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COULOMB'S LAW AND THE ELECTRIC FIELD

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COULOMB'S LAW AND THE ELECTRIC FIELD

INTRODUCTION

This module begins the study of electricity. Not only is it true that we see nature's gigantic electrical show in thunderstorm displays with lightning, but the very functioning of our smallest cells depends on the balance of electrically charged ions, and their movement through cell membranes. On a larger scale than cell membranes, water-purification studies with large membranes show promise of "electrically" removing undesired ions or debris from water. The electronic air cleaner is yet another direct application of the material to come: a 7000-V potential difference between a thin wire and flat collecting plates ionizes the air, and the "flying" electrons attach themselves to dust particles, which are then pulled to the collecting plates by strong electrical forces. Since forces that hold atoms together are ultimately electrical, the study of electricity is the study of one of nature's truly grand designs.

Later in your study of physics you will see the design unfold further; charges whose position is constant produce electric fields, charges whose velocity is constant produce magnetic fields as well as electric fields, and charges that accelerate produce that special combination of electric and magnetic fields we know as electromagnetic radiation (radio waves, x rays, microwaves, etc.).

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Add and subtract vectors (needed for Objective 2 of this module)	Dimensions and Vector Addition Module
*State Newton's law for linear motion (needed for Objectives 1 and 3 of this module)	Newton's Laws Module
*State the relation between work and energy (needed for Objective 3 of this module)	Work and Energy Module
*Analyze problems involving planar motion under constant acceleration (needed for Objective 3 of this module)	Planar Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Conductors versus insulators - Make the distinction between insulators and conductors.

2. Electric forces and fields - Calculate, for a group of point charges at rest,
   (a) the resultant force on one of the charges caused by all of the others, and/or
   (b) the total electric field at some point in space caused by all of the charges.

3. Particle motion in electric fields - Apply the definition of electric field to solve problems involving a charged particle in an electric field, where
   (a) the particle is at rest under the influence of additional forces, like gravity or tension, and/or
   (b) the particle moves in a constant electric field.

   These problems will require you to calculate any of the following quantities: force, acceleration, time, position, velocity, work, kinetic energy. For vector quantities you must be able to calculate components, magnitude, and direction.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 18, Sections 18.1 through 18.5, and then Sections 18.9 through 18.12, and General Comments 1 to 4. Then study Problems A through F before working Problems G through J. Make your own decision about working some Additional Problems before taking the Practice Test and a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	(Chap. 18)
1	Secs. 18.1, 18.9, General Comment 1		Sec. 18.11		
2	Secs. 18.2, 18.3, 18.5, General Comments 2, 3, 4	A, B, C	Sec. 18.3, Illus.[a] 18.3	G, H	1 to 10
3	Sec. 18.12	D, E, F	Illus. 18.1	I, J	11, 12, 18, 19, 20

[a]Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read over all of Chapters 22 and 23, for background. Then concentrate on a careful reading of Sections 22-2 through 22-4 in Chapter 22 and Section 23-5 in Chapter 23. Study General Comments 1 through 4 and Problems A through F before working Problems G through J and Problems 2 and 13 in Chapter 22, Problem 34 in Chapter 23. Take the Practice Test and decide whether to do some Additional Problems or take a Mastery Test. Note that calculating electric fields due to continuous-charge distributions using calculus is not an objective in this module, therefore you need not dwell on equations like (23-6) and (23-7).

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 22-2, 22-3, General Comment 1					Chap. 22, Quest.[a] 3, 6, 7
2	Secs. 22-2, 22-3, 22-4, General Comments 2, 3, 4	A, B, C	Chap. 22, Ex.[a] 2, 3, 4	G, H	Chap. 22, Probs. 2, 13	Chap. 22, Probs. 3, 4, 5; Chap. 23, Probs. 14 to 17
3	Sec. 23-5	D, E, F	Chap. 23, Ex. 7, 8	I, J	Chap. 23, Prob. 34	Chap. 22, Probs. 14, 26; Chap. 23, Probs. 31(a), 35, 36, 37

[a]Ex. = Example(s). Quest = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read over Chapter 24 and Chapter 25 through Example 1 in Section 25-2 (p. 343). Read General Comments 1 through 4 and study Problems A through F before working Problems G through J and Problem 24-1(a) and (b) in your text. Take the Practice Test, and decide whether to work some Additional Problems or take a Mastery Test.

Omit Example 4 in Section 25-1 (p. 341). Also note the text's interchangeable use of the terms "electric field," "electric field strength," and "electric intensity."

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	General Comment 1, Secs. 24-1, 24-4, 25-1		Sec. 24-5			
2	Secs. 24-6, 25-1, 25-2, General Comments 2, 3, 4	A, B, C		G, H	24-1(a), (b)	24-2(a), (b), 25-1, 25-6 to 25-9
3	Sec. 25-1	D, E, F	Sec. 25-1, Ex.[a] 1, 2, 3	I, J		24-3, 24-4, 25-2 to 25-5, 25-11

[a]Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Read over Chapter 22, except Section 22-6, then briefly read Chapter 23, except Section 23-6, without working through any examples. With this background, read General Comments 1 through 4, and study Problems A through F and Examples 22-1 and 23-3. Then work Problems G through J. Take the Practice Test and decide whether to take a Mastery Test, or work some Additional Problems. The Additional Problems on Objective 3 are likely to help you.

WEIDNER AND SELLS

Objective	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	General Comment 1, Chap. 22, Introduction, Secs. 22-1, 23-4				
2	Secs. 22-2, 22-3, 23-1, General Comments 2, 3, 4	A, B, C	Ex.[a] 22-1	G, H	22-5, 22-7, 22-15, 23-1, 23-2
3	Sec. 23-5	D, E, F	Ex. 23-3	I, J	23-12, 23-15, 23-18, 22-9

[a]Ex. = Example(s).

GENERAL COMMENTS

1. Electric Field

The concept of the electric field is introduced in this module as a force per unit charge. In your text reading, you will find conductors mentioned as materials in which charges are free to move, whereas insulators are materials in which charges are not free to move. Let us now put these two statements together: When you establish an electric field in a conductor, the electrons (which are free to move) feel a force equal to their charge times the electric field and thus initially accelerate, and a current (movement of charge) gets started. If you establish an electric field in an insulator essentially no current flows, since the charges are basically not free to move in response to the field. (In an insulator, charges can move through distances like an atomic radius before they are stopped by forces within the atom. This movement in response to an electric field sets up "dipoles," about which you may learn in a later module.)

The results of this discussion are so important when using Gauss' law in connection with conductors that we have made it an objective in this module (Objective 1). You must keep in mind that if charges are moving in a conductor, they are responding to an electric field. If we find that the charges are at rest in a conductor, however, this means that there is zero electric field in that conductor.

2. Principle of Superposition

This principle is very simple, but very important. It says that if cause A has effect a, and cause B has effect b, then A and B taken together will have effect (a + b).

In this module, where charges exert forces on each other, this principle definitely holds; if a charge Q feels a force $\vec{F}_a$ when only charge A is present, and feels a force $\vec{F}_b$ when only charge B is present, it will experience a force that is the vector sum of $\vec{F}_a$ and $\vec{F}_b$ when both A and B are present. Since the electric field at the point where a charge Q is located is the force felt by this charge divided by the charge Q, the same superposition principle holds for electric fields: if charge A by itself causes an electric field $\vec{E}_a$ at some point in space when present by itself, and if charge B causes an electric field $\vec{E}_b$ at this same point in space when only B is present, the electric field at that point when both A and B are present will be the vector sum of $\vec{E}_a$ and $\vec{E}_b$.

3. Coulomb's Law

If we ask for the total force on point charge Q_1 in the presence of point charge Q_2 and point charge Q_3, the answer may be written

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13}, \tag{1}$$

where $\vec{F}_{12}$ is the force on Q_1 due to Q_2 and $\vec{F}_{13}$ is the force on Q_1 due to Q_3. From Coulomb's law, the force is

$$|\vec{F}_{ij}| = k(Q_iQ_j/r_{ij}^2)$$

directed along the line between Q_i and Q_j. Therefore,

$$\vec{F}_{12} = (9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(Q_1Q_2/r_{12}^2)(\vec{r}_{12}/r_{12}),$$

where r_{12} is the distance between Q_1 and Q_2 and $\vec{r}_{12}/r_{12} = \hat{r}_{12}$ is a "unit" vector (length = 1) that points along the line in the direction from Q_2 to Q_1. In order to add F_{12} and F_{13} correctly, we may write

$$\hat{r}_{12} = \hat{i}(x_{12}/r_{12}) + \hat{j}(y_{12}/r_{12}) \quad .$$

When we also do this for $\vec{r}_{13}$ and add, Eq. (1) becomes

$$\vec{F}_1 = [(\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)Q_1Q_2}{r_{12}^2})(\frac{x_{12}}{r_{12}}) + (\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)Q_1Q_3}{r_{13}^2})(\frac{x_{13}}{r_{13}})]\hat{i}$$

$$+ [(\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)Q_1Q_2}{r_{12}^2})(\frac{y_{12}}{r_{12}}) + (\frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)Q_1Q_3}{r_{13}^2})(\frac{y_{13}}{r_{13}})]\hat{j}. \quad (2)$$

PROBLEMS A AND B ARE GOOD EXAMPLES OF THIS PROCEDURE, AND YOU SHOULD GIVE THEM CAREFUL STUDY.

4. Electric Fields in Space

These remarks conclude your reading on the subject of the electric field in this module. The main point to be made is that for a single charge or for a group of charges, the electric field is not everywhere represented by a single number, or even one single vector. The magnitude and direction of the electric field depend on position. In Figure 1, fields $\vec{E}_1$, $\vec{E}_2$, and $\vec{E}_3$ are all different in both

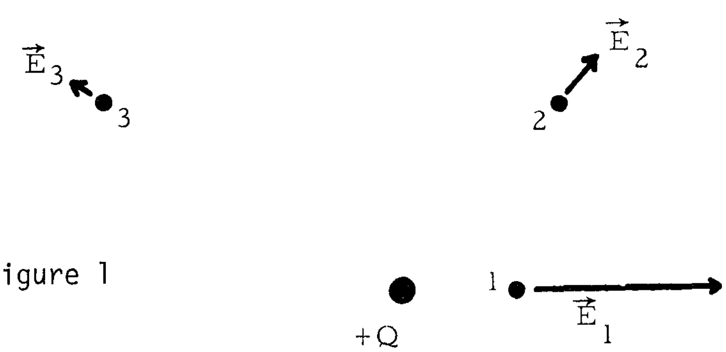


Figure 1

magnitude and direction because the points 1, 2, and 3 are in different locations in space: the electric field $\vec{E}$ is a function of coordinates like x, y, and z - it changes as x, y, and z change. This is again illustrated in Figure 2 for 6 points with respect to an assembly of 20 charges. The arrows are not rigorously correct, but they show what is going on generally: the electric field in the region between the lines of charge is fairly constant, and outside this region, the electric field is quite small. We often idealize this situation to say that the field between the "plates" is constant, and the field outside is zero. The point, however, is that even in the "ideal" situation, the electric field is a function of the coordinates: it depends on where you are located in space.

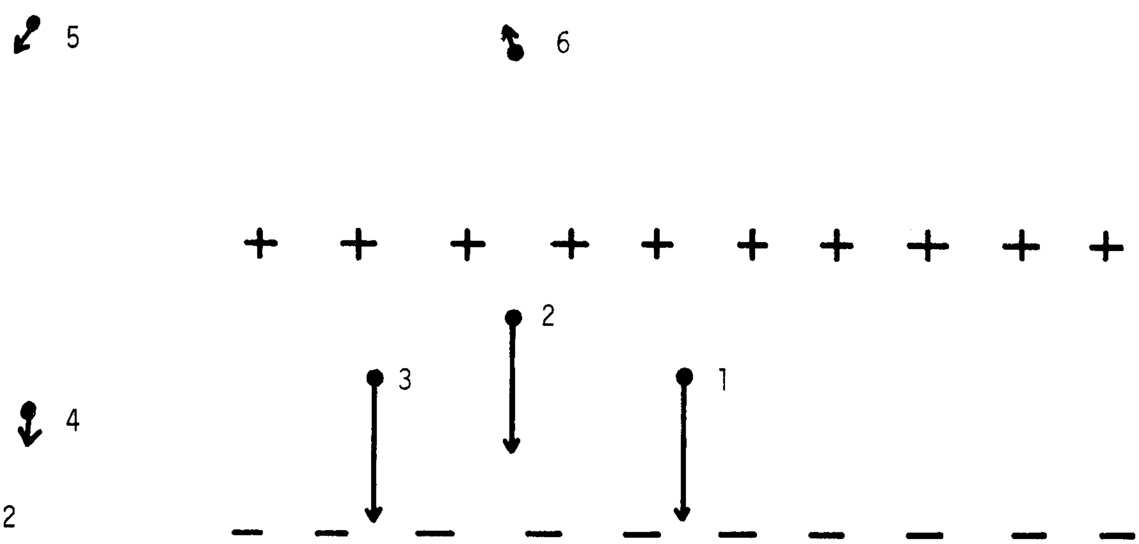


Figure 2

PROBLEM SET WITH SOLUTIONS

A(2). Calculate the total force on the -1.00-μC charge shown in Figure 3.

Solution

$$\vec{F}_{total} = \vec{F}_1 + \vec{F}_2.$$

Step 1: pick axes $\hat{i}$ and $\hat{j}$. Let $\hat{j}$ be positive along $\vec{F}_2$ and $\hat{i} \perp \hat{j}$ at q_1 in the plane of the paper.

Step 2: sketch all forces ($\vec{F}_1$ and $\vec{F}_2$). This gives the signs of the components right away:

$$\vec{F}_1 = -(F_1 \sin\theta)\hat{i} + (F_1 \cos\theta)\hat{j}, \qquad \vec{F}_2 = +F_2\hat{j}.$$

Step 3: Calculate force magnitudes from $F = q_1q_2/4\pi\varepsilon_0 r^2$:

$$F_1 = \frac{(9.0\times10^{9}\ \mathrm{N\,m^2/C^2})(2.50\times10^{-12}\ \mathrm{C^2})}{(3^2+4^2)\ \mathrm{m^2}} = 9.0\times10^{-4}\ \mathrm{N},$$

$$F_2 = \frac{(9.0\times10^{9}\ \mathrm{N\,m^2/C^2})(2.00\times10^{-12}\ \mathrm{C^2})}{4^2\ \mathrm{m^2}} = 1.13\times10^{-3}\ \mathrm{N}.$$

Step 4: add components to get the resultant:

$$F_{total} \equiv -(9.0\times10^{-4}\ \mathrm{N})(3/5)\hat{i} + [(9.0\times10^{-4}\ \mathrm{N})(4/5) + (1.13\times10^{-3}\ \mathrm{N})]\hat{j}$$
$$= -(5.4\times10^{-4}\ \mathrm{N})\hat{i} + (1.85\times10^{-3}\ \mathrm{N})\hat{j}.$$

Figure 3

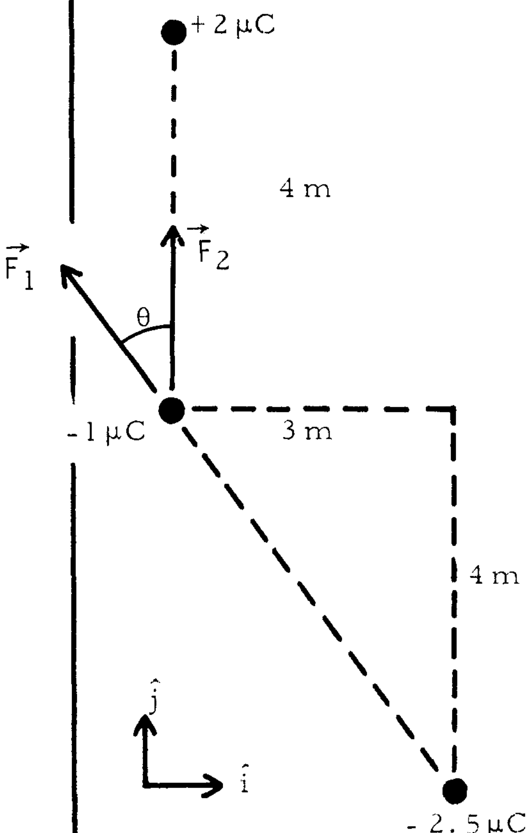


Figure 4

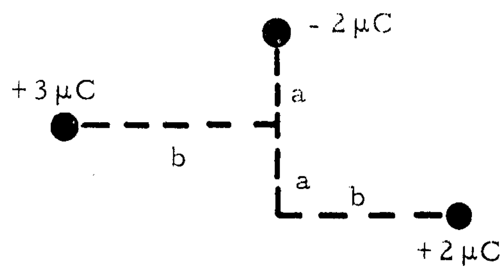
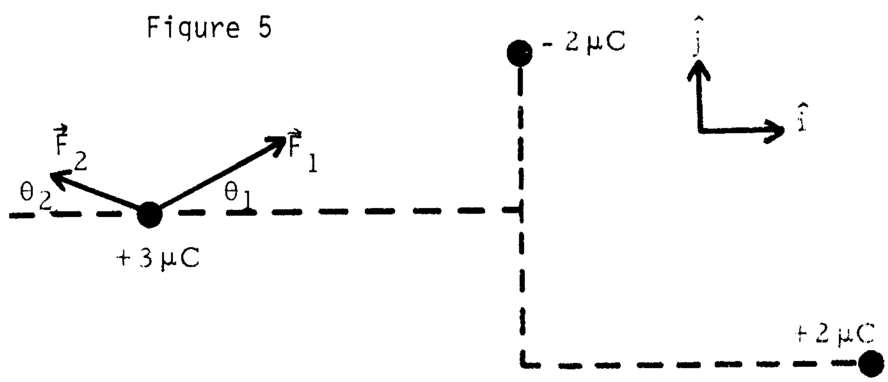


Figure 5



B(2). Calculate the total force on the +3.00-μC charge in Figure 4. (a = 1.00 m, b = 2.00 m).

Solution

Step 1: Choose axes as in Figure 5.
Step 2: Sketch in all forces. This gives component signs.

$$\vec{F}_{total} = (F_1 \cos \theta_1)\hat{i} + (F_1 \sin \theta_1)\hat{j} - (F_2 \cos \theta_2)\hat{i} + (F_2 \sin \theta_2)\hat{j}$$

$$= [F_1(2/\sqrt{1^2 + 2^2}) - F_2(4/\sqrt{4^2 + 1^2})]\hat{i} + [F_1(1/\sqrt{1^2 + 2^2}) + F_2(1/\sqrt{1^2 + 4^2})]\hat{j}.$$

Step 3: Calculate magnitudes from $F = q_1 q_2/4\pi\varepsilon_0 r^2$:

$$F_1 = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(6.0 \times 10^{-12} \text{ C}^2)}{(1^2 + 2^2) \text{ m}^2} = 1.08 \times 10^{-2} \text{ N},$$

$$F_2 = \frac{(9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(6.0 \times 10^{-12} \text{ C}^2)}{(1^2 + 4^2) \text{ m}^2} = 3.18 \times 10^{-3} \text{ N}.$$

Step 4: Add components to get the resultant:

$$\vec{F}_{total} = (0.97 \times 10^{-2} - 0.310 \times 10^{-2} \text{ N})\hat{i} + (0.475 \times 10^{-2} + 0.077 \times 10^{-2} \text{ N})\hat{j}$$
$$= (6.6 \times 10^{-3} \text{ N})\hat{i} + (5.5 \times 10^{-3} \text{ N})\hat{j}.$$

C(2). Calculate $\vec{E}$ at point P in Figure 6 due to the -1.00-μC charge (a = 2.00 m, b = 3.00 m).

Solution

Step 1: Choose axes as in Figure 7. The direction of $\vec{E}$ at P is the direction in which a positive charge would move if placed at P. The magnitude of $\vec{E}$ at P is $(1/4\pi\varepsilon_0)$[1.00 μC/(distance)^2] (which is the ratio of force $\vec{F}_q$ on a very small test charge q divided by q: $E = F_q/q$).

Figure 6

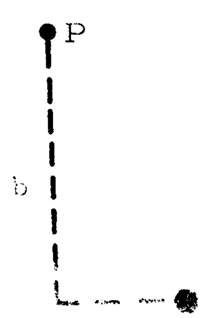
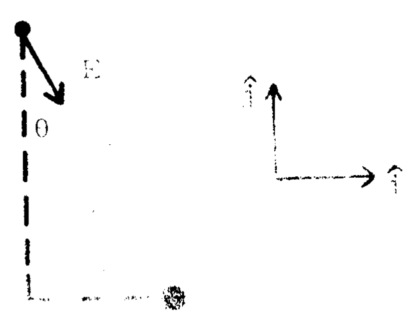


Figure 7



Step 2: Sketch $\vec{E}$ as in Figure 7 and calculate

$$\vec{E} = +(E \sin \theta)\hat{i} - (E \cos \theta)\hat{j}.$$

Step 3: Calculate the magnitude from $E = (1/4\pi\varepsilon_0)(q/r^2)$:

$$E = (9.0 \times 10^9 \text{ N m}^2/\text{C}^2)(10^{-6} \text{ C})/(2^2 + 3^2) \text{ m}^2 = 0.69 \times 10^3 \text{ N/C}.$$

Step 4: Calculate the result:

$$E = -[(0.69 \times 10^3 \text{ N/C})(2/\sqrt{13})]\hat{i} + [(0.69 \times 10^3 \text{ N/C})(3/\sqrt{13})]\hat{j}$$

$$= +(0.38 \times 10^3 \text{ N/C})\hat{i} - (0.58 \times 10^3 \text{ N/C})\hat{j}.$$

D(3). A stationary particle whose mass is 0.100 kg and whose charge is +0.300 C is suspended by a massless string under gravity in the presence of an electric field of magnitude 1.00 N/C as shown in Figure 8. Calculate the angle θ.

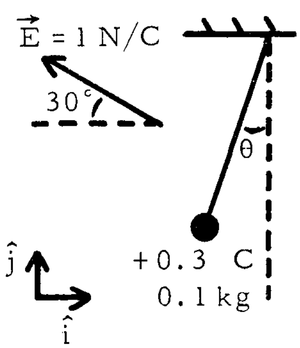


Figure 8

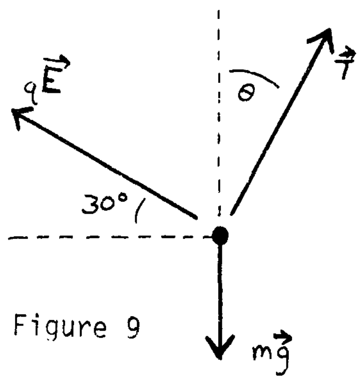


Figure 9

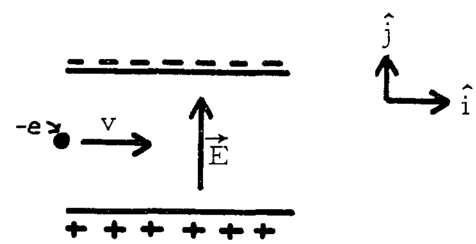


Figure 10

Solution

The sum of all forces must be zero for the particle at rest. First, we pick a coordinate system, taking +x to the right and +y upward, and draw a free-body diagram as in Figure 9. Then we add up all the forces acting on the particle:

	x component	y component
electrical ($\vec{F}_e = q\vec{E}$):	$-(0.300 \text{ C})(1.00 \text{ N/C})(\cos 30°)$	$+(0.300 \text{ C})(1.00 \text{ N/C})(\cos 60°)$
tension:	$T \sin \theta$	$T \cos \theta$
gravity:	0	$-(0.100 \text{ kg})(9.8 \text{ m/s}^2)$

Since the x and y components add to zero, we get

$$T \sin \theta = (0.300 \text{ N})(\cos 30°), \quad T \cos \theta = (0.98 \text{ N} - 0.300 \text{ N})(\cos 60°),$$

$$\tan\theta = \frac{(0.300)(\sqrt{3}/2)}{0.98 - 0.300(0.50)} = \frac{(0.300)(0.866)}{0.83} = \frac{0.260}{0.83} = 0.310, \qquad \theta = 17.0°.$$

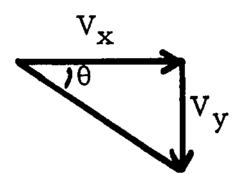
E(3). The diagram in Figure 10 shows an electron traveling with velocity 8.0×10^6 m/s in the x direction through a pair of deflecting plates 2.00 cm long. Assuming the electric field between the deflecting plates to be constant and equal to 9800 N/C in the +y direction, calculate
(a) the time the electron spends between the deflecting plates;
(b) the acceleration of the electron when between the plates;
(c) The electron's y component of velocity when it emerges from the plates;
(d) the angle between the electron's initial velocity and its velocity upon emerging;
(e) the amount the electron is deflected in the y direction when it emerges.
(f) Assuming the electron to enter the plates at x = 0, y = 0, find the equation for y(x).
The charge on an electron is -1.60×10^{-19} C; the mass of an electron is 9.1×10^{-31} kg.

Solution

(a) Since there is no acceleration in the x direction, the x component of velocity is constant at 8.0×10^6 m/s. Since only 0.0200 m needs to be traveled, the time is

$$t = \frac{2.00 \times 10^{-2}\ \text{m}}{8.0 \times 10^6\ \text{m/s}} = 2.50 \times 10^{-9}\ \text{s}.$$

Figure 11



(b) $a_x = 0$, $a_y = (\text{force})_y/m$,

$$a_y = \frac{qE_y}{m} = \frac{(-1.60 \times 10^{-19}\ \text{C})(9800\ \text{N/C})}{9.1 \times 10^{-31}\ \text{kg}} \equiv 1.70 \times 10^{15}\ \text{m/s}^2.$$

(c) $v_y = v_{0y} + a_y t = 0 + (-1.70 \times 10^{15}\ \text{m/s}^2)(2.50 \times 10^{-9}\ \text{s}) = -4.3 \times 10^6\ \text{m/s}.$

(d) See Figure 11.

$$\tan\theta = \frac{v_y}{v_x} = \frac{-4.3 \times 10^6\ \text{m/s}}{8.0 \times 10^6\ \text{m/s}} = -0.54; \qquad \theta = 28.4° \text{ below the horizontal.}$$

(e) $y = y_0 + v_{0y}t + (1/2)a_y t^2 = 0 + 0 - (1/2)(1.70 \times 10^{15}\ \text{m/s}^2)(2.50 \times 10^{9})^2 \equiv 0.53$ cm.

(f) $x = x_0 + v_{0x}t + (1/2)a_x t^2, \quad v_{0x} = 8.0 \times 10^6\ \text{m/s}, \quad a_x = 0, \quad x_0 = 0.$

$y = y_0 + v_{0y}t + (1/2)a_y t^2, \quad y_0 = v_{0y} = 0, \quad a_y = qE_y/m, \quad x = v_{0x}t,$

$$t = \frac{x}{v_{0x}}, \qquad y = \frac{1}{2}a_y t^2 = \frac{1}{2}\left(\frac{a_y}{v_{0x}^2}\right)x^2 = \left(\frac{a_y}{2v_{0x}^2}\right)x^2 = \left(\frac{qE_y}{2mv_{0x}^2}\right)x^2,$$

and

$$\frac{qE_y}{2mv_{0x}^2} = \frac{(-1.60 \times 10^{-19}\ \text{C})(9800\ \text{N/C})}{2(9.1 \times 10^{-31}\ \text{kg})(8.0 \times 10^6\ \text{m/s})^2} = \frac{-1.60}{(128)(9.1)} \times 10^4 = -13.8\ \text{m}^{-1}.$$

Thus $y = -13.8x^2\ \text{m}^{-1}$.

F(3). An electron ($m = 9.1 \times 10^{-31}$ kg and $q = -1.6 \times 10^{-19}$ C) with $\vec{v} = 10^6\hat{i}$ m/s enters a region of space with uniform electric field $\vec{E} = 5.0\hat{i}$ N/C.

(a) How much time will it take for the electron to be stopped by the electric field?
(b) How far will it have traveled in coming to rest?
(c) How much work is done on the electron in bringing it to rest?
(d) What was the kinetic energy of the electron at the start of the problem?

Solution

(a) $$a_x = \frac{qE_x}{m} = \frac{(-1.60 \times 10^{-19}\ \text{C})(5.0\ \text{N/C})}{9.1 \times 10^{-31}\ \text{kg}} = -0.88 \times 10^{12}\ \text{m/s}^2;$$

$$t = \frac{v_x}{a_x} = \frac{10^6\ \text{m/s}}{0.88 \times 10^{12}\ \text{m/s}^2} = 1.12 \times 10^{-6}\ \text{s}.$$

(b) $$x = x_0 + v_x t + a_x(t^2/2),$$

$$x - x_0 = (10^6\ \text{m/s})(1.12 \times 10^{-6}\ \text{s}) + (-0.88 \times 10^{12}\ \text{m/s}^2)(1.12 \times 10^6\ \text{s})^2 = 0.57\ \text{m}.$$

(c) For constant force

$$W = Fd = qEd = (-1.60 \times 10^{-19}\ \text{C})(5.0\ \text{N/C})(0.57\ \text{m}) = 4.6 \times 10^{-19}\ \text{J}.$$

(d) $$(KE)_i = mv^2/2 = [(9.1 \times 10^{-31}\ \text{kg})/2](10^6\ \text{m/s})^2$$

$$= 4.6 \times 10^{-19}\ \text{kg m/s}^2 = 4.6 \times 10^{-19}\ \text{J}.$$

Note that the work done equals the change in KE.

Problems

G(2). Calculate the total force on the -2.00-μC charge in Figure 12 ($a = 3.00$ m).

H(2). Calculate the electric field at point P in Figure 13 ($a = 1.00$ m, $b = 3.00$ m, $c = 2.00$ m). Hint: follow the steps in the Solution to Problem A using $E = q/4\pi\varepsilon_0 r^2$ (instead of F).

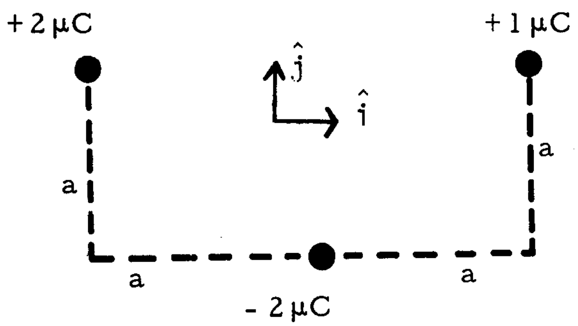


Figure 12

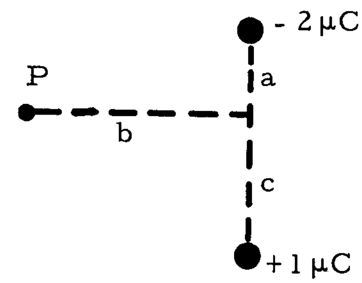


Figure 13

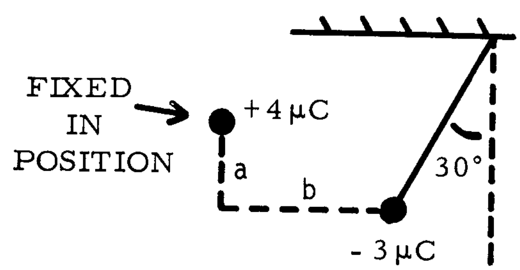


Figure 14

I(3). A particle of mass m and charge -3.00 μC is suspended at rest by a massless string as shown in Figure 14, in the presence of gravity; the fixed charge is +4.0 μC. Find the mass m (a = 2.00 m, b = 3.00 m).

J(3). An electron ($m = 9.1 \times 10^{-31}$ kg)($q = -1.60 \times 10^{-19}$ C) circles a stationary proton ($q = +1.60 \times 10^{-19}$ C) at a distance of 5.3×10^{-11} m. What is the electron's speed? Hint: Recall that particles traveling in a circle accelerate toward the center (worked out in the module Planar Motion). Solve by equating this "centripetal" acceleration times the mass to the attractive electrical force.

Solutions

G(2). $\vec{F}_{total} = [(-7.1 \times 10^{-4})\hat{i} + (2.10 \times 10^{-3})\hat{j}]$ N.

H(2). $\vec{E}_p = [(1.13 \times 10^{3})\hat{i} + (9.5 \times 10^{2})\hat{j}]$ N/C.

I(3). $m = 1.70 \times 10^{-3}$ kg.

J(3). $v = 2.20 \times 10^{6}$ m/s.

PRACTICE TEST

1. Calculate the total force on the -1.00-μC charge in Figure 15 (a = 3.00 m, b = 5.0 m, c = 2.00 m, d = 4.0 m).

2. A 0.400-kg mass has a charge +Q and is supported vertically by a massless string, with a massless spring attached on which a 3.00-N force is exerted, as in Figure 16. There is also a -5.0-μC charge, located as shown. Find the charge Q.

3. What is the main difference between a conductor and a perfect insulator?

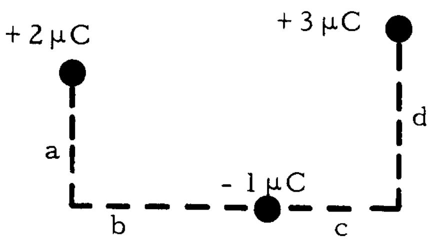


Figure 15

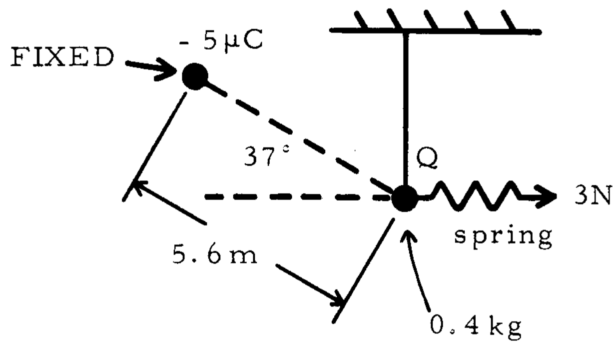


Figure 16

Practice Test Answers

1. $F_{total} = [(9.0 \times 10^{-3})(-\frac{10}{34\sqrt{34}} + \frac{6}{20\sqrt{20}})\hat{i} + (9.0 \times 10^{-3})(\frac{+6}{34\sqrt{34}} + \frac{12}{20\sqrt{20}})\hat{j}]$ N

$= [+(1.50 \times 10^{-4})\hat{i} + (1.50 \times 10^{-3})\hat{j}]$ N.

2. $3.00\text{ N} = \cos 37° \left(\frac{(5.0 \times 10^{-6}\text{ C})(9.0 \times 10^{9}\text{ N m}^2/\text{C}^2)}{(5.6\text{ m})^2}\right)Q, \quad Q = \frac{(5.6)^2(5/4)3}{45 \times 10^3}\text{ C} = 2.61\text{ C}.$

3. In a conductor, charges are free to move in response to an electric field; in a perfect insulator, charges are not free to move.

COULOMB'S LAW AND THE ELECTRIC FIELD

Date ____________

Mastery Test Form A

pass recycle

1 2 3

Name ____________ Tutor ____________

1. Determine the electric field at point P in Figure 1 (a = 3.00 m, b = c = 4.00 m) Show your work.

2. In Figure 2, a small sphere of mass 1.00 g carries a charge of 20.0 μC and is attached to a 5.0-cm-long silk fiber. The other end of the fiber is attached to a large vertical conducting plate that provides a uniform horizontal field of 10^3 N/C. Find the angle the fiber makes with the vertical.

3. What is the main difference between a conductor and a perfect insulator?

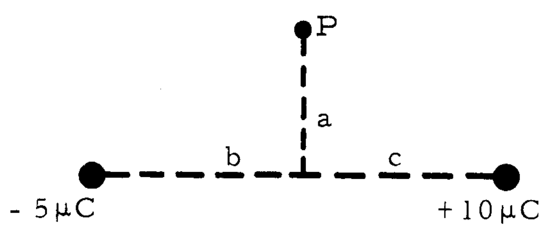


Figure 1

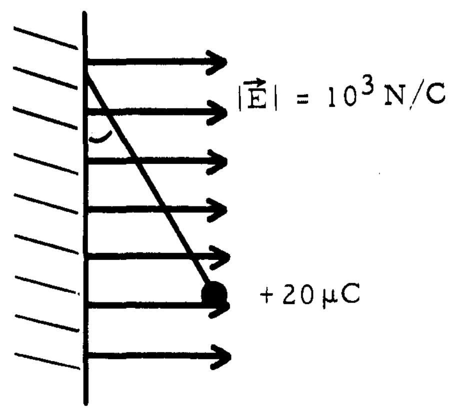


Figure 2

COULOMB'S LAW AND THE ELECTRIC FIELD Date ________________

Mastery Test Form B

pass recycle

1 2 3

Name ______________________________ Tutor ________________________

1. Determine the force on the $+2.00$-μC charge at point P in Figure 1. Show your work. ($Q = 4.0 \times 10^{-9}$ C, $a = 0.80$ m, $b = 0.60$ m.)

2. The object in Figure 2 with mass $m = 0.100$ kg and charge $q = +5.0 \times 10^{-3}$ C is fired upward at an angle of 30° with the horizontal with an initial speed of 400 m/s in a vertically downward electric field of 2.00×10^{5} N/C. How high does it rise?

3. What is the main difference between a conductor and a perfect insulator?

Figure 1

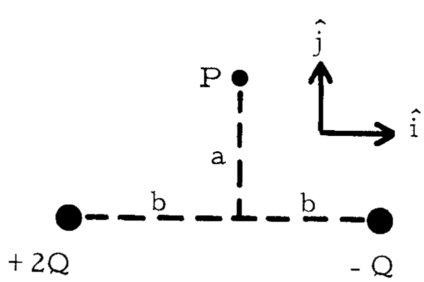
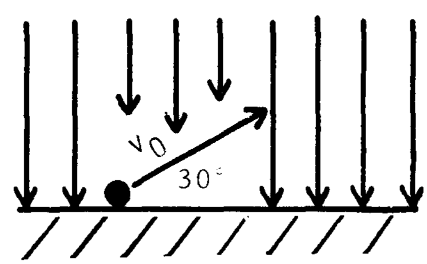


Figure 2



COULOMB'S LAW AND THE ELECTRIC FIELD Date ______________

Mastery Test Form C

pass recycle

1 2 3

Name ______________________________ Tutor ______________________

1. Determine the electric field at point P in Figure 1. Show your work. ($Q_1 = 4.0 \times 10^{-10}$ C, $Q_2 = -8.0 \times 10^{-10}$ C, $Q_3 = 10.0 \times 10^{-10}$ C.)

2. A proton in Figure 2 ($m = 1.67 \times 10^{-27}$ kg) is projected horizontally with velocity $v_0 = 10^7$ m/s into a 10^5-N/C uniform field directed vertically between the parallel plates 20.0 cm long. What is the angle the proton velocity makes with the horizontal when it emerges from the other side? Neglect any fringing field effects. (<u>Note</u>: $\tan\theta = v_y/v_x$. For small θ, $\tan\theta \simeq \theta$.)

3. What is the main difference between a conductor and a perfect insulator?

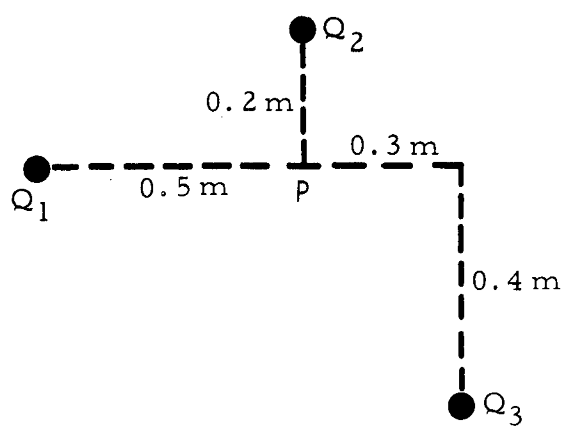


Figure 1

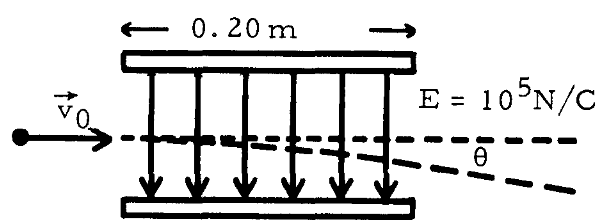


Figure 2

COULOMB'S LAW AND THE ELECTRIC FIELD   Date ________________

Mastery Test   Form D

pass   recycle

1   2   3

Name ______________________________   Tutor ______________________________

1. A -1.00-μC charge is at the center of a circular arc of radius R = 3.00 m in Figure 1. The +3.00-μC and -2.00-μC charges are located <u>on</u> the arc, as shown. Calculate the force on the -1.00-μC charge.

2. An electron ($q = -1.60 \times 10^{-19}$ C) experiences a force of ($+3.00 \times 10^{-16}\hat{i}$) N in a certain region of space.
   (a) Find $\vec{E}$ in this region.
   (b) Assuming that $\vec{E}$ is constant, and that the electron's motion takes it through the origin and also through the point x = 5.0 m, y = 4.0 m, how much work is done on the electron by the electric field in passing between these points?

3. What is the main difference between a conductor and a perfect insulator?

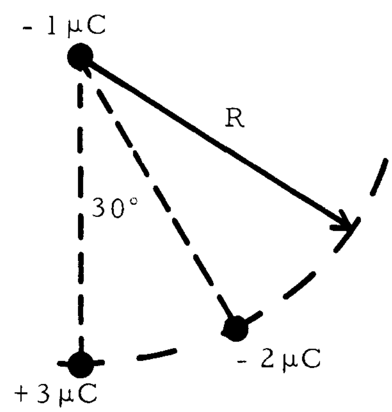


Figure 1

COULOMB'S LAW AND THE ELECTRIC FIELD Date ________________

Mastery Test Form E pass recycle

1 2 3

Name ______________________________ Tutor ______________________________

1. Find the magnitude and direction of the field $\vec{E}$ at point A in Figure 1.

2. A charged object, with $q = +1.00 \times 10^{-4}$ C and $m = 0.50$ kg, is released at rest in a uniform electric field with intensity $E = 3.00 \times 10^{4}$ N/C, directed upward. This is done near the surface of the Earth.
   (a) In which direction does it move?
   (b) How long does it take to move 4.5 m?

3. What is the main difference between a conductor and a perfect insulator?

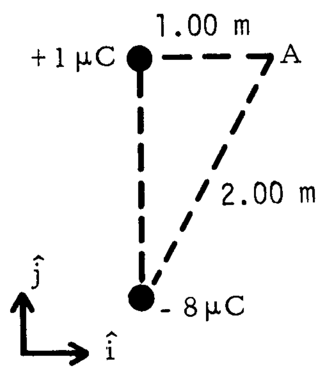


Figure 1

MASTERY TEST GRADING KEY - Form A

1. What To Look For: Invite the student to recheck his work if you spot a single sign error.

   Solution:

$$\vec{E}_p = \frac{(9.0 \times 10^9)(10^{-5})(4/5)}{25}\hat{i} + \frac{(9.0 \times 10^9)(10^{-5})(3/5)}{25}\hat{j} - \frac{(9.0 \times 10^9)(5 \times 10^{-6})(4/5)}{25}\hat{i}$$

$$- \frac{(9.0 \times 10^9)(5 \times 10^{-6})(3/5)}{25}\hat{j} \equiv (108/25 \times 10^3)\hat{i} + (27/25 \times 10^3)\hat{j}$$

$$\vec{E}_p = [-(4.3 \times 10^3)\hat{i} + (1.08 \times 10^3)\hat{j}] \text{ N/C.}$$

2. Solution: $\Sigma F_x = 0 = (2.00 \times 10^{-5}\text{ C})(10^3\text{ N/C}) - T \sin\theta = 0,$

   $\Sigma F_y = 0 = T \cos\theta - (10^{-3}\text{ kg})(9.8\text{ m/s}^2).$

   Thus $\tan\theta = 2.05$ and $\theta = 64°$.

3. Solution: In a conductor, charges are free to move in response to an electric field; in a perfect insulator, charges are not free to move.

MASTERY TEST GRADING KEY - Form B

1. Solution:

$$\vec{F}(\text{due to } 8.0\ \mu C) = \left(\frac{3}{5}\right)\frac{(9.0 \times 10^{9})(16.0 \times 10^{-15})}{1^2}\hat{i} + \left(\frac{4}{5}\right)\frac{(9.0 \times 10^{9})(16.0 \times 10^{-15})}{1^2}\hat{j}$$

$$= [(0.35/4 \times 10^{-3})\hat{i} + (0.46/4 \times 10^{-3})\hat{j}]\ \text{N}.$$

$$\vec{F}(\text{due to } -4.0\ \mu C) = \left(\frac{3}{5}\right)\frac{(9.0 \times 10^{9})(8.0 \times 10^{-15})}{1^2}\hat{i} - \left(\frac{4}{5}\right)\frac{(9.0 \times 10^{9})(8.0 \times 10^{-15})}{1^2}\hat{j}$$

$$= [(0.173/4 \times 10^{-3})\hat{i} - (0.231/4 \times 10^{-3})\hat{j}]\ \text{N}.$$

$$\vec{F}_{total} = (0.13 \times 10^{-3})\hat{i} + (0.06 \times 10^{-3})\hat{j} = [(1.3 \times 10^{-4})\hat{i} + (6.0 \times 10^{-4})\hat{j}]\ \text{N}.$$

2. What To Look For: $v_{0y} = v_0 \sin 30° = 200$ m/s. Student could also use $v_y^2 - v_{0y}^2 = 2\ a_y y$, $v_y = 0$, $y_{max} = v_{0y}^2/-2\ a_y = 2.00$ m.

Solution: The time for a_y to take v_{0y} to zero is $\left|\frac{v_{0y}}{a_y}\right|$.

$$a_y = +\frac{qE}{m} - 9.8\ \text{m/s}^2 - \frac{(5.0 \times 10^{-3}\ \text{C})(2.0 \times 10^{5}\ \text{N/C})}{0.100\ \text{kg}} - 9.8\ \text{m/s}^2$$

$$= -9.8\ \text{m/s}^2 - 10^4\ \text{m/s}^2 \simeq -10^4\ \text{m/s}^2,$$

$$t = \frac{200\ \text{m/s}}{10^4\ \text{m/s}^2} = 0.0200\ \text{s},$$

$$y_{max} = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = (200\ \text{m/s})(0.200\ \text{s}) - \frac{(10^4\ \text{m/s})(0.0200\ \text{s})^2}{2}$$

$$= 4.0\ \text{m} - 2.00\ \text{m} = 2.00\ \text{m}.$$

3. Solution: In a conductor, charges are free to move in response to an electric field; in a perfect insulator charges are not free to move.

MASTERY TEST GRADING KEY - Form C

$\hat{z}$ $\hat{j}$

1. Solution:

$$\vec{E}_P = \frac{(9.0 \times 10^9)(4.0 \times 10^{-10})}{(1/2)^2}\hat{i} - \left(\frac{4}{5}\right)\frac{(9.0 \times 10^9)(10^{-9})}{(1/2)^2}\hat{j} + \left(\frac{3}{5}\right)\frac{(9.0 \times 10^9)(10^{-9})}{(1/2)^2}\hat{i}$$

$$+ \frac{(9.0 \times 10^9)(8.0 \times 10^{-10})}{(1/5)^2}\hat{j} = (-7.2\hat{i} + 208.8\hat{j})$$

2. Solution: Travel time between plates is

$t = x/v_{0y} = 0.200 \text{ m}/10^7 \text{ m/s} = 2.00 \times 10^{-8}$ s.

$a_y = qE/m = [(1.60 \times 10^{-19})/(1.67 \times 10^{-27})] \ (-10^5) = -0.96 \times 10^{13} \text{ m/s}^2$,

$v_y = v_{0y} + a_y t = -1.90 \times 10^5$ m/s, $\tan\theta = \dfrac{1.90 \times 10^5 \text{ m/s}}{10^7 \text{ m/s}} = 1.90 \times 10^{-2}$,

$\therefore \theta = 1.90 \times 10^{-2}$ rad $= 1.10°$ below the horizontal.

3. Solution: In a conductor charges are free to move in response to an electric field; in a perfect insulator, charges are not free to move.

$\dfrac{x}{V_{0x}}$

MASTERY TEST GRADING KEY - Form D

1. Solution: Choose axes x and y.

$$\vec{F} = -\frac{(9.0 \times 10^9)(3.00 \times 10^{-12})}{9.0\ m^2}\hat{i} + \frac{\sqrt{3}}{2}\ \frac{(9.0 \times 10^9)(2.00 \times 10^{-12})}{9.0\ m^2}\hat{j}$$

$$- \frac{(9.0 \times 10^9)(2.00 \times 10^{-12})}{9.0\ m^2}\ \frac{1}{2}\hat{i}$$

$$\vec{F} = [-10^{-3}\hat{i} - (1.30 \times 10^{-3})\hat{j}]\ N.$$

2. Solution:

(a) $\vec{E} = \vec{F}/q = (3.00 \times 10^{-16})/(-1.60 \times 10^{-19})\hat{i} = -(1.88 \times 10^3)\hat{i}$ N/C.

(b) For a constant force $W = Fd = (3.00 \times 10^{-16}\ N)(5.0\ m) = 1.50 \times 10^{-15}$ J.

3. Solution: In a conductor charges are free to move in response to an electric field; in a perfect insulator, charges are not free to move.

MASTERY TEST GRADING KEY - Form E

1. Solution:

$$\vec{E} = -\left(\frac{1}{2}\,\frac{(8.0\times 10^{-6})(9.0\times 10^{9})}{4}\right)\hat{i} + \frac{(9.0\times 10^{9})(10^{-6})}{1}\hat{i}$$

$$-\ \frac{(9.0\times 10^{9})(8.0\times 10^{-6})}{4}\left(\frac{\sqrt{3}}{2}\right)\hat{j} = -(1.56\times 10^{4})\hat{j}\ \text{N/C}.$$

2. Solution:

(a) $a_y = \frac{qE}{m} + g = \frac{(10^{4}\ \text{C})(3.00\times 10^{4}\ \text{N/C})}{0.5\ \text{kg}} - 9.8\ \text{m/s}^2 = (6.0 - 9.8)\ \text{m/s}^2 = -3.8\ \text{m/s}^2.$

It moves down.

(b) $y = y_0 + v_{0y} + \frac{1}{2}a_y t^2, \quad 4.5\ \text{m} = \frac{3.8t^2}{2}, \quad t^2 = \frac{9}{3.8}, \quad t = 1.54\ \text{s}.$

3. Solution: In a conductor, charges are free to move in response to an electric field; in a perfect insulator, charges are not free to move.