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($R - I$) COLORS OF CEPHEIDS AND YELLOW SUPERGIANTS IN OPEN CLUSTERS

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Received 1975 May 29; revised 1975 July 3

ABSTRACT

($R - I$) and ($b - y$) observations of nine Cepheids which are members of open clusters, 14 field Cepheids, and 34 nonvariable yellow giants in or near open clusters are presented. Color-color relations for the Cepheids are found to differ significantly from star to star. Additionally, the nonvariables may differ from the Cepheid color-color relation. Pulsational masses are determined for the Cepheids using both ($R - I$) and ($B - V$) colors as temperature indicators. For some of the shorter period stars the ($R - I$) masses agree with evolutionary masses, which is not the case for the ($B - V$) masses. A small increase (about 0.1 mag) in the absolute magnitudes of the short-period Cepheids is sufficient to bring agreement between the ($R - I$) masses and the evolutionary masses for the bulk of the stars, but the long-period stars still have low masses. However, it is suggested that this may be due to systematic errors in the color excesses. The cluster NGC 129 is found to have a nonvariable star within the instability strip in addition to the Cepheid DL Cas. Because the absolute magnitude and intrinsic color are well determined from the cluster, this is the best established case of a nonvariable in the Cepheid strip. Several other clusters contain a number of yellow giants, and they are compared with evolutionary tracks.

Subject headings: clusters: open — stars: cepheids — stars: supergiants

I. INTRODUCTION

The intrinsic colors of Cepheids are important to a number of problems. Chief among these is the question of the masses of Cepheids calculated from the pulsation theory. For this purpose temperatures derived from the ($B - V$) colors have been almost universally used. However, it has been shown (Schmidt 1973, hereafter Paper I) that temperatures calculated from ($B - V$) yield considerably different pulsational masses from those calculated from ($R - I$).

In view of the large effects of line blanketing in the B and V bands, it is likely that ($R - I$) is a more reliable temperature indicator than ($B - V$). In addition, some evidence has been found (Schmidt 1972b) that the ($B - V$)-temperature relation is not the same for all Cepheids and that it differs between Cepheids and nonvariable supergiants in the same temperature range. Unfortunately, the widespread use of ($B - V$) is dictated by the absence of ($R - I$) colors for more than a few Cepheids. A further source of uncertainty arises from the fact that color excesses of these stars are uncertain except for those which are members of clusters. The main purpose of the present paper is to provide ($R - I$) data for a number of Cepheids, with special emphasis on those in open clusters, and to provide comparable information for nonvariable supergiants which are or may be cluster members.

* Visiting astronomer at Kitt Peak National Observatory which is operated by the Association of Universities for Research in Astronomy, Inc. under contract from the National Science Foundation.

II. THE OBSERVATIONS

Photometric observations were made of 23 Cepheids using the R and I filters of the Johnson $UBVRI$ system and the b and y filters of the Strömberg $uvby$ system. The measured colors and the R magnitudes were transformed to the standard system in the usual way. These observations were made at Kitt Peak National Observatory using the 92 cm telescope with a two-channel photometer. One channel was used with a red-sensitive photomultiplier to measure R and I while the other channel had a blue photocell and was used to obtain b and y . Table 1 contains the colors and magnitudes of the Cepheids. The phases were obtained from the ephemerides given in the third edition of the *General Catalogue of Variable Stars*.

Of the 23 Cepheids, eight are members of open clusters. Additionally, Racine (1968) has inferred the color excess and distance modulus of SU Cas from its apparent association with early-type stars near a reflection nebula. These nine stars are listed in Table 2. The distance moduli and color excesses come from the clusters of which these stars are members (Sandage and Tammann 1969). The corresponding data for the remaining 14 stars which are not members of clusters are given in Table 3. The values of E_{B-V} were taken from Parsons and Bouw (1971) for SZ Tau and RT Aur, from Fernie (1970) for S Vul, and from Sandage and Tammann (1968) for the remainder of the stars. The absolute magnitudes of the stars in Table 3 were obtained from the Sandage and Tammann (1969) calibration of the period-luminosity-color relation. In Tables 2 and 3 the magnitude mean of $R - I$, $\langle R - I \rangle$, is also given. The color curves measured here

TABLE 1
PHOTOMETRIC DATA FOR CEPHEIDS

SU Cas				EV Sct				VY Per				U Sgr			
J.D.	Phase	R	b-y	J.D.	Phase	R	b-y	J.D.	Phase	R	b-y	J.D.	Phase	R	b-y
-2441900	0.16	5.18	0.388	-2441900	0.81	8.97	0.911	-2441900	0.13	9.59	1.118	-2441900	0.60	5.87	0.757
59.89	0.71	5.44	0.469	60.62	0.14	9.14	0.854	59.88	0.31	9.77	1.169	59.63	0.75	6.02	0.749
60.89	0.71	5.40	0.509	61.63	0.45	9.01	0.823	60.87	0.31	9.77	1.213	60.61	0.90	5.83	0.697
62.92	0.22	5.17	0.446	62.60	0.78	9.00	0.814	62.89	0.86	9.54	1.084	61.62	0.04	5.52	0.602
63.90	0.78	5.51	0.509	63.60	0.04	9.07	0.852	64.89	0.04	9.58	1.089	62.60	0.19	5.58	0.643
70.86	0.30	5.27	0.479	70.60	0.36	9.07	0.822	70.84	0.11	9.68	1.050	63.60	0.23	5.58	0.617
71.87	0.81	5.45	0.501	71.60	0.69	8.94	0.802	71.85	0.30	9.87	1.134	70.60	0.38	5.64	0.691
72.86	0.32	5.31	0.477	72.59	0.01	9.11	0.827	72.85	0.48	9.98	1.193	71.59	0.53	5.82	0.696
73.86	0.83	5.42	0.503	73.58	0.33	9.04	0.833	73.85	0.66	10.08	1.193	72.59	0.67	5.96	0.757
74.85	0.36	5.32	0.482	74.59	0.66	9.00	0.838	74.84	0.84	9.73	1.084	73.58	0.82	5.97	0.794
75.88	0.87	5.36	0.476	75.59	0.98	9.11	0.851	75.86	0.02	9.55	1.105	74.58	0.12	5.65	0.602
76.88	0.37	5.34	0.479	76.58	0.30	9.06	0.819	76.86	0.20	9.68	1.133	75.59	0.56	0.647	0.654
77.85	0.37	5.34	0.486	77.59	0.15	4.79	0.346	77.84	0.38	9.82	1.173	76.58	0.33	5.64	0.690
SZ Tau				RT Aur				DL Cas				Y Sct			
J.D.	Phase	R	b-y	J.D.	Phase	R	b-y	J.D.	Phase	R	b-y	J.D.	Phase	R	b-y
-2441900	0.30	5.87	0.607	-2441900	0.33	4.98	0.411	-2441900	0.50	7.98	0.807	-2441900	0.28	8.22	1.044
60.00	0.60	5.73	0.542	60.00	0.60	5.17	0.349	59.82	0.62	8.09	0.793	61.64	0.37	8.31	1.099
60.96	0.23	5.83	0.598	61.00	0.13	4.80	0.314	60.79	0.74	8.17	0.809	62.61	0.47	8.41	1.152
62.93	0.54	5.73	0.549	62.98	0.28	4.95	0.387	61.78	0.87	7.99	0.775	63.61	0.15	8.02	1.203
63.92	0.77	5.67	0.494	72.01	0.55	5.12	0.419	62.77	0.00	7.79	0.738	70.61	0.24	8.16	0.987
70.93	0.09	5.78	0.571	73.00	0.82	4.97	0.356	63.83	0.12	7.80	0.740	71.60	0.34	8.31	1.065
71.93	0.40	5.86	0.587	74.00	0.09	4.77	0.325	64.78	0.86	8.05	0.773	72.60	0.44	8.43	1.138
72.91	0.71	5.66	0.523	75.00	0.36	5.02	0.399	70.75	0.99	7.78	0.723	73.59	0.53	8.47	1.187
73.89	0.04	5.70	0.554	75.99	0.62	5.18	0.418	71.75	0.12	7.77	0.743	74.60	0.63	8.49	1.176
74.91	0.35	5.89	0.592	76.99	0.89	4.74	0.293	72.77	0.24	7.83	0.758	75.60	0.73	8.33	1.099
75.91	0.66	5.71	0.538	77.96	0.15	4.79	0.346	73.76	0.36	7.90	0.773	76.58	0.82	8.26	1.024
76.89	0.99	5.75	0.555					74.76	0.49	8.00	0.803	77.59	0.82	8.26	1.004
77.90								76.73	0.61	8.12	0.824				
								77.73	0.71	8.20	0.820				
CF Cas				UY Per				Z Lac				VX Per			
J.D.	Phase	R	b-y	J.D.	Phase	R	b-y	J.D.	Phase	R	b-y	J.D.	Phase	R	b-y
-2441900	0.82	10.14	0.758	-2441900	0.46	9.98	1.111	-2441900	0.30	7.46	0.701	-2441900	0.54	8.20	0.709
59.78	0.01	9.87	0.752	59.88	0.64	10.19	1.149	59.68	0.40	7.59	0.724	59.86	0.64	8.14	0.723
60.71	0.22	10.00	0.745	60.88	0.02	9.69	1.027	60.88	0.30	7.72	0.769	60.85	0.91	8.15	0.736
61.75	0.42	10.13	0.816	62.92	0.20	9.77	1.078	61.69	0.49	7.80	0.766	63.88	0.00	8.22	0.732
62.72	0.64	10.30	0.891	63.89	0.50	10.05	1.038	62.71	0.58	7.74	0.730	64.80	0.55	8.31	0.726
63.77	0.94	10.04	0.772	70.85	0.69	10.23	1.133	63.65	0.67	7.52	0.803	70.83	0.64	8.26	0.722
64.78	0.06	9.91	0.763	72.85	0.87	10.08	1.076	64.77	0.77	7.52	0.868	71.84	0.74	8.20	0.717
70.74	0.27	9.99	0.821	73.86	0.06	9.73	1.021	70.66	0.31	7.52	0.724	72.84	0.83	8.13	0.724
71.74	0.48	10.16	0.843	74.85	0.24	9.90	1.073	71.69	0.41	7.67	0.758	73.84	0.92	8.27	0.765
72.76	0.68	10.27	0.860	75.87	0.44	9.99	1.112	72.71	0.50	7.66	0.766	74.83	0.01	8.24	0.796
73.76	0.89	10.03	0.777	76.87	0.62	10.14	1.133	73.71	0.59	7.79	0.765	75.85	0.10	8.33	0.813
74.75	0.99	10.06	0.777	77.85	0.80	10.21	1.122	74.70	0.68	7.69	0.727	76.86	0.19	8.50	0.831
75.74	0.29	10.06	0.817					75.69	0.77	7.48	0.657	77.83	0.19	8.50	0.831
76.72	0.50	10.22	0.853					76.74	0.87	7.48	0.657				
77.72								77.70	0.96	7.23	0.607				

TABLE 1—Continued

SV Per					RV Cas					RW Cas					CD Cyg				
J.D.	Phase	R	R-I	b-y	J.D.	Phase	R	R-I	b-y	J.D.	Phase	R	R-I	b-y	J.D.	Phase	R	R-I	b-y
-2441900					-2441900					-2441900					-2441900				
59.87	0.26	7.92	0.697	0.655	60.78	0.36	8.72	0.927	0.944	59.85	0.56	8.44	0.847	0.957	59.65	0.42	7.78	0.336	0.932
60.96	0.13	8.09	0.752	0.775	61.77	0.44	8.85	0.966	0.966	60.84	0.62	8.56	0.825	0.966	60.66	0.43	7.87	0.360	0.946
62.93	0.52	8.18	0.761	0.810	63.60	0.61	8.97	0.986	0.986	62.79	0.75	8.54	0.816	0.866	61.67	0.53	7.98	0.361	0.963
70.94	0.15	7.86	0.682	0.657	71.72	0.26	8.62	0.915	0.889	63.84	0.82	8.38	0.763	0.811	62.63	0.59	8.08	0.361	1.009
71.94	0.24	7.92	0.687	0.702	72.76	0.34	8.70	0.925	0.961	64.80	0.89	8.38	0.763	0.811	63.63	0.65	8.10	0.361	1.023
72.92	0.33	7.98	0.719	0.711	73.75	0.43	8.83	0.983	0.953	70.79	0.29	8.10	0.759	0.819	70.64	0.06	7.49	0.361	0.659
73.90	0.42	8.04	0.734	0.758	74.75	0.51	8.94	0.997	1.030	71.76	0.36	8.13	0.788	0.849	71.62	0.12	7.54	0.736	0.659
74.91	0.51	8.16	0.741	0.787	75.74	0.59	9.02	1.010	0.999	72.78	0.43	8.21	0.806	0.883	72.62	0.18	7.56	0.763	0.747
75.91	0.60	8.30	0.758	0.769	76.71	0.67	9.04	0.987	0.952	73.79	0.50	8.33	0.813	0.937	73.63	0.24	7.61	0.793	0.779
76.92	0.69	8.36	0.738	0.764	77.71	0.75	8.99	0.948	0.947	74.78	0.56	8.43	0.831	0.937	74.63	0.29	7.63	0.809	0.735
77.90	0.78	8.23	0.686	0.722						75.75	0.63	8.56	0.846	0.916	75.62	0.35	7.75	0.849	0.570
										76.85	0.70	8.56	0.819	0.944	76.61	0.41	7.73	0.853	0.307
										77.76	0.77	8.59	0.802	0.890	77.61	0.47	7.87	0.870	0.925

SZ Cas					TT Aql					CP Cep					SV Vul				
J.D.	Phase	R	R-I	b-y	J.D.	Phase	R	R-I	b-y	J.D.	Phase	R	R-I	b-y	J.D.	Phase	R	R-I	b-y
-2441900					-2441900					-2441900					-2441900				
59.87	0.84	8.62	1.036	0.938	60.64	0.49	6.18	0.836	0.958	60.67	0.90	9.33	1.033	1.018	59.64	0.95	5.70	0.703	0.720
60.86	0.91	8.58	1.019	0.894	61.65	0.56	6.30	0.870	0.981	61.68	0.96	9.33	1.017	1.021	60.65	0.98	5.77	0.708	0.690
62.87	0.06	8.42	1.004	0.902	62.62	0.63	6.39	0.870	1.004	63.64	0.07	9.00	0.906	0.936	61.66	0.00	5.76	0.737	0.702
63.88	0.13	8.32	1.001	0.906	63.62	0.70	6.42	0.861	0.995	70.65	0.46	9.16	1.044	1.142	62.63	0.02	5.82	0.733	0.782
64.88	0.21	8.45	1.011	0.941	70.63	0.21	5.84	0.759	0.810	71.63	0.52	9.26	1.096	1.146	63.62	0.04	5.82	0.742	0.791
70.84	0.64	8.68	1.059	1.025	71.61	0.29	5.92	0.800	0.863	72.63	0.57	9.30	1.104	1.211	70.64	0.20	5.98	0.821	0.937
71.85	0.72	8.66	1.042	1.013	72.61	0.36	5.87	0.815	0.779	73.64	0.63	9.44	1.138	1.197	71.62	0.22	5.91	0.843	0.942
72.84	0.79	8.72	1.051	1.027	73.62	0.43	6.11	0.834	0.951	74.64	0.68	9.44	1.125	1.194	72.62	0.24	5.90	0.845	0.961
73.84	0.86	8.67	1.020	1.005	74.60	0.50	6.19	0.856	0.983	75.63	0.74	9.49	1.107	1.173	73.63	0.26	5.94	0.855	0.962
74.84	0.94	8.61	1.017	0.968	75.61	0.59	6.32	0.870	0.994	76.63	0.80	9.44	1.084	1.158	74.63	0.29	5.93	0.852	0.993
75.86	0.01	8.45	0.993	0.919	76.59	0.65	6.39	0.872	1.000	77.62	0.85	9.40	1.049	1.122	75.62	0.31	5.98	0.863	0.997
76.86	0.08	8.41	0.988	0.924											76.60	0.33	5.98	0.870	1.011
77.84	0.16	8.38	0.969	0.930											77.61	0.35	6.01	0.877	1.012

CV Cas					TX Cyg					S Vul				
J.D.	Phase	R	R-I	b-y	J.D.	Phase	R	R-I	b-y	J.D.	Phase	R	R-I	b-y
-2441900					-2441900					-2441900				
61.76	0.09	9.85	1.197	0.971	59.72	0.26	7.54	1.345	1.222	60.64	0.89	7.61	1.151	1.226
62.73	0.16	9.90	1.200	1.049	60.66	0.32	7.63	1.340	1.229	61.66	0.90	7.57	1.154	1.207
63.78	0.23	9.91	1.250	1.082	61.67	0.39	7.70	1.398	1.010	62.63	0.92	7.55	1.138	1.248
70.72	0.72	10.49	1.305	1.259	63.64	0.53	7.87	1.430	1.412	63.62	0.93	7.52	1.131	1.231
71.72	0.78	10.30	1.243	1.163	70.65	0.09	7.43	1.246	1.096	70.63	0.04	7.31	1.078	1.134
72.75	0.86	10.34	1.239	1.139	71.63	0.07	7.40	1.257	1.105	71.62	0.05	7.32	1.090	1.123
73.74	0.93	9.76	1.105	0.886	72.63	0.14	7.49	1.303	1.200	72.61	0.06	7.30	1.084	1.136
74.74	0.00	9.80	1.123	0.958	73.64	0.21	7.54	1.328	1.215	73.62	0.08	7.31	1.082	1.126
75.72	0.06	9.82	1.144	0.993	74.64	0.28	7.55	1.350	1.263	74.63	0.09	7.28	1.085	1.128
76.71	0.13	9.90	1.199	1.059	75.62	0.34	7.65	1.378	1.295	75.61	0.11	7.30	1.084	1.110
77.71	0.20	9.90	1.231	1.074	76.61	0.41	7.71	1.403	1.361	76.60	0.12	7.29	1.090	1.125
					77.62	0.48	7.81	1.419	1.340					

$(R - I)$ COLORS OF CEPHEIDS

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TABLE 2
DATA FOR CEPHEIDS IN CLUSTERS

STAR	PERIOD	CLUSTER	TRUE DISTANCE MODULUS	E_{B-V}	$M_{<V>}$	$\langle R-I \rangle$	$\frac{\Delta(b-y)}{\Delta(R-I)}$	$\log \frac{L}{L_{\odot}}$	$\log \mathfrak{M}$		
									from $\langle B-V \rangle$	from $\langle B \rangle - \langle V \rangle$	from $\langle R-I \rangle$
SU Cas.....	1495	Reflection nebula	7.50	0.33	-2.54	0.48	0.98	2.897	0.59	0.58	0.55
EV Sct.....	3.09	NGC 6664	11.03	0.58	-2.62	0.83	...	2.950	0.49	0.50	0.69
CF Cas.....	4.88	NGC 7790	12.53	0.555	-3.08	0.82	1.37	3.148	0.53	0.45	0.67
UY Per.....	5.36	h and χ Per	11.90	0.98	-3.54	1.09	1.31	3.326	0.66	0.62	0.67
VY Per.....	5.53	h and χ Per	11.90	1.06	-3.91	1.15	1.58	3.467	0.78	0.76	0.80
U Sgr.....	6.74	IC 4725	8.98	0.55	-3.93	0.70	1.67	3.480	0.69	0.65	0.60
DL Cas.....	8.00	NGC 129	11.28	0.50	-3.84	0.77	1.63	3.459	0.65	0.63	0.74
VX Per.....	10.89	h and χ Per	11.90	0.58	-4.34	0.77	1.15	3.651	0.64	0.62	0.62
SZ Cas.....	13.63	h and χ Per	11.90	0.88	-4.71	1.03*	1.91	3.795	0.65	0.64	0.71

are not complete for 13 of the stars. In those cases, published $\langle B - V \rangle$ data were used to estimate $\langle R - I \rangle$. The procedure was to convert the published $\langle B - V \rangle$ colors to $(b - y)$ using

$$b - y = 0.654(B - V) - 0.053. \quad (1)$$

This transformation was determined from the non-variables measured in the present program and published values of $\langle B - V \rangle$. The $(b - y)$ curve so determined was then converted to $\langle R - I \rangle$ using the $(b - y) - \langle R - I \rangle$ relation for the particular star derived from the present photometry. This relation had to be extrapolated in some cases, and it was assumed to be linear. The resulting $\langle R - I \rangle$ curve was then used to extend the observations throughout the cycle, and the mean was obtained. When this was necessary, it is indicated by an asterisk following the value of $\langle R - I \rangle$ in Tables 2 and 3.

In addition to the Cepheids, 34 nonvariables which may be members of open clusters and which appear to be near the Cepheid region of the H-R diagram were also measured. The data for these stars are given in Table 4. Some are well-known cluster members while

others were included because they appeared to lie in the appropriate region of the color-magnitude diagrams given by Hoag *et al.* (1961). The first column of the table gives the number or letter identifying the star. Except for those stars with HD numbers, the designation comes from the first reference given in the last column of the table. The cluster in or near which the star is located is listed in the second column while the distance modulus and color excess of the cluster are in the third and fourth columns. For NGC 129, the cluster data were taken from Sandage and Tamman (1968) while for the others the compilation by Becker and Fenkart (1971) was used. For some of the clusters radial velocity or proper motion studies have been published which help determine membership. This is indicated in columns (5) and (6), where a P means that the data are inconclusive either way. For those stars with spectral types, we can determine whether the colors and magnitudes, together with the cluster reddening and distance modulus, are consistent with the spectral class. This is indicated in column (7). The V and $\langle B - V \rangle$ data are from published work while the R , $\langle R - I \rangle$, and $(b - y)$ data are the measurements made in the present program. Column

TABLE 3
DATA FOR FIELD CEPHEIDS

STAR	PERIOD	E_{B-V}	$M_{<V>}$	$\langle R-I \rangle$	$\frac{\Delta(b-y)}{\Delta(R-I)}$	$\log L/L_{\odot}$	$\log \mathfrak{M}$		
							from $\langle B-V \rangle$	from $\langle B \rangle - \langle V \rangle$	from $\langle R-I \rangle$
SZ Tau.....	3 ^a 15	0.31	-2.80	0.56	1.32	3.022	0.57	0.55	0.68
RT Aur.....	3.73	0.09	-3.13	0.36	1.28	3.154	0.62	0.58	0.69
Y Sct.....	10.34	0.80	-4.02	1.03*	1.81	3.546	0.69	0.63	0.71
Z Lac.....	10.88	0.48	-4.32	0.70*	1.82	3.649	0.68	0.65	0.67
SV Per.....	11.13	0.44	-4.55	0.68*	1.36	3.732	0.71	0.67	0.78
RY Cas.....	12.14	0.78	-4.66	0.89*	1.31:	3.778	0.72	0.68	0.65
TT Aql.....	13.75	0.55	-4.47	0.79*	1.76	3.726	0.74	0.66	0.68
CY Cas.....	14.38	1.10	-4.91	1.25*	1.95	3.880	0.76	0.70	0.87
TX Cyg.....	14.71	1.25	-4.90	1.33*	1.65	3.880	0.77	0.70	0.74
RW Cas.....	14.79	0.41	-4.35	0.76*	1.96	3.690	0.72	0.64	0.74
CD Cyg.....	17.07	0.64	-4.96	0.83*	1.86	3.916	0.80	0.70	0.68
CP Cep.....	17.86	1.08	-5.34	1.023*	2.04	4.046	0.79	0.74	0.32
SV Vul.....	45.04	0.64	-6.03	0.85*	2.07:	4.360	0.83	0.76	0.69
S Vul.....	68.83	0.90	-6.28	1.12*	2.40	4.473	0.78	0.75	0.68

TABLE 4
DATA ON NONVARIABLE STARS IN CLUSTERS

Star	Cluster (1)	True Distance Modulus (2)	Membership			E_{B-V} (3)	Spectral Type (7)	V (8)	$B-V$ (9)	R (10)	$R-I$ (11)	$b-y$ (12)	n (13)	References and Notes (14)
			V_R (4)	p.m. (5)	p.e. (6)									
A.....	NGC 129	11.28	Yes	...	Yes	0.50	F5 Ib	8.85	0.96	7.96	0.709	0.606	3	1
AA.....	NGC 129	11.28	No	...	No	0.50	K2 Ib	8.57	1.92	7.07	1.104	1.211	3	1
#5.....	NGC 225	9.00	...	...	...	0.29	...	9.87	0.622	9.07	0.904	0.634	2	2
HD 10494.....	NGC 634	12.02	...	...	Yes	0.89	F5 Ia	7.30	1.23	6.18	0.598	0.775	2	...
HD 11544.....	h and x Per	11.67	...	...	No	0.56	G2 Ib	6.82	1.15	5.94	0.598	0.694	2	3
208.....	NGC 752	7.90	...	Yes	P	0.03	K1 IV	8.94	1.08	8.80	0.519	0.567	2	4, 5
213.....	NGC 752	7.90	...	Yes	Yes	0.03	K0 III	9.01	0.99	8.26	0.536	0.580	2	4, 5
295.....	NGC 752	7.90	...	Yes	Yes	0.03	G9 III	9.29	0.96	8.56	0.520	0.550	2	4, 5
311.....	NGC 752	7.90	...	Yes	Yes	0.03	K0 III	9.04	1.03	8.25	0.542	0.613	2	4, 5
HD 14662.....	h and x Per	11.67	No	...	No	0.56	F7 Ib	6.34	0.90	5.56	0.524	0.557	2	3
HD 17971.....	h and x Per	11.67	...	...	P	0.56	F5 Ia	7.75	1.06	6.68	0.854	0.674	2	3
HD 18391.....	h and x Per	11.67	Yes	...	No	0.56	G0 Ia	6.89	1.95	5.30	1.200	1.210	2	3
HD 25498.....	NGC 1502	9.68	...	...	NO	0.74	K	7.93	1.19	6.94	0.745	0.716	2	...
#4.....	NGC 1528	9.66	...	...	...	0.30	...	9.97	0.92	9.20	0.608	0.588	2	2
HD 27277.....	NGC 1545	9.50	...	...	P	0.36	F8	8.09	0.99	7.31	0.568	0.591	2	...
#4.....	NGC 1647	8.70	...	...	...	0.39	...	9.34	0.90	8.63	0.492	0.510	2	...
#8.....	NGC 1647	8.70	...	...	...	0.39	...	10.33	0.90	9.57	0.525	0.550	2	2
#2.....	NGC 1664	10.46	...	Yes	...	0.20	...	7.50	0.97	6.77	0.537	0.576	2	2
#424.....	NGC 2168	9.70	...	...	...	0.23	...	10.56	0.28	10.35	0.157	0.138	2	6, 7
#2.....	NGC 2251	10.74	...	...	...	0.24	...	9.65	0.56	9.18	0.320	0.348	2	2
f.....	NGC 2287	9.60	Yes	...	P	0.00	G8 II-III	7.79	1.07	6.96	0.570	0.646	2	8
21.....	NGC 2287	9.60	Yes	...	Yes	0.00	K3 II	6.93	1.46	5.77	0.766	0.861	2	8
75.....	NGC 2287	9.60	Yes	...	Yes	0.00	KIII-III	7.46	1.18	6.53	0.612	0.705	2	8
87.....	NGC 2287	9.60	Yes	...	Yes	0.00	K4 III	7.87	1.30	6.79	0.720	0.773	2	8
97.....	NGC 2287	9.60	Yes	...	Yes	0.00	K0 II-III	7.82	1.08	6.97	0.568	0.651	2	8
14.....	NGC 6633	7.61	...	No	...	0.18	...	8.32	1.08	7.49	0.586	0.648	2	9, 10
56.....	NGC 6633	7.61	...	P	...	0.18	...	8.21	0.82	7.53	0.509	0.508	2	9, 10
116.....	NGC 6633	7.61	...	Yes	...	0.18	...	8.31	1.13	7.45	0.602	0.672	2	9, 10
122.....	NGC 6633	7.61	...	...	...	0.18	...	8.67	1.10	7.80	0.607	0.664	2	1, 10
134.....	NGC 6633	7.61	...	Yes	...	0.18	...	8.96	1.04	8.13	0.590	0.636	2	9, 10
140.....	NGC 6633	7.61	...	Yes	...	0.18	...	8.79	1.06	7.94	0.584	0.638	1	9, 10
HD 190113.....	NGC 6871	10.98	...	...	Yes	0.40	G5 Ib	7.91	1.46	6.75	0.770	0.893	2	...
#3.....	NGC 7063	9.00	...	...	...	0.08	...	9.71	0.46	9.34	0.246	0.239	3	2
HD 220770.....	NGC 7654	10.93	...	...	P	0.62	A5 Ia	7.77	0.82	6.99	0.719	0.481	3	...

REFERENCES.—(1) Arp *et al.* 1959. (2) Hoag *et al.* 1961. (3) Schild 1967. (4) Heinemann 1926. (5) Ebbighausen 1939. (6) Cudworth 1971. (7) Hoag *et al.* (1961) give $V = 10.09$, $B - V = 0.57$ for this star. However, the $(b - y)$ color measured here is more consistent with Cudworth's value of $(B - V)$. Thus this is a main-sequence star rather than an evolved star as indicated by the Hoag *et al.* photometry. (8) Cox 1954. (9) Hiltner *et al.* 1958. (10) Vasilevskis *et al.* 1958.

FIG. 1.—A plot of $(b - y)_0$ versus $(R - I)_0$ for the Cepheids and nonvariables in clusters. The solid circles represent the Cepheids, open circles are the nonvariable stars for which cluster membership is positively indicated in Table 4, and the triangles are the stars in Table 4 which may be cluster members. The solid line was fitted by eye to the Cepheids.

(13) contains the number measurements made for each star.

The accuracy of the photometry was determined from the scatter in the measurements of the non-variable stars and the standards. The standard deviations of a single measurement are 0.023 mag for R , 0.008 mag for $(R - I)$, and 0.012 mag for $(b - y)$.

III. THE COLOR-COLOR RELATIONS

In Figure 1 $(R - I)_0$ is plotted against $(b - y)_0$ for the Cepheids which are members of clusters. The

reddening was removed from the observed colors using $E_{b-y} = 0.7E_{B-V}$ and $E_{R-I} = 0.82E_{B-V}$. It can be seen that all the Cepheids tend to lie along one sequence in the plot but that individual points scatter considerably. The solid line is an eye-fitted mean. The breadth of the band of points in the diagram is about 0.17 mag in the $(b - y)$ direction which corresponds to 0.26 mag in $(B - V)$. This can be compared with the scatter in a plot of $(B - V)$ versus temperature previously published (Fig. 2 of Schmidt 1972b). In that plot the range of $(B - V)$ at a particular temperature was about 0.34 mag. Considering the errors in the

FIG. 2.—Plots of $(b-y)_0$ versus $(R-I)_0$ for various individual Cepheids. The solid line is the mean line from Fig. 1 and is included only for reference.

temperatures, this is reasonably consistent with the present result.

In Figure 2, $(b-y)_0 - (R-I)_0$ diagrams for some individual stars are presented. The solid line in each panel of Figure 2 is the mean line from Figure 1. It can be seen that the individual stars scatter significantly from the mean and that some stars obey a steeper $(b-y) - (R-I)$ relation than others or than the mean. This behavior is further illustrated by the values of the slope of the color-color relation, $\Delta(b-y)/\Delta(R-I)$, which are given in Tables 2 and 3. It can be seen that they range from 0.98 to 2.40 for various stars, compared with a slope of 1.11 for the mean relation. These quantities are plotted in Figure 3 as a function of the period. Except for three stars, SV Per, RY Cas, and VX Per, there is a good correlation between the slope of the color-color relation and the period. On the other hand, no correlation was found with the mean color, the amplitude, or the asymmetry of the light curve. Thus this changing slope is apparently related to the luminosity of the star.

Also of interest in Figure 1 are the points which represent the nonvariable stars. Unfortunately, there are only a few nonvariables which are established members of clusters (open circles). It is interesting to note, however, that they lie below the relation for the

FIG. 3.—The slope of the color-color relation, $\Delta(b-y)/\Delta(R-I)$, as a function of period for the stars in Tables 2 and 3.

Cepheids in the bluer part of the range. This amounts to about 0.07 mag in $(b-y)_0$ at $(R-I)_0 = 0.25$. On the other hand, the coolest of the nonvariables are in good agreement with the extension of the Cepheid curve. Thus the relation between $(b-y)_0$ and $(R-I)_0$ for the nonvariables is steeper than the mean Cepheid relation and has a slope of 1.31. Assuming that $(R-I)_0$ is a better temperature indicator than $(b-y)_0$, we find that this difference should change the slope of the $(B-V)_0 - \log T_e$ relation by about 0.05. This difference in slope is very close to the difference found previously (Schmidt 1972b). Thus the present data may support the previous conclusion that significant differences exist between the Cepheids and the nonvariable supergiants in the color-temperature relation. However, there are several stars in Figure 1 which could alter this conclusion if their cluster membership were established (triangles between $(b-y)_0 = 0.17$ and $(b-y)_0 = 0.28$).

We cannot examine these effects quantitatively without synthetic spectral calculations. However, we note that such calculations by Bell and Rodgers (1969) showed that $(B-V)$ colors in Cepheids are significantly affected by microturbulence and electron pressure. This arises through the strong line blanketing in the blue and visual regions of the spectrum, and presumably $(b-y)$ is similarly affected. Since the line blocking in the red part of the spectrum is considerably smaller than in the visual and blue, we will regard $(R-I)_0$ as a better temperature indicator than $(b-y)_0$ for the remainder of this paper.

IV. PULSATIONAL MASSES

Having shown that there are significant differences between the blue and the red colors, it is now of interest to consider how this affects the pulsational masses. These were calculated from the data in Tables 2 and 3 using the same calibrations and methods as in Paper I. The luminosities and pulsational masses are given in Tables 2 and 3. For $(B-V)$, two masses were calculated. One employed the temperature

FIG. 4.—The pulsational masses of the Cepheids. In the calculation of the masses in panel *a*, $\langle B - V \rangle_0$ was used as the temperature indicator while in *b* and *c*, $\langle B \rangle_0 - \langle V \rangle_0$ and $\langle R - I \rangle_0$ were used, respectively. *Solid circles*, cluster members; *open circles*, field stars; *crosses*, stars from paper I. The arrows in the lower right-hand corner of each panel indicate the effect of decreasing E_{B-V} by 0.1 mag for the field and cluster Cepheids. The evolutionary masses lie between the pair of solid lines.

derived from $\langle B - V \rangle_0$, the magnitude mean of the color, while the other used the temperature indicated by $\langle B \rangle_0 - \langle V \rangle_0$, the intensity mean. This was done to test the effect on the pulsational masses of the method of averaging over the cycle. Recent calculations by Karp (1975) indicate that the intensity mean, $\langle B \rangle_0 - \langle V \rangle_0$, is more suitable for $(B - V)$ while for $(R - I)$ the method of averaging makes little difference. We do not have sufficient data to calculate $\langle R \rangle_0 - \langle I \rangle_0$ for most of our stars so this was not attempted.

The pulsational masses are plotted against the luminosities in Figure 4. To the nine cluster Cepheids observed here, S Nor (a member of NGC 6087) has been added using the color data from Paper I. These 10 stars are plotted as filled circles. The open circles represent the stars from Table 3 while the stars from Paper I have been plotted as crosses. Points for those stars which are common to the present sample and Paper I are connected (including S Nor) by a line. The differences are almost entirely due to the different adopted color excesses. The two parallel lines in each panel of the diagram show the range of evolutionary masses as given by Iben and Tuggle (1972). In the lower right corner of each panel there are arrows which indicate the effect of errors in the color excess on the plotted points. Each arrow has a length which corresponds to 0.1 in E_{B-V} , and the arrows point in the direction a point would move if the color excess were decreased. The effect of color excess on the field-star

masses and luminosities differs from that on the cluster stars because the color enters through the period-luminosity-color relation.

We find, as did Iben and Tuggle (1972), that the pulsational mass of VY Per is greater than the masses of other stars of similar period. In fact, it is the only star in Figures 4*a* and 4*b* which agrees with the evolutionary mass-luminosity relation. Iben and Tuggle suggest that this star is an overtone pulsator on the basis of its high pulsational mass. However, the discussion below seems to point to another explanation.

Looking at the top two panels of Figure 4, we see that the use of $\langle B \rangle_0 - \langle V \rangle_0$ reduces the scatter as compared with the use of $\langle B - V \rangle_0$. However, in both cases the cluster stars scatter more than the field stars. One explanation could be that the color excesses contain errors as large as 0.1 mag. Since the field stars are moved along the sequence by errors in color excess, they will not be scattered by this effect. However, it would be surprising to find such large errors in the cluster reddenings.

Iben and Tuggle (1972) noted that the use of a period-luminosity-color (PLC) relation can act to suppress scatter in the (pulsational mass)-luminosity diagram. There are, however, two situations which must be considered in this connection. If we imagine that the temperature and luminosity of a Cepheid uniquely define its mass, then (following the arguments

of Sandage and Gratton 1963) there is a unique PLC relation which all stars obey. On the other hand, scatter in mass at a particular luminosity and temperature will scatter stars about a mean PLC relation. If we consider two stars of equal temperature and luminosity but with masses $\mathfrak{M}_1$ and $\mathfrak{M}_2 = (1 + \delta)\mathfrak{M}_1$, we can see how this works. From the pulsational calculations of Fricke *et al.* (1972) we have the relation

$$\log \mathfrak{M} = 1.27 \log L - 1.5 \log P + \text{const.} \quad (2)$$

Applying this to the two stars we find

$$\log P_2 = \log P_1 - \log(1 + \delta). \quad (3)$$

Using this period with the Sandage and Tammann (1969) PLC relation gives

$$M_{\langle V \rangle 2}(\text{PLC}) = M_{\langle V \rangle 1}(\text{PLC}) + 3.425 \log(1 + \delta), \quad (4)$$

or

$$\log L_2(\text{PLC}) = \log L_1(\text{PLC}) - 1.37 \log(1 + \delta). \quad (5)$$

Finally, entering this luminosity and the period from equation (3) in equation (2) yields

$$\log \mathfrak{M}_{Q2} = \log \mathfrak{M}_{Q1} - 0.24 \log(1 + \delta). \quad (6)$$

Thus star 2, which should appear directly above star 1 in the pulsational mass-luminosity diagram, will be shifted toward lower luminosity and mass, and the two stars will lie on a line of slope

$$\Delta(\log \mathfrak{M})/\Delta(\log L) = 0.24/1.37 = 0.175. \quad (7)$$

Since the slope of the evolutionary mass-luminosity relation, about 0.25, is similar to this value, it is apparent that the scatter is greatly suppressed by the use of the PLC relation. As an example consider two stars which are located at the same point in the H-R diagram. If one star is more massive by, say, 25 percent according to equation (5) the PLC relation would indicate a luminosity which is fainter than the less massive star by about 36 percent. This in turn, using equation (6), would result in the pulsational mass of the more massive star being *less* than that of the other star by slightly more than 1 percent. If the mean mass-luminosity has a slope of 0.25, the star which is over-massive by 25 percent will be only about 2 percent above the mean line. Hence nothing can be learned about mass scatter at a particular location in the H-R diagram from pulsational masses based on the PLC relation. However, this does not apply to stars which obey the PLC relation. That is, if any scatter in mass at a given luminosity is related only to the temperature, then the PLC relation should allow a correct determination of the pulsational mass (provided the color dependence of the PLC relation has been correctly calibrated). Thus the width of the band of points for field Cepheids in Figure 4 is an indication of mass

spread at constant luminosity across the instability strip. In view of this we might suppose the greater scatter of the points for cluster Cepheids indicates a scatter in mass at a given luminosity and temperature. This could arise from varying amounts of mass loss, but clearly a final conclusion must await further investigation.

Figure 4c differs from the other two diagrams in several respects. In the first place, we now find that for luminosities below $\log L/L_\odot = 3.55$, nearly half of the stars fall within the evolutionary mass range. We also see that there is little or no tendency for more luminous stars to have greater pulsational masses than less luminous stars. Thus, there is still a large mass discrepancy among the long-period stars, although much of it has disappeared among the short-period stars. We should note that the greater scatter in the $(R - I)$ diagram is probably due to the stronger dependence of the calculated masses on the color excesses. It is interesting that the mass of VY Per in Figure 4c is little changed from that in Figure 4a and 4b while the other stars of similar luminosity have moved up near it. This suggests that the interpretation of Iben and Tuggle that this star is pulsating in the first overtone is incorrect. It is possible that errors in the color excess have influenced the mass of this star. However, if that is the case it is surprising that $(R - I)$ agrees with $(B - V)$. Rather, it appears that for some reason $(B - V)$ gives the correct temperature only for VY Per.

There are several possible explanations for the results presented in Figure 4. Taking Figure 4c at face value, we might conclude that Cepheid masses do not increase with luminosity. This would then suggest that mass loss occurs in such a way that by the time stars reach the Cepheid strip all have nearly the same mass. However, such an extreme conclusion is unwarranted until these results are more firmly established. Another possibility is that the $(R - I)$ photometry contains systematic errors. However, the agreement of the present photometry with that of Wisniewski and Johnson (1968) is excellent for the stars in common, and any difference between the crosses (based mainly on the Wisniewski and Johnson photometry) and the circles in Figure 4c is attributable to differences in the selection of color excesses. It is possible that errors in $E_{(B-V)}$ affect the results. There is, however, good agreement between the cluster Cepheids and the field Cepheids for luminosities below $\log L/L_\odot = 3.7$. Since the cluster color excesses should be reliable, this tends to indicate that the trends at lower luminosity are not the result of color-excess errors. Additionally, the fact that there is no dependence of the masses on E_{B-V} indicates that the conversion to E_{R-I} is reasonably correct. In the case of the more luminous stars, however, there may well be systematic errors which contribute to the low masses. For X Cyg and T Mon, observations of field stars (Schmidt 1975) showed that the G band color excesses *may* be too high by as much as 0.15 mag. Parsons and Bell (1975) also found evidence supporting decreased color excesses for the long-period stars. This would move these stars near

FIG. 5.—The location of some stars from this program in the temperature-luminosity diagram. The evolutionary tracks of Iben (1967) are shown for various masses, and the dark solid line is the edge of the instability strip from a previous paper (Schmidt 1972a). The Cepheids RY Cas, U Sgr, SU Cas, and DL Cas, and star A in NGC 129 are indicated by solid circles. Open circles represent the stars in NGC 2287, crosses those in NGC 6633 and plus signs those in NGC 752.

the evolutionary mass-luminosity relation. We also note that errors in the calibration of the PLC relation as suggested by Iben and Tuggle (1972) could play a role. However, until the question of the color excesses is resolved, it seems premature to suggest changing the PLC relation to force agreement. A change in the zero point of the Cepheid luminosity scale might still be considered. Increasing the absolute magnitudes of the Cepheids by about 0.1 mag combined with decreased color excesses for the long period stars would suffice to bring agreement between the pulsational masses and the evolutionary masses in Figure 4c. It is clear that much of the burden of resolving this discrepancy lies in better establishing the temperatures and luminosities of the Cepheids. The need for accurate and reliable color excesses is obviously basic to this.

V. THE LOCATIONS OF THE NONVARIABLE YELLOW GIANTS IN THE HR DIAGRAM

The cluster NGC 129 is of special interest because it contains both a Cepheid, DL Cas, and a nonvariable supergiant, star A, which is slightly hotter than the Cepheid. The values of M_V and $(R - I)$ in Tables 2 and 4 have been used to locate these stars in the theoretical H-R diagram. This is shown in Figure 5. The luminosities of the two stars are identical. Also plotted are three other Cepheids, SU Cas, U Sgr, and RY Cas, the blue edge of the instability strip determined previously (Schmidt 1972a), and the evolutionary tracks of Iben (1967) for several masses. It can be seen that star A is significantly cooler than the blue edge of the instability strip and is cooler than U Sgr. Additionally, the Cepheids UY Per, VX Per, and VY Per (all cluster members) have absolute magnitudes similar to star A, but are bluer in $(R - I)$. Thus this

star is clearly among the Cepheids when $(R - I)$ is used to determine its temperature. If $(B - V)$ had been used, star A would still have been in the Cepheid strip but would have been quite close to the blue edge. However, for the reasons discussed above we consider the temperature determined from $(R - I)$ to be more reliable than that from $(B - V)$. We note that the cluster NGC 129 is not uniformly reddened over its area. The color excesses of several stars near DL Cas and several near star A were determined from the photometric data of Arp *et al.* (1959). It was found that the difference in E_{B-V} between the two regions was only 0.023 ± 0.020 mag (s.e.). Thus the use of the same color excess for both stars is valid and their relative locations in the H-R diagram are well determined. Because the color excess and the absolute magnitude of star A are well established from the cluster, it appears that this is the best established case of a nonvariable star in the Cepheid strip. It therefore lends further strength to the case for the existence of such stars. Additionally, finding such a star in a cluster opens the possibility of understanding the phenomenon of nonvariables in the Cepheid strip through the investigation of other cluster members.

There are three clusters in Table 4 which each contain four or more late giants or supergiants. It is of interest to compare these stars with evolutionary calculations. The requisite information is contained in Table 5 and is plotted in Figure 5. Open circles represent the stars from NGC 2287, the crosses those from NGC 6633, and the plus signs those from NGC 752. In the case of NGC 2287, the stars are too cool for the Schmidt (1973) $(R - I)$ -temperature calibration, and the calibration of Johnson (1966) was used. The evolutionary times of various crossings of the yellow giant region differ considerably, and when there are

TABLE 5
MASSES AND AGES OF NONVARIABLE STARS IN THREE CLUSTERS

Cluster	Star	$\log L/L_{\odot}$	$\log T_e$	Evolutionary Mass	Age (10^7 yr)
NGC 752.....	208	1.58	3.692	2.4	49.
	213	1.54	3.685	2.3	51.
	295	1.43	3.692	2.2	55.
	311	1.54	3.683	2.3	51.
NGC 2287.....	F	2.74	3.666	4.3	11.
	21	3.32	3.660	6.3	4.8
	75	2.92	3.664	4.8	8.6
	87	2.82	3.661	4.6	10.
	97	2.74	3.666	4.3	11.
NGC 6633.....	56	1.89	3.749	2.7	36.
	116	1.89	3.710	2.7	36.
	122	1.74	3.707	2.5	43.
	134	1.62	3.714	2.4	48.
	140	1.69	3.717	2.5	45.

multiple crossings the second is the longest. We will therefore assume that all the stars are on the same crossing. The ages and masses in Table 5 were obtained by interpolating among the evolutionary tracks assuming the second crossing. The spread in luminosity in a cluster can be interpreted in terms of a mass spread which in turn implies a range of ages. The stars in NGC 752 and NGC 2287 then have an age spread of about 6×10^7 years while those in NGC 6633 have a spread of about 10^8 years. Iben and Talbot (1966) estimated the time over which stars formed in NGC 2264 to be about 6.5×10^7 years. Unfortunately, their interpretation is uncertain owing to the presence of dust shells around many of the pre-main-sequence stars (Strom *et al.* 1971). Meyer-Hofmeister (1969) found that the scatter of red and yellow giants in NGC 1866 could be accounted for in terms of a range in age of about 1.5×10^7 years. Thus the range found here is somewhat higher than previous estimates.

Clearly, a definitive discussion of the locations of

the red and yellow giants in the H-R diagrams of these clusters is not possible at present. Various theoretical uncertainties have been ignored. For example, the location of the red-giant branch and the length of the loops in the evolutionary tracks are sensitive to the mixing length adopted, the composition, and the details of the evolutionary calculation. Thus some of our stars could be on the ascending red-giant branch of the evolutionary tracks. Additionally, the evolution may be affected by rotation, and mass loss may play a significant role. Hence our present conclusions must be tentative. However, as the evolutionary tracks become more certain and various complicating factors are better understood, the clusters with several yellow giants will provide a good check on the theory.

This work was initiated while the author held a Science Research Council Research Fellowship at the Royal Greenwich Observatory. This support is gratefully acknowledged.

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