Module 11: Suggested Reading

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Module 11 Suggested Reading

Introduction

This module contains reprints of several articles related to the ideas of stages of development and self-regulation and a bibliography of books and articles that you may wish to study after you complete the workshop.

Objectives

To provide you with examples of applications of the instructional techniques that you were introduced to in the workshop, and to make available a bibliography that you can use for further study.

Procedure

If you would like further background information on Piaget's theory as related to physics instruction, read one or more of the three reprints selected from AJP and TPT that are included in the instructional materials for this module. If you would like additional information on Piaget's theory in general, read the article by Piaget reprinted here or consult the books and articles listed in the bibliography -- most are available in paperback and many can be obtained in any college or university bookstore.

INSTRUCTIONAL MATERIALS

This module contains the following materials:

1. Reading list of suggested books and articles.


Module 11 Instructional Materials

Books


Selected Articles


A survey of four computer dictionaries gave no definition for minicomputers. From The New York Times, 5 April 1970, Sec. 3, p. 1:

Maxi Computers Face Mini Conflict, by William D. Smith.

Mini vs Maxi, the reigning issue in the glamorous world of fashion, is strangely enough also a major point of contention in the definitely unsexy realm of computers.

The definition of a minicomputer depends on to whom you are speaking. Descriptions range from electronic calculators to the IBM System 3 that sells for $42 000.

A consensus opinion would probably include as minicomputers machines that cost less than $25 000 and that include some type of input-output device such as a teleprinter, a memory of about 4000 words, and circuitry capable of performing calculations under the control of stored programs written in some form of higher-level computer language such as FORTRAN or BASIC.

The major manufacturer of minicomputers is the Digital Equipment Corporation. Other major makers include the Hewlett-Packard Corporation, the Data General Corporation, Varian Associates, Honeywell, Computer Automation, Inc., Motorola, the Raytheon Corporation and Mini-Computer Systems, Inc.

Are Colleges Concerned with Intellectual Development?

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(Received 14 December 1970; revised 8 March 1971)

The assumption is often made by college professors that incoming freshman students think logically. Using tests designed by the Swiss psychologist Jean Piaget to evaluate logical thought processes, the authors found that 66 of 131 freshmen exhibited characteristics of the concrete operational thinker, while another 26 did not meet the criteria for formal operations. Professors further compounding the problem by failing to recognize the kinds of experiences incoming freshmen students must have to move toward more logical thought. McKinnon, using a newly developed inquiry-oriented science course based upon Piagetian criteria, found a highly significant difference between those students who were exposed to the course and like students who were not. The authors concluded that secondary and elementary teachers do not take advantage of inquiry-oriented techniques so necessary to the development of logical thought because college professors do not provide examples of inquiry-oriented teaching.

INTRODUCTION

Are colleges and universities making inadequate evaluations of student ability to think logically? Is the unrest today in many universities caused by student evaluation of problems based upon emotion rather than logic? Do student claims that curriculums are irrelevant, trivial, and inadequate in terms of the magnitude of the problems facing mankind today have substance, or are these students unable to evaluate logically the structure and necessity of those curriculums? These questions, together with suspicions voiced by various professors of science about the inability of their freshman students to think logically about the simplest kind of problems, led the authors to question whether or not most college freshmen do think logically. This doubt about the ability of the entering freshman to think logically led to the following hypothesis: The majority of entering college freshmen do not come to college with adequate skills to argue logically about the importance of a given principle when the context in which it is used is slightly altered.

Since these students have been accepted by boards of admission that based their decisions upon high school transcripts and various established entrance examinations such as the American

September 1971

AJP Volume 39 / 1047
Table I. A comparison of operational level of 131 students on Piagetian data.

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total number</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formal</td>
<td>25</td>
<td>8</td>
<td>33</td>
<td>25</td>
</tr>
<tr>
<td>Post-concrete</td>
<td>12</td>
<td>20</td>
<td>32</td>
<td>25</td>
</tr>
<tr>
<td>Concrete</td>
<td>16</td>
<td>50</td>
<td>66</td>
<td>25</td>
</tr>
<tr>
<td>Mean Piagetian score</td>
<td>12.82</td>
<td>9.45</td>
<td>Average 10.74</td>
<td></td>
</tr>
</tbody>
</table>

College Test (ACT) and the Scholastic Aptitude Test (SAT), a different means of evaluation was sought. The evaluative system used is one based upon the ability of the student to think critically about problems, the answers to which would be found in his experiential background and could not be derived from memorized data.

**WHEN DO STUDENTS BEGIN TO THINK LOGICALLY?**

The scheme of evaluation of the ability to think logically which was used has been developed and verified by a Swiss psychologist, Jean Piaget, during many years' research with children. There is, however, no indication that his work has been extended to include entering college students, particularly American students. In addition, no work can be found with American children which verifies his conclusions that children begin to think logically between ages 11-15.

Piaget found that children progress through various stages of mental manipulation and that these steps cannot be circumvented. Prior to thinking about abstract ideas, a student must undergo a period of physical manipulation of objects using the basic principles upon which the abstraction to be developed depends. This stage Piaget identifies as the **concrete stage of thought**. A student may handle concepts quite adequately, but until he has had many manipulative experiences he cannot recognize those concepts in the context of a broader generalization, of which the manipulative experiences and the concepts are simply a subset. Inhelder and Piaget found that from 11-15 years of age most Swiss children should become **formal operational**, i.e., capable of abstract logical thought. The concern of this research was whether or not this was true for American college freshmen, i.e., had those students become formal operational?

**A STUDY OF THE ABILITY OF COLLEGE FRESHMEN TO THINK LOGICALLY**

McKinnon studied responses to tasks given 131 members of the freshman class at an Oklahoma university in which students had to think logically about problems of volume conservation, reciprocal implication of two factors, the elimination of a contradiction, the separation of several variables, and the exclusion of irrelevant variables from those relevant to problem solutions. These tasks had initially been developed by Inhelder and Piaget for determining the patterns of thought of children and the ages at which changes in those thought patterns occur.

Table I presents the test results for these 131 students using the foregoing tasks and the criteria specified by Inhelder and Piaget for demonstrating formal operational thought. Each student was graded from 0 through 4 on each of the tasks. Should a student score a total of 14 or more points on the five tasks, he was judged as definitely being at the formal operational stage. To achieve 14 points, he had to score at least 3 points on the
tasks for which 4 points were possible. If a student scored an average of 2 points or less on each of the five tasks, he was judged to be at the concrete stage of operations. Those students who scored more than 10 but less than 14 points were judged to be moving from the concrete stage to the formal stage of thought.

The findings, as shown in Table I, are that 50% of the entering college students tested were operating completely at Piaget’s concrete level of thought and another 25% had not fully attained the established criteria for formal thought. The average score for all students was 10.74, with the males scoring significantly higher than females. An examination of the performance of the students on the various tasks used follows:

1. Of the college freshmen tested, 17% of them did not conserve quantity (the result of a change of form), while another 10% failed to recognize equivalence of volume. Thus, 27% of those students tested were at the lowest concrete operational state or less.

2. Reciprocal implication involved the student in the problem of reflecting a ball and the necessity to relate incident and reflected angles. This task was second only to the problem of density in the number of failures recorded—64% scored 2 or less.

3. The elimination of a contradiction involved the student in relating weight and volume of floating and sinking objects in a meaningful way. More than 3 of those tested did not relate weight and volume. Typically, they recognized weight only. Seldom was there a proportionality expressed; 67% of the students tested on this task were concrete operational.

4. The separation of variables task gave evidence that 50% of entering college freshmen could not recognize the action of a potential variable and find a way to prove the action of that variable.

5. The task of excluding irrelevant variables showed that 33% of the students tested could not eliminate variables of no consequence in a swinging pendulum, while another 18% could do no more than order the effects of weight.

In the research, a comparison was made of the score obtained by each student on the various Piagetian tasks given him and this score was correlated with his ACT composite score. (See Fig. 1.) A graph of these two scores shows that Pearson product–moment correlations were high for those students scoring at the average ACT composite of 22 or better, but correlations of −0.05 were found for students scoring less than that average. The university where this study was made ranks high in terms of the average ACT scores when compared with all other colleges and universities in Oklahoma and is well above average for all regions of the United States. Almost 75% of that university’s entering freshmen, however, were either partially or completely concrete operational. What evidence exists, therefore, to demonstrate that logical thought can be promoted among all levels of students?

CAN INQUIRY-ORIENTED COURSES PROMOTE LOGICAL THOUGHT?

The University of Oklahoma Science Education Center has, for some time, been investigating the effects of inquiry-oriented teaching upon both teachers and pupils. Various new courses in science which utilize the inquiry approach have been evaluated. Porterfield compared teachers of reading who had inquiry educational experiences in science with those who had not. He found that the former tended to use more questions requiring analysis and synthesis and other high-level cognitive thought patterns than did the latter group. Wilson found much the same in a study of 30 classes of elementary children when fifteen of the teachers had been exposed to inquiry experiences in science and fifteen had not. He found that the former tended to use more questions requiring analysis and synthesis and other high-level cognitive thought patterns than did the latter group. Wilson found much the same in a study of 30 classes of elementary children when fifteen of the teachers had been exposed to inquiry experiences in science and fifteen had not. Schmidt found similar results by investigating the teaching in social studies done by teachers who had and had not been involved with inquiry in science. Friel found in a study of seventh, eighth, and ninth grade science that courses placing emphasis upon the inquiry approach allowed students to be able to function at a much higher level of logical thought than those courses in which students did not have that inquiry experience.

Stafford used the development of conservation reasoning in children as an evaluative tool to determine whether or not inquiry-oriented science experiences move first graders toward the acquisition of concrete operational thought. The specific unit he used was Material Objects. Stafford found: “...those first grade children who have experiences with the unit achieved the ability to
TABLE II. A comparison of the growth in logical thought processes of the experimental and control groups.

<table>
<thead>
<tr>
<th>Group</th>
<th>Stage</th>
<th>Pre-test Females</th>
<th>Pre-test Males</th>
<th>Post-test Females</th>
<th>Post-test Males</th>
<th>Net gain Females</th>
<th>Net gain Males</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Formal</td>
<td>4</td>
<td>11</td>
<td>14</td>
<td>16</td>
<td>10</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>Post-concrete</td>
<td>14</td>
<td>6</td>
<td>17</td>
<td>8</td>
<td>3</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Concrete</td>
<td>24</td>
<td>10</td>
<td>11</td>
<td>3</td>
<td>-13</td>
<td>-7</td>
<td>-20</td>
</tr>
<tr>
<td>Control</td>
<td>Formal</td>
<td>4</td>
<td>14</td>
<td>7</td>
<td>17</td>
<td>3</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Post-concrete</td>
<td>6</td>
<td>6</td>
<td>11</td>
<td>7</td>
<td>5</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Concrete</td>
<td>26</td>
<td>6</td>
<td>18</td>
<td>2</td>
<td>-8</td>
<td>-4</td>
<td>-12</td>
</tr>
</tbody>
</table>

conserve much more rapidly than did those children who did not have these experiences. M1

Material Objects is an inquiry-centered unit and Stafford concluded: "...children so taught do show more rapid intellectual development than do those children not having such experiences." M1

Finally, McKinnon, M1 in a study of the effect of an inquiry-centered science course on entry into the formal operational stage of concrete operational freshman college students, found a highly significant difference between those students enrolled in the course and a like group who had not been exposed to the course.

The data of Table I gave evidence of the ability of students to think logically. The data of Table II show the effect of the inquiry-centered course upon freshman students' ability to think logically. A net gain in favor of the experimental group resulted in 15 students moving into the formal stage of thought—compared with six for the control group. The post-concrete gain was, respectively, five and six, with the experimental group showing a net movement of 20 out of this category compared with 12 for the control group, a net gain of more than 50% for the group exposed to the influence of the new science course. The material of the science course did not include references to the tasks which were part of the test instruments; therefore, changes in ability to think logically were caused by added opportunities for inquiry. Another comparison in terms of the mean Piagetian scores for the two groups is shown in Table III.

After obtaining individual pre-test–post-test differences and summing them up for each group, an F ratio of 6.24 was obtained. This value is significant in favor of the test group at the 0.001 level of confidence; therefore, the hypothesis must be accepted that a properly designed course in science for freshman college students does enhance their logical thought patterns by increasing their ability to hypothesize, verify, restructure, synthesize, and predict.

The preceding research gives evidence that students do not think logically. However, research carried out on newly developed courses does give evidence that the logical thought processes can be enhanced. Therefore, who is at fault and what steps must be taken to alleviate the situation?

AN EVALUATION OF EDUCATIONAL RESPONSIBILITY USING THE INQUIRY APPROACH

If students do not think logically when they enter college, who has not discharged his responsibility? The immediate answer to the foregoing question is, the high school. That answer, however, needs to be examined.

Piaget states formal operations begin to emerge around 11 years of age. But Friot found that 82% of eighth and ninth grade children (ages 13 and 14 years) were still concrete operational. Thus, children probably enter senior high school two to three years behind the age set by Piaget for

TABLE III. Pre-test and post-test Piagetian mean scores for both experimental and control groups.

<table>
<thead>
<tr>
<th>Group</th>
<th>Experimental</th>
<th>Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>Piaget score</td>
<td>n</td>
</tr>
<tr>
<td>Pre-test</td>
<td>69</td>
<td>10.77</td>
</tr>
<tr>
<td>Post-test</td>
<td>69</td>
<td>12.32</td>
</tr>
</tbody>
</table>
Colleges and Intellectual Development

entering into formal operation. While some of this age difference might be attributed to differences in the samples of Piaget and Friot, the entire 82% cannot be. The answer to the question of who is responsible for the lag in intellectual development seems to be the elementary school. But that answer, too, needs to be examined.

Begin that examination with another question. Who is teaching in the elementary and secondary schools? Teachers who have been educated in the existing colleges and universities. Those teachers have been subjected to four years of mainly listening experience. They have been lectured to, told to verify, given answers, and told how to teach. Lest you think the foregoing happens entirely in the colleges and/or departments of education, remind yourself that all the content taken by a teacher (which represents a substantially greater number of credit hours than do courses in education) is taken in other colleges and/or departments. Teachers, in other words, not having the kinds of experiences with inquiry which Piaget says they must have in order to allow logical thought processes to develop. Future teachers are not having learning experiences in college which will permit them to learn the value of inquiry in educating a child. The foregoing rather dogmatic statement was substantiated by Gruber when he found that only 25% of those attending NSF Institutes showed interest in inquiry-oriented science teaching, while Torrance found that only 1.4% of elementary and 8.4% of secondary social studies teachers listed independent and critical thinking as important educational objectives. These statistics suggest that pre-college teachers place little value upon logical thought as an outcome of 12 years of schooling. Considering the paucity of research on implementation of logical thought as an educational objective, these educators' values will not change. The responsibility, then, for the small percentage of high school students attaining formal operations rests in part at the door of the institutions of higher education. They have assumed that their role is to tell. Future teachers, therefore, assume that telling is teaching and when they get their first class, they tell, tell, tell! All the while, very little, if any, intellectual development is going on. If, then, a college student develops logical thought, such development is more by accident than design.

One of the criteria Piaget cites for intellectual development is that of social transmission. Just possibly more intellectual development goes on in dorms, fraternities, sororities, and student hangouts than in the classroom because social transmission occurs in these places and little occurs in classes. To test our assertions, walk down the hall of any building on any campus and stop outside any classroom door and listen to who is talking. In most instances only information is being transmitted by the instructor.

Stafford and Renner hypothesized that "...specialized educational experiences in inquiry-centered science teaching encourage a teacher to become sensitive to children, functionally aware of the purposes of education, and equipped to lead children to learn how to learn in all subject areas." The importance of this hypothesis is in the phrase "...all subject areas," for inquiry methodology is not only the province of science, but all the other disciplines as well. Unfortunately, few other teaching areas have recognized the importance of the inquiry approach.

With the exception of a few new courses in the social science areas, most educators have chosen to ignore the lead taken by science and mathematics in devising new courses from kindergarten through the 12th grade. In many cases, the colleges have failed to use inquiry even when teaching the new curricula. This point was well illustrated by Gruber. Therefore, the blame must, in the last analysis, be placed, at least partially, upon the shoulders of those who teach at the college level and who insist upon ignoring the rapidly accumulating evidence in favor of the inquiry approach.

Renner and Stafford also pointed to the necessity of the teacher becoming "...functionally aware of the purposes of education..." which in far too many cases they are not now. Unless teachers are aware of the primary purpose of education being the development of the learner's intellectual ability, they will not pursue teaching by giving the student opportunities for exploration using all his senses. Rather, they will continue to teach students what the teacher wants them to know and not what the students want to learn.

Finally, the total accumulation of research to date leads to the following hypotheses: (1) The secondary educational experience does not now
promote logical thinking in most students. (2) An abundance of inquiry-oriented courses taught by teachers who are products of college and university professors who practice and profess inquiry must come into being in the secondary schools before an alternative to the first hypothesis can be accepted. Those experiences will have to be developed by many colleges.

Those hypotheses have profound educational implications since a serious problem has been shown to exist and the means for its alleviation have also been shown to be available to the profession. If colleges and universities do not try to solve the problem by assuming the responsibility for the intellectual development of their students, but continue to look at their primary purpose as the transmission of information about the several disciplines, the elementary and secondary schools will continue to fail in their mission of truly educating students. The needed changes, however, can come only through acceptance of inquiry by all of those who teach the teachers.

Radiation Field of a Charge Moving on a Straight Line

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(Received 11 September 1970; revised 21 April 1971)

A derivation of the radiation field of a charge accelerating on a straight line is presented that makes use of Gauss' law in a direct manner and does not make use of the concept of lines of force.

We derive the radiation field of an accelerating point charge from the following assumptions: (1) electric effects are transmitted with the velocity c; (2) Gauss' law holds good in all inertial frames of reference; (3) the electric field of a charge moving uniformly is known.

These are the assumptions made by J. R. Tessman and J. T. Finnell to derive the radiation field of a point charge moving on a straight line. However, we shall not make use of the concept of lines of force, and Gauss' law shall be used in a most direct way.

Consider the following kinematic sequence on a straight line of a particle with the charge q:

(a) The charge moves with constant velocity \(v_1\) until \(t=t_0\). At \(t=t_0\) we designate its position by \(O\).

(b) The charge moves with constant velocity \(v_2\) thereafter. We only suppose that \(v_1, v_2\) are less than c.
Piagetian Theory
and Instruction in Physics

John W. Renner and Anton E. Lawson

Jean Piaget and his associates have been gathering data and formulating important theoretical observations about the intellectual development of children since 1927. Although it has taken American psychologists and educators a relatively long time to become acquainted with his work, it is becoming apparent that we can gain much by a careful evaluation of his efforts and their educational implications.

Numerous texts have become available in recent years attempting to explain Piaget’s theory and its educational significance. The primary purpose of this paper is similarly to explain his ideas, and further to expand a scheme of instruction and classroom procedures that arise as a consequence of that theory. When possible these ideas will be put forth using examples in physics context in an effort to elucidate difficult ideas.

Mental Structures

A central idea in Piaget’s work and fundamental in understanding his theory is the concept of mental structure. It would be satisfying to be able to indicate the physiological and chemical nature of these structures, but at this point in the study of human mental functioning that is not possible. Instead their existence in the brain is hypothesized from observable behavior; determination of their exact nature awaits further research. These hypothesized mental structures function to organize the environment so that the organism can function effectively. In this sense the construction of these structures carries adaptive value for the individual. An analogous situation is found in the genetic adaptation of evolving species. Basically, then, mental structures represent a more or less tightly organized mental system to guide behavior.

During development of the human infant to adulthood, these structures must be built within the brain. A complete developmental sequence of the structures is not genetically given to the child; they must be learned. According to Piaget, the building and rebuilding of these mental structures is what underlies the process of intellectual development. These structures control how and what we think and guide behavior. In other words, structures actually represent our knowledge.

Since science educators are deeply concerned with intellectual development and the building of mental structures about everything from the metric system to the theory of relativity, two questions need to be asked: (1) How are structures built? (2) Once the structure is built is it static or can it be altered?

These two questions are not mutually exclusive, and we will answer the second one first. Structures can be altered, and that may be a more than

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Anton E. Lawson (M.A., University of Oregon) is a Graduate Assistant of Science Education. He has taught in elementary and junior high schools in California, and is currently investigating selected aspects of Piagetian theory and their educational implications. (Dept. of Physics, University of Oklahoma, Norman, Oklahoma 73069.)
adequate definition of education—the building and rebuilding of structures. The answer to the first question should then give us good insights into how learning takes place and how instruction should be planned.

The Building of Mental Structures — A Problem

An important point must be made before examining the process by which mental structures are formed according to Piaget. Structures do not come from simply making a mental record of the world by keeping eyes and ears open. Unfortunately, it would appear that many teachers subscribe to this view. Work done by Van Senden with congenitally blind persons provides an interesting example of this point. These persons, who had gained sight after surgery, could not identify objects without handling them. They were unable to distinguish a key from a book, when both lay on a table. Also they were unable to report seeing any difference between a square and a circle. The important idea to note is this: Whether the task is to simply distinguish objects in the environment or complex relationships such as \( F=ma \), acceleration, or velocity, the ability to develop the understandings requires much more than a simple photographing of the environment.

According to Piaget a person is unable to perceive things until his mind has a structure which enables its perception. Without the development of a mental structure things which seem obvious to an adult, such as the difference between a key and a book, a square and a circle, are simply not perceived by beginners. But this leads us to a fundamental problem. If learning is the building or rebuilding of mental structures, and if structures are needed in order to perceive and learn and are not derived from simply copying the external world, then where do they come from?

Plato’s answer to this question was simple. The structures were innate and developed through the passage of time and the growth of the brain. Of course at the other end of the spectrum is the belief that these structures derived directly from the environment. This is the classical empiricist’s view; but we have already seen that this view is untenable.

Piaget rejects the Platonic view, except to admit that certain very primary structures must be present at birth. Piaget’s view is that the development of structures derives from a dynamic interaction of the organism and the environment which he calls equilibration.

The Building of Mental Structures — Equilibration

From birth, basic structures enable the child to begin interacting with his surroundings. As long as that interaction is successful the basic structures continue to guide behavior. However, owing to the child’s inborn drive to interact with his environment he meets contradictions, i.e., things which do not fit his present mental structures. These contradictions produce a state of disequilibrium. In other words, his present mental structures are disrupted and must be replaced. Through continued investigation and guidance from others, the child alters or accommodates his disrupted mental structure. Once this is accomplished he is then able to assimilate the new situation. The new structure that is developed is then tried. If the structure guides behavior so that the child’s efforts are rewarded (reinforced) the structure is also reinforced. In this manner the child builds new mental structures and adapts to new situations.

The above-described process underlies all development according to theory. The entire process of development of mental structures is viewed as a process of equilibration or self-regulation. This process results in the development of progressively more complex and useful mental structures.

The Building of Mental Structures — Contributing Factors

The role of three main factors, experience, social transmission, and maturation can be isolated in the process of equilibration. It is apparent that experience is a necessary part of learning. With no contact with the environment, no contradictions of present structures arise and no possibility for further exploration into the situation that produced the contradiction is possible.

There are basically two kinds of experience — physical, and logical—mathematical. This distinction is important because the different experiences lead to different kinds of mental structures.

Physical experience is exactly what the phrase connotes — actual physical action on the objects in the world. This physical experience leads to the development of structures about objects. At some point, however, the learner begins to see more in his interaction with the world than just objects. He sees that his actions with objects produce some kind of order themselves. An example of this is when a learner discovers that ten objects, when counted left to right provide the same result as when counted right to left. In other words, the action itself has properties. The learner now can make the generalization that the sum of any set of objects is independent of their order. Now the student has a mental structure that he can utilize in many situations and that is a logical—mathematical structure. The structures then enable the learner to operate logically within his environment. The basic behavioral patterns directed by the mental structure are called operations. In the early structure-building stages the opportunity for the learner to interact with concrete material is mandatory.

Piaget has not projected to what academic level the necessity for interaction with material exists; he says, “...coordination of actions before the stage of operations needs to be supported by concrete material.” A literal interpretation of that statement would be that, regardless of age, the student must have materials to perform actions with until he can begin to utilize logical—mathematical operations. Our research with kindergarten and elementary school children, junior high school students, and college freshmen, all studying science, supports our interpretation of the foregoing quotation.

The factor of experience, then, helps students to build operational-structures which can ultimately lead them to think abstractly about the world around them. In other words, it is experience with the materials of the discipline that produces the person who can understand abstract content and not studying abstract content which produces students who can interact with the materials and invent abstract generalizations. This says to science teachers that the laboratory must precede the introduction of an abstract generalization.
Piaget's second factor, social transmission, also provides a basis for structure building. The very young child — and some not so young — operate from a very egocentric frame of reference. He cannot see things objectively because he always looks at them as related to himself. Such a thinker cannot objectively view and/or evaluate anything. In order to shake the learner from an egocentric view of anything, he must experience the viewpoints and thoughts of others. He must, in other words, interact with other people. If he does not, he has no reason to alter the mental structures which he gained from an egocentric frame of reference. Social interaction can lead to conflict, debate, shared data, and the clear delineation and expression of ideas. All of these require that the student carefully examine his present beliefs which will, according to the Piagetian model, develop and change structures. In order to have all of this happen, however, students must be encouraged to talk with each other and their teachers. Data from an experiment must be shared, discussed, retaken, and rediscussed. Students, "...should converse, share experience, and argue." The factor of social interaction is valuable in building and rebuilding structures, but it is insufficient because the learner can receive valuable information via language or via education directed by an adult only if he is in a state where he can understand this information. That is, to receive this information he must have a set of experiences that enables him to assimilate this information.

Maturation, the third factor, must also be considered. Evidence indicates that these structures require time to develop. Old structures cannot be accommodated to new experiences all at once. The process of development is slow, as any teacher can attest.

Perhaps this personal example will help clarify how these three factors interact in the process of equilibration to change structures. Our first contact with \( V=IR \) was a rather traumatic experience. We vaguely understood that it involved the conservation of energy, but concentrated upon memorizing what the symbols meant and how to juggle the formula. In short, an advanced state of disequilibrium was our lot! When meter readings were substituted for the very abstract terms of potential difference and current, the symbols began to have meaning, and after a good deal of thinking equilibrium was achieved. Then a series circuit with one source and more than one resistor and parallel circuit was introduced. The notion that in a series circuit the total potential difference, \( V_t \), of the source equaled the sum of all voltage drops, \( V_i, i=1,2,3,\ldots,n \), around the circuit brought on another disequilibrium. Once again meter reading (objects) were salvation; we began to really understand that

\[
V_t = \sum_{i=1}^{n} V_i, \quad i=1,2,3,\ldots,n
\]

really was a conservation of energy statement. Now \( V=IR \) was a concept which was available for use and once again equilibrium was achieved. Parallel circuits presented no problem and Kirchhoff’s laws were nearly obvious.

This example demonstrates that the science laboratory clearly has a place in promoting equilibration and disequilibrium. Data from an experiment can be very threatening, because they too often produce disequilibrium. But to the sensitive, concerned science teacher, disequilibrium is an opportunity; he can now introduce the student to the major conceptualizations of the discipline which will produce equilibrium. This sequence of events suggests that perhaps the principal role of the teacher is to promote disequilibrium and equilibrium, because through the process of equilibration structures are built and rebuilt. Equilibration proceeds through experience with the materials worked with and the social interaction of those around us.

The Learning Cycle

An instructional technique incorporating much of Piagetian theory has been developed and refined by the Science Curriculum Improvement Study, University of California,
Berkeley. Their procedure is basically a three-phase process: (1) exploration, (2) invention, and (3) discovery.

*Exploration* involves the students in concrete experience with materials. As a consequence of these initial explorations, which sometimes may be highly structured by the teacher or on other occasions relatively free, the learner encounters new information which does not fit his existing structures. This produces disequilibrium. At the appropriate time, determined by the teacher, he suggests a way of ordering the experiences. In essence, the teacher invents a new structure which often involves a new concept. This phase, termed *invention*, is analogous to Piaget’s structure building and promotes a new state of understanding or equilibrium. The question now is: Can the new situation be applied in other situations? During phase three, *discovery*, further application of the inventions are discovered by the students. Discovery experiences serve to reinforce, refine, and enlarge the content of the invention.10

Again an example from physics may help to clarify these points. Experience in the laboratory with voltage and resistance, seeing the effect these have on current, and recording all these data is exploration. These exploratory experiences, if provided at the appropriate time, will promote disequilibrium and lead students to question relationships. Since it would take a brilliant student to invent the notion that \( V = IR \), the formal statement of that relationship is left up to the teacher. The teacher, having explained the relationship, has in effect provided a way of ordering the student’s experience. This is invention. Now the student is in a position to make discovery with this new concept. He can apply it to various types of circuits, magnitudes of voltage, current, and resistance, practically any type of situation he can design. That is the true notion of discovery. Exploration, invention, and discovery are the three phases of the learning cycle and represent a process which will lead the learner to move from physical action to abstract mental operations. Science in general — and in our opinion physics in particular — has a unique opportunity to lead students to build structures. Are we utilizing it? There is much evidence to suggest we are not.11

**Levels of Thinking**

Piaget’s theory has gone further than describing how mental structures are formed. He has outlined the basic structures that dictate behavior from birth to adulthood. The structures fall roughly into four categories. Each category or stage incorporates and adds to the structure of the previous stages. If Piaget is correct, it becomes imperative for educators to understand these stages of development. They provide a possible key for adapting instruction to the learner’s capabilities. They further suggest types of activities which could promote intellectual development.

The child at birth is in a state Piaget calls *sensory-motor*. During this period, which lasts until about 18 months, the child acquires such practical knowledge as the fact that objects are permanent. The name of the second stage describes the characteristics of the child — *preoperational*, the stage of intellectual development before mental operations appear. In this stage, which persists until around seven years of age, the child does not, for example, reverse his thinking; he exhibits extreme egocentricism, centers his attention upon a particular aspect of a given object, event, or situation, reasons transductively, and does not demonstrate conservation12 reasoning. In other words, the child’s thinking is very rigid.

At about seven years of age the thinking stages of children begin to “thaw out” — they show less rigidity. The stage the child has entered is called *concrete operational*. Those structures which permit the reversal of thinking *et al.*, which are denied a pre-operational thinker, begin to show themselves as the child moves more and more deeply into the concrete operational stage. The child can now perform what Piaget calls mental experiments — he can assimilate data from a concrete experience and arrange and rearrange them in his head. In other words, the concrete operational child has a much greater mobility of thought than when he was younger.

The name of this stage of development — concrete operational — is representative of the type of thinking of this type of learner. As Piaget explains this stage: “The operations involved...are called ‘concrete’ because they related directly to objects and not yet to verbally stated hypotheses.”13 In other words, the mental operations performed at this stage are “object bound” — operations are tied to objects. This point must be firmly entrenched in the minds of teachers, because when working with students who are moving through this stage they must focus their teaching on the object — the actuality — and not on the abstract. Density, for example, is an abstraction — lenses are concrete.

As the child begins to emerge from the concrete operational stage of thought, according to the Piagetian model, he enters the last stage called *formal operational*. According to Piaget, this occurs between 11 and 15 years of age. A person who has entered that stage of formal thought “...is an individual who thinks beyond the present and forms theories about everything, delighting especially in considerations of that which is not.”14 Formal operational thought is capable of reasoning with propositions only and has no need for objects. It should be pointed out, however, that for this type of thought to occur it must be developed through the use of objects. For that reason this type of thought can be described as propositional logic. An analysis of formal operations reveals that they “...consist, essentially of ‘implication’... and ‘contradiction’ established between propositions which themselves express classifications, seriations, etc.”15 The formal thinker can form hypotheses and test them. To do this, he must isolate and control variables and exclude irrelevant ones. This type of thought can truly be described as abstract.

The maximum educational gain that comes from the study of science is derived from the isolation and investigation of a problem. Quite obviously this involves the formulation and stating of hypotheses and using a form of thinking which can be described as, if..., then..., therefore. That is, of course, propositional logic. In other words, science teaching should promote formal thought. But it cannot do so if concrete operational thinkers are asked to interact with science on a formal operational level and their teacher teaches them as though they think formally. Concrete operational learners must interact with science at that level; they cannot do otherwise. Only then will they build the struc-
tures that promote their intellectual development toward formal thought.

Where are today's science students in the development of formal thought? If the programs of study available for high school physics are examined, for example, the fact that they require the use of abstract thinking is immediately apparent. The same can be said for most of the new curricula developments in science. As Kohlberg and Gilligan recently said: "Clearly the new curricula assumed formal operational thought rather than attempting to develop it."16 Is such a statement justified? Can science taught at the pre-collegiate and college levels promote formal thought? What can teachers do, if anything, as they select and arrange curricula and interact with students to promote formal thought? A later article in this journal will address itself to those questions.

[The second part of this article will appear in the May issue of The Physics Teacher.]

References


2. This scheme of instruction also incorporates theoretical observations detailed in Chester A. Lawson, Brain Mechanisms and Human Learning (Houghton Mifflin, Boston, 1967).

3. For hypothesized neural mechanisms see Lawson, Ref. 2, pp. 9–16.


15. See Ref 14, p. 149.

16. Lawrence Kohlberg and Carol Gilligan, Daedalus 100, No. 4, 1051 (Fall 1971).
Promoting Intellectual Development Through Science Teaching

John W. Renner and Anton E. Lawson

The previous article in this series, ["Piagetian Theory and Instruction in Physics," Phys. Teach. 11, 165 (1973)] discussed the process of intellectual development and the intellectual level concepts of Jean Piaget and briefly commented upon the relation of those ideas to teaching and learning physics. The purpose of this article is to comment upon the thought patterns of secondary school and first-year college students and to suggest types of experiences students need to have to enable them to move toward acquiring formal thought.

We start with the assumption that all students deserve the opportunity to develop the capacity to think with the "If..., then..., therefore..." form — in other words, to develop formal thought. Three questions immediately arise:

1. What type(s) of thought do secondary school and first year college students use?
2. How can the student's level-of-thought be assessed?
3. What can educational institutions do to change the type(s) of thinking students do?

Levels of Thought, Students, and Content

If you reflect back to the first article we prepared on the topic of learning, you will recall that we pointed out that learners begin to leave the pre-operational stage at around seven years of age. At this point, they enter the concrete operational stage of thought and, according to Piaget, move more and more deeply into that stage until somewhere between 11 and 15 years of age. That is the time when they begin to move into the last stage of intellectual development — formal operational thought.

Now the transition from concrete to formal thought is of the utmost importance to teachers who work with students in grades 10-12 in the secondary schools and in their first years of college. If students have achieved the ability to think formally, the teacher can proceed to lead them to deal in the great abstractions of science because they can think with form, "if..., then..., therefore...," or propositional logic. These teachers need not be as concerned with providing students direct experience with the materials of the discipline as those teaching concrete operational thinkers. But if students are concrete operational, they cannot think with propositional logic and all they learn will come from interacting with the materials of the discipline. These statements carry with them serious implications for science teaching, indeed for all types of teaching which deal with abstractions. Therefore, the validity of these statements must be carefully evaluated. At this particular time such an evaluation has not been carried out to any satisfactory extent. However, to any teacher who has had the experience of having his students simply not comprehend what to him seemed eminently clear, Piaget's hypothesis becomes extremely compelling.
Basically one can grasp why Piaget asserts that "if..., then..., therefore..." thinking is required to understand abstract concepts if you understand the nature of the abstract concepts themselves. The abstractions in physics, as well as in biology and chemistry, are in actuality models created by scientists to explain observable data. These models do not arise directly from the observations; rather, they simply represent attempts to construct an explanation or model which implies what is observed. The scientist creates the model (we do not know how) and reasons if his model is true, then consequences should be found. If the predicted consequences are indeed found, he has therefore supported his model. The process is hypothetico-deductive or in the if..., therefore... form. For a student to fully grasp the meaning of the abstract models he, too, must be able to think in the if..., therefore... form. The inertia principle, for example, has to be deduced and verified from its implied consequences. Strictly speaking, it does not give rise to observable empirical evidence.

Consider Newton's second law, \( F = ma \). That law is always stated (and properly so) in terms of the mass of a body. Now mass is not a concrete concept -- it is an abstraction. All matter that students have experienced exists in a gravitational field. Therefore what students have experienced is not mass but weight. This point is of little consequence to a formal operational thinker; mass is an abstract concept he can comprehend and do mental experiments with. To succeed in understanding \( F = ma \) (particularly when identifying its units) however, the learner must be able to do mental experiments with abstract concepts. Now look at acceleration -- a rate of change of a rate of change. A rate of change is a concrete concept; miles/hour, pounds/second, and pounds/foot are all situations with which a learner can have concrete experiences. But when you change that rate of change so that you are referring to miles/hour/second, providing experience which will lead a student to that is nearly impossible. (To make acceleration even more abstract, it is usually written, for example, as \( ft/sec^2 \).) About the best that can be done is to let the student experience the fact that as an object slows down, the time intervals required to travel equal distances get progressively longer.

Now consider the experience students have had with forces. Those experiences have no doubt been pushes and pulls and have probably been measured in pounds. Now a student takes an abstract quantity (mass) which he has not experienced and multiplies it by a second very abstract quantity (acceleration) and produces a third quantity called force. But here the force is not measured in pounds but in kilogram-meters/second\(^2\) and is called a newton. There is nothing concrete about that entire process. It is a complete abstraction. Now if a student is a formal thinker, he can probably handle that abstraction -- he can't if he is concrete operational. Do not misread can't to mean "doesn't want to"; it means exactly what it says, can't.

Couple Newton's second law with the calorie, transverse waves, the particle theory of light, the gauss and maxwell, and the second law of thermodynamics and you have a pretty good sampling of a first-year physics course. You also have a fair list of abstractions. Those are abstract topics for which formal operations are a necessity. How does a teacher determine whether or not his class can handle such abstract topics?

Assessing Student Level of Thought

What we have done in the area of determining student success with tasks which reflect formal operational thought has been greatly influenced by four sources:


The foregoing sources contain many more tasks than will be described here, and you are urged to try them. Here are two tasks which we have used quite extensively.

(1) The Conservation of Volume (Source 2, above). This task requires two cylinders of exactly the same size but having different weight (we have used one made of brass and the other of aluminum); those properties of the cylinders are pointed out to the student. He is next presented with two identical tubes partially filled with water and allowed to adjust the water levels until he is convinced that each tube contains exactly the same amount. The student is then asked if when the cylinders are put in the tubes, the heavy cylinder will push the water up more, if the lighter cylinder will push the level up more, or if the cylinders will push the levels up the same. The examiner requires the student to explain his answer, and often it is the explanations and not the initial responses that are most revealing of thought patterns. If the student completes the task successfully, he has provided evidence of beginning formal operational thought.

(2) The Exclusion of Irrelevant variables\(^2\) (Source 1, above). The student is presented with a pendulum whose length can be easily changed and three different sized weights which can be used for the pendulum bob. He is told to do as many experiments as he needs to, using many different lengths of string and all the various-sized weights until he can explain what he needs to do to make the pendulum go fast or slow. Again, note that the examiner bases his evaluation on the student's explanations. The variables of string length, angle, and push are also pointed out to the student. If the examinee recognizes that length is the only relevant variable, he is about to enter into the formal operational thought period. If he not only excludes the irrelevant variables but hypothesizes a solution to the problem and demonstrates his solution, he has entered the formal period. If the student can state a general rule about pendula in such a way that it can be tested, he is probably capable of working with propositional logic. Although the concept of an oscillating pendulum and its period is not an abstract concept itself (its discovery and construction related directly to a concrete physical experiment), solution of the pendulum problem does indicate the use of propositional
logic and that is a prerequisite to the understanding of abstractions.9

Student Performance on the Tasks

Physics is normally taught in the high schools to students in grades eleven and twelve. We administered these tasks, therefore, to 99 eleventh graders and 97 twelfth graders from Oklahoma public schools. The schools were randomly selected, and students in each selected school were also randomly selected. Table I shows what we found.

Table I. Performance of formal operational tasks by a random sample of high school students.

<table>
<thead>
<tr>
<th>Population</th>
<th>Conservation of volume</th>
<th>Exclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>11th Grade (N=99)</td>
<td>19</td>
<td>14</td>
</tr>
<tr>
<td>Females (N=54)</td>
<td>26</td>
<td>13</td>
</tr>
<tr>
<td>Males (N=45)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12th Grade (N=97)</td>
<td>18</td>
<td>16</td>
</tr>
<tr>
<td>Females (N=47)</td>
<td>34</td>
<td>20</td>
</tr>
<tr>
<td>Males (N=50)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The data in Table I suggest that out of the population from which physics students are drawn, not many are formal operational. You are urged to administer these tasks to your students. If you are interested in doing some group evaluations of your students, study sources three and four listed earlier. Source three deals with determining student ability to reason abstractly by presenting a problem and then providing one clue at a time. The clues and the original statement of the problem must then be analyzed and used to draw conclusions. Source four assesses student ability to apply the concept of ratio. When using ratios, the student is utilizing proportional thinking which is an essential component of formal thought. Please do not make the assumption that by the time students get to physics in high school only those who think formally enroll. Our high school data from those enrolling in high school physics, though not extensive enough to make a definite statement, suggest that such is not the case. Data will be presented later which show that many concrete operational thinkers are found at the first year college level.

Kohlberg and Gilligan 4 report that in a study of the ability of 265 persons to perform successfully on the pendulum task (exclusion), these results were obtained:

- age 10-15 — 45%; age 21-30 — 65%
- age 16-20 — 53%; age 45-50 — 57%

If you assume that performance on the pendulum task is an indication that formal operational thought is present, the foregoing data suggest what our data do — a large percentage of the adolescent population is not formal operational. Unfortunately, our age ranges and those of Kohlberg and Gilligan do not coincide exactly, and so no more definite statement can be made from those two groups of data.

The conservation of volume and the pendulum tasks were taken by college freshmen. The results shown in Table II were obtained.

Table II. Performance of college freshmen for formal operational tasks.

<table>
<thead>
<tr>
<th>Number of college freshmen</th>
<th>Conservation of volume</th>
<th>Exclusion</th>
</tr>
</thead>
<tbody>
<tr>
<td>185</td>
<td>133</td>
<td>77</td>
</tr>
</tbody>
</table>

The data shown in Table II clearly reflect that the majority of college freshmen have not moved deeply into the formal operational stage of thought — 77 of 185 experiencing success on the exclusion task is not too impressive. We do not mean to infer that performance on the pendulum task is an absolute measure of the achievement of formal operational thought. We do mean to infer that performance on these tasks is a strong indication of student ability to use propositional logic. We tested our inference that these two tasks do help isolate formal thinkers — those that use thought patterns which are "the stock in trade of the logician, the scientist, or the abstract thinker." 15 In searching for a test population we ruled out all quantitative fields because the tasks are quantitative in nature. We were reminded that the "if..., then..." construct is also the stock in trade of the lawyer. In order to survive in the study of law, students have to think mainly on the abstract level. We asked several groups of second and third year law students to react to the two tasks we just described. Table III reflects our results. A total of 66 students reacted to the tasks and 50 of them demonstrated formal operational thought. We feel, therefore, that these two tasks have a good probability of identifying formal thought.

Table III. Performance of second and third year law students on two formal operational tasks.

<table>
<thead>
<tr>
<th>Conservation of volume (N=22)</th>
<th>Concrete Operational</th>
<th>Formal Operational</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>19</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exclusion of irrelevant variables (N=44)</th>
<th>Concrete Operational</th>
<th>Formal Operational</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>13</td>
<td>31</td>
</tr>
</tbody>
</table>

What Educational Institutions Can Do to Foster Formal Thought

Our research has shown us that the level of thought of junior high school students 6 and college freshmen 7 can be changed by providing them inquiry-centered experiences in science. We believe that the principal reason our research has shown an increase in the thought levels of students is because we accepted that most of them participating in the experiments were concrete operational. That put squarely
upon us the responsibility for providing concrete experiences with the objects and ideas of the discipline. These students were involved in actually creating some knowledge of their very own. We know that this was the first time some of them had been given that opportunity. We believe that actual involvement with the materials and ideas of science and being allowed to find out something for themselves accounts for the movement toward and into formal thought which we found.

Science teachers in general and physics teachers in particular have a vehicle at their command that makes active student involvement convenient. That vehicle is the laboratory. Both of our research studies had the laboratory at its nerve center. In the case of the college study that laboratory did not too frequently involve hardware and chemicals, but it was a place where data were gathered, ideas were honed, hypotheses were made and tested, and verifications were carried out. That is the true laboratory.

In teaching the majority of physics courses (both college and high school) the laboratory can be used to lead students, through inquiry, to develop understandings of the concepts to be learned. The teacher, then, has three responsibilities to discharge before ever meeting a class:

(1) Isolate those concepts which, when learned, will provide students with an accurate and adequate understanding of the discipline. The teacher must use his understanding of the structure of the discipline in order to select the concepts, and his goal is to provide the learner with his own understanding of the discipline's structure. Textbooks are of little help here.

(2) Find those laboratory investigations which when cast in an inquiry framework will, upon completion, allow the student to develop an understanding of the concept being considered. Textbooks are of no help here.

(3) Make sure the investigations are cast into an inquiry framework and be sure the necessary materials are available.

Now classes start. The teacher becomes an asker of questions, a provider of materials, a laboratory participant, and a class chairman and secretary. Perhaps most importantly, he is a discussion leader. He gathers the class together (chairman) and solicits the data they have gathered (secretary). He then leads a discussion on what the data mean (discussion leader). He also makes the necessary conceptual inventions at the proper time, decides when discovery can take place, and when the present concept needs to be related to the next one by exploration. He must also decide when exploration of a completely new concept must begin. This teacher is not a teller, he is a director of learning.

Traditional teaching methods embrace the notions that (a) teaching is telling, (b) memorization is learning, and (c) being able to repeat something on an examination is evidence of understanding — those points are the antithesis of inquiry.

The development of formal thought must become the focus of attention of every teacher in the country. The Educational Policies Commission said, in 1961, that the central purpose of the school must be to teach students to think and they operationally defined thinking. Such good advice! We would add that the central role of the school must be to teach children to think with form not objects — in other words, to move students into the stage of formal operational thought. Science has the structure to enhance greatly the achievement of this objective. We must not blow our chances to make a maximum contribution to education in general and education in science in particular! Let's establish an environment in our classrooms that encourages and promotes formal thought!

References

2. Renner and Stafford, Ref. 1, p. 294.
4. Lawrence Kohlberg and Carol Gilligan, Daedalus 100, 1051 (Fall 1971).
8. Refer to John W. Renner and Anton E. Lawson, Phys. Teach. 11, 165 (1973), under the section “Learning Cycle" for an explanation of this term and its phases of exploration, invention, and discovery.
My dear colleagues, I am very concerned about what to say to you, because I do not know if I shall accomplish the end that has been assigned to me. But I have been told that the important thing is not what you say, but the discussion which follows and the answers to questions you are asked. So this morning I shall simply give a general introduction of a few ideas which seem to me to be important for the subject of this conference.

First I would like to make clear the difference between two problems: the problem of development in general and the problem of learning. I think these problems are very different, although some people do not make this distinction.

The development of knowledge is a spontaneous process, tied to the whole process of embryogenesis. Embryogenesis concerns the development of the body, but it concerns as well the development of the nervous system and the development of mental functions. In the case of the development of knowledge in children, embryogenesis ends only in adulthood. It is a total developmental process which we must re-situate in its general biological and psychological context. In other words, development is a process which concerns the totality of the structures of knowledge.

Learning presents the opposite case. In general, learning is provoked by situations—provoked by a psychological experimenter; or by a teacher, with respect to some didactic point; or by an external situation. It is provoked, in general, as opposed to spontaneous. In addition, it is a limited process—limited to a single problem, or to a single structure.

So I think that development explains learning, and this opinion is contrary to the widely held opinion that development is a sum of discrete learning experiences. For some psychologists development is reduced to a series of specific learned items, and development is thus the sum, the accumulation of this series of specific items. I think this is an atomistic view which deforms the real state of things. In reality, development is the essential process and each element of learning occurs as a function of total development, rather than being an element which explains development. I shall begin, then, with a first part dealing with development, and I shall talk about learning in the second part.

To understand the development of knowledge, we must start with an idea which seems central to me—the idea of an operation. Knowledge is not a copy of reality. To know an object, to know an event, is not simply to look at it and make a mental copy or image of it. To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed. An operation is thus the essence of knowledge; it is an interiorized action which modifies the object of knowledge. For instance an operation would consist of joining objects in a class to construct a
classification. Or an operation would consist of ordering, or putting things in a series. Or an operation would consist of counting, or of measuring. In other words, it is a set of actions modifying the object, and enabling the knower to get at the structures of the transformation.

An operation is an interiorized action. But, in addition, it is a reversible action; that is, it can take place in both directions, for instance, adding or subtracting, joining or separating. So it is a particular type of action which makes up logical structures.

Above all, an operation is never isolated. It is always linked to other operations, and as a result it is always a part of a total structure. For instance, a logical class does not exist in isolation; what exists is the total structure of classification. An asymmetrical relation does not exist in isolation. Seriation is the natural, basic operational structure. A number does not exist in isolation. What exists is the series of numbers which constitute a structure, an exceedingly rich structure whose various properties have been revealed by mathematicians.

These operational structures are what seem to me to constitute the basis of knowledge, the natural psychological reality, in terms of which we must understand the development of knowledge. And the central problem of development is to understand the formation, elaboration, organization, and functioning of these structures.

I should like to review the stages of development of these structures, not in any detail, but simply as a reminder. I shall distinguish four main stages. The first is a sensory-motor, pre-verbal stage, lasting approximately the first 18 months of life. During this stage is developed the practical knowledge which constitutes the substructure of later representational knowledge. An example is the construction of the schema of the permanent object: For an infant, during the first months, an object has no permanence. When it disappears from the perceptual field it no longer exists. No attempt is made to find it again. Later, the infant will try to find it, and he will find it by localizing it spatially. Consequently, along with the construction of the permanent object there comes the construction of practical or sensory-motor space. There is similarly the construction of temporal succession, and of elementary sensory-motor causality. In other words, there is a series of structures which are indispensable for the structures of later representational thought.

In a second stage, we have pre-operational representation—the beginnings of language, of the symbolic function, and therefore of thought, or representation. But at the level of representational thought, there must now be a reconstruction of all that was developed on the sensory-motor level. That is, the sensory-motor actions are not immediately translated into operations. In fact, during all this second period of pre-operational representations, there are as yet no operations as I defined this term a moment ago. Specifically, there is as yet no conservation which is the psychological criterion of the presence of reversible operations. For example, if we pour liquid from one glass to another of a different shape, the pre-operational child will think there is more in one than in the other. In the absence of operational reversibility, there is no conservation of quantity.

In a third stage the first operations appear, but I call these concrete operations because they operate on objects, and not yet on verbally expressed hypotheses. For example, there are the operations of classification, ordering, the construction of the idea of number, spatial and temporal operations, and all the fundamental operations of elementary logic of classes and relations, of elementary mathematics, of elementary geometry, and even of elementary physics.

Finally, in the fourth stage, these operations are surpassed as the child reaches the level of what I call formal or hypothetic-deductive operations; that is, he can now reason on hypotheses, and not only on objects. He constructs new operations, operations of propositional logic, and not
simply the operations of classes, relations, and numbers. He attains new structures which are on the one hand combinatorial, corresponding to what mathematicians call lattices; on the other hand, more complicated group structures. At the level of concrete operations, the operations apply within an immediate neighborhood: for instance, classification by successive inclusions. At the level of the combinatorial, however, the groups are much more mobile. These, then, are the four stages which we identify, whose formation we shall now attempt to explain.

What factors can be called upon to explain the development from one set of structures to another? It seems to me that there are four main factors: first of all, maturation, in the sense of Gesell, since this development is a continuation of the embryogenesis; second, the role of experience of the effects of the physical environment on the structures of intelligence; third, social transmission in the broad sense (linguistic transmission, education, etc.); and fourth, a factor which is too often neglected but one which seems to me fundamental and even the principal factor. I shall call this the factor of equilibration or if you prefer it, of self-regulation.

Let us start with the first factor, maturation. One might think that these stages are simply a reflection of an interior maturation of the nervous system, following the hypotheses of Gesell, for example. Well, maturation certainly does play an indispensable role and must not be ignored. It certainly takes part in every transformation that takes place during a child's development. However, this first factor is insufficient in itself. First of all, we know practically nothing about the maturation of the nervous system beyond the first months of the child's existence. We know a little bit about it during the first two years but we know very little following this time. But above all, maturation doesn't explain everything, because the average ages at which these stages appear (the average chronological ages) vary a great deal from one society to another. The ordering of these stages is constant and has been found in all the societies studied. It has been found in various countries where psychologists in universities have redone the experiments but it has also been found in African peoples for example, in the children of the Bushmen, and in Iran, both in the villages and in the cities. However, although the order of succession is constant, the chronological ages of these stages varies a great deal. For instance, the ages which we have found in Geneva are not necessarily the ages which you would find in the United States. In Iran, furthermore, in the city of Teheran, they found approximately the same ages as we found in Geneva, but there is a systematic delay of two years in the children in the country. Canadian psychologists who redid our experiments, Monique Laurendeau and Father Adrien Pinard, found once again about the same ages in Montreal. But when they redid the experiments in Martinique, they found a delay of four years in all the experiments and this in spite of the fact that the children in Martinique go to a school set up according to the French system and the French curriculum and attain at the end of this elementary school a certificate of higher primary education. There is then a delay of four years, that is, there are the same stages, but systematically delayed. So you see that these age variations show that maturation does not explain everything.

I shall go on now to the role played by experience. Experience of objects, of physical reality, is obviously a basic factor in the development of cognitive structures. But once again this factor does not explain everything. I can give two reasons for this. The first reason is that some of the concepts which appear at the beginning of the stage of concrete operations are such that I cannot see how they could be drawn from experience. As an example, let us take the conservation of the substance in the case of changing the shape of a ball of plasticine. We give this ball of plasticine to a child who changes its shape into a sausage form and we ask him if there is the
same amount of matter, that is, the same amount of substance as there was before. We also ask him if it now has the same weight and thirdly if it now has the same volume. The volume is measured by the displacement of water when we put the ball or the sausage into a glass of water. The findings, which have been the same every time this experiment has been done, show us that first of all there is conservation of the amount of substance. At about eight years old a child will say, "There is the same amount of plasticene." Only later does the child assert that the weight is conserved and still later that the volume is conserved. So I would ask you where the idea of the conservation of substance can come from. What is a constant and invariant substance when it doesn't yet have a constant weight or a constant volume? Through perception you can get at the weight of the ball or the volume of the ball but perception cannot give you an idea of the amount of substance. No experiment, no experience can show the child that there is the same amount of substance. He can weigh the ball and that would lead to the conservation of weight. He can immerse it in water and that would lead to the conservation of volume. But the notion of substance is attained before either weight or volume. This conservation of substance is simply a logical necessity. The child now understands that when there is a transformation something must be conserved because by reversing the transformation you can come back to the point of departure and once again have the ball. He knows that something is conserved but he doesn't know what. It is not yet the weight, it is not yet the volume; it is simply a logical form—a logical necessity. There, it seems to me, is an example of a progress in knowledge, a logical necessity for something to be conserved even though no experience can have lead to this notion.

My second objection to the sufficiency of experience as an explanatory factor is that this notion of experience is a very equivocal one. There are, in fact, two kinds of experience which are psychologically very different and this difference is very important from the pedagogical point of view. It is because of the pedagogical importance that I emphasize this distinction. First of all, there is what I shall call physical experience, and, secondly, what I shall call logical—mathematical experience.

Physical experience consists of acting upon objects and drawing some knowledge about the objects by abstraction from the objects. For example, to discover that this pipe is heavier than this watch, the child will weigh them both and find the difference in the objects themselves. This is experience in the usual sense of the term—in the sense used by empiricists. But there is a second type of experience which I shall call logical mathematical experience where the knowledge is not drawn from the objects, but it is drawn by the actions effected upon the objects. This is not the same thing. When one acts upon objects, the objects are indeed there, but there is also the set of actions which modify the objects.

I shall give you an example of this type of experience. It is a nice example because we have verified it many times in small children under seven years of age, but it is also an example which one of my mathematician friends has related to me about his own childhood, and he dates his mathematical career from this experience. When he was four or five years old—I don't know exactly how old, but a small child—he was seated on the ground in his garden and he was counting pebbles. Now to count these pebbles he put them in a row and he counted them one, two, three, up to ten. Then he finished counting them and started to count them in the other direction. He began by the end and once again he found ten. He found this marvelous that there were ten in one direction and ten in the other direction. So he put them in a circle and counted them that way and found ten once again. Then he counted them in the other direction and found ten once
more. So he put them in some other arrangement and kept counting them and kept finding ten. There was the discovery that he made.

Now what indeed did he discover? He did not discover a property of pebbles; he discovered a property of the action of ordering. The pebbles had no order. It was his action which introduced a linear order or a cyclical order, or any kind of an order. He discovered that the sum was independent of the order. The order was the action which he introduced among the pebbles. For the sum the same principle applied. The pebbles had no sum; they were simply in a pile. To make a sum, action was necessary—the operation of putting together and counting. He found that the sum was independent of the order, in other words, that the action of putting together is independent of the action of ordering. He discovered a property of actions and not a property of pebbles. You may say that it is in the nature of pebbles to let this be done to them and this is true. But it could have been drops of water, and drops of water would not have let this be done to them because two drops of water and two drops of water do not make four drops of water as you know very well. Drops of water then would not let this be done to them, we agree to that.

So it is not the physical property of pebbles which the experience uncovered. It is the properties of the actions carried out on the pebbles, and this is quite another form of experience. It is the point of departure of mathematical deduction. The subsequent deduction will consist of interiorizing these actions and then of combining them without needing any pebbles. The mathematician no longer needs his pebbles. He can combine his operations simply with symbols, and the point of departure of this mathematical deduction is logical—mathematical experience, and this is not at all experience in the sense of the empiricists. It is the beginning of the coordination of actions, but this coordination of actions before the stage of operations needs to be supported by concrete material. Later, this coordination of actions leads to the logical—mathematical structures. I believe that logic is not a derivative of language. The source of logic is much more profound. It is the total coordination of actions, actions of joining things together, or ordering things, etc. This is what logical—mathematical experience is. It is an experience of the actions of the subject, and not an experience of objects themselves. It is an experience which is necessary before there can be operations. Once the operations have been attained this experience is no longer needed and the coordinations of actions can take place by themselves in the form of deduction and construction for abstract structures.

The third factor is social transmission—linguistic transmission or educational transmission. This factor, once again, is fundamental. I do not deny the role of any one of these factors; they all play a part. But this factor is insufficient because the child can receive valuable information via language or via education directed by an adult only if he is in a state where he can understand this information. That is, to receive the information he must have a structure which enables him to assimilate this information. This is why you cannot teach higher mathematics to a five-year-old. He does not yet have structures which enable him to understand.

I shall take a much simpler example, an example of linguistic transmission. As my very first work in the realm of child psychology, I spent a long time studying the relation between a part and a whole in concrete experience and in language. For example, I used Burt's test employing the sentence, "Some of my flowers are buttercups." The child knows that all buttercups are yellow, so there are three possible conclusions: the whole bouquet is yellow, or part of the bouquet is yellow, or none of the flowers in the bouquet are yellow. I found that up until nine years of age (and this was in Paris, so the children certainly did understand the French language) they
replied, "The whole bouquet is yellow or some of my flowers are yellow." Both of those mean the same thing. They did not understand the expression, "some of my flowers." They did not understand this of as a partitive genitive, as the inclusion of some flowers in my flowers. They understood some of my flowers to be my several flowers as if the several flowers and the flowers were confused as one and the same class. So there you have children who until nine years of age heard every day a linguistic structure which implied the inclusion of a subclass in a class and yet did not understand this structure. It is only when they themselves are in firm possession of this logical structure, when they have constructed it for themselves according to the developmental laws which we shall discuss, that they succeed in understanding correctly the linguistic expression.

I come now to the fourth factor which is added to the three preceding ones but which seems to me to be the fundamental one. This is what I call the factor of equilibration. Since there are already three factors, they must somehow be equilibrated among themselves. That is one reason for bringing in the factor of equilibration. There is a second reason, however, which seems to me to be fundamental. It is that in the act of knowing, the subject is active, and consequently, faced with an external disturbance, he will react in order to compensate and consequently he will tend towards equilibrium. Equilibrium, defined by active compensation, leads to reversibility. Operational reversibility is a model of an equilibrated system where a transformation in one direction is compensated by a transformation in the other direction. Equilibration, as I understand it, is thus an active process. It is a process of self-regulation. I think that this self-regulation is a fundamental factor in development. I use this term in the sense in which it is used in cybernetics, that is, in the sense of processes with feedback and with feedforward, of processes which regulate themselves by a progressive compensation of systems. This process of equilibration takes the form of a succession of levels of equilibrium, of levels which have a certain probability which I shall call a sequential probability, that is, the probabilities are not established a priori. There is a sequence of levels. It is not possible to reach the second level unless equilibrium has been reached at the first level, and the equilibrium of the third level only becomes possible when the equilibrium of the second level has been reached, and so forth. That is, each level is determined as the most probable given that the preceding level has been reached. It is not the most probable at the beginning, but it is the most probable once the preceding level has been reached.

As an example, let us take the development of the idea of conservation in the transformation of the ball of plasticene into the sausage shape. Here you can discern four levels. The most probable at the beginning is for the child to think of only one dimension. Suppose that there is a probability of 0.8, for instance, that the child will focus on the length, and that the width has a probability of 0.2. This would mean that of ten children, eight will focus on the length alone without paying any attention to the width, and two will focus on the width without paying any attention to the length. They will focus only on one dimension or the other. Since the two dimensions are independent at this stage, focusing on both at once would have a probability of only 0.16. That is less than either one of the two. In other words, the most probable in the beginning is to focus only on one dimension and in fact the child will say, "It's longer, so there's more in the sausage." Once he has reached this first level, if you continue to elongate the sausage, there comes a moment when he will say, "No, now it's too thin, so there's less." Now he is thinking about the width, but he forgets the length, so you have come to a second level which becomes the most probable after the first level, but which is not the most probable at the point of departure. Once he has focused on the
width, he will come back sooner or later to focus on the length. Here you will have a third level where he will oscillate between width and length and where he will discover that the two are related. When you elongate you make it thinner, and when you make it shorter, you make it thicker. He discovers that the two are solidly related and in discovering this relationship, he will start to think in terms of transformation and not only in terms of the final configuration. Now he will say that when it gets longer it gets thinner, so it’s the same thing. There is more of it in length but less of it in width. When you make it shorter it gets thicker; there’s less in length and more in width, so there is compensation—compensation which defines equilibrium in the sense in which I defined it a moment ago. Consequently, you have operations and conservation. In other words, in the course of these developments you will always find a process of self-regulation which I call equilibration and which seems to me the fundamental factor in the acquisition of logical-mathematical knowledge.

I shall go on now to the second part of my lecture, that is, to deal with the topic of learning. Classically, learning is based on the stimulus-response schema. I think the stimulus-response schema, while I won’t say it is false, is in any case entirely incapable of explaining cognitive learning. Why? Because when you think of a stimulus-response schema, you think usually that first of all there is a stimulus and then a response is set off by this stimulus. For my part, I am convinced that the response was there first, if I can express myself in this way. A stimulus is a stimulus only to the extent that it is significant, and it becomes significant only to the extent that there is a structure which permits its assimilation, a structure which can integrate this stimulus but which at the same time sets off the response. In other words, I would propose that the stimulus-response schema be written in the circular form—in the form of a structure which is not simply one way. I would propose that above all, between the stimulus and the response, there is the organism, the organism and its structures. The stimulus is really a stimulus only when it is assimilated into a structure and it is this structure which sets off the response. Consequently, it is not an exaggeration to say that the response is there first, or if you wish at the beginning there is the structure. Of course we would want to understand how this structure comes to be. I tried to do this earlier by presenting a model of equilibration or self-regulation. Once there is a structure, the stimulus will set off a response, but only by the intermediary of this structure.

I should like to present some facts. We have facts in great number. I shall choose only one or two and I shall choose some facts which our colleague, Smedslund, has gathered. (Smedslund is currently at the Harvard Center for Cognitive Studies.) Smedslund arrived in Geneva a few years ago convinced (he had published this in one of his papers) that the development of the ideas of conservation could be indefinitely accelerated through learning of a stimulus-response type. I invited Smedslund to come to spend a year in Geneva to show us this, to show us that he could accelerate the development of operational conservation. I shall relate only one of his experiments.

During the year that he spent in Geneva he chose to work on the conservation of weight. The conservation of weight is, in fact, easy to study since there is a possible external reinforcement, that is, simply weighing the ball and the sausage on a balance. Then you can study the child’s reactions to these external results. Smedslund studied the conservation of weight on the one hand, and on the other hand he studied the transitivity of weights, that is, the transitivity of equalities if \( A = B \) and \( B = C \), then \( A = C \), or the transitivity of the inequalities if \( A \) is less than \( B \), and \( B \) is less than \( C \), then \( A \) is less than \( C \).

As far as conservation is concerned,
Smedslund succeeded very easily with five- and six-year-old children in getting them to generalize that weight is conserved when the ball is transformed into a different shape. The child sees the ball transformed into a sausage or into little pieces or into a pancake or into any other form, he weighs it, and he sees that it is always the same thing. He will affirm it will be the same thing, no matter what you do to it; it will come out to be the same weight. Thus Smedslund very easily achieved the conservation of weight by this sort of external reinforcement.

In contrast to this, however, the same method did not succeed in teaching transitivity. The children resisted the notion of transitivity. A child would predict correctly in certain cases but he would make his prediction as a possibility or a probability and not as a certainty. There was never this generalized certainty in the case of transitivity.

So there is the first example, which seems to me very instructive, because in this problem in the conservation of weight there are two aspects. There is the physical aspect and there is the logical–mathematical aspect. Note that Smedslund started his study by establishing that there was a correlation between conservation and transitivity. He began by making a statistical study on the relationships between the spontaneous responses to the questions about conservation and the spontaneous responses to the questions about transitivity, and he found a very significant correlation. But in the learning experiment, he obtained a learning of conservation and not of transitivity. Consequently, he successfully obtained a learning of what I called earlier physical experience (which is not surprising since it is simply a question of noting facts about objects), but he did not successfully obtain a learning in the construction of the logical structure. This doesn't surprise me either, since the logical structure is not the result of physical experience. It cannot be obtained by external reinforcement. The logical structure is reached only through internal equilibration, by self-regulation, and the external reinforcement of seeing that the balance did not suffice to establish this logical structure of transitivity.

I could give many other comparable examples, but it seems useless to me to insist upon these negative examples. Now I should like to show that learning is possible in the case of these logical–mathematical structures, but on one condition—that is, that the structure which you want to teach to the subjects can be supported by simpler, more elementary, logical–mathematical structures. I shall give you an example. It is the example of the conservation of number in the case of one-to-one correspondence. If you give a child seven blue tokens and ask him to put down as many red tokens, there is a preoperational stage where he will put one red one opposite each blue one. But when you spread out the red ones, making them into a longer row, he will say to you, "Now, there are more red ones than there are blue ones."

Now how can we accelerate, if you want to accelerate, the acquisition of this conservation of number? Well, you can imagine an analogous structure but in a simpler, more elementary situation. For example, with Mlle. Inhelder, we have been studying recently the notion of one-to-one correspondence by giving the child two glasses of the same shape and a big pile of beads. The child puts a bead into one glass with one hand and at the same time a bead into the other glass with the other hand. Time after time he repeats this action, a bead into one glass with one hand and at the same time a bead into the other glass with the other hand and he sees that there is always the same amount on each side. Then you hide one of the glasses. You cover it up. He no longer sees this glass but he continues to put one bead into it while at the same time putting one bead into the other glass which he can see. Then you ask him whether the equality has been conserved, whether there is still the same amount in one glass as in the other. Now you will find that very small children, about four years old, don't want...
to make a prediction. They will say, "So far, it has been the same amount, but now I don't know. I can't see any more, so I don't know." They do not want to generalize. But the generalization is made from the age of about five and one-half years.

This is in contrast to the case of the red and blue tokens with one row spread out, where it isn't until seven or eight years of age that children will say there are the same number in the two rows. As one example of this generalization, I recall a little boy of five years and nine months who had been adding the beads to the glasses for a little while. Then we asked him whether, if he continued to do this all day and all night and all the next day, there would always be the same amount in the two glasses. The little boy gave this admirable reply. "Once you know, you know for always." In other words, this was recursive reasoning. So here the child does acquire the structure in this specific case. The number is a synthesis of class inclusion and ordering. This synthesis is being favored by the child's own actions. You have set up a situation where there is an iteration of one same action which continues and which is therefore ordered while at the same time being inclusive. You have, so to speak, a localized synthesis of inclusion and ordering which facilitates the construction of the idea of number in this specific case, and there you can find, in effect, an influence of this experience on the other experience. However, this influence is not immediate. We study the generalization from this recursive situation to the other situation where the tokens are laid on the table in rows, and it is not an immediate generalization but it is made possible through intermediaries. In other words, you can find some learning of this structure if you base the learning on simpler structures.

In this same area of the development of numerical structures, the psychologist Joachim Wohlwill, who spent a year at our Institute at Geneva, has also shown that this acquisition can be accelerated through introducing additive operations, which is what we introduced also in the experiment which I just described. Wohlwill introduced them in a different way but he too was able to obtain a certain learning effect. In other words, learning is possible if you base the more complex structure on simpler structures, that is, when there is a natural relationship and development of structures and not simply an external reinforcement.

Now I would like to take a few minutes to conclude what I was saying. My first conclusion is that learning of structures seems to obey the same laws as the natural development of these structures. In other words, learning is subordinated to development and not vice-versa as I said in the introduction. No doubt you will object that some investigators have succeeded in teaching operational structures. But, when I am faced with these facts, I always have three questions which I want to have answered before I am convinced.

The first question is: "Is this learning lasting? What remains two weeks or a month later?" If a structure develops spontaneously, once it has reached a state of equilibrium, it is lasting, it will continue throughout the child's entire life. When you achieve the learning by external reinforcement, is the result lasting or not and what are the conditions necessary for it to be lasting?

The second question is: "How much generalization is possible?" What makes learning interesting is the possibility of transfer of a generalization. When you have brought about some learning, you can always ask whether this is an isolated piece in the midst of the child's mental life, or if it is really a dynamic structure which can lead to generalizations.

Then there is the third question: "In the case of each learning experience what was the operational level of the subject before the experience and what more complex structures has this learning succeeded in achieving?" In other words, we must look at each specific learning experience from the point of view of the spontaneous operations...
which were present at the outset and the operational level which has been achieved after the learning experience.

My second conclusion is that the fundamental relation involved in all development and all learning is not the relation of association. In the stimulus–response schema, the relation between the response and the stimulus is understood to be one of association. In contrast to this, I think that the fundamental relation is one of assimilation. Assimilation is not the same as association. I shall define assimilation as the integration of any sort of reality into a structure, and it is this assimilation which seems to me to be fundamental in learning, and which seems to me to be the fundamental relation from the point of view of pedagogical or didactic applications. All of my remarks today represent the child and the learning subject as active. An operation is an activity. Learning is possible only when there is active assimilation. It is this activity on the part of the subject which seems to me to be underplayed in the stimulus–response schema. The presentation which I propose puts the emphasis on the idea of self-regulation, on assimilation. All the emphasis is placed on the activity of the subject himself, and I think that without this activity there is no possible didactic or pedagogy which significantly transforms the subject.

Finally, and this will be my last concluding remark, I would like to comment on an excellent publication by the psychologist Berlyne. Berlyne spent a year with us in Geneva during which he intended to translate our results on the development of operations into stimulus–response language, specifically into Hull’s learning theory. Berlyne published in our series of studies of genetic epistemology a very good article on this comparison between the results obtained in Geneva and Hull’s theory. In the same volume, I published a commentary on Berlyne’s results. The essence of Berlyne’s results is this: Our findings can very well be translated into Hullian language, but only on condition that two modifications are introduced. Berlyne himself found these modifications quite considerable, but they seemed to him to concern more the conceptualization than the Hullian theory itself. I am not so sure about that. The two modifications are these. First of all, Berlyne wants to distinguish two sorts of response in the S-R schema: (a) responses in the ordinary, classical sense, which I shall call “copy responses;” (b) responses which Berlyne calls “transformation responses.” Transformation responses consist of transforming one response of the first type into another response of the first type. These transformation responses are what I call operations, and you can see right away that this is a rather serious modification of Hull’s conceptualization because here you are introducing an element of transformation and thus of assimilation and no longer the simple association of stimulus–response theory.

The second modification which Berlyne introduces into the stimulus–response language is the introduction of what he calls internal reinforcements. What are these internal reinforcements? They are what I call equilibration or self-regulation. The internal reinforcements are what enable the subject to eliminate contradictions, incompatibilities, and conflicts. All development is composed of momentary conflicts and incompatibilities which must be overcome to reach a higher level of equilibrium. Berlyne calls this elimination of incompatibilities internal reinforcements.

So you see that it is indeed a stimulus–response theory, if you will, but first you add operations and then you add equilibration. That’s all we want!

Editor’s note: A brief question and answer period followed Professor Piaget’s presentation. The first question related to the fact that the eight-year-old child acquires conservation of weight and volume. The question asked if this didn’t contradict the order of emergence of the pre-operational and operational stages. Piaget’s response follows:

The conservation of weight and the conservation of volume are not due only to
experience. There is also involved a logical framework which is characterized by reversibility and the system of compensations. I am only saying that in the case of weight and volume, weight corresponds to a perception. There is an empirical contact. The same is true of volume. But in the case of substance, I don’t see how there can be any perception of substance independent of weight or volume. The strange thing is that this notion of substance comes before the two other notions. Note that in the history of thought we have the same thing. The first Greek physicists, the pre-socratic philosophers, discovered conservation of substance independently of any experience. I do not believe this is contradictory to the theory of operations. This conservation of substance is simply the affirmation that something must be conserved. The children do not know specifically what is conserved. They know that since the sausage can become a ball again there must be something which is conserved, and saying “substance” is simply a way of translating this logical necessity for conservation. But this logical necessity results directly from the discovery of operations. I do not think that this is contradictory with the theory of development.

I think that we must distinguish within the cognitive function two very different aspects which I shall call the figurative aspect and the operative aspect. The figurative aspect deals with static configurations. In physical reality there are states, and in addition to these there are transformations which lead from one state to another. In cognitive functioning one has the figurative aspects—for example, perception, imitation, mental imagery, etc.

The operative aspect includes operations and the actions which lead from one state to another. In children of the higher stages and in adults, the figurative aspects are subordinated to the operative aspects. Any given state is understood to be the result of some transformation and the point of departure for another transformation. But the pre-operational child does not understand transformations. He does not have the operations necessary to understand them so he puts all the emphasis on the static quality of the states. It is because of this, for example, that in the conservation experiments he simply compares the initial state and the final state without being concerned with the transformation.

In exercising perception and memory, I feel that you will reinforce the figurative aspect without touching the operative aspect. Consequently, I’m not sure that this will accelerate the development of cognitive structures. What needs to be reinforced is the operative aspect—not the analysis of states, but the understanding of transformations.

Editor’s note: The second question was whether or not the development of stages in children’s thinking could be accelerated by practice, training, and exercises in perception and memory. Piaget’s response follows:

I am not very sure that exercise of perception and memory would be sufficient.
Physics Problems and the Process of Self-Regulation

Anton E. Lawson and Warren T. Wollman

In two previous articles\textsuperscript{1,2} Jean Piaget’s theory of intellectual development and its general implications for physics teaching were discussed. The purpose of this article is to examine more closely one aspect of that theory and discuss its implications for designing and using homework problems. We will briefly describe the process of self-regulation (the process Piaget hypothesizes governs all intellectual growth) and suggest a way in which homework problems can be used to provide students an opportunity for self-regulation. Further, we will discuss deficiencies of typical homework problems and provide a number of example problems which we believe can initiate self-regulation. Through the process of self-regulation initiated by thought-provoking problems, we believe students will not only be able to develop understandings of the concepts involved but will also progress from relatively concrete (or limited) to more abstract (or generalizable) modes of thinking.

The process of self-regulation

The process by which Piaget hypothesizes that patterns of reasoning are refined, extended, or combined with other patterns of reasoning is called self-regulation. Initially, basic reasoning patterns serve to guide an individual’s actions within his surroundings. As long as those actions promote satisfactory interaction, the basic patterns continue to guide behavior. However, owing to the individual’s extended interaction with his environment he meets contradictions, that is, situations for which his initial patterns of reasoning do not serve as effective guides to behavior. These contradictions produce a state of disequilibrium. In other words, his patterns of reasoning are found wanting and must somehow be changed. If the disequilibrium is not too great, he will spontaneously begin to alter his patterns of reasoning in an attempt to assimilate the new situation. The process by which an individual actively seeks to reestablish equilibrium is termed self-regulation. The altered reasoning patterns which develop are then tried. If the patterns guide behavior successfully so that the person’s efforts obtain positive feedback the patterns are reinforced. Continued positive feedback then produces an increasingly stable set of reasoning patterns. In this manner the person gradually builds new reasoning patterns and adapts to new situations.

Homework problems can initiate self-regulation

The gradual process of reestablishing equilibrium through self-regulation affords the possibility of initiating interactions between students and subject matter with the use of homework problems provided the following two factors are present: Problems must be chosen so that the student can partially but not completely understand...
Typical homework problems seldom require a student to examine his own thinking.

them in terms of old ideas (i.e., a moderate state of disequilibrium must result from the problem); and sufficient time must be allowed for the student to grapple with the new situation, possibly with appropriate "hints" to direct his thinking, but allowing him to put the ideas together himself.

An important facet then in selecting problems which encourage self-regulation is to obtain a careful match between what the student knows and the kind of problem he is asked to work through. The ideal situation would seem to be one in which the problems are challenging but are felt to be solvable. The hypothesis is that a challenging but solvable problem will place a student into an initial state of disequilibrium. However, through his own efforts at bringing together what he has done in the laboratory, read in the textbook, heard in lectures, learned from other past experiences, and obtained from teacher or peer discussions he will gradually organizes his thinking about this information and successfully solve the problem. This success will then establish a new and more stable equilibrium. The new state of equilibrium will be one with increased understanding of the subject matter and increased problem-solving capability. Before giving examples of the kind of problem we believe can initiate self-regulation a few comments will be made regarding deficiencies of standard homework problems.

What's wrong with typical homework problems?

Typical homework problems seldom require a student to examine his own thinking, make comparisons, and raise questions which, in fact, are crucial to scientific inquiry. These problems usually require students to apply an equation or sometimes two or three equations to obtain a solution. Students quickly come to realize that the name of this game is "Can you discover the correct equation?" This is a game of recognition—a sort of high order matching process involving little thought. Although this process can be an important one, we believe that little if any self-regulation takes place in this way. Typical homework problems do not require the student to think about:

1. The data of the problem. Usually there is just the right amount, no more nor less, whereas in real situations there is either a dearth or superfluity of information and the problem is to discover what is relevant.
2. The approach to the problem. Usually this is determined by the chapter heading. If, for example, a mechanics problem can be solved either by Lagrange's equations, Newton's laws, or energy conservation, the choice is dictated by irrelevant considerations, e.g., the problem comes from the chapter on Lagrange's equations. It is important for students to learn that many approaches may seem reasonable and the problem is to decide whether one is particularly appropriate.
3. The tacit assumptions of a problem-solving strategy, for example deciding between use of Boyle's law or the Van der Waals equation. This decision is usually made for the student, not by the student.
4. The physical arguments involved in the problem as opposed to the mathematical ones. Too often problems are only exercises in using mathematical tools (a necessary exercise) without ever demanding that the student try either to arrive at or qualitatively justify the mathematical result by physical (phenomenological) arguments utilizing both principles and order of magnitude calculations. Indeed, the physical or intuitive argument often precedes the mathematical in real research.
5. The statement of a problem. Problems are tailored to fit the text when, in fact, the real problem is doing the tailoring by conceptualizing a real situation in terms of a model. This involves all of the above points.

How to encourage self-regulation

A few points should be kept in mind when designing, discussing, using, and scoring problems to encourage self-regulation:

1. Open-ended problems (problems with no single solution) are often excellent tools to encourage thinking.
2. Problems which present an apparent paradox produce disequilibrium and can initiate self-regulation. Paradox problems by their nature are generally short and incisive. Leighton in his foreword to the exercise workbook written to accompany The Feynman Lectures in Physics discussed the kinds of problems which appeared most suitable to him. He suggested that problems of a kind that are numerically or analytically simple, yet incisive and illuminating in content were particularly useful.
3. To encourage self-regulation it is often helpful to ask students to record and hand in all the various ideas they tried and found unsuccessful as well as the ones which were successful in arriving at the problem.
"Real" problems should, and indeed must, involve a certain amount of trial and error.

solution. Discussions of these steps in an atmosphere in which these ideas are recognized not only as worthwhile but as necessary, clue students into the fact that "real" problems should and indeed must involve a certain amount of trial and error, albeit informed trial and error.

4. Have the students search for necessary data so they examine their conceptualization of the problem. Either give superfluous data or omit necessary data. To account for the latter, students should have to make plausible assumptions or introduce suitable symbols for quantities that are needed to solve the problem.

5. Require students to draw a diagram of the physical situation. To do this students have to think deeply about the spatial relationships of the interacting objects, and may find discrepancies as they compare their preconceptions with the diagram.

6. Provide for a "problem clinic" or tutorial service where students can get help with problems while they are solving them, and before they have to be turned in. Interaction with other persons can be very helpful and is often even necessary if students are to conceptualize, then critically analyze their own thinking.

7. For problems designed to engage a student over a period of, say, two weeks, the teacher should consult with the student several times in order to:
   A. Discuss with him his initial approach. If this approach is reasonable but known in advance to be inappropriate, the teacher should not intervene at this point, but rather let the student discover for himself why the approach will not work.
   B. Discuss with the student alternative approaches both when the initial approach is appropriate and when it is reasonable but not appropriate. In either case, let the student first discover which approach will work. Then discuss alternatives, even if the first approach worked. It may be that he will accept inappropriate alternatives as reasonable. He may then discover on his own why they are not.
   C. Discuss both semi-quantitative (order of magnitude) and qualitative arguments anticipating the outcome of more rigorous approaches. Limiting cases should be used as a check when solutions to simpler problems are already known.
   D. Discuss alternatives to an inappropriate and time-consuming approach. This is to avoid having the student spend too much time discovering the inadequacies of an approach. Overall, the student should get from the teacher a feeling for the general considerations appropriate to choosing and comparing strategies, i.e., a feeling for the process of inquiry.

8. Although solutions (numerical or algebraic) should be provided for all problems (not just the "odd-numbered" ones), students must understand that a premature glance at a solution will surely affect their conception of the problem and distort the problem solving procedure. Knowledge of the solution can provide stimulating feedback after the student has completed and carried through a formulation of a solution.

Examples of problems that can promote self-regulation

Problem 1 Since the net force on the spring scale shown in Fig. 1 is zero how can the scale register a non-zero reading? What does the scale register? Why isn't it 20 since it is pulled by 10 lbs at each end?

Comment: This example, which is especially useful when associated with a demonstration, illustrates how a little knowledge can go a wrong way. At first, concepts are only vaguely grasped and thus over-extended. Here we obviously have two forces whose sum is equal to zero and yet the scale does not read zero. Or, we might think that each force contributes 10 lbs of tension to the scale to give 20 lbs. These two approaches use unrestricted (over-extended) concepts which must be coordinated, via self-regulation, with other concepts, e.g., free-body diagrams and action-reaction, in order to resolve the discrepancy.

Fig. 1. Spring balance and suspended weights.
Problem 2 A capacitor and resistor are connected in a circuit as shown in Fig. 2. The values are \( C = 250 \, \mu\text{F} \), \( R = 10000 \, \Omega \), and \( E = 400 \, \text{V} \). Initially the switch is closed and then it is opened suddenly. Use two methods to calculate the energy dissipated in the resistor after the switch is opened. Do both methods give the same result? Should they give the same result? If so, why? If not, why not?

**Comment:** This problem calls for two quantitative analyses of the same situation. If the student is able to think of two methods of solution and obtain the same answer using both methods no disequilibration will result. However, if two different answers are obtained the student should check his own work. The discrepancy could be resolved quickly if the source of the difference was an error in calculation. If, however, the difference was due to difficulty in conceptualization, then the check will promote self-regulation.

![Circuit diagram showing the capacitor, resistor, switch, and battery.](Fig. 2)

Problem 3 The gas temperature at one level of the upper atmosphere is about 1000°K. The temperature at the surface of a burning match is about the same. Yet a person would be very cold in the upper atmosphere. How can that be?

**Comment:** This problem presents a paradox because 1000°K is a very high temperature and yet it is “cold up there.” Resolution through self-regulation leads to a more scientific and less everyday notion of the relation between temperature and “cold” or “hot.”

Problem 4 A glass is exactly full of water at 0°C and has a cube of ice floating in it. When the ice melts (still at 0°C) the water will not overflow, because the ice displaced a volume of water equal to the volume of the water into which the ice melted. OK. Let us look at some fine points. In what direction would each of the following affect the result? Give only the direction.

(a) The ice cube contained some grains of sand.
(b) The ice cube contained some air bubbles.
(c) The water (and the glass) were not at 0°C to start with, but were at room temperature.
(d) The “water” is not water at all, but is a Martini which is close to 0°C but, due to its alcoholic content, has density less than that of water.

**Comment:** This problem originally appeared in an article by Richard Crane. It, as well as other problems in that article (for example, problems 8, 17, 18, 26-29), are excellent examples of problems which will promote self-regulation. Problems 34, 41, 42, and 48 which appeared in a second article by Crane also are thought provoking and should encourage self-regulation.

Problem 5 If internal energy is partly molecular motion, what is the difference between a hot, stationary golf ball sitting on a tee and a cold golf ball rapidly moving off the tee?

**Comment:** Of course, the molecular motion part of internal energy refers to random motion. Thus, self-regulation refines or sharpens a global or relatively diffuse concept. It is typical of students that they only assimilate parts of a concept at first. By provoking them to discover or recover all the parts, the concept becomes more sharply defined.

Problem 6 When a cylinder, open at one end, is placed over a burning candle which is sitting in a container of water the candle flame goes out and water rises into the cylinder. Why does the flame go out and why does the water rise? Note: Not all observations are mentioned in the description. What other observations do you think you would make if the phenomenon was observed? Obtain the necessary materials and try the experiment yourself. Try the experiment varying the number of candles used, the amount of water in the container, the size and shape of the cylinder, the speed with which you place the cylinder over the candle, and anything else you can think of.

**Comment:** This problem is one which often yields a quick but erroneous solution. Most students will hypothesize that the candle goes out because it burned up all the oxygen in the cylinder and the water then came in to replace the oxygen. Selected items of information or questions could be supplied at this point to provoke students to abandon this idea and continue their search. For example: What is produced when a flame consumes oxygen? Two burning candles make more water rise than one. Small bubbles were observed escaping from the bottom of the cylinder. Why might this have occurred? These observations contradict the initial explanation and should provoke disequilibrium. Once other explanations are offered they can be analyzed to determine their suitability. They may lead...
Problem 7  Everyone 'knows' that to win a tug of war, a team has to pull harder than the other team. What everyone doesn't know is that, in fact, each team always pulls equally hard, even the winning team. Under these circumstances, how can one team ever win (short of the other team just letting go)?

Problem 8  Polishing surfaces reduces friction between them unless you polish them extremely well, then friction will increase. How can that be true?

Problem 9  (a) See Fig. 3a. The focal lengths of two identical, thin, convex lenses are the same and measured to be 20 cm each ($F_1 = 20$ cm, $F_2 = 20$ cm). The two lenses are placed next to each other as shown in Fig. 3b and taped together at their edges only. The focal length of this combination, $F_c$, is 10 cm. Write an equation that gives the focal length of a lens combination that consists of two lenses having identical focal lengths.

(b) One of the 20 cm focal length lenses is replaced by one having a focal length ($F_3$) of 5 cm. The focal length of the resulting combination is measured to be 4 cm. Write an equation that can be used to calculate the focal length of a lens combination that consists of two lenses of unequal focal lengths.

(c) Now check your two equations. Are they the same? Do you think they should be the same. If so, why? If not, why not? If you believe they should be the same but you have two different equations rethink the problem and try to reduce the two situations to one equation.

Problem 10  A student measures his weight by climbing onto the large platform of a big spring scale. He takes a step to one side and notices that just as he started to do this, the scale registered less than his weight. Before he could puzzle this through, he noticed that just as he completed the step, the scale now registered more than his weight. If there is nothing wrong with the scale, then what was going on?

Problem 11  A brick is supported by a string A from the ceiling, and another string B is attached to the bottom of the brick. If you give a sudden jerk to B it will break, but if you pull on B steadily, A will break. Since the force is the same both ways how could this occur?
force is force and so equal forces have equal effects. So how can the string break in one instance and not in the other? Again, common sense is in conflict with observation and this use of physics to set the world straight is likely to be retained.

Acknowledgment

The authors wish to express sincere appreciation to Professor Robert Karplus and Professor Lester Paldy for their helpful suggestions in the formulation and presentation of the ideas put forth in this manuscript. Credit is due also to Professor John Renner for the ideas used in problems 6 and 9 and to Robert Karplus for problem 2. Ideas for some of the other problems came from D. Halliday and R. Resnick, Physics (Wiley, New York, 1966). In all cases the problems were edited and modified.

AESOP (Advancement of Education in Science Oriented Programs) is supported by a grant from the National Science Foundation.

projection pointers

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The Oersted effect on the overhead

It is well known that the effect on a magnetic compass needle of being deflected when placed near a current-carrying wire was discovered by Hans Oersted in 1820. An elementary demonstration of this effect is usually presented in any course dealing with electricity and magnetism, and it is a very convincing proof that moving electric charges produce magnetic fields. Several apparatus manufacturers* sell a simple device to demonstrate the Oersted effect to small classes. The apparatus consists of a metallic bar bent into a rectangular loop and mounted on an insulated base with a compass needle suspended at the middle of the loop. When a large current is sent through the loop the compass needle will deflect and line up perpendicular to the loop; i.e., tangent to the magnetic field line at that position. Reversing the current direction results in the needle reversing its direction, showing how the magnetic field direction is related to the current direction (right-hand rule).

In a large or auditorium-size lecture class it is difficult for all the students to see the effect demonstrated by this small apparatus. Since the overhead projector is used extensively in such situations it is natural to try to adapt this demonstration to the overhead. This is simply accomplished by replacing the opaque base with one made of Lucite and securing to it an inverted-U-shaped metal bar with screw terminals at each end for connection to a current source. The same compass needle that is used in the commercial apparatus is suspended under the bar by a needle point in the same manner as is found in the commercial device (see Fig. 1). When the apparatus is operated on the overhead the compass needle deflection is easily viewed by all. A small piece of paper can be taped to one end of the compass needle as a visible reference. A further modification (not shown in the figure) uses a smaller raised Lucite platform to place the compass needle above the metal bar for demonstrating the circular symmetry of the magnetic field.

Fig. 1. The Oersted effect demonstrated in place on the overhead.

References


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