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Productivity Measurement in the Presence of "Poorly Priced" Goods

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Interest in productivity measurement was originally motivated by interest in the effect of additional output on consumers' standard of living, but traditional productivity studies have focused on the effect of shifts in the production frontier itself rather than changes in welfare. There are two reasons for this (*a*) the economy's production frontier is a constraint on consumers' utility so its expansion is of inherent interest; and (*b*) if there are no market failures, measures of shifts in the production frontier and welfare change coincide. This reasoning collapses when market failure causes a divergence between consumers' marginal rates of substitution and producers' marginal rates of transformation, i.e., when some goods are poorly priced.

The case of unpriced environmental goods is an obvious case in point. There is general concern that if new technology fouls the environment, then measures of shift in the production frontier such as total factor productivity (TFP) cannot really reflect what we want to measure because the value of goods such as clean air or water are not included in the TFP calculations.

The objective of this paper is to review some of the literature related to this issue, and to suggest more general approaches to productivity measurement in the presence of market failure.

Productivity and Welfare Change: A Graphical Interpretation

The problem of measuring technological progress in the presence of poorly priced goods can be illustrated by a two-good economy at distorted equilibrium A illustrated in figure 1. Good v is poorly priced relative to the *numéraire* good x, because the price w supporting production is much less than the price p that supports consumption. Good v might represent clean air, for example, while x represents steel, with consumers' marginal rate of substitution being higher than producers' marginal rate of transformation. New technology shifts the production possiblities curve from ppf to ppf', and the economy moves from equilibrium point A to A', (much more steel, a little less clean air) with an increase in p relative to w. A common measure of TFP for this case is the initial-share-weighted change in netputs or percentage increase in profit at initial prices [$TFP = 1 + \Sigma s_i d \ln x_i = \Pi(w, \tau_1)/\Pi(w, \tau_0)$], or on the graph, $O\Pi'_1/O\Pi_A$. On the other hand, a measure of improvement in consumer welfare is Hicks's equivalent variation, the minimum increase in expenditures needed for consumers to achieve welfare level u' as opposed to u, given initial prices p. EV is measured in this case as the percentage increase in expenditures, Opx'/OE_A.[1] The difference between these two measures of technological progress is simply the set of prices used to evaluate the two bundles A and A'.

Which of these two measures of technological progress is appropriate? The tradition of productivity measurement has sought to measure the relative shift in the production function at observed points A and A'. Use of prices w, as the supporting hyperplane for the ppf, is completely consistent with this tradition. These prices are also consistent with a distance function measure of productivity change between A and A'. However, it is the consumers who are ultimately served by the productivity change, and the change in consumer welfare due to the technological change is approximated by EV, which uses price set p.

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[1] Strictly speaking, EV is less than $px' - EA$ because substitution permits achievement of u' with a less expensive bundle than A' For purposes of this paper, we ignore this substitution effect

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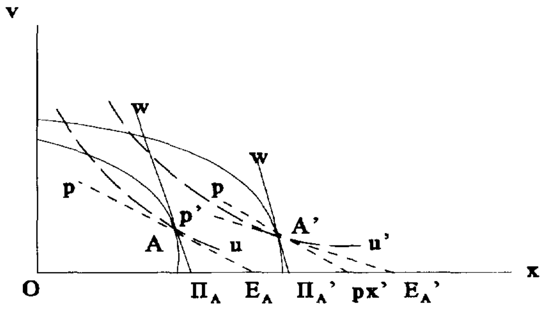

Figure 1. Technological change with poorly priced goods

Do the two measures differ? If there are no economic distortions so that $w = p$, the two measures will coincide. Otherwise, the two measures will generally differ. In the extreme case, one can imagine TFP measured with w as a vertical line (clean air has no value to the firm) and p as a nearly horizontal line (steel has very little marginal value to consumers relative to clean air), so that the technological change results in a substantial increase in TFP but virtually no increase in consumer welfare.

In this connection it is useful to recall the First Law of Thermodynamics, which states that neither matter nor energy can be created or destroyed, but that each can be transformed into a different form. There can be no increase in outputs per unit of input if all mass and energy are accounted for. We record increases in productivity because we only measure those inputs and outputs that we value. We submit that because productivity is thus a value-laden concept, it should be consumer values that are used in productivity measurement, rather than marginal rates of transformation or producer values, as is the usual practice.

Productivity Adjustments for Poorly Priced Goods

The literature contains two types of studies on adjustments in productivity estimates to account for situations where market prices do not reflect shadow prices. One group of studies has focused on shadow prices to measure shifts in the production frontier with the goal of refining existing productivity measures. In a set of papers in Cowing and Stevenson the impact of imperfect competition is incorporated by estimating the difference between price and marginal costs. The October/November 1986 issue of the *Journal of Econometrics* focused on the existence of short-run fluctuations in capacity utilization and the resulting implications for the use of shadow values of fixed inputs instead of market prices which may not reflect the contribution of the input. Other extensions include adjustments to productivity to account for the existence of pollution abatement regulation (Crandall, Pittman, Färe et al., Conrad and Morrison[2]). Current U.S. Department of Agriculture efforts to adjust agricultural productivity measures for water pollution use the distance function approach of Färe et al.

The other set of studies has focused on the estimation of shadow prices to measure shifts in consumer well-being. Among these are efforts to adjust real national product to reflect nonmarketable commodities. This involves the valuation of goods and services that do not pass from seller to buyer in a market (stay-at-home spouse, police force service, services of the environment and services of natural resources) or for which market prices do not accurately reflect supply and demand conditions. Once these adjustments are incorporated, changes in this index would reflect changes in social well-being. Along this line we find Dasgupta and Mäler who mention two ways of assessing changes in social well-being. One would be to measure the value of changes in the constituents of well-being (utility and freedoms), and the other would be to measure the value of the alterations in the commodity determinants of well-being (goods and services that are inputs in the production of well-being). This approach uses a key theorem in resource allocation theory. It states that, provided certain technical restrictions are met, for any conception of social well-being, and for any set of technological, transaction, information, and ecological constraints, there exists a set of shadow prices of good and services that can be used in the estimation of real national product.[3] The shadow prices can be conceived either as the increase in the maximum value of aggregate well being if a unit more of the resource were made available costlessly. They are Lagrange multipliers in an intertemporal general equilibrium welfare maximizing exercise. Alternatively, shadow prices can in some cases be estimated from the prevailing structure of production and con-

[2] Also see Conrad and Morrison for a summary of earlier literature

[3] The technical restrictions amount to the requirement that the Kuhn-Tucker Theorem is usable The assumption of convexity is dubious for some pollution problems, see Dasgupta and Maler for a suggestion in these cases

sumption (not from the optimum). For commodities that affect consumers' welfare, shadow prices could be obtained from restricted expenditure functions or from distance functions. It is possible to estimate shadow prices of nonmarketed goods (use value, if not intrinsic or option value) in terms of the shadow prices of marketed goods. In some situations the nonmarket commodity is a substitute for a marketable commodity; in others it is a complement. Contingent valuation methods are an alternative approach offering estimates of consumer shadow prices. Examples of this approach can be found in Dasgupta and Mäler and in Milon and Shogren.

A General Equilibrium Model of Technological Change with Poorly Priced Goods

The net welfare effect of technological change in the presence of poorly priced goods depends upon the nature of the distortion and just how equilibrium quantities are determined. In this section we use a general equilibrium model of a closed economy to examine the case in which producers determine the quantity of the unpriced or poorly priced good. This represents an externality or public good situation such as clean air, in which firms choose the level of the good in accord with a price of zero, but the level of clean air also enters into consumer utility functions.

The economy consists of an unspecified number of consumers, $n + 1$ goods, and m resources. The foundation for the general equilibrium model consists of the expenditure and profit functions in equation (1):

$$(1)\quad E(\mathbf{p}, \mathbf{v}, u) \equiv \min_{x_0,\mathbf{x}}[x_0 + \mathbf{px} | u(x_0, \mathbf{x}, v) \geq u]$$

and

$$\Pi(\mathbf{p}, \mathbf{v}, \tau) \equiv \max_{y_0,\mathbf{y}}[y_0 + \mathbf{py} | (y_0, \mathbf{y}, v, \tau) \in T]$$

where x_0 and y_0 are quantities of the *numéraire* good chosen by consumers and producers, respectively; $\mathbf{x}$ and $\mathbf{y}$ are $n \times 1$ vectors of quantities of other goods chosen by consumers and producers, respectively, negative values indicating quantities supplied by consumers or demanded by producers; $\mathbf{v}$ is an $m \times 1$ vector of quantities of poorly priced goods, $\mathbf{p}$ is the $n \times 1$ vector of prices for $\mathbf{x}$ and $\mathbf{y}$; τ is an index of technological change; and T is the feasible technology set.

Here, E may be interpreted as the minimum expenditure required to maintain consumers at individual utility levels indicated by vector $\mathbf{u}$, or as the minimum expenditure required to attain some welfare function at scalar value u. The profit function can be interpreted either as the maximum profit from an aggregate economy, or the aggregate of individual firm profit functions.

The general equilibrium in the economy is represented by the three equations in equation (2)

$$(2a)\quad E = \Pi$$

$$(2b)\quad E_{\mathbf{p}} = \Pi_{\mathbf{p}}$$

$$(2c)\quad \Pi_{\mathbf{v}} = \phi$$

where ϕ is an $m \times 1$ vector of zeroes (or other exogenous values for $\mathbf{v}$).

Equation (2a) specifies the budget constraint, while (2b) requires quantities chosen by consumers to be equal to those chosen by producers (equality of the *numéraire* good chosen by consumers and producers is insured by Walras's Law). Equation (2c) specifies that producers choose quantities of poorly priced goods $\mathbf{v}$ so that their shadow price equals ϕ (due to lack of property rights, for example). The exogenous values ϕ may be zero as suggested above, or may reflect some nonzero value if producers perceive some internal or moral cost or benefit related to the netput not directly reflected in firm profits.

Before examining the comparative statics of this general equilibrium, let us explicitly define four scalar measures of interest in measuring technological progress:

(3) dual rate of technological change (DRTC)

$$\equiv \Pi_\tau(\mathbf{p}, \mathbf{v}, \tau)/\Pi(\mathbf{p}, \mathbf{v}, \tau)$$
$$= \sum_{i=0}^{n} k_i d \ln x_i(\mathbf{p}, \mathbf{v}, \tau)$$
$$\equiv \delta;$$

total factor productivity (TFP)

$$\equiv (x_0' + \mathbf{px}')/\Pi(\mathbf{p}, \mathbf{v}, \tau)$$
$$= 1 + \sum_{i=0}^{n} k_i d \ln x_i^e,$$

where x^e are equilibrium values;

equivalent variation (EV)

$$\equiv (x_0' + \mathbf{px}')/E(\mathbf{p}, \mathbf{v}, u)$$
$$= 1 + \sum_{i=0}^{n} s_i d \ln x_i^e,$$
$$= TFP + \sum_{i=0}^{n} (s_i - k_i) d \ln x_i^e;$$

welfare change $= u'/u$.

The dual rate of technological change measures the relative change in profits due to technological change, under constant prices. Total factor productivity is the conventional definition, except that here we use intial instead of average prices or profit shares as weights. Equivalent variation we define in this context as the consumer-price-weighted counterpart of TFP, which can be expressed as TFP plus the share-difference-weighted logarithmic change in quantities. Welfare change is defined here as the relative change in welfare due to the technological change. Although the relationship between TFP and EV is evident here, the relationships between them and DRTC and welfare change can only be made explicit by examining the comparative statics of the general equilibrium conditions.

The logarithmic differentials of the equilibrium equations (2) can be expressed as

(4)
$$a'.\ d \ln u + (\mathbf{s}_v - \mathbf{k}_v) d \ln \mathbf{v} = \delta d\tau$$
$$b'.\ \eta d \ln u + (\mathbf{H}_{xp} - \Sigma_{xp}) d \ln \mathbf{p} + (\mathbf{H}_{xv} - \Sigma_{xv}) d \ln \mathbf{v} = (\mathbf{B}_x + \iota\delta) d\tau$$
$$c'.\ -\Sigma_{vp} d \ln \mathbf{p} - \Sigma_{vv} d \ln v = \mathbf{B}_v d\tau$$

where $\mathbf{sv} = (1/E)\mathbf{E}_v\mathbf{D}_v$, shadow values of v as shares in expenditures; $\mathbf{k}_v = (1/\Pi)\Pi_v\mathbf{D}_v$, shadow values of v as shares in profits; ι is an $n \times 1$ unit vector; $\mathbf{Dz}$ is a matrix with vector $\mathbf{z}$ on the diagonal; $\eta = u\mathbf{D}_x^{-1}\mathbf{E}_{pu}$, $n \times 1$ vector of real income expenditure elasticities; $\mathbf{H}_{xp} = \mathbf{D}_x^{-1}\mathbf{E}_{pp}\mathbf{D}_p$, $n \times n$ compensated demand elasticity matrix; $\Sigma_{xp} = \mathbf{D}_x^{-1}\Pi_{pp}\mathbf{D}_p$, $n \times n$ compensated supply elasticity matrix; $\mathbf{B}_x = \mathbf{D}_x^{-1}\Pi_{p\tau} - \iota\delta$, $n \times 1$ vector of x-biases of technological change; $\mathbf{B}_v = \mathbf{D}_v^{-1}\Pi_{v\tau}$, $m \times 1$ vector of v-biases of technological change.

Here the nature of the technological change is expressed in parametric form: δ representing the rate of technogical change; the $\mathbf{B}_x$ bias vector reflecting the difference between δ and the fraction each optimal netput would change (under constant prices); and the $\mathbf{B}_v$ bias vector indicating the increase in shadow prices of v. The effect of a change in technology on this equilibrium can be expressed as

$$(5)\quad \begin{bmatrix} d \ln u \\ d \ln p \\ d \ln v \end{bmatrix} = \begin{bmatrix} 1 & 0 & s_v - k_v \\ \eta & H_{xp} - \Sigma_{xp} & H_{xv} - \Sigma_{xv} \\ 0 & -\Sigma_{vp} & -\Sigma_{vv} \end{bmatrix}^{-1} \begin{bmatrix} \delta \\ B_x + \iota\delta \\ B_v \end{bmatrix} d\tau$$

$$= \begin{bmatrix} 1 + (s_v - k_v)V_{Bx}\eta & -(s_v - k_v)V_{Bx} & -(s_v - k_v)V_{bv} \\ -P_{Bx}\eta & P_{Bx} & P_{Bv} \\ -V_{Bx}\eta & V_{Bx} & P_{Bv} \end{bmatrix} \begin{bmatrix} \delta \\ B_x + \iota\delta \\ B_v \end{bmatrix} d\tau$$

$$= \begin{bmatrix} \delta - (s_v - k_v)(d \ln v/d\tau) \\ P_{Bx}[B_x + (\iota - \eta)\delta] + P_{Bv}B_v \\ V_{Bx}[B_x + (\iota - \eta)\delta] + V_{Bv}B_v \end{bmatrix} d\tau$$

where $V_{Bv} = \{\Sigma_{vp}(H_{xp} - \Sigma_{xp})^{-1}[H_{xv} - \Sigma_{xv} - \eta(s_v - k_v)] - \Sigma_{vv}\}^{-1}$, elasticity of v with respect to v-biases in technological change; $V_{Bx} = V_{Bv}\Sigma_{vp}(H_{xp} - \Sigma_{xp})^{-1}$, elasticity of v with respect to x-baises in technological change; $P_{Bv} = -(H_{xp} - \Sigma_{xp})^{-1}[H_{xv} - \Sigma_{xv} - \eta(s_v - k_v)]V_{Bv}$, elasticity of prices with respect to v-biases in technological change; and $P_{Bx} = (I + P_{Bv}\Sigma_{vp})(H_{xp} - \Sigma_{xp})^{-1}$, elasticity of prices with respect to x-biases in technological change.

Using these results, for this general equilibrium system, the relationships between the measures of technological progress defined in equation (3) can now be expressed as follows:

(6) total factor productivity (TFP)
$$= \delta + \mathbf{k}\Sigma_{xx}\, d \ln \mathbf{p}/d\tau + \mathbf{k}\Sigma_{xv} d \ln \mathbf{v}/d\tau;$$

equivalent variation (EV)
$$= \delta + \mathbf{s}\Sigma_{xx}\, d \ln p/d\tau + \mathbf{s}\Sigma_{xv}\, d \ln \mathbf{v}/d\tau,$$
$$= \text{TFP} + (\mathbf{s} - \mathbf{k})\Sigma_{xx} d \ln \mathbf{p}/d\tau + (\mathbf{s} - \mathbf{k})\Sigma_{xv} d \ln \mathbf{v}/d\tau;$$

$$\text{welfare change} = \delta - (\mathbf{s}_v - \mathbf{k}_v)d \ln \mathbf{v}/d\tau,$$
$$= \delta - (\mathbf{s}_v - \mathbf{k}_v)\{V_{B_x}[B_x + (\iota - \eta)\delta] - V_{B_v}B_v\}.$$

Each measure consists of DRTC plus a price effect [$d \ln \mathbf{p}/d\tau$ and $d \ln \mathbf{v}/d\tau$ are as determined in equation (5)]. All three measures of technological progress are equal to one another and to δ if (*a*) there are no biases in the technological change ($\mathbf{B}_x$ and $\mathbf{B}_v$ are null vectors), and (*b*) welfare preferences are homothetic (the vector of income elasticities η is the unit vector). If these conditions are not met, then the equilibrium response to the technological change will include either a price effect [the middle row of the last matrix in equation (5)] or a poorly priced good effect (the last row of that matrix), or both, as indicated in equation (6).

While the equations defining these measures appear complicated, all that is required to evaluate them is knowledge of income elasticities, compensated demand and supply elasticities, and initial shares, plus the three technological change parameters, namely the dual rate of change δ and the bias parameters B_x and B_v.

Conclusions

In this paper we present a general equilibrium approach to productivity measurement in the presence of departures from Pareto optimality. This framework makes it obvious that it is consumers that are ultimately served by the technological change. In the presence of market failure it is therefore more appropriate to use a measure of change in consumer welfare due to technological change than the traditional TFP indicator.

We define four scalar measures of technological progress: the dual rate of technological change, total factor productivity, equivalent variation, and welfare change. In a general equilibrium context these measures are interrelated, differing only by the price effect of the technological change. They are equal to one another under conditions of homothetic preferences and unbiased technological change.

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