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Published in *Insurance: Mathematics and Economics* **42**:2 (April 2008), pp. 628-637; doi 10.1016/j.insmatheco.2007.06.005
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Submitted March 4, 2006; revised June 13, 2007; accepted June 14, 2007; published online July 18, 2007.[1]

Securitization of Catastrophe Mortality Risks

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Abstract
Securitization with payments linked to explicit mortality events provides a new investment opportunity to investors and financial institutions. Moreover, mortality-linked securities provide an alternative risk management tool for insurers. As a step toward understanding these securities, we develop an asset pricing model for mortality-based securities in an incomplete market framework with jump processes. Our model nicely explains opposite market outcomes of two existing pure mortality securities.

1. Introduction

Securities with mortality risk as a component have been around a long time. These securities arise as securitization of portfolios of life insurance or annuity policies. The risks underlying a life insurance or annuity portfolio include interest rate risk and policyholder lapse risk, as well as mortality or longevity risk. In these transactions, the positive future net cash flow from the policies is dedicated to pay the bondholders. Therefore, they are similar to asset securitization. Cowley and Cummins (2005) surveys recent life insurance securitization transactions, including these asset-type securities.

However, securitization of *pure* mortality or longevity risk is a recent and potentially important innovation in financial markets. Pure mortality or longevity securitization is more like property-linked catastrophe bonds than the common asset-type life insurance securitizations. This is because, like that of a property-linked catastrophe bond based on earthquake or hurricane losses, the payment of a mortality security is only subject to a well-defined risk. In the case of a mortality bond, the event might be a sudden spike in death rates, which may be caused by a flu epidemic.

Catastrophes impose a big potential problem for a life insurer's solvency since fatalities from natural and man-made disasters may be tremendous. For example, the earthquake and tsunami in southern Asia and eastern Africa in December 2004 killed 182,340 people and made 129,897 missing (Guy Carpenter, 2005). Although most of the victims did not purchase life insurance, the life insurance industry may not have enough capacity to cover this type of catastrophe losses if such an event were to occur in a more economically developed region where most of people buy life insurance. Cummins and Doherty (1997) noted that "a closer look at the industry reveals that the capacity to bear a large catastrophic loss is actually much more limited than the aggregate statistics would suggest."

Longevity risk is the other side of mortality risk. Although mortality improves over time, future rates of improvement are uncertain. At the same time we are seeing, especially in the US, a trend to shift longevity risk to individuals. In the US, defined benefit pension plans are converting to defined contribution plans. Proposed Social Security reforms further shift mortality risk to individuals. Thus, there should be an increased demand for individual annuities. As the demand for annuities increases, the annuity insurers' need for risk management of potential mortality improvements will increase.

[1] This paper was presented at the 2005 World Risk and Insurance Economics Congress Meeting, the 2006 Asia-Pacific Risk and Insurance Association Annual Meeting, the 2006 American Risk and Insurance Association Annual Meeting, and the 2006 Financial Management Association Annual Meeting. It won the 2006 Asia-Pacific Risk and Insurance Association *Harold D. Skipper Best Paper Award*. We appreciate helpful comments from Patrick Brockett, Richard MacMinn, Naresh Bansai, and other participants at these meetings. We are especially grateful to detailed and insightful comments from an anonymous referee.

As a new risk management tool, mortality securitization enhances the capacity of the life insurance industry by transferring its catastrophic losses to financial markets. Jaffee and Russell (1997) and Froot (2001) argue that insurance securitization offers a potentially more efficient mechanism for financing catastrophe losses than conventional insurance and reinsurance. Securitization brings more capital and provides innovative contracting features for the life insurance industry to bear potential mortality shocks, thus avoiding the market disruptions caused by prohibitive reinsurance prices and availability cycles. Moreover, because mortality securities may be uncorrelated with financial markets, they provide a valuable new source of diversification for market participants (Cox et al., 2000; Litzenberger et al., 1996; Canter et al., 1997). Finally, Cowley and Cummins (2005) categorizes securitization as arbitrage opportunities or new classes of risk that enhance market efficiency.

The first pure death-risk linked deal was the three-year Swiss Re bond issued in December 2003 (Swiss Re, 2003; MorganStanley, 2003; The Actuary, 2004). Almost one year later, the European Investment Bank (EIB) issued the first pure longevity-risk linked deal — a 25-year 540 million pound (775 million euro) bond as part of a product designed by BNP Paribas aimed at protecting UK pension schemes against longevity risk.[2] In 2004, the Swiss Re Company issued two more mortality bonds. The Swiss Re deal is a hedge against catastrophic loss of insured lives that might result from natural or man-made disasters in the US or Europe. The EIB deal is a security to transfer the other tail of mortality risk, longevity risk, to bondholders.

Interestingly, the market outcomes of the first two mortality bonds are opposite. According to MorganStanley, "the appetite for this security [the Swiss Re bonds] from investors was strong." This is the same reaction investors have had to the so-called "catastrophe bonds" based on portfolios of property insurance. The strong appetite for mortality securities may indicate a growing potential market for mortality securities (Lin and Cox, 2005). On the other hand, the EIB bond has not sold very well at all.

It is important to understand why investors viewed the Swiss Re bond price favorably, yet do not buy the EIB bond. To evaluate these two bonds, we need a model that can capture and price mortality risks. We notice there are only a few preliminary papers in this area. Developing asset pricing theory in this area is important since it will help market participants better understand this new financial instruments. Most of the existing mortality securitization pricing papers have two major shortcomings. First, they ignore mortality jumps (Lee and Carter, 1992; Lee, 2000; Renshaw et al., 1996; Sithole et al., 2000; Milevsky and Promislow, 2001; Olivieri and Pitacco, 2002; Dahl, 2003; Cairns et al., 2004) to model death-linked securities. Mortality jumps should not be ignored in mortality securitization modeling especially on the death side since the rationale behind selling or buying mortality securities is to hedge or take catastrophe risks.[3] Second, they use the complete market pricing methodology. At this point, it looks like mortality risk bonds cannot be replicated with traded securities. Therefore, we propose to price mortality bonds with the Wang transform, a technique that allows pricing new securities relative to securities whose prices are known.

The Wang transform is a market-based equilibrium pricing method that unifies the finance and insurance pricing theories (Wang, 2002). Wang (2004) recently applied the Wang transform to the property-linked catastrophe securities. Apparently our application of the Wang transform to mortality bonds is new. We also use a jump model for mortality dynamics to price the Swiss Re bond linked to catastrophic death risk. Moreover, we improve the existing literature (Lin and Cox, 2005) on pricing longevity risk by taking into account parameter uncertainty. Our models enable investors to better understand why the Swiss Re deal is an attractive investment despite the uncertainty associated with catastrophic mortality risks while the EIB bond has not sold very well.

The paper proceeds as follows. Section 2 describes the current mortality securitization market and the designs of the first two mortality bonds — the Swiss Re and EIB bonds. The two-factor Wang transform as an incomplete market pricing method is introduced in Section 3. In Section 4, we propose a mortality stochastic model with jumps to price the Swiss Re bond linked to catastrophic death risk. We show that the jump process plays an important role in mortality securitization modeling. We also improve the model in Lin and Cox (2005) to price longevity risk imbedded in the EIB bond by taking into account parameter uncertainty. Our models nicely explain the opposite market outcomes of these two bonds. Section 5 is a final discussion and conclusion.

2. Mortality securitization markets

Lane and Beckwith (2005) describe recent activity in the insurance securitization market. In the past ten years, insurance and capital market are converging. Capital market investors search for uncorrelated risk for diversification and the risk-adjusted excess return "α". Insurance-linked securities have low or no correlation with financial markets, providing diversification. Moreover, the existing insurance-linked securities provide high risk-adjusted excess return. They attract more and more investors. For example, hedge funds increased their investment in insurance-linked securities. At the same time, insurers are looking for new sources of risk financing in the capital markets. Before 1999, there were only seven insurance-linked securities in a total dollar amount of only $886.1 million. However, insurance securitization increased in both dollar amounts and the number of issues especially in the last two years. There are 27 and 21 transactions in 2004 and 2005 with dol-

[2] From http://www.IPE.com on November 8, 2004.

[3] Catastrophic death events like earthquakes or epidemics, in most cases, occur unanticipatedly and only last a short time period (e.g. one year). On the other hand, longevity risk dynamics are a more gradual process and span a long time period. Therefore, modeling mortality jumps seems more important for securities linked to death risk.

lar amounts \$1894.60 million and \$1803.30 million respectively (Lane and Beckwith, 2005). The insurance securitization includes three "pure" mortality bonds in the US and one in the Europe since December 2003. The first two publicly known pure mortality securities are the Swiss Re bond issued in December 2003 (Swiss Re, 2003; MorganStanley, 2003; The Actuary, 2004) and the European Investment Bank (EIB) longevity risk bond issued in November 2004.

2.1. Design of the Swiss Re bond

The financial capacity of the life insurance industry to pay catastrophic death losses from hurricanes, epidemics, earthquakes, and other natural or man-made disasters is limited. To expand its capacity to pay catastrophic mortality losses, the Swiss Reinsurance Company, the world's second-largest reinsurance company, obtained \$400 million in coverage from institutional investors after its first pure mortality security. Swiss Re issued the bond in late December 2003. It matured on January 1, 2007. So it is a three-year deal. The principal is exposed to mortality risk. The mortality risk is defined in terms of an index q_t in year t based on the weighted average annual population death rates in the US, UK, France, Italy and Switzerland. If the index q_t (t = 2004, 2005, or 2006) exceeds 130% of the actual 2002 level, q_0, then the investors will have a reduced principal payment. The following equation describes the principal loss percentage, X_t, in year t:

$$\text{loss}_t = \begin{cases} 0 & \text{if } q_t \le 1.3q_0 \\ 1 - (1.5q_0 - q_t)/0.2q_0 & \text{if } 1.3q_0 < q_t \le 1.5q_0 \\ 1 & \text{if } q_t > 1.5q_0 \end{cases} \quad (1)$$

where q_t = weighted average population mortality in the US, UK, France, Italy, and Switzerland in year t. q_0 = Year 2002 level. Therefore, the payment at maturity is equal to

$$\text{Payment at Maturity} = 400{,}000{,}000 \times \begin{cases} 100\% - \sum_{t=2004}^{2006} \text{loss}_t & \text{if } \sum_{t=2004}^{2006} \text{loss}_t < 100\% \\ 0 & \text{if } \sum_{t=2004}^{2006} \text{loss}_t \ge 100\% \end{cases} \quad (2)$$

Since the probability of having two mortality catastrophes in a row is negligible, Equation (2) approximates to

$$\text{Payment at Maturity} \approx 400{,}000{,}000 \times \begin{cases} 1 & \text{if } q \le 1.3q_0 \\ (1.5q_0 - q)/0.2q_0 & \text{if } 1.3q_0 < q \le 1.5q_0 \\ 0 & \text{if } q > 1.5q_0 \end{cases} \quad (3)$$

where $q = \max(q_{2004}, q_{2005}, q_{2006})$.

2.2. Design of the EIB longevity bond

About one year after the Swiss Re bond issue, in November 2004, the EIB issued a longevity bond to provide a solution for financial institutions looking for instruments to hedge their long-term systematic longevity risks. This bond is the result of the co-operation between the BNP Paribas as structurer/manager, the European Investment Bank (EIB) as issuer and the PartnerRe as the provider of analysis, expertise and risk-taking capacity. The total value of the issue was £540 million (775 million euro). It was primarily intended for purchase by UK pension plans (Cairns et al., 2005). The term of the EIB bond is 25 years. Potential *buyers* of the EIB bond are pension plans, as the bond transfers longevity risks to investors.

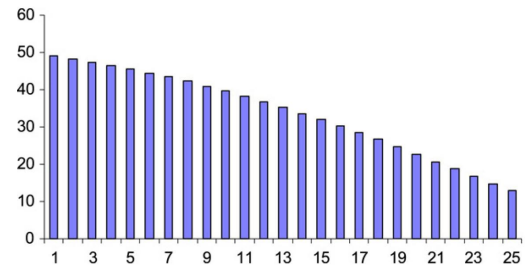

Figure 1. Projected coupons of the EIB longevity bond. The vertical axis shows projected cash flows (millions of pounds) and time is on the horizontal axis.

Here is how the EIB bond works: The bond's cash flows will be based on the actual longevity experience of the English and Welsh male population aged 65 years old, as published annually by the Office for National Statistics. The future cash flows of the bond will be equal to the amount of a fixed annuity, £50 million, multiplied by the percentage of the reference population still alive at each anniversary. The cash flows, therefore, decline over time. Figure 1 shows projected coupons (payable annually) based on the projected survival rates produced by the UK Government Actuary's Department (GAD).

3. Incomplete market pricing method—the two-factor Wang transform

Pricing derivative securities in complete markets involves replicating portfolios. For example, if we have a traded bond and stock index, then options on the stock index can be replicated by holding bonds and the index, which are priced. The analogy for the Swiss Re bond does not work. The bond is a mortality derivative, but we have no efficiently traded mortality index with which to create a replicating hedge. Situations like this are called incomplete markets. Pricing in this situation must rely on some other assumption — there is no traded underlying security.

Wang (1996, 2000, 2001, 2002) develop a method of pricing risks that unifies financial and insurance pricing theories. We can apply it to the incomplete market situation. Wang's method transforms the underlying distribution in such a way that prices are discounted expected values. The transform has many desirable properties. It has a clear economic interpretation since it can recover the capital asset pricing model (CAPM) for underlying assets and the Black–Scholes formula for options.

3.1. The Wang transform

Consider a random payment X paid at time T. If the cumulative density function is $F(x)$, then a transform is called

the Wang transform if the "distorted" or transformed distribution $F^*(x)$ is determined by the market price of risk λ according to the equation

$$F^*_X = \Phi[\Phi^{-1}(F_X(x)) + \lambda], \tag{4}$$

where $\Phi(x)$ is the standard normal cdf. $F^*_X(x)$ in Equation (4) is called the one-factor Wang transform of $F_X(x)$. In the insurance world, the market price of risk λ in the Wang transform reflects the level of market systematic risk and firm-specific unhedgeable risk. The idea is that, after transformation, the fair price of X should be the discounted expected value using the transformed distribution.

The one-factor Wang transform assumes that the true distribution is known. However, in reality, we can at best estimate the parameters of a probability density function by a sample out of the population. To account for this, Wang (2002) suggests the two-factor transform:

$$F^*_X(x) = Q[\Phi^{-1}(F_X(x)) + \lambda], \tag{5}$$

where Q is the t-distribution. As in the one-factor model in Equation (4), the discounted expected value under the transformed distribution $F^*_X(x)$ in Equation (5) is the price of X.

We can determine λ from the retail market for life insurance or annuities by Equation (5) so that at time 0 the price of a life insurance or annuity contract with payment X at time T is its discounted expected value using the transformed distribution. Therefore the formula for the price is

$$v_T \mathrm{E}^*(X) = v_T \int x \mathrm{d}F^*(x) \tag{6}$$

where v_T is the discount factor determined by the market for risk-free bonds at time 0. Thus, for an insurer's given liability X with cumulative density function $F(x)$, the Wang transform will produce a "risk-adjusted" density function $F^*(x)$. The discounted expected value under $F^*(x)$, denoted by $v_T\mathrm{E}^*[X]$, is the price of X at time 0. Wang (1996, 2000, 2001, 2002) describe the utility of this approach, showing that it generalizes well-known techniques in finance and insurance.

Suppose an insurer transfers its longevity risk to the financial market by issuing a longevity bond. We can derive the market price of risk λ based on the annuity retail market, then use the same λ to price the longevity bond. If there are no transaction costs between the insurance market and the financial market and annuities were actually traded, our method guarantees no arbitrage opportunity between these two markets. There is another way to view this. Insurers price these life and annuity obligations using a distribution (usually privately held) of future life time. We get to observe the insurers' prices and use an industry market distribution (known to all) $F(x)$. From the prices, we are able to derive the market price of risk λ so that the observed retail life insurance or annuity prices are discounted expected values using $F^*(x)$. Then we transfer the same λ to the bond market and use the $F^*(x)$ to price mortality-linked bonds. As a result, the insurer uses the same mortality assumptions to price the mortality bond as it uses in pricing retail life insurance or annuities.

3.2. Why transform?

According to the classical CAPM with the complete market assumption, the risk premium of an asset should be zero if its payoffs are uncorrelated with those of the market portfolio. The insurance market is an incomplete market which violates the complete market assumption in the CAPM. Therefore, the CAPM cannot explain the positive and very high risk premium of insurance-linked securities whose risk has no or low correlation with that of financial markets. By maximizing the *ex post* value of the firm, Froot and O'Connel (1997), Froot and Stein (1998) and Froot (2007) suggest that the high risk premium of insurance-linked securities or reinsurance reflects the risk aversion of insurers or investors when they face unhedgeable insurance risks. Risk aversion of the insurers may arise from the fact that the true economic capital requirements of insurance/reinsurance business are not straightforward and potential financial distress costs are very high (Minton et al., 2004). On the other hand, risk aversion of investors may arise from their loss aversion and/or default risk, potential moral hazard behavior and basis risk of the insurance-linked security issuing firm (Doherty, 1997). Consistent with the theories of Froot and O'Connel (1997), Froot (2007) and Doherty (1997), the transformed distribution in the Wang transform reflects the risk aversion of insurers and investors to unhedgeable risks. In Section 4, we show that the transformed mortality distribution has a longer tail (i.e. higher probability of having catastrophes) than the physical distribution. Evidently insurers and investors are risk averse to catastrophic mortality events.

4. Mortality securitization modeling

Mortality securitization modeling depends on two indispensable parts: (1) the mortality forecasting theory and (2) the incomplete market pricing theory. First, the principal or coupons of a mortality security are determined by future mortality levels. For example, the principal of the Swiss Re bond will be reduced if the future population mortality index increases by more than 30% relative to the 2002 level. Therefore we need a model to describe future mortality stochastic processes. Second, the insurance market is an incomplete market. If we transfer insurance risks to the financial market by selling insurance-linked securities, we should use an incomplete market pricing method to price these securities.

4.1. Existing mortality securitization modeling

4.1.1. Existing mortality forecasting literature

Mortality securitization modeling is based on the analysis of future mortality dynamic processes. The mortality dynamics include "normal" deviations from the trend and "unanticipated" mortality shocks. Since the rationale behind selling or buying mortality securities is to hedge or take catastrophe mortality risks, a good mortality stochastic model should take into account mortality jumps—which might be caused by epidemics, wars, or natural catastrophes such as tsunamis.

Most of the existing mortality forecasting papers do not explicitly model mortality jumps when they describe fu-

ture mortality stochastic processes. Dahl (2003) and Milevsky and Promislow (2001) model the force of mortality as an Itô-type stochastic process in continuous time with continuous sample paths. Cairns et al. (2004) use a more sophisticated approach, but still it is a continuous time model with continuous sample paths. Econometric methods, like Renshaw's method (Renshaw et al., 1996; Sithole et al., 2000) and the Lee–Carter model (Lee and Carter, 1992; Lee, 2000), do not explicitly take into account mortality jumps either. We propose a model with a jump, or regime switch, to indicate the presence of an event such as a flu epidemic. Our model can be much more simple than those cited here, since we need a model of the population mortality, a single number each period, rather than an entire mortality table in each period. Therefore we choose a simple model, a two-state regime-switching log-normal model. We estimate the model and illustrate it by pricing a death-linked security, the Swiss Re bond.

There are only a few papers that show how to price mortality-linked securities. Cairns et al. (2004) price mortality securities in the context of a complete market measure, providing a very important advance in modeling, but we do not have enough data to apply it. And we are not sure how one would estimate the model. On the other hand, we use the Wang transform, which does not require a complete market, only a reference price.

4.2. Our model for Swiss Re bond

The Swiss Re bond links bondholder payoffs to a population mortality index. This provides transparency, reducing moral hazard, since the population index is a weighted average of government-produced mortality statistics.

4.2.1. Model

Our approach combines a Brownian motion and a discrete Markov chain with log-normal jump size distribution with parameters m and s. Most of mortality jumps, e.g. the 1918 flu or 2004 earthquake and tsunami, are one-time events. It is true for the death side in most cases. However, mortality usually improves over a long period. We use another method to price the EIB bond which is discussed in Section 4.3. Catastrophes push up the whole population death rate just for that bad year. The mortality curve after the shock is independent of that during the shock. Our model takes this into account.

The discrete Markov chain counts the number of events N_t during years $t = 0, 1, 2, \ldots$ with $N_0 = 0$ and transition

$$N_{t+1} = \begin{cases} N_t + 1, \text{ with probability } p \\ N_t, \text{ with probability } 1 - p. \end{cases} \tag{7}$$

We describe the US population mortality index $\tilde{q}_t$ dynamics at time t as a geometric Brownian motion q_t when there are no events:

$$\mathrm{d}q_t / q_t = \alpha \mathrm{d}t + \sigma \mathrm{d}W_t. \tag{8}$$

The variable α is the instantaneous expected force of the US population mortality index. σ is the instantaneous volatility of the mortality index, conditional on no jumps. W_t is a standard Brownian motion with mean 0 and variance t. The equivalent explicit description of the index, assuming no events in $(t - h, t)$, is

$$q_t = q_{t-h} \mathrm{e}^{(\alpha - \sigma^2/2)h + \sigma(W_t - W_{t-h})}. \tag{9}$$

The percentage change in the mortality rate due to a random jump is denoted by $Y - 1$. The Markov chain and the jump size Y distributions are independent (and independent of W). A jump event occurs during the period $[t - h, t]$ with probability p. When a jump occurs, there is a shock effect Y_t on q_t. We express the mortality index, including the jump, as $\tilde{q}_t = q_t Y_t$ for $t = 0, h, 2h, 3h, \ldots$. Our data consist of annual observations so $h = 1$.

If there is no jump in $[t - h, t]$, then $Y_t = 1$ and $\tilde{q}_t = q_t$. When there is a jump, we assume that Y_t is log-normally distributed with parameters m and s. That is, $Y_t = \mathrm{e}^{m + sU_t}$ where the U_t is a standard normal variable. In summary, if the index assuming no event is

$$q_t = q_0 \mathrm{e}^{(\alpha - \sigma^2/2)h + \sigma W_t} \text{ for } t \geq 0,$$

then the US population mortality index $\tilde{q}_t$ can be explicitly described as follows:

$$\tilde{q}_t = \begin{cases} q_t Y_t & \text{with probability } p \\ q_t & \text{with probability } 1 - p \end{cases} \quad \text{for } t = h, 2h, 3h, \ldots \tag{10}$$

where $Y_h, Y_{2h}, \ldots$ are either 1 (with probability $1 - p$) or independent log-normal variables with parameters m and s.

The mortality index $\tilde{q}_t$, independent of previous mortality jumps, will be continuous most of the time with finite jumps of differing signs and amplitudes occurring at discrete points of time. Let $\mathcal{F}_t$ denote the events determined by the processes W_s, N_s, and Y_t for $s \leq t$. We apply Equation (10) to get and expression for the index at the end of the period $[t - h, t]$ in terms of the index without jumps.

$$\tilde{q}_{t+h} \mid \mathcal{F}_t = q_t \exp\left[(\alpha - \sigma^2/2)h + \sigma(W_{t+h} - W_t)\right] Y_{t+h}. \tag{11}$$

In effect, a jump impulse during $[t - h, t]$ has no effect during $[t, t + h]$.

Now we apply the maximum likelihood method to estimate the parameters α, σ, p, m, and s from the data. The term $\sigma(W_{t+h} - W_t)$ describes the continuous part of the unanticipated "normal" mortality index change and $Y - 1$ describes the percentage change due to an "abnormal" mortality shock with probability p when $N_{t+h} - N_t = 1$. Appendix shows the derivation of our maximum likelihood function from Equation (11).

4.2.2. Data

Our data are obtained from the Vital Statistics of the United States (VSUS).[4] The VSUS reports the United States age-adjusted death rates per 100,000 standard million

[4] Source: http://www.cdc.gov

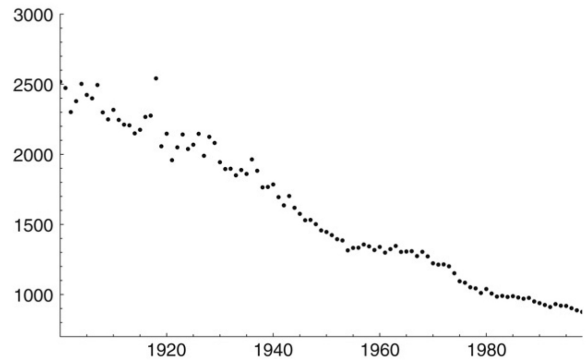

Figure 2. 1900–1998 US total population death rate per 100,000 (= $100{,}000q_t$ where t = 1900, 1901, …, 1998).

Table 1. Maximum likelihood parameter estimates based on the US population mortality index 1900–1998

Parameter	Estimate	Parameter	Estimate
α	−0.0096	m	0.1492
σ	0.0310	s	0.0404
p	0.0115		

The model without jumps is rejected at the significance level of 0.1%.

population (2000 standard) for selected causes of death. Age-adjusted death rates are used to compare relative mortality risks across groups and over time; they are the indexes rather than the direct measures. We plot our data from 1900 to 1998 in Figure 2.

Figure 2 shows that the mortality stochastic process does not follow a mean-reverting process.[5] Moreover, there are several jumps in the US population mortality evolution which should be captured by a good mortality stochastic model. Mortality shocks may cause financial distress or bankruptcy of insurers or pension plans and they are also the risks underlying the mortality securities.

4.2.3. Estimation results

Based on the US population mortality index q_t from 1900 to 1998 shown in Figure 2, Table 1 reports our maximum likelihood estimation results. The instantaneous expected force of mortality index α is equal to −0.0096. The negative sign of α suggests the US population mortality improves over time. The instantaneous volatility of the mortality index, conditional on no jumps, σ, is equal to 0.0310. The probability of a jump event each year is equal to 1.15%. Our likelihood ratio test rejects the model without jumps at the significance level of 0.1%.

4.2.4. Market price of risk

In this section, we show how to estimate the market price of risk of the Swiss Re bond. First, we simulate Equation (11) with the estimates shown in Table 1 to get cumulative probabilities $F(q)$, where q is the maximum of the simulated US population mortality index q_t from 2004 to 2006. We run 10,000 simulations.

Second, the physical cumulative probabilities $F(q)$ are transformed by a starting value of the market price of risk

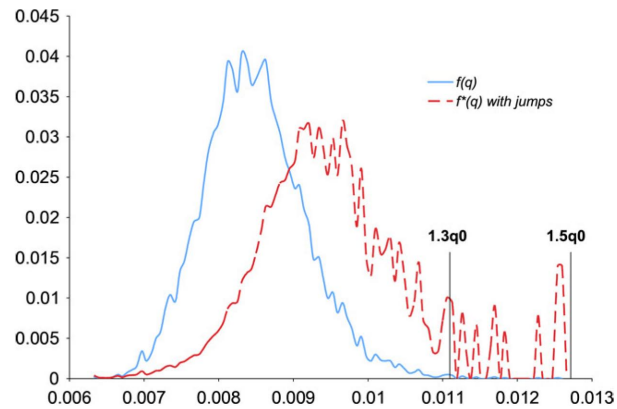

Figure 3. Two-factor Wang transformed probability distribution of $q = \max(q_{2004}, q_{2005}, q_{2006})$ with λ_{SR} = –1.3603 and six degrees of freedom (shown as a dashed line) and the physical probability distribution of q (shown as a solid line). The horizontal axis is the death rate and the vertical axis stands for the probability.

λ_{SR} (λ_{SR} will be solved later) to get the pricing probabilities $F^*(q)$ as shown in Equation (12):

$$F^*(q) = Q[\Phi^{-1} F(q) - \lambda_{SR}]. \tag{12}$$

Q is the Student's t-distribution with six degrees of freedom following Wang (2004). The transformed probabilities $f^*(q)$ can easily be derived from cumulative probabilities $F^*(q)$.

Third, after obtaining simulated q values and their transformed probabilities $f^*(q)$, we calculate the payment at maturity following Equation (3). Based on Equations (11) and (12), the par spread of the Swiss Re bond 1.35% (Swiss Re, 2003; MorganStanley, 2003; The Actuary, 2004), the US population mortality index from 1900 to 1998 and the US Treasury yield rates on December 30, 2003, our estimated market price of risk λ_{SR} of the Swiss Re deal is −1.3603.

In Figure 3, the dashed line denotes the transformed probability density function (PDF) of q with λ_{SR} = –1.3603 by using the two-factor Wang transform. It lies on the right of the PDF of simulated US population mortality index "$f(q)$". After transforming the data, we put more weight on the right tail. This implies that the market expects a higher probability of having a big loss than the actual probability suggests.

4.2.5. Is the jump process important?

Most of the existing mortality stochastic models do not consider the jump process. In Section 4.2.1, we use Brownian motion and the Markov chain to model the dynamics of the US population mortality index. To prove that the jump process is important in the mortality securitization modeling, we compare the market price of risk *without* mortality jumps with that *with* jumps.[6]

The mortality stochastic model *without* jumps is shown as follows:

$$dq_t/q_t = \alpha_n dt + \sigma_n dW_t, \tag{13}$$

where α_n and σ_n are the expected force and volatility of the

[5] But it could be a process that is mean reverting around a trend.

[6] We thank Patrick Brockett for his suggestion to add this part to the paper.

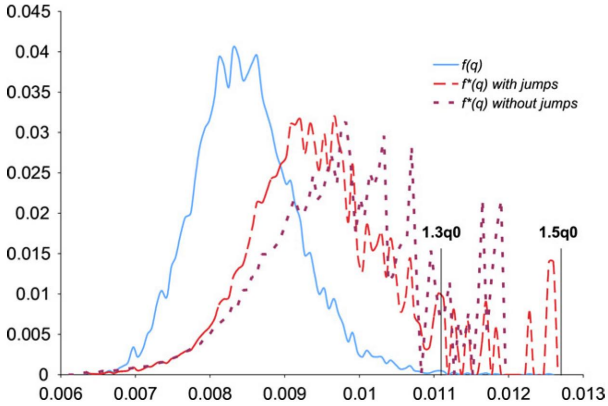

Figure 4. Two-factor Wang transformed probability distribution of q=max(q_{2004},q_{2005},q_{2006}) with λ_{SR} = –1.3603 and six degrees of freedom (shown as a dashed line) in the jump model and that with λ_{SR} = –1.5545 and six degrees of freedom (shown as a dotted line) in the model without jumps. The x-axis is the death rate and the y-axis stands for the probability.

US population mortality index with subscript n indicating no jumps. Based on the same data shown in Section 4.2.2, our maximum likelihood estimate of α_n is −0.0100 and 0.0388 for σ_n. Without jumps, our estimated market price of risk for the US-based Swiss Re bond is equal to −1.5545, which is 14% higher than the market price of risk −1.3603 in terms of absolute value when we model the US population mortality index with jumps. Moreover, we plot the transformed distributions with and without jumps of q in Figure 4. Figure 4 shows that the model without jumps underestimates the probability of having a catastrophe death event since the model with jumps has a longer tail. Failing to model jumps leads to a notable deviation from the right market price of risk and the correct transformed distribution. We conclude that the jump process plays an important role in the mortality securitization modeling.

4.2.6. Is the Swiss Re bond a good deal for investors?

Wang (2004) reports that the average market price of risk of property catastrophe bonds is about −0.45. Applying the two-factor Wang transform with λ = −0.45, our calculated par spread for the Swiss Re bond is 0.39%, which is lower than that of the Swiss Re bond, 1.35%. The difference may arise from the fact that we use the US population index as the benchmark while the Swiss Re deal is based on the weighted average of five developed countries. If we use the weighted index, we expect that our calculated par spread will be even lower because of the diversification effect of mortality risks among these five countries. Bantwal and Kunreuther (1999) found the spread of property catastrophe bonds to be too high to be explained by standard financial theory. Here we find the market price of risk of the Swiss Re bond, −1.3603, is even higher than that of the property catastrophe bonds, −0.45 in terms of absolute value. Although the high risk premium of the Swiss Re deal may suggest high transaction costs of the first mortality security, it may also be interpreted as the Swiss Re overcompensating the investors for their taking its mortality risks. So it explains why the "appetite" for the Swiss Re bond was strong.

Why did the Swiss Re company pay such a high risk premium to the investors? The Swiss Re Company's life reinsurance business accounted for 43% of its group revenues in 2002, up from 38% in 2001 (MorganStanley, 2003). Although capital is crucial for a firm to absorb mortality shocks, the true economic capital requirements of life reinsurance business is not straightforward. Moreover, the costs of potential financial distress are high. Minton et al. (2004) conclude that securitization of financial institutions is a contracting innovation aimed at lowering financial distress costs. Therefore, MorganStanley (2003) concludes that Swiss Re must be taking a view that the cost of capital that is relieved via this transaction exceeds the effective net cost of servicing the bond. Moreover, insurance companies pay a high risk premium to develop the mortality securitization market. If catastrophes deplete the traditional reinsurance risk-taking capacity, the insurers can turn to the mortality security market for protection. In all, the Swiss Re mortality bond is a good deal to the investors.

4.3. Our model for the EIB bond

Compared with mortality deterioration caused by catastrophes like epidemics or earthquakes, mortality improvement spans a much longer period. Therefore, we adopt Lin and Cox (2005)'s method to price a longevity bond. Two differences of our method from Lin and Cox (2005) are as follows. First, we use the two-factor Wang transform to account for parameter uncertainty while Lin and Cox (2005) use one-factor model; second, we use the EIB bond price to derive the market price of risk while Lin and Cox (2005) use the US single premium immediate annuity (SPIA) market quotes.

4.3.1. Our model

We show how to estimate the market price of risk of a longevity bond based on the two-factor Wang transform. We defined our transformed distribution F^* for the EIB bond in year t as

$$F^*({}_tq_{65}) = {}_tq^*_{65} = Q[\Phi^{-1}({}_tq_{65}) - \lambda_{EIB}] \tag{14}$$

where ${}_tq_{65}$ is the probability that a person aged 65 dies before age 65 + t and t = 1, 2, …, 25. To calculate the values of ${}_tq_{65}$ for the English and Welsh male population aged 65, we use the realized mortality rates of English and Welsh males aged 65 and over in 2003. Q is the Student's t-distribution with six degrees of freedom following Wang (2004). The interest rates are the gilt STRIPS on November 18, 2004.

The total value of the EIB bond was £540 million. Its 25-year bond annual payout is equal to a fixed annuity, £50 million, multiplied by the percentage of the reference population still alive at each anniversary. Assuming the expense factor 6%, we solve the market price of risk λ_{EIB} for the English and Welsh male by the following equation:

$$540{,}000{,}000 \times (1 - 0.06) = 50{,}000{,}000\, a^*_{65\overline{25}|},$$

where $a^*_{65\overline{25}|}$ is the present value of a life annuity of 1 per year, payable in instalments at the end of each year while the annuitant survives for 25 years. The mortality rates in

$a^*_{65\overline{25}|}$ are the distorted mortality rates based on Equation (14).

4.3.2. Market price of risk

Our calculated market price of risk for the EIB bond λ_{EIB} is equal to 0.2408. It is not surprising that the market price of risk of the EIB bond (0.2408) is lower than that of the Swiss Re bond (−1.3603) in terms of absolute value since longevity risks have much less immediate and dramatic impact on annuity business or pension plans (25 years or longer) than catastrophic death risks caused by disasters like flu (less than a year) on the life insurance business. The interesting question here is whether the market price of risk of the EIB bond, λ_{EIB} = 0.2408, is too high for the potential bond buyers—the UK pension plans.

Let us compare the market price of risk of the EIB bond λ_{EIB} = 0.2408 with that of private annuities. Based on the same idea in Equation (14), our calculated market price of risk of the average SPIA for the US male annuitants aged 65 is only 0.1524.[7] The market price of risk reflects the costs of adverse selection. Adverse selection is the tendency of persons with a higher-than-average chance of loss to seek insurance at standard rates, resulting in higher-than-expected loss levels (Rejda, 2005). For example, healthy people purchase more annuities while those in poorer health buy more life insurance. Prior literature concludes that pension plans have much less adverse selection problem than commercial annuity insurers because both healthy and less healthy employees participate in pension plans. Therefore the longevity risk for a pension plan should be lower than that for a commercial annuity insurer. Since mortality experiences of US population and English and Welsh population are similar, theoretically, the market price of risk for the EIB bond should be lower than that for the annuity business, 0.1524. However, this is not true for the EIB bond. Therefore, it explains why the UK pension plans, the potential *buyers* of the EIB bond, are not willing to buy the EIB bond since they can get the cheaper protection from the annuity or reinsurance markets.

5. Discussion and conclusions

We are learning every day how important mortality forecasts are in the management of life insurers and private pension plans. Securitization and development of mortality bonds can be an important part of capital market solution to these problems. Before the Swiss Re bond was issued at the end of 2003, life insurance securitization was not designed to manage mortality risk; rather they are exactly like asset securitization. In these cases, the insurers convert future life insurance profits into cash to increase liquidity rather than manage mortality risk. The new securitization we study in this paper focus on the other side of an insurer's balance sheet—liabilities on future mortality payments. The introduction of mortality securities to transfer the insurer's risks in the liabilities side increases its capacity and maintains its competitiveness.

A market for mortality-based securities will develop if the prices and contracting features make the securities attractive to potential buyers and sellers. The Swiss Re bond has sold well but the EIB bond has not. We explain these opposite market outcomes by looking at their risk premiums. To calculate the risk premiums we need models, analogous to the term structure on interest rate models. The mortality bond market will be richer in that, in addition to default free zero coupon bonds, it will have bonds which will be redeemed at face value only if a specified number of lives survives or dies to the maturity date. We find only a few preliminary papers on this topic. Development of the theory in this direction is important as an extension of traditional bond market models and it would be very useful in explaining mortality market risk to potential market participants.

Our model shows that the Swiss Re mortality bond offers a higher risk premium to investors than the property-linked catastrophe bonds. However, the EIB charges a very high risk premium to take longevity risks in the UK pension plans. Since the price of the EIB bond is not attractive, no UK pension plan has bought this bond until now!

Moreover, the EIB bond provides "ground up" protection, covering the entire annuity payment. But the plan can predict the number of survivors to some extent, especially in the early contract years. The EIB bond price includes coverage the plan does not need (including rates, commissions, etc.). A more attractive contract might cover payments to survivors in excess of some strike level. The price would be much lower, as shown in Lin and Cox (2005).

Someone may argue that the index-linked mortality securities are subject to unacceptable levels of basis risk. The basis risk is lower for the Swiss Re bond than for the EIB bond. The population index of the Swiss Re bond accounts for basis risk by using different weights in five countries to match Swiss Re's business. However, the EIB bond does not provide a good hedge for a pension plan: there exists a significant basis risk between the reference population mortality and that of an individual pension plan. The EIB deal does not allow pension plans to decide their own mortality references (e.g. by using different weights in the Swiss Re bond) to reduce the basis risk. The basis risk problem further reduces the attractiveness of the EIB bond.

In summary, we contribute to the mortality securitization literature by proposing a mortality stochastic model with jumps for death-linked insurance securities and pricing the Swiss Re and EIB bonds in an incomplete market framework. Our models nicely explain the opposite market outcomes of these two deals. Finally we comment on the structure and basis risk problem of these two bonds. Again, it shows the attractiveness of the Swiss Re deal but not for the EIB bond.

[7] We get the market quotes of the non-qualified SPIA in August 1996 from Kiczek (1996). Kiczek (1996) reports the male and female SPIA monthly payout rates of 102 companies with the $100,000 lump-sum premium at the issue age 65. We assume that an insurer sells its SPIA at the market average payout rates $764 for male. We also assume an expense factor of 6%. Our US Treasury yield rates on August 15, 1996 are obtained from the *Wall Street Journal*. Our mortality rates are from the 1996 Male IAM 2000 Basic Mortality Table.

APPENDIX. Maximum likelihood estimation of mortality Stochastic model with jumps

After taking the logarithm of both sides of Equation (11), we obtain

$$\log \tilde{q}_{t+h} = \log q_t + (\alpha - \sigma^2/2)h + \sigma\,\Delta W_t + \log Y_{t+h} \tag{15}$$

where Δ denotes the change over $[t, t+h]$. Let $Z_t = \Delta \log \tilde{q}_t$ so that

$$\begin{aligned} Z_t &= \Delta \log \tilde{q}_{t+h} - \log \tilde{q}_t \\ &= (\alpha - \sigma^2/2)h + \sigma\,\Delta W_t + \log Y_{t+h} - \log Y_t \end{aligned}$$

The conditional distribution of Z_t, given $\mathcal{F}_t$, is denoted $Z_t \mid \mathcal{F}_t$. We have observations of the index $\tilde{q}_t$ for $t = h, 2h, \ldots Kh$.

If no mortality jump event occurs during the periods $[t-h, t]$ and $[t, t+h]$, then $Y_t = Y_{t+h} = 1$ and $Z_t \mid \mathcal{F}_t$ is normally distributed with mean $M_{nn} = (\alpha - \sigma^2/2)h$ and variance $S^2_{nn} = \sigma^2 h$. If a jump event occurs during the period $[t-h, t]$ but not $[t, t+h]$, then $Y_t = \exp(m + sU_t)$ and $Y_{t+h} = 1$. In this case, $Z_t \mid \mathcal{F}_t$ is normally distributed with mean $M_{yn} = (\alpha - \frac{1}{2}\sigma^2)h - m$ and variance $S^2_{yn} = \sigma^2 h + s^2$. There are two other possibilities: no mortality jump during the period $[t-h, t]$ but a mortality jump during $[t, t+h]$; a mortality jump during the period $[t-h, t]$ and another mortality jump during $[t, t+h]$. We summarize the conditional means and variances of Z_t as follows. Given $\mathcal{F}_t$, we will know if jumps occurred in $[t-h, t]$ and $[t, t+h]$. Let nn denote the event "no jumps occurred"; yn the event "a jump occurred in $[t-h, t]$ but no jump occurred in $[t, t+h]$," and so on. In summary, the distribution is described as follows:

Jump events	$E[Z_t \mid \mathcal{F}_t]$	$Var[Z_t \mid \mathcal{F}_t]$	Probability
nn	$(\alpha-\sigma^2/2)h$	$\sigma^2 h$	$(1-p)^2$
yn	$(\alpha-\sigma^2/2)h-m$	$\sigma^2 h+s^2$	$p(1-p)$
ny	$(\alpha-\sigma^2/2)h+m$	$\sigma^2 h+s^2$	$(1-p)p$
yy	$(\alpha-\sigma^2/2)h$	$\sigma^2 h+2s^2$	p^2

The probability density function of Z_t, $f_Z(z)$, can be written in terms of the conditional density of $Z_t \mid \mathcal{F}_t$, denoted $fz(z \mid \mathcal{F}_t)$, which has a conditionally normal distribution:

$$\begin{aligned} f_Z(z_t) = {} & f_Z\,(z_t | N_t - N_{t-h} = 0 \text{ and } N_{t+h} - N_t = 0) \\ & \times \Pr\,(N_t - N_{t-h} = 0 \text{ and } N_{t+h} - N_t = 0) \\ & + f_Z\,(z_t | N_t - N_{t-h} = 1 \text{ and } N_{t+h} - N_t = 0) \\ & \times \Pr\,(N_t - N_{t-h} = 1 \text{ and } N_{t+h} - N_t = 0) \\ & + f_Z\,(z_t | N_t - N_{t-h} = 0 \text{ and } N_{t+h} - N_t = 1) \\ & \times \Pr\,(N_t - N_{t-h} = 0 \text{ and } N_{t+h} - N_t = 1) \\ & + f_Z\,(z_t | N_t - N_{t-h} = 1 \text{ and } N_{t+h} - N_t = 1) \\ & \times \Pr\,(N_t - N_{t-h} = 1 \text{ and } N_{t+h} - N_t = 1) \\ = {} & \left(\frac{1}{S_{np}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_t - M_{nn}}{S_{nn}}\right)^2}\right)(1-p)(1-p) \\ & + \left(\frac{1}{S_{yn}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_t - M_{yn}}{S_{yn}}\right)^2}\right) p(1-p) \\ & + \left(\frac{1}{S_{ny}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_t - M_{ny}}{S_{ny}}\right)^2}\right)(1-p)p \\ & + \left(\frac{1}{S_{yy}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_t - M_{yy}}{S_{yy}}\right)^2}\right) pp. \end{aligned} \tag{16}$$

We have a time series of K observations of $\tilde{q}_t$, where $t = 0, 1, 2, \ldots, K-1$, so the step size is $h = 1$. We have $K-1$ observations z_t of $Z_t = \log \tilde{q}_{t+1} - \log \tilde{q}_t$. The correlations between the z_t's are zero except when jump events occur. The probability of having a mortality catastrophe event is very low. Based on the historical data, the correlation introduced by mortality catastrophes to the likelihood function is only around −0.005. Therefore, we use independence as an approximation and estimate the parameters p, α, σ, m, and s by maximizing the logarithm of the likelihood function (17) based on the observations $z_1, z_2, \ldots, z_{K-1}$. The likelihood is

$$L = \prod_{t=1}^{K-1} fz(z_t) \tag{17}$$

Taking the logarithm of Equation (17), we get

$$\begin{aligned} \log L &= \log \prod_{t=1}^{K-1} f_Z(z_t) = \sum_{t=1}^{K-1} \log f_Z(z_t) \\ &= \sum_{t=1}^{K-1} \log \left\{ \sum_{j=0}^{1}\sum_{\ell=0}^{1} f_Z(z_t | N_t - N_{t-h} = j \right. \\ &\quad \text{and } N_{t+h} - N_t = \ell) \\ &\quad \left. \cdot \Pr\,(N_t - N_{t-h} = j \text{ and } N_{t+h} - N_t = \ell) \right\} \\ &= \sum_{t=1}^{K-1} \log \left(\frac{1}{S_{nn}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_t - M_{nn}}{S_{nn}}\right)^2}\right)(1-p)(1-p \\ &\quad + \left(\frac{1}{S_{yn}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_t - M_{yn}}{S_{yn}}\right)^2}\right) p(1-p) \\ &\quad + \left(\frac{1}{S_{ny}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_t - M_{ny}}{S_{ny}}\right)^2}\right)(1-p)p \\ &\quad + \left(\frac{1}{S_{yy}\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{z_t - M_{yy}}{S_{yy}}\right)^2}\right) pp. \end{aligned} \tag{18}$$

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