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A bilinear integral equation for the cylinder is derived within the meson sector of the theory of dual topological unitarization. The equation is more general than conventional linear cylinder equations since it includes regions of phase space in which produced particles overlap in rapidity. The equation also permits a simple treatment of phase space which corresponds to that of the planar bootstrap problem. Two classes of solutions are found, only one of which results in the Pomeron-f identity. This treatment also indicates that the residue of the Pomeron may be twice as large as that suggested by earlier calculations but in agreement with a more recent calculation.

I. INTRODUCTION

At the present time both quantum chromodynamics (QCD) and dual topological unitarization (DTU)[1] are being actively studied as possible theoretical foundations for the strong interactions. Recently, the relationship between QCD and DTU has begun to be clarified. The self-consistency requirements of unitarity framed in terms of topology suggest that it may be possible to bootstrap the underlying quarks and gluons.[2] Advances in the DTU theory also indicate the presence of topological gluons.[3] Light is also being thrown on the problem of flavor, which is not well understood from the point of view of QCD. Confinement, still uproven within QCD, seems natural from the DTU viewpoint.[1,2] On the other hand, the DTU approach at the moment provides no way of treating e^+e^- phenomenology.

One particular advantage of the DTU approach is the understanding it gives of Regge-pole properties. Nonvacuum Regge poles with $\alpha(0)\approx\frac{1}{2}$ have been generated by self-consistency.[4] The properties of the Pomeron appear to be explained by means of the cylinder term in DTU.[5] In QCD the properties of the Pomeron are described in terms of gluon exchange,[6] which can also be diagrammatically pictured as a cylinder. So far the discussion of the Pomeron from the DTU viewpoint has not explicitly involved gluons. However, there are regions of phase space in the cylinder discontinuity, heretofore mostly neglected, where clusters of particles emitted from different sides of the incoming quark lines overlap in rapidity (see Fig. 2). As can be seen from Fig. 2, the diagram can be sliced perpendicular to the cylinder axis in such a way that no quark lines are cut. It is possible that such terms may have some correspondence with gluon exchange although we shall make no attempt to relate them precisely to gluon exchange in QCD. Until recently, a systematic treatment of such effects had been lacking.[7] Such "gluon" or overlap effects are important not only because understanding them could clarify the relation between QCD and DTU but also because the existence of the phenomenon referred to as the Pomeron-f identity[1,5] appears to be sensitive to such effects. With only quark exchanges in the cylinder, the Pomeron-f identity is a natural result, but if "gluon" or overlap effects are included, the Pomeron-f identity may be avoided.[7]

This paper presents a new equation for the cylinder contribution which is suggested by the work of Tuan.[8] The advantages of this equation are that (1) it is more general than most previous equations for the cylinder since it permits internal "gluon" exchange (or overlap contributions) in the cylinder, and (2) the equation, being nonlinear in the cylinder term, provides the possiblity of a more direct study of the relation between the Reggeon and Pomeron residues. There has been some controversy over whether Pomeron residue in the cylinder is the same as for the planar or a factor of 2 greater.[9] The present more precise treatment favors the latter result.

II. EQUATION FOR THE CYLINDER

We derive an equation in the meson sector for the discontinuity of the cylinder in the s channel where the cylinder axis is in the t channel. Thus for fixed t and large s this contribution gives the usual high-energy behavior of the "bare" Pomeron.[5] We designate by C the cylinder discontinuity in the s channel, which is given in terms of products of planar amplitudes. In order to formulate our particular equation for the discontinuity of C it is necessary to decompose it into four parts:

$$C=C_{uu}+C_{dd}+C_{ud}+C_{du}\,. \tag{2.1}$$

Each C_{ij} in (2.1) represents a discontinuity of C in a different region of phase space for the particles in the intermediate state. In Fig. 1 these separate regions of phase space are diagrammatically indicated for the forward direction $t=0$, where A and B are the incident particles and the other particles participate in the unitarity integral. Each C_{ij} is

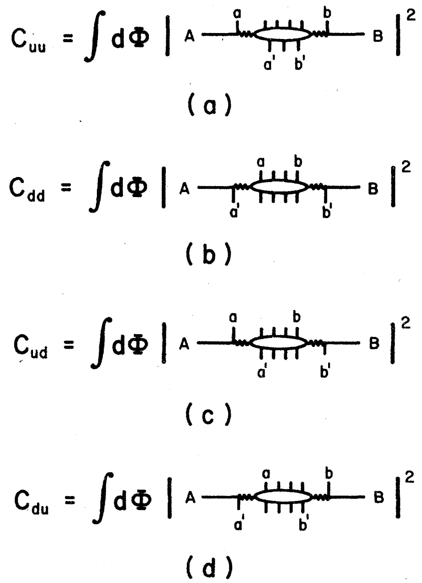

FIG. 1. Four regions of phase space for the full cylinder discontinuity in Eq. (2.1).

proportional to a squared ordered amplitude as shown. For each C_{ij} we include only the region where the momentum transfers indicated by the wiggly lines are small. In C_{uu}, for example, we include only the phase-space region where the momentum transfer between A and the next particle a clockwise in the ordered amplitude from A is small and the momentum transfer between B and the next particle b counterclockwise in the ordered amplitude from B is small. We shall assume, as usual, that regions of phase space with large momentum transfers are sharply damped. (A related discussion of the contributions of different phase-space regions but employing the more conventional treatment of a linear cylinder equation has been given recently by Freeman and Jones.[7])

We now wish to draw attention to an important subtlety associated with the cylinder discontinuity decomposition shown in Fig. 1. It is possible for there to be phase-space regions where the momentum transfers between A and *both* a and a' are simultaneously small. Such an event could then be classified either as 1(a) or 1(b) leading to the possibility of double counting. Such events, however, in the language of rapidity would mean that a and a' have nearly the same or overlapping rapidities. We shall assume throughout that for a given specific point in rapidity space for the ordered amplitude (in this case on the right and left ends), it is unlikely that two or more particles will have that exactly the same rapidity and such events are neglected. We thus ignore the possibility that a and a' or b and b' have the same rapidity.

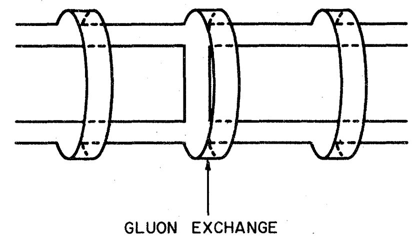

FIG. 2. Cylinder with interior overlap and possible gluon exchange.

However, we do not exclude in general the possibility of overlap in rapidity for particles in the interior of the cylinder. Thus in Fig. 1 the particles emitted from above and below the blob may in general overlap in rapidity. This makes our cylinder equation more general than conventional ones, which by their very construction exclude such overlap. As we shall see this makes possible the avoidance of the Pomeron-f identity.[10] Furthermore, one possible physical interpretation of including overlap contribution in the interior of the cylinder is that of "gluon" exchange as mentioned earlier. Depicted in Fig. 2 is a cylinder with interior overlap drawn in terms of quark lines. The diagram can be clearly sliced in the t channel in such a way that no quark lines are cut.

We are now ready to formulate the unitarity equations obeyed by the four components C_{ij} of C. To simplify the discussion we shall now use the concept of rapidity to divide up the particles in the intermediate states of Fig. 1 into two blobs. We assume (although we are confident that a more thoughtful use of the ordered-amplitude concept alone could eliminate the need for this assumption) that at high energy the ordered amplitudes in Fig. 1 only give important contributions where the sequence of particles in both rapidity and order is the same. We then divide the intermediate particles in Fig. 1 into two blobs according to rapidity—a simple way to accomplish this is to divide the total rapidity interval in half putting those particles in the first half in the left blob and those in the second half in the right blob. We shall assume, in line with our previously discussed treatment of particles a and a', that for the specific point that is half the total rapidity, it is improbable that two or more particles have that same rapidity. It will be recognized that this procedure for separating the intermediate particles into two clusters is exactly the same one often used for the planar bootstrap.[11] This means that phase-space integration for two blobs will be the same for the planar and cylinder cases.[8] This will permit a close comparison between the planar bootstrap and cylinder calculations.

The complete equation for the C_{uu} and C_{ud} ampli-

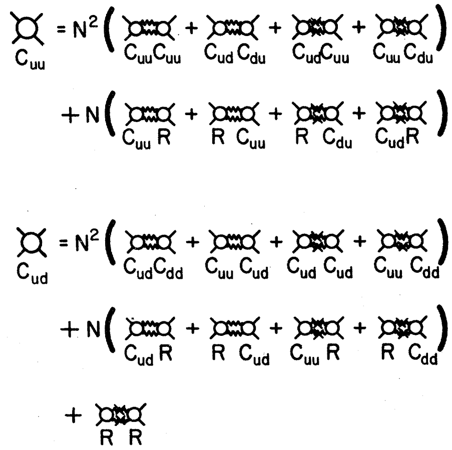

FIG. 3. Unitarity equation for cylinder components.

tudes is shown in Fig. 3 where we have assumed SU(N) symmetry and the N and N^2 factors indicate one and two closed quark loops, respectively, and R represents the planar ordered amplitude. The equations of Fig. 3 are to be applied at high energy and the exchanged wiggly lines are Reggeons. The crosses on the Reggeon lines occur when the right-most internal particle in the left cluster and the left-most internal particle in the right cluster are emitted from opposite sides of the Reggeon line. In this case the Reggeon propagator carries no phase (the so-called twisted propagator). The equations for C_{dd} and C_{du} are obtained by interchanging $u \longleftrightarrow d$ in the equations for C_{uu} and C_{ud}, respectively.

III. THE PLANAR BOOTSTRAP

We review here briefly the planar bootstrap equation[11] for the Reggeon trajectory $\alpha_R(t)$ in order to compare it with the cyliner equation in Fig. 3. The bootstrap equation for the discontinuity of the planar ordered amplitude R is shown in Fig. 4. Here the rule by which the intermediate particles in the unitarity relation are divided up between the clusters is exactly the same as that described earlier for the cylinder equations of Fig. 4.

Canceling from both sides of Fig. 4 the common external Regge couplings one has the usual result[11]

$$\frac{1}{N} = \int d\phi_{12} \frac{g_{RRR}{}^2(t_1 t_2 t)\cos\pi[\alpha_R(t_1) - \alpha_R(t_2)]}{[\alpha_R(t) - \alpha_c(t_1 t_2)]^2}, \qquad (3.1)$$

where g_{RRR} is the triple-Reggeon coupling and α_c is the Regge-Regge cut. It is from a study of the

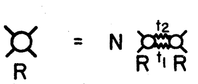

FIG. 4. Planar bootstrap equation.

bootstrap condition (3.1) that Schaap and Veneziano[4] conclude $\alpha_R(0) \approx \frac{1}{2}$.

IV. RESIDUE OF REGGEON POLE IN THE CYLINDER AND P-f IDENTITY

We wish now to examine the important question that relates directly to the phenomenon of the Pomeron-f identity, namely, what is the residue of the α_R pole in the cylinder? It is known that in general the α_R pole must appear in the cylinder term as well as in the planar contribution, but its exact residue in C determines whether or not a cancellation occurs in the singlet exchange for $C+R$ leading to the P-f identity.[1,2,12] If the residue of α_R in C is $(-2/N)$ times the planar α_R residue then the P-f identity holds and the Pomeron and f trajectories are the same.[13] Otherwise, there are two distinct vacuum trajectories.

Most previous studies of the cylinder employ equations that are linear in C and which explicitly exclude overlap, and such equations imply the P-f identity[2] (Ref. 7 is a recent paper which is an exception and which systematically treats overlap). Our equation for C in Fig. 3 permits a self-consistent analysis of the residue of α_R. To proceed we assume that in each C_{ij} amplitude, the residue of the α_R pole is $-\lambda_{ij}/2N$ times its residue in the planar amplitude. We shall assume that while the factor λ_{ij} very conceivably depends upon t, it does not depend upon t_1 and t_2 or the particular planar channel coupled to the ends of the C_{ij}. (This seems reasonable since by definition the C_{ij} have only planar couplings on each end.) Nonetheless factorization properties can be subtle and as Veneziano[14] has pointed out factorization of the cylinder poles is much cleaner from the point of view of t-channel unitarity, whereas we are working with s-channel unitarity.

Thus starting with

$$\mathrm{Res}(C_{ij})_{\alpha_R} = -\frac{\lambda_{ij}}{2N}\,\mathrm{Res}(R)_{\alpha_R} \qquad (4.1)$$

and recalling (3.1) we find from Fig. 3, taking the residue of each side of the equation at α_R, the self-consistency equations

$$-\frac{\lambda_{uu}}{2N} = \frac{1}{4N}[\lambda_{uu}{}^2 + \lambda_{ud}\lambda_{du} + \bar{\lambda}\lambda_{uu}(\lambda_{ud} + \lambda_{du})] - \frac{1}{N}[\lambda_{uu} + \tfrac{1}{2}(\lambda_{du} + \lambda_{ud})\bar{\lambda}], \qquad (4.2)$$

$$-\frac{\lambda_{ud}}{2N} = \frac{1}{4N}[\lambda_{ud}(\lambda_{dd} + \lambda_{uu}) + (\lambda_{ud}{}^2 + \lambda_{uu}\lambda_{dd})\bar{\lambda}] - \frac{1}{N}[\lambda_{ud} + \tfrac{1}{2}(\lambda_{uu} + \lambda_{dd})\bar{\lambda}] + \frac{\bar{\lambda}}{N}. \qquad (4.3)$$

The factor $\bar{\lambda}(t)$ in (4.2) and (4.3) is just the ratio of the Reggeon loop integral with twists to the loop integral without twists as given by (3.1). We recall that the Reggeon loop integral with twists is just (3.1) with the additional factor $\cos\{\pi[\alpha_R(t_1) - \alpha_R(t_2)]\}$ omitted in the integrand.[1,11] Thus we see that in the forward direction

$$\bar{\lambda}(t=0)=1\,. \qquad (4.4)$$

Finally, we note that by interchanging $u \leftrightarrow d$ in (4.2) and (4.3) we obtain two more equations that must be satisfied corresponding to taking the residue at the α_R pole in C_{dd} and C_{du}.

It is clear that (4.2) and (4.3) together with the two additional equations that result from (u,d) interchange have, in general, more than one solution. The symmetry of the equations for C_{ij} suggests that the interesting physical solutions occur when

$$\begin{aligned} \lambda_{uu} &= \lambda_{dd} = \lambda\,, \\ \lambda_{ud} &= \lambda_{du} = \lambda\,. \end{aligned} \qquad (4.5)$$

This instinct is supported by the interesting fact that (4.2) and (4.3) (together with the $u \leftrightarrow d$ interchanged equations) can be readily shown to be incapable of generating unique solutions unless (4.5) is satisfied. If (4.5) is satisfied the equations resulting from $u \leftrightarrow d$ interchange are equivalent to (4.2) and (4.3). In the case wherein (4.5) is satisfied there are four solutions that divide into two pairs of solutions characterized by the value of the sum $\lambda+\gamma$. The solutions are

$$\left.\begin{aligned} &\lambda=2, \quad \gamma=0 \\ &\lambda=\frac{2\bar{\lambda}-1}{\bar{\lambda}-1}, \quad \gamma=\frac{-1}{\bar{\lambda}-1} \end{aligned}\right\}\lambda+\gamma=2\,, \qquad (4.6)$$

$$\left.\begin{aligned} &\lambda=\frac{2\bar{\lambda}+1}{\bar{\lambda}+1}, \quad \gamma=\frac{-1}{\bar{\lambda}+1} \\ &\lambda=\frac{2\bar{\lambda}^2}{1-\bar{\lambda}}, \quad \gamma=\frac{2\bar{\lambda}(1+\bar{\lambda}^2}{1-\bar{\lambda}^2} \end{aligned}\right\}\lambda+\gamma=\frac{2\bar{\lambda}}{\bar{\lambda}+1}\,. \qquad (4.7)$$

The total cylinder coupling to the Reggeon is $-(\lambda+\gamma)/N$ times the planar residue. Thus both solutions in (4.6) give the Pomeron-f identity. The first solution in (4.6) has a simple interpretation in terms of the standard expansion of the cylinder with no overlap. The point is that in the standard expansion for C_{ud} or C_{du}, which is proportional to $\lambda_{du}=\gamma$, starts with one pair of twists, then three twists, five twists, etc. The sum of the series is of the form

$$s^{\alpha_R}\left(\beta \ln s + \beta^3 \frac{\ln^3}{3!}s + \cdots\right) = s^{\alpha_R}\sinh(\beta \ln s)\,, \qquad (4.8)$$

where β includes the N dependence. Thus the sum has the asymptotic behavior $s^{\alpha_R+\beta}$ and there is no asymptotic term of the form s^{α_R}, hence $\gamma=0$.

Here the Pomeron pole α_P is just

$$\alpha_P=\alpha_R+\beta\,. \qquad (4.9)$$

By contrast the expansion for C_{uu} or C_{dd} has terms with even numbers of twist pairs starting with two and, hence, even powers of $\ln s$. Then the series sum, which is analogous to (4.8), will have both a term proportional to $s^{\alpha_R+\beta}$ and a term proportional to s^{α_R} (since in this case a one must be added and subtracted to the series to develop an exponential behavior in $\ln s$).

The solutions (4.7) do not lead to the Pomeron-f identity, which requires $\lambda+\gamma=2$. In these cases the overlap regions (or "gluon exchange") have destroyed the effect. In particular we note that in the forward direction $(\bar{\lambda}=1)$, the solutions (4.7) give a cylinder coupling to the Reggeon which is $-1/N$ times the planar residue and thus in the singlet exchange for $C+R$ an α_R trajectory survives with half the residue it had in the singlet part of the planar amplitude.

One point of possible concern is that the surviving Reggeon in the "complete cylinder" (cylinder plus planar) is unshifted from its planar position whereas from t-channel unitarity one does not expect a pole of the complete cylinder at the position of a pole in the planar amplitude.[1] While a shift in the Reggeon-pole position is not expressly required, such a shift is not unlikely.

Our approach explicitly utilizes s-channel unitarity but through duality we may anticipate that our amplitude contains the main features required by t-channel unitarity. A subsequent imposition of exact t-channel unitarity would then only "fine tune" the complete cylinder, i.e., induce only slight changes in the positions and residues of the Reggeon and Pomeron. Thus, the Reggeon in the complete cylinder would no longer lie at the position of the planar pole.

V. RESIDUE OF THE POMERON

In conventional and less general treatments of the cylinder than the one given here, the residue of the Pomeron pole is determined at once in terms of the Reggeon residue in the planar amplitude. The usual result is that the Pomeron residue is the same as the Reggeon residue of the planar singlet exchange. However, recently Freeman[9] has suggested that the Pomeron residue may be a factor of 2 greater. He bases his result on a detailed study of two-blob phase space.

Our more general cylinder equation provides a unique opportunity to study this question since we can formulate a self-consistency equation for the Pomeron residue analogous to that found for planar Reggeon residues, viz., (3.1). The point is our

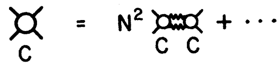

FIG. 5. Equation for the total cylinder in the forward direction.

equation for C depicted in Fig. 3 is bilinear in C. Furthermore, as we have emphasized, the two-C blob phase space in our cylinder equation is the same as that for the planar bootstrap equation.

To simplify the discussion we go to the forward direction $t=0$. The twists in the Reggeon exchanges shown in Fig. 3 effectively go away at $t=0$ and the equation for the total $C=C_{uu}+C_{dd}+C_{ud}+C_{du}$ is shown in Fig. 5. We have not shown the cross terms in Fig. 5 because at high energy they will be smaller than the term bilinear in C.

The self-consistency relation for the Pomeron residue implied by Fig. 5 is [in analogy with (3.1), only now in the forward direction $t=0$]

$$\frac{1}{N^2}=\int d\phi_1 \frac{g_{PRR}{}^2(t_1t_10)}{[\alpha_P(0)-\alpha_c(t_1,t_1)]^2}, \tag{5.1}$$

where g_{PRR} is the Pomeron-Reggeon-Reggeon coupling and $\alpha_P(0)$ is the Pomeron trajectory at $t=0$. Conventional calculations of the cylinder yield

$$g_{PRR}{}^2=\frac{2r}{N}g_{RRR}{}^2, \tag{5.2}$$

where $r=1$, i.e., the residue of the Reggeon in the single planar term is the same as the residue of the Pomeron. We can, however, by a comparison of (5.1) with (3.1) evaluated at $t=0$, deduce a restriction on r. Using (5.2) we arrive at

$$\int d\phi_1 \frac{g_{RRR}{}^2(t_1t_10)}{[\alpha_R(0)-\alpha_c(t_1t_1)]^2} = 2r\int d\phi_1 \frac{g_{RRR}{}^2(t_1t_10)}{[\alpha_P(0)-\alpha_c(t_1t_1)]^2}. \tag{5.3}$$

If the integrals in (5.3) are dominated by the forward direction $t_1\approx 0$ then we have

$$2r\approx\left(\frac{\alpha_P(0)-2\alpha_R(0)+1}{\alpha_R(0)-2\alpha_R(0)+1}\right)^2. \tag{5.4}$$

With $\alpha_P(0)=1$, $\alpha_R(0)=\frac{1}{2}$, we get $r=2$. Thus the coupling of the Pomeron pole appears to be about a factor of 2 larger than the standard result. This agrees with Freeman's result but our argument is totally different from his.

It should be emphasized that our result on the strength of the Pomeron coupling relative to the Reggeon coupling is quite general and is unrelated to the question of the Pomeron-f identity or upon which of the solutions (4.6) or (4.7) is appropriate.

ACKNOWLEDGMENT

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