}^{s_2} \vec{F} \cdot d\vec{s}.$$

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 7-6	A			
2	Secs. 7-4, 7-5, 7-7, 7-8	B, C		M	7-11, 7-12, 7-13, 7-34
3	Secs. 7-4, 7-5, 7-7, 7-8	D, E	Sec. 7-4, Ex.* 1, 2, 4, 5 Sec. 7-8, Ex. 3	N, O, P	7-22, 7-23a, 7-26, 7-29
4	Secs. 7-4, 7-5, 7-7, 7-8	F	Sec. 7-4, Ex. 3	Q, R	7-19, 7-24, 7-27

*Ex. = Example(s).

In Section 7-4, the text introduces the potential energy, associated with the ability of a body in a force field to do work, by considering as an example the energy of a body in a gravitational field. The text then generalizes the concept by introducing elastic poential energy. This treatment leads easily to the conservation-of-energy law, but obscures a useful relationship between force and potential energy. As you read Section 7-4, note that if $W' = 0$ in Eq. (7-7), then (rearranging terms)

$$mgy_2 - mgy_1 = -(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2),$$

or in general $E_p = - E_k$, and from Work and Energy

$$E_p = - \int_{s_1}^{s_2} F(\vec{s}) \cdot d\vec{s}.$$

Alternatively, this relationship may be stated in differential form:

$$F(s) = -dE_p(s)/ds.$$

You will be assisted in mastering this objective by studying Problems B and C.

Read the General Comments; study the solutions to Problems A through F; and solve Problems M through R. If you need more practice you should work some of the optional problems listed in the Table. Try the Practice Test before attempting a Mastery Test.

GENERAL COMMENTS

The texts vary somewhat in their presentation of formulation for conservation of mechanical energy and total energy convenient for solving problems. We suggest the following for problems involving single springs and single masses (or multiple masses moving at the same velocity).

(1) Mechanical energy conservation: $K_i + U_i = K_f + U_f$ or

$$(1/2)mv_i^2 + (1/2)kx_i^2 + mgh_i + U_i = (1/2)mv_f^2 + (1/2)kx_f^2 + mgh_f + U_f,$$

where the subscripts i and f refer to initial and final states of the motion.

(2) Total energy conservation where friction is the only nonconservative force:

$$K_i + U_i = K_f + U_f + E_{nc},$$

$$(1/2)mv_i^2 + (1/2)kx_i^2 + mgh_i = (1/2)mv_f^2 + (1/2)kx_f^2 + mgh_f + \int_{x_1}^{x_2} f\ dx.$$

The absolute magnitude sign has been used for the work of the frictional force to avoid sign confusion, which arises because

$$E_{nc} = -W = -\int_{x_1}^{x_2} \vec{f}\cdot d\vec{x}.$$

But since f and dx are always oppositely directed, $\vec{f}\cdot d\vec{x} = -f\ dx(\cos\theta)$; Hence $E_{nc} = f \cos\theta\ dx$. It should be pointed out that because one is always concerned only with differences in potential energy, the choice of the zero of potential energy is arbitruary. If the force associated with the potential energy is constant, as in the case of the gravitational force on an object close to the earth, we may choose any convenient horizontal level (usually the lowest, or ground level) as the zero for gravitational potential energy. If, on the other hand, the force varies with displacement, as in the case of the spring, it is customary to choose the zero of potential energy as that displacement for which the force is zero.

Objective 1 will be satisfied by the following statement: A force is <u>conservative</u> if the work done by it on a particle that moves between two points depends only on these points and not on the path followed. A force is <u>nonconservative</u> if the work done by that force on a particle that moves between two points depends on the path taken between those points. (See Problem A for an alternative definition.)

PROBLEM SET WITH SOLUTIONS*

A(1). Define a nonconservative force and give an example of one.

Solution

A force is nonconservative if the work done by the force on a particle that moves through any round trip is not zero. (See General Comments for alternative definitions.)

Example: friction

A block sliding on a flat surface is projected against a spring mounted on the wall. If the surface is frictionless, the block will be brought to rest momentarily by the spring. The work done by the spring on the body is

$$W_1 = \int_i^f \vec{F} \cdot d\vec{x} = -\frac{1}{2}kx^2.$$

The motion reverses; on the outward path the work done is $W_2 = +(1/2)kx^2$, and the total work $W_1 + W_2 = 0$. If the surface is not frictionless, then the work done by the frictional force is

$$W_f = \int_i^f \vec{f} \cdot d\vec{x},$$

which is negative in both directions, and $W_1 + W_2 \neq 0$ for the round trip. Frictional forces, therefore, are an example of nonconservative forces. Conversely, for a conservative force the round-trip work is zero.

B(2). A body moving along the x axis is subject to a force given by $F(x) = -kx + cx^2$. Find the potential energy function for this force. Let $U(x) = 0$ at $x = 0$.

Solution

The potential energy is defined as $U(x) = -\int_{x_0}^{x} F(x)dx + U(x_0)$.

In this case $x_0 = 0$ and $U(x_0) = 0$.

$$U(x) = \int_0^x kx\,dx - \int_0^x cx^2\,dx, \qquad U(x) = \frac{1}{2}(kx^2) - \frac{1}{3}(cx^3).$$

C(2). What force corresponds to a potential $U(x) = -k_1m_1m_2/x + k_2x$?

*Problems satisfying Objective 4 also satisfy Objective 1 (i.e., Problems F, Q, R).

Solution

For one-dimensional motion $F(x) = -dU(x)/dx$. Therefore,

$$F(x) = \frac{d}{dx}\left(\frac{k_1 m_1 m_2}{x}\right) - \frac{d}{dx}(k_2 x) = -\left(\frac{k_1 m_1 m_2}{x^2}\right) - (k_2).$$

D(3). A 1.00-kg object coasts along a smooth horizontal surface at 2.00 m/s and strikes a spring with force constant k = 25.0 N/m whose right end is firmly attached to the wall. What is the maximum amount by which this spring is compressed? (See Fig. 1.)

Solution

Use conservation of mechanical energy, taking i to be the point of first contact with the spring and f as the point of maximum compression:

$$K_i + U_i = K_f + U_f, \quad (1/2)mv^2 + 0 = 0 + (1/2)kx^2,$$

$$x = (m/k)^{1/2}(v) = (1.00\ \text{kg}/25.0\ \text{N/m})^{1/2}\ (2.00\ \text{m/s}) = 0.40\ \text{m}.$$

Of course, when the object is at rest, the spring continues to push to the left, and the object is accelerated to the left. When it returns to its initial point the energy conditions will be the same as in i above, although v will be oppositely directed.

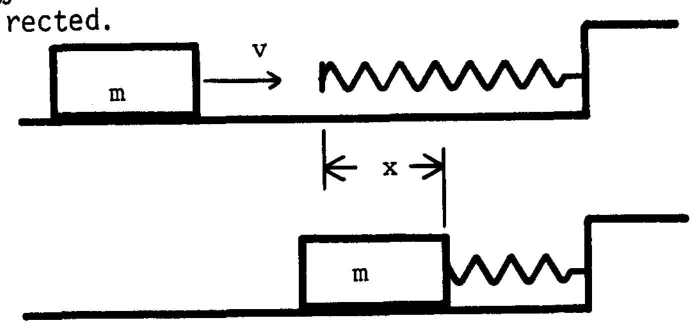


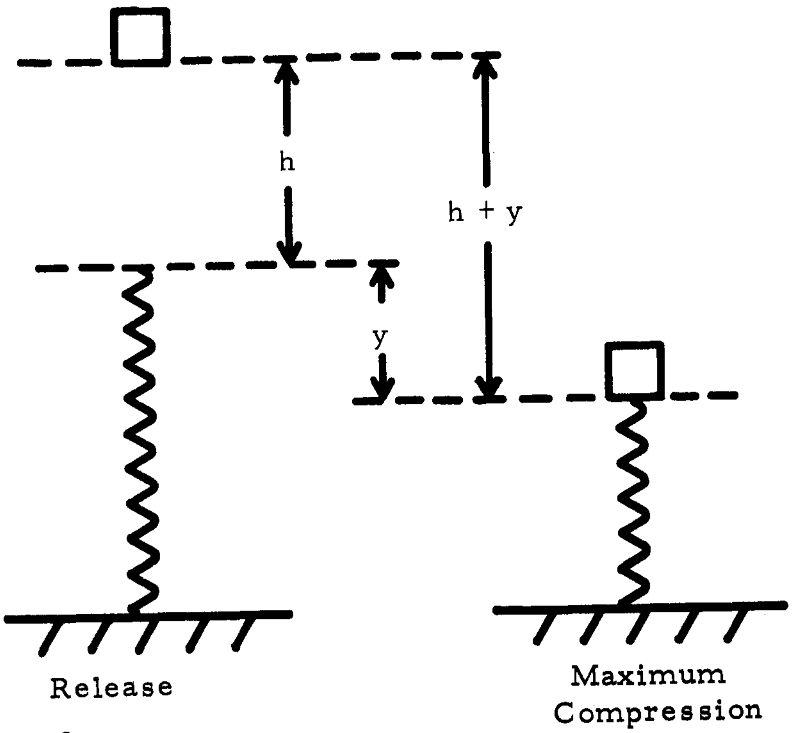
Figure 1

E(3). A block of mass 1.00 kg, initially at rest, is dropped from a height h = 1.00 m onto a spring whose force constant is k = 50 N/m. Find the maximum distance y that the spring will be compressed. (See Fig. 2.)

Solution

This is a process for which the principle of the conservation of mechanical energy holds. At the moment of release, the kinetic energy is zero. At the moment when maximum compression occurs, there is also no kinetic energy. Hence, the loss of gravitational potential energy of the block equals the gain of elastic potential energy of the spring:

Figure 2



$$mg(h + y) = (1/2)ky^2 \quad \text{or} \quad y^2 - 2mgy/k - 2mgh/k = 0.$$

Therefore,

$$y = \frac{mg}{k} \pm \frac{1}{2}\left[\left(\frac{2mg}{k}\right)^2 + \frac{8mgh}{k}\right]^{1/2}, \qquad \frac{mg}{k} = \frac{(1.00\ \text{kg})(9.8\ \text{m/s}^2)}{50\ \text{N/m}} = 0.196\ \text{m},$$

$$y = 0.196\ \text{m} \pm (1/2)(0.154 + 1.568)^{1/2}\ \text{m} = 0.196\ \text{m} \pm 0.656\ \text{m} = 0.85\ \text{m}.$$

(The negative solution corresponds to stretching the spring and is an unphysical solution since the block is not attached to the spring.)

F(4).

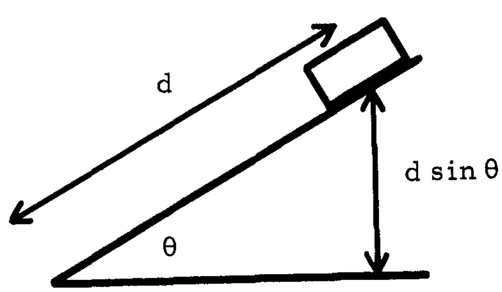


Figure 3

A 5.0-kg body is given an initial speed up an incline plane of 4.0 m/s. It coasts up the plane, comes momentarily to rest, and then coasts down again, its speed at the base of the incline being 3.00 m/s on return. (a) How much energy is dissipated in friction? (b) If the angle of the incline is 37°, what distance does the body travel up along the incline, assuming one-half of the energy found in part (a) is expended as the block goes up the plane?

Solution

(a) We find the loss in mechanical energy by comparing the initial and final mechanical energies. The potential energy is the same in the initial and final states, the body being, both initially and finally, at the base of the incline. Therefore, the decrease in the mechanical energy is simply the change in the kinetic energy:

$$E_{nc} = (k_i + U_i) - (K_f + U_f) = (1/2)m\,(V_i^2 - V_f^2)$$
$$= (1/2)(5.0\text{ kg})(16.0\text{ m}^2/\text{s}^2 - 9.0\text{ m}^2/\text{s}^2) = 17.5\text{ J}.$$

(b) Now we choose as our final state that at which the body is momentarily at rest on the incline. From part (a) we know that 9.0 J of mechanical energy have been converted into nonmechanical energy as the body increases its vertical displacement by $y = d \sin \theta$, as shown in the figure.

$$E_{nc} = (K_i + U_i) - (K_f + U_f),$$
$$8.75\text{ J} = [(1/2)\,mv_i^2 + 0] - (0 + mgd \sin \theta)$$
$$= (1/2)(5.0\text{ kg})(16.0\text{ m}^2/\text{s}^2) - (5.0\text{ kg})(9.8\text{ m/s}^2)d(\sin 37°),$$
$$d = (40 - 9.0)/29.4 = 1.06\text{ m}.$$

Problems

M(2). Find the potential energy function corresponding to this force field often used to describe the interaction of two atoms in a molecule:

$$F(r) = -A/r^7 + B/r^{13}$$

(r is the distance between atoms, A and B are constants). Let the potential energy be zero when the atoms are infinitely far apart. Draw a rough sketch of the force and potential energy function.

N(3). A particle is suspended at one end of a taut string whose upper end is fixed in position (a simple pendulum). The string's length is 12.5 m, and the particle passes through the lowest point at a speed of 7.0 m/s. What is the angle between the string and the vertical when the particle is momentarily at rest?

O(3). A 2.00-kg block and a 1.00-kg block are attached to opposite ends of a massless cord 2.00 m long. The cord is hung over a small frictionless and massless pulley a distance of 1.50 m from the floor, with the 1.00-kg block initially at floor level as shown in Figure 4. Then the blocks are released from rest. What is the speed of either block when the 2.00-kg block strikes the floor?

Figure 4

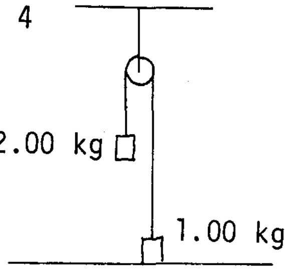
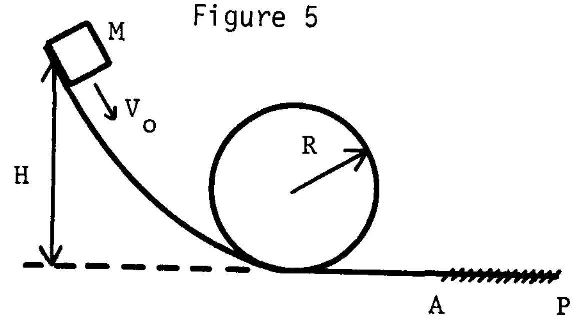


Figure 5



P(3). A 2.00-kg block is dropped from a height of 40 cm onto a spring whose force constant k is 1960 N/m. Find the maximum distance the spring will be compressed.

Q(4). A block of mass M slides on a frictionless track that is bent as in Figure 5. The radius of the loop is R. The block starts its journey at a height H above the floor with an initial speed (at height H) of v_0.

(a) How fast is the block traveling when it is upside down at the top of the loop?
(b) How fast is the block traveling after it has com leted the loop?
(c) At point A the block starts sliding on a rough portion of the floor. The force of friction is F. How far beyond A does it travel before it stops at point P?

R(4). A 4.0-kg block starts up a 30.0° incline with 128 J of kinetic energy. How far will it slide up the plane if the coefficient of friction is 0.300?

Solutions

M(2). $U(r) = - A/6r^6 + B/12r^{12}$.

N(3). 37°.

O(3). 2.60 m/s.

P(3). 10.0 cm.

Q(4). (a) $[v_0^2 + 2g(H - 2R)]^{1/2}$.

(b) $(v_0^2 + 2gH)^{1/2}$.

(c) $(mv_0^2 + 2mgH)/2F$.

R(4). 4.3 m.

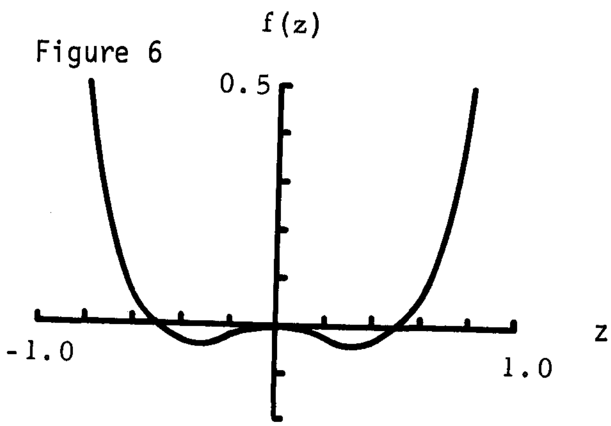
PRACTICE TEST

1. A particle of mass 16.0 kg constrained to move along the z axis is subject to a conservative force field given by $F(z) = -Az^3 + Bz$, as in Figure 6. (F is in newtons; the numerical values of A and B are A = 8.0, B = 1.00.)
   (a) What are the dimensions of A and B?
   (b) Find the potential energy as a function of z and sketch it.
   [U (0) = 0.]
   (c) With what speed will the particle arrive at z = 0 if it starts from rest at z_0 = 4.0 m?
   (d) Do the same for the particle starting at z_0 = 0.100 m.

2. What is meant by a conservative force?

3. A 16.0-kg block traveling at 6.0 m/s in a horizontal direction collides with a horizontal weightless spring of force constant 5.0 N/m. The block compresses the spring a distance s. When the spring is back to the uncompressed position, the block is traveling with a speed of 2.00 m/s. If the coefficient of friction between the block and surface is 0.40, determine the energy expended by nonconservative forces. (See Fig. 7.)

Figure 6



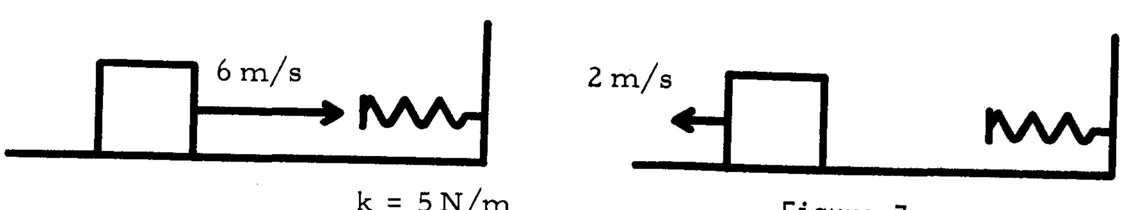


Figure 7

Practice Test Answers

1.(a) A: M/L^2T^2 B: M/T^2, since F: ML/T^2.

(b) $U(z) = -\int_0^z F(z)dz = (\frac{A}{4})z^4 - (\frac{B}{2})z^2$,

$$U(z) = 2z^4 - (1/2)z^2.$$

(c) $(1/2)mv^2 + U(z) = E = \text{const}$, $E = U(4) = 512 - 8.0 = 504$ J,
at $z = 0$, $U(0) = 0$, $E = (1/2)mv^2$, $v = \sqrt{E/m} = \sqrt{63} = 7.9$ m/s

(d) $(1/2)mv^2 + U(z) = E = \text{const}$, $E = U(0.100) = (2 \times 10^{-4}) - (0.50 \times 10^{-2})$
$= -0.0048\text{J} < 0$.

Since $U(0) = 0$, and $(1/2)mv^2 \geq 0$, the particle will not arrive at origin (it is repelled).

2. A force is conservative if the work done by the force on a particle that moves through any round trip is zero.

3. $K_i + U_i = K_f + U_f + E_{nc}$, $E_{nc} = (K_i - K_f) + (U_i - U_f) = (1/2)M(v_i^2 - v_f^2)$

$+ (0 - 0) = (1/2)(16.0\text{ kg})(36 - 4.0)\text{m}^2/\text{s}^2 = 256$ J.

Date ______________________

CONSERVATION OF ENERGY

pass recycle

Mastery Test Form A

1 2 3 4

Name ______________________ Tutor ______________________

1. (<u>Note</u>: The following experiment takes place in an orbiting space ship.) A 1.00-kg body is hurled against a special spring that exerts a restoring force given by the function $F(x) = -k_1 - k_2x^2$ when deformed from equilibrium. What was the body's initial speed if it compresses the spring by 0.200 m before being stopped? The constants in the force function are $k_1 = 100$ N and $k_2 = 2000$ N/m^2.

2. Two 10.0-kg blocks are connected together by a massless rope strung over a massless, frictionless pulley. The table exerts a 20.0-N frictional force on m. The blocks start from rest at t = 0 and are allowed to accelerate (see Fig. 1).
(a) Using only energy considerations, calculate the speed of M after it has fallen a distance of 2.00 m.
(b) Calculate the kinetic energy of the two blocks after M has fallen a distance of 2.00 m.
(c) Give a definition of a nonconservative force. Identify all forces that do not work in this situation. Which are conservative? Which are nonconservative?

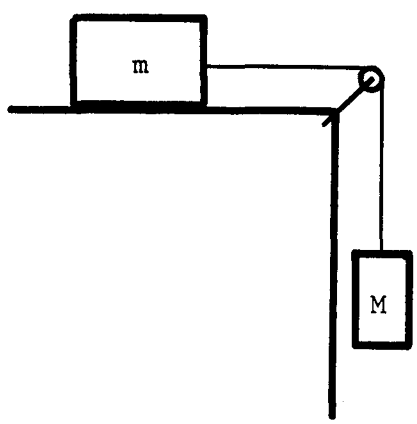


Figure 1

Date ______________

CONSERVATION OF ENERGY

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Mastery Test Form B

1 2 3 4

Name______________________ Tutor______________________

1. A 2.00-kg object, initially at rest, slides down a frictionless segment of the track from A to B, as illustrated in Figure 1.
(a) Calculate the speed of the object at B.
(b) The track between B and C is sufficiently frictional that the object, after continuing past B, comes to rest at point C. Calculate the work done by friction in slowing the object.
(c) Define a conservative force.
(d) Calculate the net work done by conservative forces for travel between points A and C.
(e) Calculate the net work done by nonconservative forces for travel between points A and C.

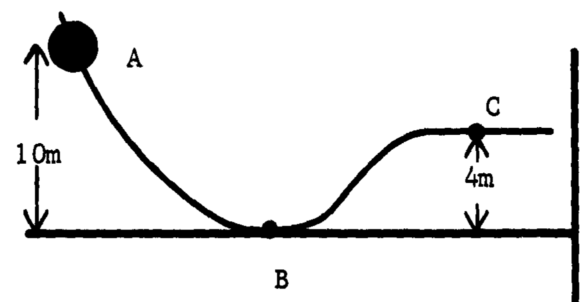

Figure 1

2. The potential energy of a particle of mass m constrained to move along the z axis is $U(z) = Az^2 + B$.
(a) What is the physical significance of B?
(b) Find the force field F(z) experienced by the particle, and sketch it as a function of z.
(c) With what speed will the particle arrive at the point z = 1.00 m if it starts with zero speed at the point z = 6.0 m? Let A = 2.50 kg/s^2; m = 7.0 kg.

Date ____________

CONSERVATION OF ENERGY

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Mastery Test Form C

1 2 3 4

Name____________ Tutor____________

1. Define a nonconservative force, and give an example.

2. A block of 0.200 kg is released from rest from its position of being pressed against a spring whose length is initially 0.100 m shorter than its relaxed length and whose spring constant is 1200 N/m. The block slides without friction along the horizontal and up a ramp that makes an angle of 30.0° with the horizontal and whose top edge is 1.30 m above the level of the horizontal surface. (See Fig. 1.) Determine the velocity of the block as it flies off the ramp.

3. A certain peculiar spring obeys the force law $F_x(x) = -Ax - Bx^2$, where $A = 22.0$ N/m and $B = 18.0$ N/m^2. (See Fig. 2. Note this is not Hooke's law!)

(a) Compute the potential energy function $U(x)$, taking $U(0) = 0$.

(b) One end of this spring is fastened to a wall and the other end is fastened to an object of mass $M = 1.20$ kg resting on a rough horizontal surface. The object is moved to the right, stretching the spring by 1.00 m, and then released. If $\mu_k = 0.50$, what is the speed of the object when it reaches the point at which the spring is unstretched?

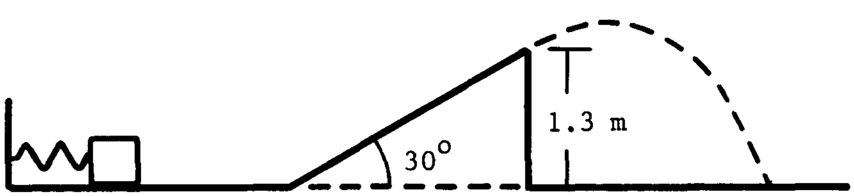

Figure 1

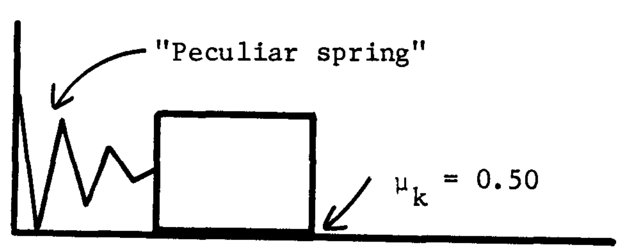

Figure 2

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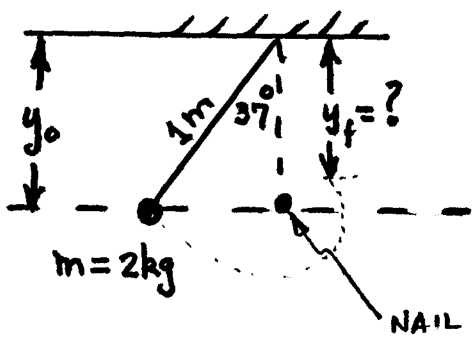
Conservation of Energy pass recycle

Mastery Test Form D 1 2 3 4 5

Name ____________________________________ Tutor __________________________

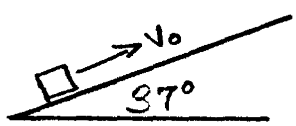
1. A 1.0 kg object is acted upon by a conservative force given by $F = +3.0x-5.0x^2$ where F is in Newton and x is in meters. Find the potential energy of the object at x=2m.

2.



A 2 kg mass is fastened to a 1m string to make a pendulum as shown. The pendulum is released at an angle of 37°. At the center of its swing the string hits a nail and swings on a shorter arc. Using energy conservation, find the distance, y_f, between the ceiling and the top of the swing on the right hand side.

3.



A 5 kg body is given an initial speed up an incline plane of 6.0 m/s. It coasts up the plane, comes momentarily to rest, and then coasts down again, its speed at the base of the plane being 4.0 m/s on return.

(a) How much energy is dissipated in friction?

(b) How far up the plane does the body travel assuming one-half of the energy found in part (a) is expended as the block goes up the plane.

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Conservation of Energy pass recycle

Mastery Test Form E 1 2 3 4 5

Name_______________ Tutor_______________________

1. Two particles interact by a repulsive force whose magnitude varies with distance r between them as k/r^3, where k is a constant. What is the potential energy of the two particles? (Take the potential energy to be zero for the separation distance at which the force is zero.)

2. A 0.2 kg block is dropped from a height of 2.6 m above the end of a vertical unstretched spring. It strikes the spring, compresses it, and then rebounds with the spring in contact with it. The spring constant is 5 N/m.

   (a) Find the speed of the block as it rebounds through the original position of the end of the spring.

   (b) How far will the spring compress?

   (c) Are there any nonconservative forces in this problem?

3. A block and a spring are placed on a flat table. The block is pushed back against the spring compressing it 0.20 m. When released, the block moves 0.7 m before coming to rest. How large is the friction force between block and table? (k = 30 N/m)

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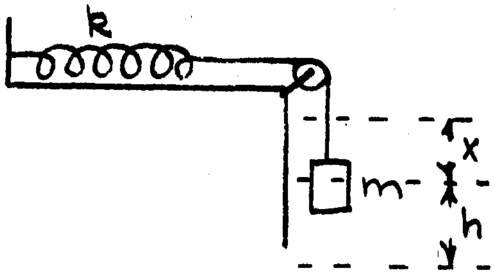
Conservation of Energy pass recycle

Mastery Test Form F 1 2 3 4 5

Name_____________________ Tutor_____________________________

1. (a) Given that $F(x) = 3x - x^3$, find the potential energy, u(x), when x = 2. (You may neglect units.)

   (b) F(x) a conservative or nonconservative force?

2. 

The block in the figure is x from rest when the spring has its relaxed length. How far does the block descend before coming momentarily to rest. Data: x = 0.2 m, $k = 30 \frac{N}{m}$, m = 1 kg.

3. A 1.0 kg object dropped from a height of 2.0 m bounces back to a height of 1.5 m. How much energy is dissipated in the collision with the floor?

MASTERY TEST GRADING KEY - Form A

What To Look For

1. (a) First step is to find the potential. Check the minus sign.

(b) Make sure all terms in the conservation equation are present.

Solutions

1. (May be done in two steps.)
(a) Find the potential energy from the force

$$U = - \int_{x_1}^{x_2} F(x)\,dx = \int_0^x (k_1 + k_2x^2)\,dx,$$

$$U(x) = k_1x + (k_2/3)\,x^3.$$

Gravitational forces do not enter into this "weightless" environment.

(b) Using conservation of mechanical energy, find v:

$$(U_i + K_i) = (U_f + K_f),$$

$$k_1x + (1/3)k_2x^3 + 0 = 0 + (1/2)mv^2,$$

$$v^2 = 2k_1x/m + 2k_2x^3/3m$$

$$= \frac{(2)(100\ \text{N})(0.200\ \text{m})}{(1.00\ \text{kg})} + \frac{(2)(2000\ \text{N/m}^2)(0.200\ \text{m})^3}{(3)(1.00\ \text{kg})}$$

$$= 40 + 10.7 = 50.7 = 7.1\ \text{m/s}.$$

What To Look For

2. (a) Complete energy balance, i.e., make sure all terms are present. Common error is omission of $(1/2)mv^2$. Check free-body diagram.

Solutions

2. (a)

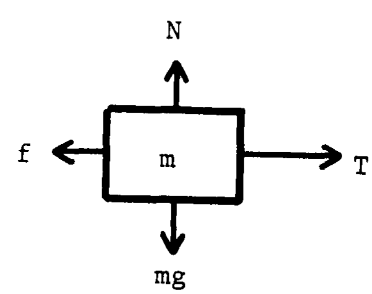


Figure 12

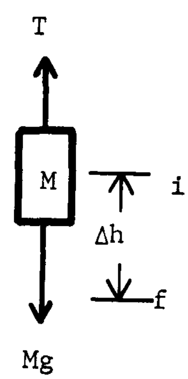


Figure 13

What To Look For	Solutions
	Conservation of total energy: $U_i + K_i = U_f + K_f + E_{nc}$, $mgh' + Mgh_1 + 0$ $= mgh' + Mgh_2 + (1/2)mv^2 + (1/2)Mv^2 + fh$, $(1/2)(m + M)v^2 = (mg - f)h$, $v^2 = 2(Mg - f)h/(m + M) = 16.0\ m^2/s^2$, $v = 4.0$ m/s. (b) $K = (1/2)(m + M)v^2 = (1/2)(10 + 10)16$ $= 160$ J.
(c) Look for alternative definitions of non-conservative force.	(c) For a nonconservative force, the work done in a round trip is not zero: Mg - conservative, f - nonconservative. Alternative definition: nonconservative force - a force is nonconservative if work done by it integrated around a closed path is not zero.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
	1. (a) By conservation of total energy: $U_i + K_i = U_f + K_f + E_{nc}$. There are no nonconservative forces so $E_{nc} = 0$, $mgh_1 + 0 = 0 + (1/2)mv^2$, $V = (2gh_1)^{1/2} = [(2)(9.8\ m/s^2)(10.0\ m)]^{1/2} = 14.0\ m/s$.
1. (b) The energy used up in friction is equivalent to a negative work on the particle by frictional force.	(b) $U_i + K_i = U_f + K_f + E_{nc}$, $0 + (1/2)mv^2 = mgh_2 + 0 - W_{nc}$, $W_{nc} = mgh_2 - (1/2)mv^2 = (2.00\ kg)(9.8\ m/s^2)(4.0\ m) - (1/2)(2.00\ kg)(196\ m^2) = 1.20 \times 10^2$ J.
(c) Look for acceptable alternate definitions of conservative force.	(c) If the total work done by a force in a round trip is zero, the force is conservative.
	(d) $W_{consv} = -\Delta U = U_i - U_f = mg(h_1 - h_2) = -1.20 \times 10^2$ J.
	(e) $W_{nc} = -1.20 \times 10^2$ J; see part (b).
2. (a) Question the student to see if he is aware of the concept but does not relate it to the question in this form.	2. (a) Physical significance - an arbitrary constant can always be added to the potential energy. Note it cancels in part (c).
(b) Look for lack of negative sign in definition.	(b) $F = -dU(z)/dz = -(d/dz)(Az^2 + B) = -2\ Az$.

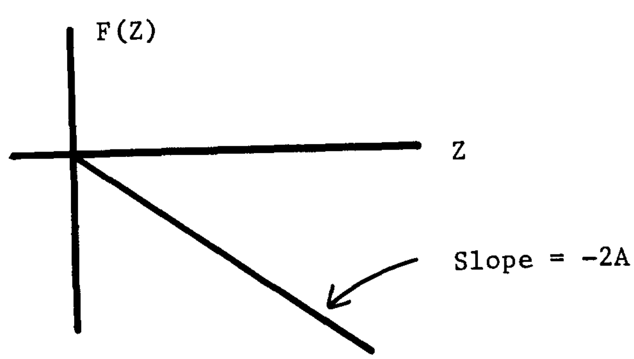


Figure 14

What To Look For	Solutions
	(c) Conservation of mechanical energy:
	$U_i + K_i = U_f + K_f$,
	$Az_1^2 + B + 0 = Az_2^2 + B + (1/2)mv_f^2$,
	$v_f^2 = (2A/m)(z_1^2 - z_2^2) = [(2(2.50)/7.0)](36 - 1)$
	$= 5.0(35)/7.0 = 25.0$,
	$v_f = 5.0$ m/s.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions

1. If the work done by a force in a round trip is not zero, the force is nonconservative. Friction and air resistance are common examples.

2. Releasing the spring transfers the potential energy stored in the spring to kinetic energy of the block. By conservation of mechanical energy:

   $U_i + K_i = U_f + K_f$,

   $(1/2)kx^2 + 0 = 0 + (1/2)mv^2$, $v^2 = kx^2/m$.

   As the block goes up the ramp, kinetic energy is transformed to gravitational potential energy:

   $U_i + K_i = U_f + K_f$,

   $0 + (1/2)mv^2 = mgh + (1/2)mv_f^2$, $v_f^2 = v^2 - 2gh$.

   From above, $v^2 = kx^2/m$.

   $v_f^2 = kx^2/m - 2gh = (1200\ \text{N/m})(0.100\ \text{m})^2/(0.200\ \text{kg}) - 2(9.8\ \text{m/s}^2)(1.30\ \text{m}) = 60 - 25.5 = 34.5$,

   $v = 5.9$ m/s.

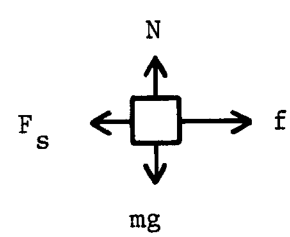
3. (a) Using $U(x) = -\int_{x_1}^{x_2} F(x)\, dx$,

   Find $U(x)$ for $F = -Ax - Bx^2$:

   $U(x) = +\int_0^x (Ax + Bx^2)\, dx = \frac{Ax^2}{2} + \frac{Bx^3}{3} + U_0(0)$,

   $U(x) = 11x^2 + 6x^3$.

(b) Free-Body Diagram:



$N = mg$, $F_s = f = \mu mg$. By conservation of total energy;

$U_i + K_i = U_f + K_f + E_{nc} = U_f + K_f - W_{nc}$, $11x^2 + 6x^3 + 0 = 0 + K_f - \int_x^0 f\, dx$,

$11x^2 + 6x^3 = K_f + \mu mgx$, or $K_f = 11x^2 + 6x^3 - \mu mgx = 11 + 6 - (1/2)(1.20\ \text{kg}) \times (9.8\ \text{m/s}^2)(1.00\ \text{m}) = 17 - 5.88 = 11.1\ \text{J}$,

$v = [2(11.1\ \text{J})/(1.20\ \text{kg})]^{1/2} = 4.3$ m/s.