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Mathematical Communication, Conceptual Understanding, and Students' Attitudes Toward Mathematics

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Math in the Middle Institute Partnership
Action Research Project Report

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Department of Mathematics
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Mathematical Communication, Conceptual Understanding, and Students' Attitudes Toward Mathematics

Abstract

This action research study of my 8th grade classroom investigated the use of mathematical communication, through oral homework presentations and written journals entries, and its impact on conceptual understanding of mathematics. This change in expectation and its impact on students' attitudes towards mathematics was also investigated. Challenging my students to communicate mathematics both orally and in writing deepened the students' understanding of the mathematics. Levels of understanding deepened when a variety of instructional methods were presented and discussed where students could comprehend the ideas that best suited their learning styles. Increased understanding occurred through probing questions causing students to reflect on their learning and reevaluate their reasoning. This transpired when students were expected to write more than one draft to math journals. By making students aware of their understanding through communicating orally and in writing, students realized that true understanding did not come from mere homework completion, but from evaluating and assessing their own and other's ideas and reasoning. I discovered that when students were challenged to communicate their reasoning both orally and in writing, students enjoyed math more and thought math was more fun. As a result of this research, I will continue to require students to communicate their thinking and reasoning both orally and in writing.

Introduction

For twelve years, I have taught mathematics to students in grades ranging from fourth to eighth. During this tenure, homework was assigned nearly daily, with the expectation it be completed and handed in the next day. The homework had been checked for accuracy, assigned a percentage grade, and handed back to the students. It did not take long to figure out what types of homework problems were easiest to grade and what types of problems were not so easily graded. Assigning more of the “easy to grade” types of problems had become the norm. “Easy to grade” problems were usually computation problems that had one exact answer. These problems seldom involved problem solving or features that pushed my students to think. Problems like this required instrumental understanding, versus conceptual understanding. They were neat and succinct.

On the contrary, “hard to grade” problems were messy. By messy, I mean more difficult to teach, understand, and get students to like and complete. These types of problems were also more challenging to grade, and resulted in the majority of students missing them, especially when they were focused on the “right” answer. They were messy because to truly assess a student’s level of understanding, the student must explain their thinking and justify their solution. “Hard to grade” problems had more than one correct answer and are generally more difficult for students. Problems like this were time consuming.

Ideally, my classroom would be so rich with discussion, engagement and interaction that homework is not necessary. Students would be confident problem solvers and willing to share their ideas, conjectures and mistakes. Students would learn from other students as much as from the teacher. Students would be empowered to learn, excited to experiment, and disciplined

enough to do extra practice as needed. Students' motivations would not be fueled by grades, but by intrinsic factors. Students would be curious. Students would be hungry for knowledge.

My Math in the Middle journey has encouraged self assessment of my mathematical teaching and learning. Being a student again has made me realize the power of communication and learning. Many times I have been pushed out of my comfort level and been required to explain my thinking, justify my solution, and admit my insecurities and misunderstanding. Often times, I have learned more from being wrong than being confidently right. As a learner, I have been empowered by communication. As a teacher, I am intrigued about the relationship between communication and learning.

For these reasons, the investigation of mathematical communication and conceptual understanding was chosen. My goal was to increase mathematical communication both orally and in writing through homework presentations, discussion, and journal writing, and then explore whether or not it impacted conceptual understanding. I was also interested in discovering how this change in expectations might affect students' attitudes toward mathematics.

Problem Statement

According to the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics (2000), communication is an essential part of mathematics. The communication process helps build meaning. When students are challenged to think and reason and then communicate their ideas orally or in writing, true conceptual understanding develops. Listening to others' explanations provides students opportunity to clarify their understanding and consolidate mathematical ideas.

In thinking of how to assign homework, many of NCTM's principles apply. The teaching principle states that mathematics teaching requires understanding what students know and need

to learn, then challenging and supporting them to do it well. According to the learning principle, students must learn mathematics with understanding, while the assessment principle maintains that assessments should support learning and furnish useful information.

In evaluating my homework practices prior to this research, I was not meeting any the above principles and expectations. My homework assignments did not necessarily reveal what students really knew, nor did they challenge all students. The types of problems I assigned did not promote learning with understanding. My homework assignments did not furnish useful information. My grading practices did not support or promote understanding very well.

NCTM's principle of equity states that mathematics education should require high expectations for all. For these reasons, I wanted to alter the expectations of my classroom. In order for homework to furnish useful information, it must be intended for true conceptual understanding, not regurgitation of facts from the lesson. Increased focus on assigning open-ended types of problems that truly assess understanding is vital. This should be followed by an opportunity for discussion with the expectation that students explain their thinking through writing in "problem solving journals." This happens by creating a classroom community that empowers learning for understanding, not solely for a grade. Creating a community of learners, empowered by learning through communication, allows everyone to assume ownership of high expectations – for themselves and one another.

All teachers, regardless of the subject or discipline, prefer students who want to learn. Creating a classroom that values understanding, not just grades, benefits all teachers and learners. Creating a classroom climate that thrives on communicating mathematical thinking is relevant to all teachers because being able to convey one's ideas leads to true, conceptual understanding. Forcing students to communicate often forces them to think and reflect. With

much practice, students can become confident communicators and thinkers. These traits are ideal characteristics needed in this era of high stakes testing and No Child Left Behind. By having students communicate their ideas and thinking, teachers have a better understanding of what students know and do not know. Teachers will more confidently recognize when students are prepared for standards-based testing and can demonstrate true understanding of concepts.

Literature Review

One of the biggest obstacles for mathematics instruction is teachers use an inappropriate approach to the nature of mathematics and learning. Few teachers expect and require mathematics to be meaningful; fewer still see mathematics as a creative undertaking. Rather, teachers seem to interpret mathematics as acquiring and memorizing facts and algorithms. Typical teachers too often are satisfied with teaching mathematics as manipulating symbols and doing routine problems, without ever ensuring that their students acquire deep, conceptual understanding. Students usually learn and believe what they are taught. I believe student attitudes are shaped by their teachers. Therefore, a shift must occur in mathematics instruction from rote memorizing and performing algorithms, to critical thinking and conceptual understanding. This should result in improved student attitudes toward mathematics. I believe this can occur by implementing oral and written communication into the mathematics classroom.

According to NCTM, communication is an essential part of mathematics and mathematics education. When students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally or in writing, they learn to be clear and convincing (NCTM, 2000). Challenging students to think and reason about mathematics allows students to construct their own meaning for mathematics and in turn

develops deep, conceptual understanding. The use of communication and problem solving has been encouraged in mathematics education as stated by the National Research Council (1989).

> Education research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: “examine,” “represent,” “transform,” “solve,” “apply,” “prove,” “communicate.” This happens most readily when students work in groups, engage in discussion, make presentations, and in other ways take charge of their own learning (pp. 58-59).

Creating opportunities for students to examine, apply, prove and communicate mathematics will not only give meaning to the discipline, but also develop a deeper understanding of mathematics.

In my review of the research on oral and written communication and conceptual understanding of mathematics, my analysis revealed five major themes. The first theme in the literature is that teachers must value the constructivist perspective in order to construct their own mathematical understanding. Secondly, teachers must have sound mathematical knowledge and be masters of their content in order to create a learning environment that develops meaning for mathematics. A third theme is that teachers must base instructional decisions on what they learn about their students’ understanding from their students’ oral and written communication in order to develop curriculum that best fits student needs. A fourth theme revealed that students can attain deeper conceptual understanding by reflection on their own learning, as well as listening to others’ thinking than from only feedback from their teacher. The final theme I identified is that students can learn as much or more by communicating conflicting ideas and argument, as they can by only being presented with correct solutions. The following sections will examine each of

these themes in more depth and also explain the ways in which my research differs from the published literature in these areas.

Constructivist Perspective

From the constructivist perspective, meaning is not inherent in written or spoken language. Meaning is constructed by each individual on the basis of his or her own experience and involves individual interpretation (Yackel, Cobb, & Wood, 1993). Words mean nothing by themselves. It is only when a thinker makes use of them that words stand for anything or have meaning. By creating a classroom climate that promotes thinking and encourages communication of mathematical thinking, students can construct their own mathematical meanings.

Wood (1999) performed a research project on reform-oriented classrooms in which teaching and teachers' thinking and reasoning were of the central interest. In this 18-month investigation of teacher's actions during class discussions in a second grade classroom, teachers designed their lessons around the constructivist perspective. According to Wood, "Children understand mathematics best if they are actively involved in their own learning" (p. 173). Requiring students to express their thinking either orally or in writing will allow students to take ownership of their learning as well as be actively involved in their own learning.

Hoyles (1985) researched the implications of using pupil to pupil discussion within mathematics classrooms. Holyes' research suggested that discussion promotes mathematical understanding. According to Hoyles, classroom communication is important in students' development of conceptual understanding. Encouraging students to communicate about their conceptions and justify their own strategies of exploration represents a shift in the social relations in the classroom from a "teacher-centered" to a more "pupil-centered" approach (Hoyles, 1985). By shifting to more constructivist and child-centered approaches to learning,

emphasis is placed on children's ways of knowing and the inventiveness of their mathematical thinking. When students form the intention of making sense of their experiences, they become inquirers and effectively take charge of their own learning.

In my research project, students constructed their own mathematical meaning by orally presenting their mathematical thinking and solutions to homework problems, as well as explaining and justifying their mathematical understanding in a writing journal. In Yackel, Cobb, and Wood's (1993) study, students constructed meaning in spoken and written language as well. Their study observed the work of two students, while I observed the work of my entire class. In Yackel, Cobb and Wood's study, as well as in mine, meaning was constructed by each individual on the basis of his or her own experience and involved individual interpretation.

Teacher Knowledge

It has been argued that the use of oral and written communication in mathematics learning should promote deeper conceptual understanding. However, such a change implies major shifts in the teaching practices most teachers use. There is increasing evidence that knowledge of children's thinking is a powerful influence on teachers as they consider instructional change (Fennema et al., 1996). Teachers must first develop mathematical knowledge before instituting instructional change. Fennema, Carpenter, Franke, Jacobs, and Empson performed a longitudinal study of twenty-one first, second, and third grade teachers who received professional development in Cognitively Guided Instruction (CGI). In this study, as the teachers learned new ideas about mathematics, they were able to understand students' thinking better and reflect more on the teaching and learning processes in which the students were engaged in.

In Wood, Cobb, and Yackel's (1991) case study of a second grade classroom, their intent was to focus an investigation on individual children's construction of mathematical knowledge. However, in the process of creating environments for children to learn mathematics with meaning, they became aware that the classroom had simultaneously and unintentionally become a learning environment for the teacher as well. They learned that the knowledge of teachers was significantly related to student achievement. If teachers have little understanding of mathematical principles and concepts, then their teaching is based on rule-bound procedures. For instructional change is to occur, teachers should emphasize the conceptual nature of mathematics and the active role of students in the development of their mathematical meanings. Teachers should also understand the crucial role that classrooms play as learning environments for students, as well as teachers.

My graduate work and research has proven that mathematics is far more than algorithms and rules. Mathematics instruction should focus on constructing meaning and developing true understanding. In Wood et al. (1991) case study, there was a change in teaching mathematics through professional development and increasing the knowledge of the teachers. Mathematics instruction and focus has changed in my classroom as a result of professional development and action research. My research project was designed to promote conceptual understanding through changing my teaching practices. In Fennema et al.'s four year study of teachers' beliefs and instruction (1996), teachers learned about children's thinking and decided how to use the knowledge to make instructional decisions. My study did not investigate teachers, but rather students and how they learned from expressing their own thinking. As the teacher and the researcher, my instructional knowledge of mathematics played a vital role in my students' deeper understanding.

Instructional Decisions

In a communication-rich classroom, the teacher's role is to facilitate discussion, encourage critical thinking, and clarify misconceptions when necessary. In Shield and Galbraith's (1998) study of the eighth grade level, the authors concluded that there is a relationship between students' ability to communicate mathematically and their level of meaningful learning. According to Shield and Galbraith, "The teaching of mathematics, particularly at the middle and high school levels, has often been characterized as a "tell-show-do" approach led by the teacher" (p. 32). By designing instruction so that students have to communicate their thinking, teachers can create meaningful learning for their students. In Fennema et al.'s (1996) longitudinal study, the teachers came to believe that their role was not to tell children how to think, but to provide an environment in which children's knowledge could develop as the children engaged in problem experiences and reported on solution strategies.

By allowing students to share their thinking verbally or in writing, teachers can make instructional decisions based on children's understanding. Teachers can then change their instructional practices to better meet their students' needs. Wheatley (1992) studied and researched elementary age students and problem-centered learning. According to Wheatley, teachers need to simply listen to students. Teachers must adapt to the students rather than expecting students to do the adapting. It only makes sense that teachers should base curriculum and lesson plan decisions on the students' current mathematical knowledge rather than on a fixed curriculum.

Instructional decisions should be made to target student needs. By reading student journals, teachers can adapt their instruction to "fit" the students. In Borasi and Rose's (1989) investigation of journal writing and mathematics instruction, the journals provided the teachers

with a wealth of information about the students and the course which would have been lost otherwise. The journals allowed the teacher as a reader to get to know students individually and to realize their students' specific problems and difficulties. Written communication and journal writing can be a vital tool for teachers and their instructional decisions.

In Fennema et al.'s (1996) study, the teachers realized the important criteria for making decisions about what and how to teach were children's understandings. The teachers discovered that children should not be asked to practice procedures that the students did not understand. The teachers also realized a way to find out if the students understood was to ask them to explain their thinking. Oral and written communication can and should play a powerful role in instructional design and decisions. By students communicating their mathematics thinking and ideas, teachers can better assess their true understanding and design curriculum that best suits the students' needs. By specifically targeting and meeting students' needs, deeper conceptual understanding inevitably occurs.

Although my research study was intended to examine oral and written communication and conceptual understanding, my instructional decisions were guided by student understanding. As in Borasi and Rose's (1989) study, my students expressed their mathematical thinking in writing. By requiring my student to present their mathematics thinking and solutions, I was able to better assess my students' understanding, just as in Fennema et al.'s (1996) study. My research study involved problem-centered learning as students worked in groups and pairs to solve problems and presented in groups similar to Grayson's (1992) study. However, my study was performed with middle school students, while the above mentioned studies all took place with elementary children.

Reflection

In mathematics learning, reflection is characterized by distancing oneself from the action of doing mathematics (Grayson, 1992). Grayson states that evidence has been presented that encouraging reflection results in greater mathematics achievement. Encouraging students to express their thinking orally or in writing forces students to reflect on their own thinking. According to Hoyles (1985), who has researched group discussion in mathematics, “The ideas of others can suggest modifications to one’s own thoughts, clarify half-worked out predictions or explain half-understood processes” (p. 207). Creating opportunities where students have to reconstruct their solutions and explain and justify their ideas orally or writing deepens student understanding.

However, in Wood et al.’s (1991) case study of change in teaching mathematics through professional development, teachers realized that they had to create opportunities for learning in which the children not only expressed their mathematical thoughts, but also listened to solutions of others. By listening to other’s thinking, individuals can reconstruct their own thinking, as well as extend and deepen their reasoning and understanding. Therefore, student can deepen their understanding of mathematics by listening, just as teachers can deepen their understanding of students by listening.

Reflection occurs while explaining one’s thinking and by listening to others thinking, but also through writing. In Borasi and Rose’s (1989) study of journal writing, they reported that “restating concepts and rules in one’s own words can in fact facilitate internalization” (p. 355). Writing journals were used to reflect on material learned in class. While reflecting in writing,

students were actively engaged in the deliberate structuring of meaning. When students take ownership of mathematical meaning, conceptual understanding of mathematics develops.

The articles mentioned above relate to my study in many ways. By requiring my students to orally present homework solutions, my students reflected on their own thinking, as did fellow students by listening just as in Wood, Cobb and Yackel's (1991) study. Like in Borasi and Rose's study (1989), my students were required to reflect in a math journal. However, the intent was not to have them reflect on each day's learning, but rather have them justify their mathematical solutions to open-ended math problems related to our curriculum.

Conflicting Ideas and Argument

By listening to and reflecting on others' mathematical ideas, conflicting ideas and views will obviously arise. In Yackel, Cobb, and Wood's (1993) study on mathematical communication in small groups, the researchers found unique opportunities for learning when discrepancies were discovered between partners. In order for the students to be aware that there was a discrepancy, the students had to extend their own conceptual thinking to try to make sense of the partner's interpretation and then formulate an argument or explanation that might be meaningful to the partner.

In Wood's (1999) 18-month investigation of a second grade classroom in which students' disagreement was resolved by argumentation, students were able to experience mathematics as a subject that relies on reasoning for the justification of ideas. In Wood's study, listeners were expected to follow the thinking and reasoning of others to determine whether what was presented was logical and made sense. As listeners, students were also expected to voice their disagreement and to provide reasons for disagreeing. According to Wood, "The significance of argument to conceptual understanding in mathematics is related to the development of students' thinking and

reasoning that occurs during the acts of challenge and justification" (p. 189). By creating a safe environment where students feel comfortable to challenge other's ideas and understand the value in giving reason for their disagreement, all students can create mathematical meaning through their own thinking and reasoning.

The research I reviewed relates to my study because my students worked with partners or in small groups, just as they did in Yackel, Cobb, and Wood's (1993) study. Discrepancies in thinking often occurred between partners. My students were encouraged to reason through their conflicting views and share conflicting views during homework presentations. Unlike Wood's (1999) study of second graders, my focus group was eighth grade students. However, as in both studies, my students were actively involved in their own and others' learning.

Conclusion

Although each of the research studies analyzed had different purposes and unique findings, each study provided valuable information that can be used by mathematics teachers interested in implementing oral and written communication into their classrooms. It is evident from these studies that articulating their own thinking, whether in writing or verbally, was challenging and empowering for students. In reviewing Clark, Waywood & Stephens's (1993) research on mathematical communication through journal writing in grade seven through 11, the study reiterated the link between communication and learning. "Classroom environments that place communication demands on students can facilitate the construction and sharing of mathematical meaning and promote student reflection on the nature of the mathematical meanings they are required to communicate" (Clarke, Waywood, & Stephens, 1993, p. 235). Requiring students to communicate their thinking either orally or in writing, to justify and reflect on their mathematical ideas, and to reason and argue discrepancies or conflicting views, allowed

students multiple opportunities to construct their own mathematical meaning. Deeper conceptual mathematic understanding should occur by collaborating on mathematics ideas and presenting mathematics solutions. As an old proverb states, “The one who teaches learns twice.” In my study, students collaborated on ideas and constructed meaning together through homework presentations and journal writing.

Purpose Statement

The purpose of my study was to investigate how oral and written mathematical communication impacts conceptual understanding and students’ attitudes. Additionally, my research assessed how listening to student presentations, discussions and reading students’ journals affected my teaching. Data collection took place during the spring semester, 2008 in my eighth grade mathematics classroom. This study attempted to answer the following research questions:

- What will happen to student conceptual understanding of mathematics when students are challenged to orally communicate their reasoning?
- What will happen to student conceptual understanding of mathematics when students are challenged to communicate their reasoning and thinking in writing?
- What will happen to student attitudes toward mathematics when students are challenged to communicate their reasoning both orally and in writing?
- How does challenging my students to communicate mathematics orally and in writing affect my teaching?

Method

A variety of data collection methods were used to examine my research questions. Each research question was analyzed, using three pieces of data collection for triangulation of evidence. For my first research question, individual interviews, quizzes, and assessment of homework presentations were used to analyze what happened to student conceptual understanding of mathematics when students were challenged to orally communicate their reasoning. Two students were interviewed every other week and expected to communicate orally their reasoning on open-ended problems (see Appendix A for interview questions). In these interviews, I chose problems similar to those presented in class. I could then assess students' level of understanding, as well as analyze if the homework presentations aided in concept mastery. Student understanding of the math topics was determined by oral presentations, discussions, and bimonthly quizzes. Finally, I formally assessed students' oral presentations of homework assignments with a rubric (Appendix B). Each student was also required to present weekly.

Math journals, quizzes, and student interviews were used to analyze students' conceptual understanding of mathematics when challenged to communicate their reasoning and thinking in writing. Originally, students were to journal solutions to open-ended math problems over the topics presented and discussed in class weekly. Jounaling a different problem every other week resulted, as weekly became too frequent. Grading weekly journals was too time consuming and the quality was improved when more than one week was used to draft and rewrite them. I formally assessed these journals with a rubric (Appendix C). I also used bimonthly quizzes to assess students' understanding of math topics that we had journaled on. The quizzes were open-ended test items; similar to the problems that the students had previously journaled where they

had to briefly explain their thinking. Finally, I interviewed two students bimonthly (see Appendix A for interview questions). At first, it seemed unnatural to sit and watch a student write a solution to a problem similar to those previously journaled about. Soon it became apparent that the discussion we had after the student finished writing was powerful. By reading what the student had written during the interview together, not only could I assess the student's level of understanding, but also ask questions to probe the student's thinking.

The third research question was aimed at learning what would happen to students' attitudes toward mathematics when they are challenged to communicate their reasoning both orally and in writing. I evaluated this by giving a survey, group interviews, and recording observations in a personal journal. Student attitudes were assessed with a pre- and post-survey (Appendix D). Groups of three students were interviewed at the beginning and the end of my research over how their attitudes towards mathematics had changed over the research period (Appendix E). I also recorded anecdotal notes in my personal journal almost daily in regards to student behavior and attitudes.

Initially, the personal journal seemed tedious and time consuming. However, my journal eventually became the most vital tool in the data collection. I was fortunate to have a twenty minute break each day after my eighth grade math class to make quick notes about what went on in class each day. I recorded these daily reflections in a notebook. On the weekend, I used these notes and the questions outlined in Appendix F to write a weekly journal/reflection. These weekly journals became a powerful tool not only for reflection, but also as a teaching tool in which I made instructional decisions for the next week.

There were several obstacles in my data collection. It was very difficult to "fit" everything in. Spring is such a busy time of year, and it seemed students were out of the

classroom more than they were in it. At one point during data collection, in twenty days of teaching my students and I were in class together only eight days. However, the biggest obstacle was organizing the data I collected. I organized all paperwork in a file crate. At the end of each interview, I tried to immediately transcribe the interviews. I wrote in a daily journal. I graded quizzes and journals as quickly as possible and recorded grades. However, it seemed like I was always behind in my data collection. I think this happened in part because my research was too broad. Focusing on oral and written communication, students' attitudes and understanding seemed like too much. It seemed my research revealed more breadth of many topics, rather than depth of one. However, the data I did collect was useful, interesting, and intriguing overall.

Findings

At Garden County Junior High math classes are grouped homogeneously. I teach the average to above average students. My eighth grade math class of thirteen students lasts fifty minutes, five days a week. My classroom is set up in hexagon shaped pods. Each pod is equipped with a pencil holder of utensils, such as dry erase markers, erasers, pencils, etc. There is also a small dry erase board for each student at the pod. Typically, class begins with a problem of the day that is displayed on my Smart Board as students enter the room. An example would be:

> *A multiple-choice test has 5 questions, each with 5 possible answers. Find the probability of guessing the correct answers to all of the questions.*

The students work on the problem of the day for about five minutes, and then one student or sometimes a group of students share their solution. In some cases, students show how they derived their solution. On Fridays, class begins with a math vocabulary quiz: ten words are drawn from the vocabulary jar, and students define the words in writing and/or with diagrams and examples. Occasionally instead of a problem of the day, students play an interactive math

game on the Smart Board or take a quiz with the Qwizdom remotes. Fortunately, my classroom is rich with technology.

When the warm up is over, I usually answer any questions from the previous night's homework. Then, homework presentations are presented. Typically, two people present solutions each day. The students can use any of the technology in the room to present. Generally, they use the ELMO and show the work they prepared for their presentation. Sometimes students use the Smart Board to share work they did on the computer, such as graphs made on Excel. Homework presentations generate a lot of discussion and usually take longer than the intended fifteen minutes.

Next, I present the day's lesson. Since students are generally only attentive for one minute times their age, I try to limit my "lecture" to fifteen minutes or less. Often times during my "lecture," students work problems on dry erase boards, so that I can check for understanding. If time permits, I have students share these solutions. Sometimes I simply put students' dry erase boards on the ELMO to share correct solutions. I usually assign homework at the end of each "lecture." Homework assignments are generally ten to fifteen problems involving both practice problems over the day's lesson, as well on open-ended problem solving types problems that they present solution to the next day. Students sign up to present the next day. Once a week, students are expected to explain their thinking and solution to an open-ended problem in writing in their math journal. The last ten minutes of class are usually student work time, when I walk around and answer questions and check for understanding. During this work time, it is normal for students to work together. I encourage collaboration and discussion. In fact, each week students have a different "problem solving partner" that they sit with and are expected to work with during math class.

An “average” day in my classroom emphasizes the point that collaboration and communication are and have always been important to the community of my classroom. Although I have always encouraged discussion, debate and collaboration in my math classes, I had never considered how it impacted student understanding and attitudes about mathematics. This is how and why my research questions and inquiry evolved.

Even with our limited research window of three months, I believe I discovered many worthwhile findings that will definitely impact my future teaching. Overall, my research concluded that students’ understanding was impacted by increased communication. In most cases, levels of understanding were deepened through oral and written communication, but no data collection guaranteed mastery or true conceptual understanding. In terms of attitudes, the data suggested that students enjoyed math more because of increased oral and written communication. The data also concluded that students realized homework alone is not the best indicator of their understanding. I will confirm these finding with evidence for each research question.

Oral Communication

An obvious finding in terms of oral communication was if students completed the homework problems that were to be presented in class prior to the presentations, a richer class discussion was generated. Engaging discussion aided in deepening student understanding as students discovered their own mistakes as well as others.

Evidence from group interviews supported this finding. One of my group interview questions was, “Will it be easy or difficult to understand other students’ explanations during homework presentations?” Students gave these responses:

- *"Because everyone thinks differently, I think everyone needs to work the problem first so we can understand the presentation."* (Chad[1])

- *"I think it will be easy to understand other's explanations only if I've already worked the problem."* (Jill)

- *"I think the presentation will only help me understand if I can follow along with what they are doing."* (Amy)

Another one of my group interview questions was, "Do you think communicating mathematics orally will increase your understanding of math topics?" All students answered "yes." I then asked them to explain why they said "yes", and these are a few of the responses:

- *"When I work the problems myself, I don't always know if I'm right, but when I listen to the presentations, I can figure out what I did right or wrong."* (Jason)

- *"I learn more from mistakes. The presentations point out where I went wrong."* (Evan)

- *"If I can talk through a problem, I know I know what I'm doing."* (Brian)

- *"To me, finding the mistakes of others is learning. I can only find the mistakes if I've already done the problems too."* (Wendy)

I found it interesting that even though this question addressed communicating mathematics orally, the students perceive communicating orally as listening and responding to the presentations, as well as presenting. All responses indicated increased understanding of math concepts.

I had also written about two incidents in my personal journal, which made me realize the power of all students working the problems prior to the presentations. Both entries commented on students' discussion during math class.

> *Also, I'm wondering what the discussion would have been like today if the students hadn't already done the problems themselves. Meaning…would the students have been able to point out Chad's error and make suggestions, had they not already worked the*

[1] All names are pseudonyms.

> *problem? The students' "background knowledge" of this problem definitely generated a richer discussion (Personal Journal, January 31, 2008).*

It is obvious from this entry that I noticed a change in the class discussion when students had previously worked the problems. Evidence of this finding was also apparent in a second journal entry.

> *Three students presented today. All had correct answers, but after Evan's presentation, Amy asked if he could share how he approached the problem differently. Many students better understood Amy's method. How exciting!! Would the students have realized there were many methods to solving this problem if they hadn't worked the problems previously and then listened to the presentations (Personal Journal, February 6, 2008)?*

Likewise, my teacher observations and notes confirmed that classroom discussion was much richer when students had previously worked the problems.

I also found that when challenging students to orally communicate their reasoning through homework presentations and class discussions, the students were introduced to more than one way to solve mathematics problems. Students identified which method made sense to them, which lead to a deeper understanding of mathematics.

The idea that students were familiarized with more than one way to solve problems was supported by this journal entry about a student wanting to demonstrate how he solved a problem differently than what was presented.

> *It is so fun to sit back and listen to the conversation generated by the presentations. On Wednesday, Brian, Evan, and Jill presented on unit rates and ratios. Again, very interesting. All had correct answers. After Evan presented, Amy wanted to share how he approached the problem differently. How awesome – a volunteer to present!!! At first, many students did not understand Amy's thinking, but as I encouraged her to keep talking her way through it and suggested drawing a picture, most students "got it." It was way cool to hear "oh yeah" and "now I get it, and that makes so much more sense" (Personal Journal, February 8, 2008).*

Although it took a great deal explanation and processing, students learned more than one way to solve the presented problem and in some cases the new ideas made much more sense.

Evidence from student interviews also supported that students learned more than one method to solving problems. I asked students, "How has doing homework presentations and listening to them improved your mathematics understanding?" Students commented as follows:

- *"Usually I don't "get it" until I listen to classmates solving the problem, then its like "ah-ha." My classmates' ideas always make more sense than mine."* (Chad)
- *"During the presentations, I see more ways to figure out problems. For example, I always guess and check to find the answers and never think of solving problems algebraically until someone shares that during presentations."* (Jason)
- *"Seeing a wide variety of ways to solve problems makes me think deeper. I wouldn't push myself to think that way without hearing others' ideas."* (Amy)

Often times I had my students complete a journal for homework and write a second draft after listening to the presentations. Below is an example of a student's first journal, and then his second entry after the presentation. For homework, I had students complete a journal on this problem: *"A survey indicated that 8 out of 9 doctors used brand X aspirin. If 2700 doctors were surveyed, how many used brand X? Explain*." Then in class we discussed and presented different ways to solve this problem.

This is an example of a Jason's first journal:

> *To solve this, I just noticed that 9 times 300 equals 2700, 8 times 300 is 2400. The answer is 2400.*

At the end of class, I asked the students to write a second journal solving this same problem, including methods they learned from the presentations and discussion.

This is the same student's second journal:

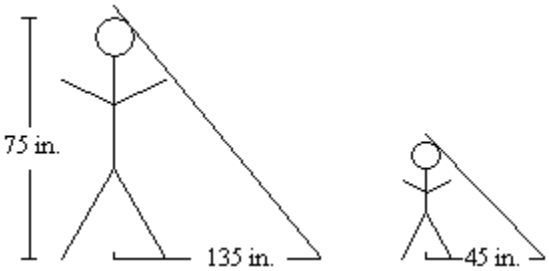
> *I learned several ways to solve this problem. In this problem, 9 doctors were surveyed originally and now they're going to survey 2700 total doctors, so I know both of these numbers are representing the whole. 8 is the part of doctors that like Brand X, so I have two ratios, 8/9 and I don't know/2700. Besides finding a common multiplier like I did in my first journal, I could set the ratios equal to each other and solve for X. Like this:*
> *8/9 = X/2700*
> *8*2700 = 9X*

21600 = 9X
2400 = X
I could also think about simplifying the ratios. 8/9 is in simplest form, so I thought about what I could divide 2700 by to get 9, and came up with 300. So then I thought what number can be divided by 300 to get 8. I knew 2400/300 was 8. Also, to check to see if this was right, I could change both ratios to decimals and see if they are the same. 8/9 = 0.888888889 and 2400/2700 = 0.888888889. Both decimals are .8 repeating. These are all the ways I can solve this problem.

In comparing both journal entries, it is obvious that the student's level of understanding deepened. In the second journal entry, the students demonstrated several methods to solving this problem.

Unfortunately, evidence concluded that challenging students to orally communicate their reasoning through homework presentations and class discussions did not guarantee conceptual understanding of mathematics. At times, students just regurgitated what they had learned from presentations with no conceptual understanding. This was very obvious during student interviews. I had students solve the following problem in individual interviews:

At the same time of day, a woman who is 75 inches tall casts a 135-inch shadow and her daughter casts a 45-inch shadow. What is the daughter's height?



All students answered the problem correctly, and solved the problem with a proportion. I then asked them these questions: "Why did you use a proportion?" "What is a proportion?" "Should there be an equal sign in your proportion?" These are some of the students' answers:

- *"I used a proportion because that is the way that it is easiest to look at problems comparing two things with something missing. I know this because the formula says to do it."* (Addison)

- *"I set up a proportion because that is what we've been doing in class."* (Jen)
- *"A proportion is a way to find x."* (Pat)
- *"I set up a proportion because that would show me what I needed to multiply and divide the numbers by."* (Jill)

It was obvious from these responses that the majority students had no conceptual understanding of proportions; rather they just know how to solve them.

This finding is also supported in a journal I wrote towards the beginning of my research.

> *The interviews were frustrating, but very educational. I asked the students to solve a problem that involved figuring the height of a girl using the given information of height and the shadow length. This unit has been on ratios and proportion, so I obviously thought they'd set up a proportion and solve for the girls height. This is exactly what each student did; all students getting the correct answer. Here lies my frustration…not one student could tell me "why" they set up a proportion. Not one student put an equal sign between their ratios. Obviously, my students have memorized a method to setting up and solving proportions, but with no true conceptual understanding. My research question asks: what will happen to student conceptual understanding of mathematics when students are challenged to orally communicate their reasoning? Now, by no means, do I expect the presentation alone to increase conceptual understanding. But, it is obvious from these interviews that my students are mastering a method/algorithm, but not mastering the "why" (Personal Journal, February 19, 2008).*

My frustration with the fact that students could not explain "why" they solved the problem the way they did was obvious in this journal entry. Again, it is apparent students memorized a method, clearly with little conceptual understanding.

Overall, my findings on oral communication and its impact on understanding were positive. Engaging discussions aided in deepening students understanding during homework presentations when students previously worked the problems. Class discussion and homework presentations introduced students to more than one method to solving mathematics problems. However, my analysis revealed that homework presentations and class discussions did not guarantee conceptual understanding of mathematics.

Written Communication

During this research, I quickly realized that oral and written communication intertwined and benefited one another. I found that requiring students to justify their solution in writing prior to presenting their solution better prepared them for their presentation and developed a clearer understanding of mathematics topics.

This was strongly supported during student interviews. I asked students the following question during individual interviews: "Would it better prepare you for your homework presentations if you were required to journal that same problem first?" Eleven out of twelve students said yes. The following are some student responses:

- *"Yes, because we could see if we really know what we're doing."* (Chase)
- *"It would help me understand the problem more and be able to explain it."* (Jason)
- *"Writing organizes my thinking, so I usually do "journal" but not as deep before I present so I know what I am going to talk about."* (Wendy)
- *"I think my presentation would be more organized if I had to write a journal first."* (Ross)
- *"If you can write about your thinking, you have to know what you're doing, so that would probably help my presentations."* (Brian)

These student quotations testified that students saw the value in justifying their solution in writing prior to presenting their solutions. Students understood that written communication of mathematics ideas organized their thinking and clarified their understanding.

Often times my students presented solutions to problems they had solved in their written journals. In this entry from my personal journal, I described a positive experience with journal presentations.

> *On Thursday, students presented their journal writing. Overall the presentations were well done, some of the best yet. Most students scored higher on the scoring rubric with*

> *this presentation. It was also interesting that many students thought it was much easier to present after they had written the journal because writing "made them be more organized and think deeper." I plan to explore this further. Should I be making them journal every problem that they present (Personal Journal, February 8, 2008)?*

This entry concluded that students' presentation quality was much improved when they presented topics they had previously written about.

Eventually, I had each student present twice. Their first presentation was over a homework problem and their second presentation was over a problem they had journaled. The first time I did this, ten of 13 students scored higher on their journal presentation. Two out of 13 scored the same on both presentations. One student scored lower on his journal presentation. This indicates that writing out solutions before presenting them provided better presentations, which led to a deeper understanding.

I also found that requiring students to write a second draft to journal assignments, as they reflected on their peers' presentation and/or teacher's comments and probing questions from their first draft, forced students to think deeper and better understand. This was supported in student interviews. I interviewed students, asking them, "Does having to respond back to my comments and questions in your journal on a second draft make you learn more and have a deeper understanding?

- *"Yes, your (the teacher's) questions tell me where and what I need to think more about."* (Addison)
- "*Yes…like…it makes me think of different and better ways to solve the problem and how to explain it. The second draft is on why we solve the problem that way and how we know that is correct."* (Wendy)
- "*The questions force you to think deeper because you know the teacher won't give up on probing until she gets what she wants, but in the end I know this make me learn more*." (Chad)
- "*The probing questions make me work harder and want to find the right answer and explain it in the best way. It's kind of like a mystery you just have to solve.*" (Jason)

- *"When I have to write a second response, I have to think more, and then I usually really get it."* (Pat)

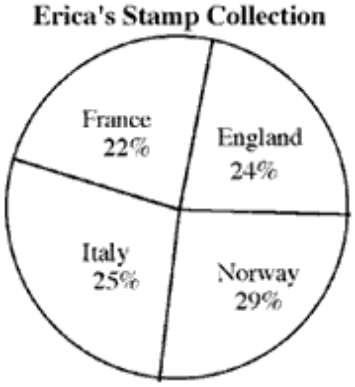
It was apparent from these responses that students believed writing second drafts to their mathematics solutions did make them think harder and understand more, deepening their understanding.

At times, I had my students write a second draft to their journaled solutions after they had listened to a presentation of the solution. In many cases, students' understanding deepened as a result of listening to presentations and rewriting their solution. I clarified this in my personal journal.

> *Jason and Wendy's journals were fantastic. Wendy even sectioned her journal entry into "how" and "why." I chose to have these students share/present their journal entry to the class on Tuesday. I heard many "ahahs." I decided then to have every student rewrite their journal entry; hoping that they could include what they learned from the girls' presentations in their final draft. The students' final drafts were much, much better and showed deeper understanding. This tells me that a one time shot at journaling a topic is not enough, especially if conceptual understanding is the goal. When journaling, students need a second chance to revise and think deeper. I also think if I write questions on their first draft that probes their thinking; the students will think deeper, try harder and learn more (Personal Journal, March 14, 2008).*

This entry concluded that it was necessary for students to revisit their journaled solutions and rewrite solutions which better clarified their level of understanding.

Student journals definitely supported this finding as well. I have included a first response, my probing questions and a second response to one of our journal assignments from two different students. Although this problem is simple, the growth in understanding from the first entry to the second was very obvious.



One of the journal problems read: Erica has a collection of 400 stamps. The graph below shows that percentage of stamps she has from each country. How many more stamps are there from Norway than from England?

In the student sample below, the student clearly solved the problem correctly in her first solution. However, there was a lack of evidence of conceptual understanding, so I asked her to answer the following questions in her second response:

- Why did you set up proportions?
- I'm not sure I understand how you used a proportion to solve this problem. Can you go into more detail?
- Are you confident your answer is reasonable?
- How could you make this a better representation of your understanding?

Wendy's first journal response:

> *There are 20 more stamps. I set up proportions. After I found out how many stamps Norway and England had, I subtracted them to find the difference. Norway = 116 stamps. England = 96 stamps. 116-96 = 20*

Wendy's second journal response:

> *First, I set up a proportion. X over 400 because that is the number of England stamps over all the stamps in her collection. That ratio is equal to the ratio 24% out of 100%. When you solve that, it comes out to be 96 stamps in her collection that are from England.*
>
> *Then, I did the same thing with Norway's stamps, but used 29% over 100 instead of 24%. That turned out to be 116 stamps of the 400 in her collection. I knew both these were reasonable because they are around 25%, and 25% of 400 is 100 and both of my numbers are in that area.*
>
> *After that, I took 116 stamps (Norway) and subtracted 96 stamps (England). I got 20 stamps. That is how many more Norway stamps there are in her collection than England stamps.*
>
> *In conclusion, there are 20 more Norway stamps than England stamps in Erica's collection.*
>
> .

Obviously, the second journal entry gave a clearer picture of the student's understanding and demonstrated much improvement from the first entry.

Another student also calculated the correct answer to this problem, but his notation was somewhat unclear, as was his justification. I probed his thinking with the following questions:

- Are your calculations proportions? If, so what is wrong with your set up?
- If they are proportions, I'm not sure I understand how you used proportion to solve this problem. Can you go into more detail?
- How could you make this a better representation of your understanding?

Pat's first journal response:

> *There are 20 more stamps than England. Here's why…*
> *24/100 96/400*
> *29/100 116/400*
> *116-20 = 20*

Pat's second journal response:

> *The 1st thing I identified was the graphs were percents, so I knew all of the % were out of 100. I also knew the total amount of stamps was 400. So 100% of the collection was 400 stamps. So the 1st collection I did was Norway which was 29% or 29 over 100 equal to X over 400. X is the amount of stamps from Norway. The reason I put an equal sign between them is because they are 2 equal proportions. So I did 29 times 400 divided by 100, which equaled 116 stamps from Norway. After that, I did England's. I put X over 400 again equal to 24 over 100. Now I did the math again which is 24 times 400 divided by 100, which equaled 96 stamps for England. So to find how many more were in Norway than England, I minused Norway from England which was 20 more stamps in Norway than England.*
>
> $$(X/400 = 29/100) – (X/400 = 24/100)$$
> $$116 – 96 = 20$$

In both of these situations, I had a better understanding of the student's level of understanding after reading their second response. Requiring students to write a second draft to their journal assignment while using teacher's questions to probe deeper thinking created a clearer picture of the students' understanding. This requirement also forced the students to think more deeply and conceptually to understand.

Overall, my findings on how written communication impacted conceptual understanding were very positive. Requiring students to journal mathematic solutions prior to presenting their

solution better prepared them for their presentation and deepened their level of understanding. Also when students were required to write a second draft to written mathematics solutions, students were probed to think deeper and better understand.

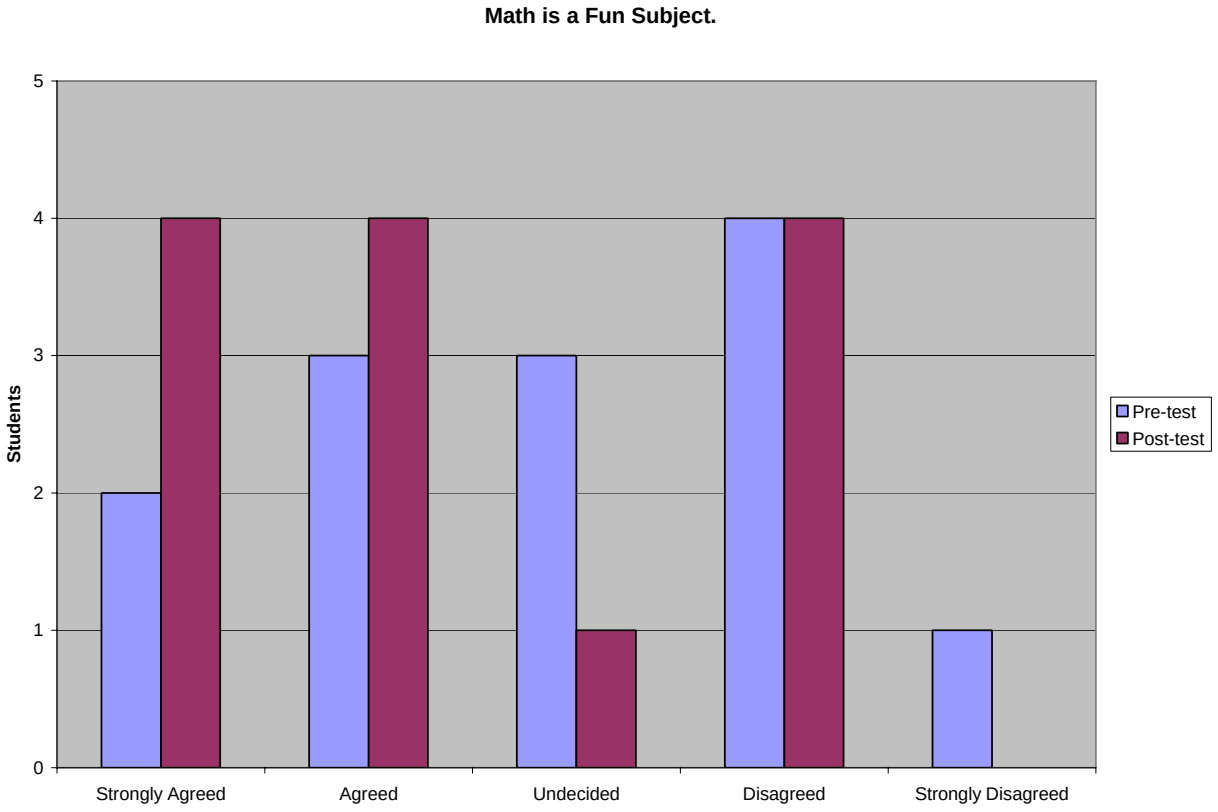
Student Attitudes

Students found math class more enjoyable when oral and written communication was incorporated on a daily basis. My first finding was when students were challenged to communicate their reasoning both orally and in writing, students enjoyed math more and thought math was fun. Student interviews provided evidence for this finding. I asked students during individual interviews if they enjoyed having to justify their thinking orally and in writing. Ten out of 13 said yes. Below are some of their responses:

- *"I really enjoy the discussions we have in math class during the presentations. I learn so much more from them."* (Brian)
- *"I probably shouldn't admit that I enjoy math more because we'll have to do more presentations and journals, but I actually do like it more."* (Chase)
- *"Math is way more fun, especially our debates over right/wrong answers."* (Amy)
- *"Even though it is more work, I know I'm learning more, so that makes it exciting."* (Chad)

From the student comments, it was evident that students preferred mathematics class when homework presentations and journals were incorporated.

In the attitude survey, students had to rate the statement, "Math is a fun subject." In the pre-survey two students strongly agreed; three agreed; three were undecided; four disagreed; and one student strongly disagreed. In the post-survey, four students strongly agreed; four agreed, one was undecided, and four disagreed (Appendix G). This data was best displayed in a double bar graph.



The graph shows that in the pre-survey five students agreed that math was fun, while in the post survey eight students agreed math was fun. Over the course of this research project, more students enjoyed mathematics class.

I also gave a mid-survey and had the students answer the question, "Is math class more enjoyable since we've incorporated more oral and written communication?" All but two students answered yes. One student answered no. One student answered kind of. Below are some of their reasons why:

- *"It is more fun because we get to talk about math."* (Chad)
- *"I don't get so stressed about homework because I know I'll learn more from the presentations, so I actually learn something."* (Jill)
- *"It is a more enjoyable because I get a clearer understanding of the problems we are working on rather than being lost."* (Jason)

- *"It is more fun because you get to explain what you're thinking and how others are thinking."* (Amy)

It is obvious from these student responses that students enjoyed math not only because they like presenting their solutions, but also because they believed they were actually learning with clearer understanding.

My personal journal referenced observations of students enjoying math more. This particular entry explained my thoughts and observations after analyzing a survey.

> *I gave a mid-survey this week. One of the questions was "is math class more enjoyable since we've incorporated more oral and written communication?" Most students responded "yes." Some commented that they feel they are getting a clearer understanding. One student commented that she doesn't get so stressed out about homework because she knows she'll learn more from the presentations and actually learn something. One student commented that he enjoyed listen to others and not Mrs. Cotton all the time. Another student commented that it is really interested to hear how others solved problems. I guess although I'm frustrated that the level of my students' understanding hasn't changed a lot, I'm excited that my students are enjoying the changes we've made in instruction, homework and delivery. I would assume if they enjoy coming to class and have positive attitudes, then only positives outcomes can occur with their learning of mathematics (Personal Journal, March 7, 2008).*

This entry concluded that my students enjoyed the changes made in instruction, homework and delivery.

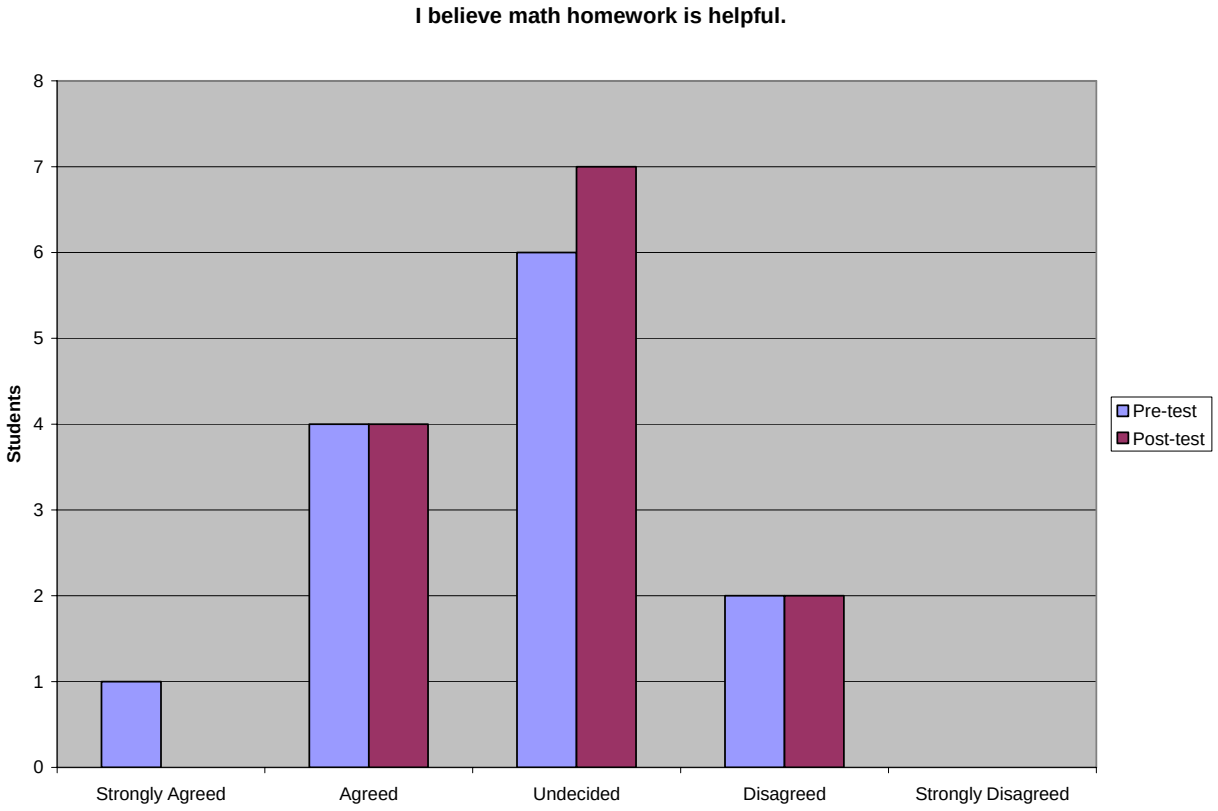
A second finding in terms of students' attitudes was that students now understand that homework alone is not the best indicator of their understanding. My students usually hand their homework in after presentations and discussion. I asked students this question during individual interviews, "Do you think it makes any sense to grade your homework after you listen to presentations? Are the grades an accurate picture of your learning and understanding?" Here are some of their responses:

- "*No, the grade is really the grade for the group, not individually."* (Misty)
- *"The grade is just if we paid attention, not an indicator of our understanding."* (Evan)

- *"Not really, most of the time I copy down the answers I didn't get right."* (Chase)
- *"I think we should get points for doing the homework, but not a percentage grade."* (Brian)
- *"You should probably just grade the problems we don't discuss or present if you want a grade for us individually. The grades we get on tests and quizzes and on our journals probably describe our performance best."* (Wendy)

It is obvious from these responses that my students question the validity of grading homework after listening to presentations. I believe it is paramount that they recognize that grades are not always a true indicator of their understanding.

Students had to rate the following statement on the attitude pre- and post- survey: "I believe math homework is helpful." On the pre-survey one student strongly agreed; four agreed; six were undecided; two disagreed; none strongly disagreed. The only change on the post-survey was that no one strongly agreed and seven were undecided. I displayed this data in the graph below.



By looking at this data, I concluded that before starting this project more students agreed homework was helpful. The post-survey illustrated that more students were undecided about the helpfulness of homework. From my perspective I believed students began to realize that homework alone was not the only means to understanding mathematics concepts. The students began to figure out that presentations, discussions and journals were other strategies besides homework that supported mathematical understanding.

I also wrote in my personal journal about a revelation I had about grading homework after presentations and my frustrations with grading homework in general.

> *We began class today as usual with homework discussions and presentations. It took many attempts to find students that could present their solutions, primarily because many students didn't have their homework done. This probably shouldn't have been a surprise being that is was Monday. We did eventually have enough brave souls, and class carried on as it usually does. Later in the day, I had time to look over the students' homework and I realized that all students handed in the homework. It dawned on me (and I'm not sure why for the first time) that my students had merely copied down the answers from the presentations. Most of the questions on the homework we discussed, so it was a perfect way for my students to get a 100%. Once again, I'm realizing that grading homework gives me NO indication of student understanding. Grading today's homework basically just indicated if the student was listening or not. Why am I still grading homework? Especially the problems we've presented and discussed? I learn so much more from listening to their interviews, presentations and reading their journal entries. But I do feel students need to practice their math skills and how else can we besides homework? Will students do the homework if I don't grade it? (Personal Journal, March 21, 2008).*

In this entry, my frustrations with grading homework were apparent. Grading homework after students had listened to the presentation gave me no indication of their true understanding, instead just a grade on how well the listened.

In the ending group interviews, I asked students, "How much do you think math homework helps students learn math?" Although many students commented that homework was necessary to practice math skills, nearly all students commented that they learned more from

communicating their thinking and reasoning either orally or in writing. The following are some student comments:

- *"Although I had to do my homework to prepare for the presentations, I learned most from giving presentations and listening to them. I especially knew I understood when I could correct others mistakes during presentations."* (Jason)

- *"I think math homework can help to a certain point. Too much homework just caused me to be overwhelmed and then I learned nothing from it. I learned most from having to prepare a couple problems for presentations or journals than from doing lots of practice problems. I learned a lot from rewriting journals, presentations and discussions."* (Jill)

- *"Group work (presentations) makes me understand better than working individually on homework."* (Chase)

- *"Homework does not help unless you get how to do it. If you get how to do it, then why have to do the homework? I learned mostly from the work we did in class."* (Chad)

It is clear from these student comments that they understand that homework alone is not a true indicator of their learning. Obviously students realized that presentations, journals, discussions and cooperative group work aided in their understanding as well.

In conclusion, my students' attitudes were impacted by our communication rich classroom environment. Students enjoyed math class more, thought math was fun, and realized that homework alone is not the best indicator of their understanding. It is obvious from this evidence that my students became more aware of their mathematics understanding and how to attain it by communicating their thinking and reasoning. As their teacher, I also learned so much more about each student just by listening to their thinking and reasoning.

Teacher Discoveries

As a teacher, I have been empowered by what I have learned about my students from evaluating their homework presentations, reading their math journals, and listening to their student interviews. My classroom truly came "alive" with communication. Assessing students' mathematics understanding through oral and written communication allowed me to better

facilitate individual students' needs. This type of assessment is a far more accurate indicator of students' level of understanding and was a valuable tool for instructional design. The following excerpts from my personal journal explain this discovery and passion.

I wrote at length in my personal journal about how much more I was learning about my students by listening to their presentations and discussions and reading their journals.

> *What was most powerful about grading the presentations was that by listening to the student present, I had a clearer picture of their true understanding. For example, Jill had an incorrect answer to her presentation problem, but by listening to her presentation, I knew she understand how and why to use Pythagorean Theorem; she had just made a mistake with her calculations. If I had only graded her homework as I always do, only focusing on the right answer, Jill would have gotten the question entirely wrong. By using the presentation rubric, this student was advanced in her explanation of using Pythagorean Theorem, just not proficient in her calculations due to minor inaccuracies – not jut WRONG! My research questions focus mostly on if student understanding will increase by communicating their ideas, but I'm thinking that teacher awareness of student understanding will be even more powerful (Personal Journal, February 1, 2008).*

> *On Friday, I gave a quiz over problems similar to those that students had presented and written journals about to see if the communication is promoting understanding. Five of the questions were multiple choice and five were short answer where they had to explain their thinking. Overall all, the student did well. One student in particular did not do well. The nice thing about having students explain their thinking on some of the problems is that it allowed me to know exactly where they went wrong and could easily target their needs. So, the written communication helps me as a teacher better facilitate student needs. I'm beginning to think the knowledge I'm gaining of my students' understanding is profound* (Personal Journal, February 15, 2008).

A powerful piece of my data collection was student interviews. I was shocked at how much I learned about my students' understanding (or lack there of) by just having a one on one conversations with them. Although individual interviews were very time consuming, they were a beneficial assessment piece. This journal excerpt described my reactions after student interviews and my profound observations.

> *The interviews were frustrating, but very educational. I asked the students to solve a problem that involved figuring the height of a girl using the given information of height and shadow length. This unit has been on ratios and proportion, so I obviously thought they'd set up a proportion and solve for the girls height. This is exactly what each student*

> *did; all students getting the correct answer. Here lies my frustration…not one student could tell me "why" they set up a proportion. Obviously, my students had memorized a method to setting up and solving proportions, but with no true conceptual understanding. My research question asks: what will happen to student conceptual understanding of mathematics when students are challenged to orally communicate their reasoning? Now, by no means, do I expect the presentation alone to increase conceptual understanding. But, it is obvious from these interviews that my students are mastering a method/algorithm, but not mastering the "why." I don't blame the presentation for this, but can confidently say that the presentations may not bet deepening the level of understanding. As a teacher, this really bums me out. How can I change my teaching, so the conceptual understanding occurs? How can I convince my students that it is necessary and beneficial to understand the "why" (Personal Journal, February 19, 2008)?*

Assessing student understanding through presentations, journal writing and interviews not only confirmed students' level of understanding, but also allowed me to make instructional decisions. Often times when students were unable to write a second draft to their journal entry, it was a true indicator that they did not understand. In these situations, I needed to intervene and remedy the misunderstandings. I wrote about this in my personal journal as well.

> *I handed back students journals with my comments and questions in them. I tried very hard to ask my students questions that would probe their thinking and deeper their understanding. My students have become masters at describing how to solve the problem. My intention was to get them to continue thinking about why. Most students hate it, but some are seeing that is does make them thinking deeper. As a teacher, it is very interesting to see which students can actually write a second journal that answers my questions and those that can't. This not only confirms their level of understanding, but identifies their needs. In some cases, the students can't answer my questions and then I know that I must intervene and help the student. The most amazing thing about this communication is that I would have never specifically identified individual student needs by merely grading homework. I am a better teacher for my students because of it (Personal Journal, April 1, 2008).*

It is obvious from these excerpts from my personal journals that I am a better teacher because of my awareness of their needs through evaluating their communication. Although I do believe my research did impact my students and their understanding of mathematics, I am confident that this research has allowed me to grow as a teacher so that I can continue to impact my future students as well.

Conclusions

I believe that challenging my students to communicate mathematics, both orally and in writing, deepened my students' understanding of the mathematics. Levels of understanding deepened when a variety of methods were presented and discussed where students could "grab onto" the ideas that best suited them. Deeper understanding occurred when students were probed with questions that made them reflect on their learning and reevaluate their reasoning. This occurred when students were expected to write more than one draft to math journals. By making students aware of their understanding through communicating orally and in writing, students realized that true understanding does not come from merely completing homework problems, but from evaluating and reevaluating their own and other's ideas and reasoning.

These conclusions are consistent with the research done by Wood, Cobb and Yackel's (1991) case study. Teachers realized in this study that they had to create opportunities for learning in which the students expressed their mathematical thinking, as well as listened to others. Just as I revealed in my research, the authors of this study concluded that by listening to other's thinking, students reconstructed their own thinking and deepened their understanding. Teachers in Wood et al.'s study also believed the teachers' understanding of students deepened from listening to their students.

In Wood's (1999) 18-month investigation in which students' disagreement was resolved by argumentation, students were able to experience mathematics as a subject that relies on reasoning for the justification of ideas. Students were expected to follow the thinking and reasoning of others, then voice their disagreements. Challenging one's reasoning was an indication of deeper understanding. This occurred with my students as well. However, I am not certain that the argumentation always resulted in deeper understanding for all students. Often

times, my students commented that they were more confused at the end of the debate. These situations were beneficial to student understanding, but needed to be addressed by the teacher to clarify any misunderstanding. Overall, my research suggested that deeper mathematical understanding occurred when my students collaborated on ideas and constructed meaning through homework presentations and journal writing.

Implications

In considering how I will use the conclusions from my research with regard to future classroom practice, it seems I should listen to my students. In the ending interview, I asked students if they prefer to communicate their mathematical thinking and understanding orally or in writing. Nine students preferred oral communication; three students preferred written communication; and one student preferred both oral and written communication. I then asked students if I should require homework presentations and math journals next year. Seven students commented that I should require both oral and written communication; two students recommended just oral; three suggested just journals; and one student didn't advise using either oral or written communication. I found this very interesting. Over half of the class preferred only oral communication, yet most of the class advised requiring both oral and written communication for next year. I plan to incorporate both oral and written communication again next year. I will reevaluate the frequency of presentations and journal entries as many students commented that it was too much at times. I would also like to continue assessing students with individual interviews. Although time consuming, I've been empowered by how much I learned about my students from interviews.

I strive to do the best by my students. The findings from my research tell me that creating a communication rich environment for my students is what we both need. My students can

construct their own mathematical meaning in a communication rich classroom, which promotes a deeper understanding of mathematics. Most importantly, I can identify students' level of understanding and needs by listening to student presentations and discussions and by reading student journals. I can then tailor instruction to those individual needs.

This research journey has opened my eyes and energized my teaching. Martin Dansky said, "Teaching is useless unless you can learn from your students." I am empowered and excited to continue learning from my students' presentations, discussions and written journals. Both my students and I will learn and benefit from a communication-rich mathematics classroom.

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Appendix A

Math Performance Interviews

Oral Communication

Say to student:

I would like you to work on this problem, saying aloud whatever it is you are thinking as you work through the problem. I especially want to hear you talk about how you decide what to do to solve the problem.

(Chose an appropriate open-ended problem that focuses on similar topics that are being covered in class at time of interview. Score student with presentation rubric. Record my observations in my personal journal).

Written Communication

Say to student:

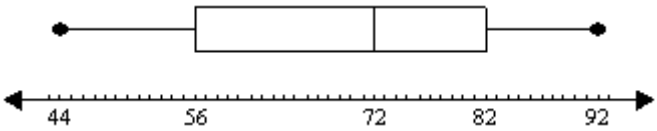
I would like you work on another problem, only this time, explain your thinking and justify your solution in writing. I especially want you to write about how you decide what to do to solve the problem.

(Chose an appropriate open-ended problem that focuses on similar topics that are being covered in class at time of interview. Score student with writing rubric. Record my observations in my personal journal).

Example problems that could be used for math performance interviews:

1. Roger spent $\frac{3}{5}$ of his money on stereo equipment and $\frac{1}{6}$ of the money he had left on lunch. He left the house with $24 and the only money he spent was on stereo equipment and lunch. How much money did Robert return with?
2. A store had run out of backpacks on Monday. The school store received an order of backpacks on Wednesday. During that day, the store sold 22 backpacks. On Thursday, the store sold one third of the backpacks that were left. When the store opened on Friday, there were 112 available. How many backpacks did the order contain?
3. John made a doll house model of his home for his daughter Mindy. The doll house and everything in it are $\frac{1}{12}$ the size of the real house and all of its furnishings. If a doll house model of a painting is 1.5 inches tall, how tall is the actual painting?
4. The holiday season is over. Mega Shop is reducing the price of its video movies from $30 to $18. What is the rate of discount?

5. Philip is a salesperson in a retail store and earns $90 per week plus 15% of his weekly sales. If Philip made $480 one week, what were his sales that week?
6. A square public fountain measures 30 feet on each side. One-foot-square tiles will surround the fountain to make a public walkway that measures 10 feet wide. What is the area to be covered by the tiles?
7. Tori is planning a banquet. She can afford to spend $66.00 for food, $26.50 for gifts, and $17.50 for flowers. She plans to sell the banquet tickets for $17.00 per guest. What is the minimum number of people she can invite and still make a profit?
8. When Lakeesha opens and lays a cereal box out flat, she sees that the top and the bottom of the box both measure 2 in. by 8 in., the sides of the box both measure 2 in. by 12 in., and the front and back of the box both measure 8 in. by 12 in. What is the surface area of the cereal box?
9. Ms. Alison drew a box-and-whisker plot to represent her students' scores on a midterm test. Thom received 56 on the test. Describe how her score compares with those of her classmates.



10. A survey about the music program at a school found that 160 students wanted an after-school music program, 400 students wanted music classes during the day, and 240 students thought that the school should have no music program. Create a circle graph to represent these student responses.
What would be the measure of the central angle for the "in-school" group?
11. A number cube is rolled 450 times and the results are recorded as follows: 73 ones, 78 twos, 67 threes, 76 fours, 75 fives, and 81 sixes. What is the experimental probability of NOT rolling a five?
12. A piglet weighs 9 lb at birth. It gains 1.5 lb per day. At this rate, how old will the piglet be when it reaches double its birth weight?
13. The cost of a school banquet is $70 plus $13 for each person attending. Determine the function rule that describes this situation. What is the cost for 66 people?
14. Three twelfths of the students attending Main Street Junior High are members of the Student Council. If there are 384 students in the school, how many students do NOT belong to the Student Council?
15. One florist can make 3 centerpieces in 50 minutes. The flower shop received an order for 180 centerpieces. How many hours will it take 4 florists to fill the order if all the florists work at the same rate?

Appendix B

Homework Presentation Rubric

	Explanation	**Mathematics**	**Readiness**
Advanced Superior **4**	Accurate response that is communicated clearly; answers questions correctly	Student uses correct mathematical language and symbols	Student is ready to present
Proficient Satisfactory, with Minor Flaws **3**	Main ideas are accurate, with some minor inaccuracies; answers most questions correctly	Student uses correct mathematical language and symbols with minor errors	Student is ready to present
Progressing Nearly Satisfactory, with Serious Flaws **2**	Response has minimal accuracy and explanation is minimal and/or unclear; answers few questions correctly	Student attempts, but does not use mathematical language and/or symbols correctly	Student is not entirely ready to present
Beginning Unsatisfactory **1**	Response is inaccurate and explanation doesn’t cover problem; no questions answered	No use of mathematical language and/or symbols	Student is not ready to present

Appendix C

Problem Solving Journal Rubric

	0	1	2	3	4
Mathematical Knowledge	Shows no understanding of the problem.	Shows very limited understanding of the problem, major computational errors.	Shows understanding of some of the problems mathematical concepts, may contain computational errors.	Shows nearly complete understanding of the problems mathematical concepts and principles, no computation errors.	Shows complete understanding of the problems mathematical concepts and principles, no computation errors.
Communication	Words do not reflect the problem, may include drawings which completely misrepresent the problem.	Has some satisfactory elements, but fails to complete or may omit significant parts of the problem.	Makes significant progress towards completion of problem, but ambiguous and unclear.	Gives a fairly complete response with reasonably clear explanations or descriptions.	Gives a complete response with clear and appropriate diagrams.
Writing	No/little attempt to write a paragraph with 3-5 complete sentences; excessive mechanics and convention errors.	Some complete sentences, but not a 3-5 sentence paragraph; many mechanics and convention errors.	Complete sentences, but not a 3-5 sentence paragraph; some mechanics and convention errors.	3-5 complete sentence paragraph; minor mechanics and convention errors.	More than 5 complete sentences with accurate mechanics and conventions.
Solution	No solution stated.	Solution is not stated, and is incorrect.	Solution is stated, and is incorrect.	Incorrect solution is stated, but is reasonable.	Correct solution is stated.

Appendix D

Student Mathematical Attitude Survey

Date: _____ Name:_____

Please respond to the following items by drawing a circle around the response that most closely represents your opinions right now: Strongly Agree (SA), Agree (A), Undecided (U), Disagree (D), or Strongly Disagree (SD).

1. Mathematics is enjoyable and stimulating to me.

   SA A U D SD

2. Communicating with other students helps me have a better attitude towards mathematics.

   SA A U D SD

3. I am interested and willing to acquire further knowledge of mathematics.

   SA A U D SD

4. The skills I learn in this class will help me in other classes.

   SA A U D SD

5. I learn mathematics well from lectures.

   SA A U D SD

6. I am sure I can learn mathematics.

   SA A U D SD

7. Knowing mathematics will help me earn a living.

   SA A U D SD

8. Math is a worthwhile, necessary subject.

SA A U D SD

9. Math is a fun subject.

SA A U D SD

10. My teachers give me examples of how math is important in life.

SA A U D SD

11. I believe math homework is helpful.

SA A U D SD

12. I believe teachers assign math homework to help us learn better.

SA A U D SD

13. I am good at completing my math homework assignments.

SA A U D SD

14. I would be better at math if I understood how math relates to my life.

SA A U D SD

Appendix E

Group Interview Questions

I will interview groups of three students at the beginning and the end of the research project to determine how their attitudes towards mathematics will change or has changed by being challenged to reason both orally and in writing, and which method they prefer.

Beginning Interview:

- Why do you think teachers assign homework?
- How much do you think doing math homework helps students learn math? Please explain.
- Why do you think I have started having students do homework presentations?
- Why do you think I have started having students justify their solutions in writing?
- Do you think communicating mathematics orally and in writing will increase your understanding of math topics?
- Will it be easy or difficult to understand other students' explanations during homework presentations?
- Do you think you could learn more by listening to your peers solving problems?
- Do you think it would be easier to present a solution if you first explained your solution in a math journal?
- Do you think you'd prefer to communicate your mathematically thinking and understanding orally or in writing? Why?
- Do you have any questions about increasing your mathematical understanding through oral and written communication?

Ending Interview:

- Why do you think teachers assign homework?
- How much do you think doing math homework helps students learn math? Please explain.
- Why do you think I required students do homework presentations?
- Why do you think I required students to justify their solutions in writing?
- Do you think communicating mathematics orally and in writing has increased your understanding of math topics?
- How easy or difficult is it to understand other students' explanations during homework presentations?
- Do you think you learned more by listening to your peers solving problems?
- Do you think it was easier to present a solution if when you first explained your solution in a math journal?
- Do you prefer to communicate your mathematically thinking and understanding orally or in writing? Why?

- As I consider if I will require homework presentations and math journals next year, what advice would you give me?
- Think back to when we first began doing homework presentations and math journals:
  - How have the presentations and journal entries changed throughout the semester?
  - Describe the pros and cons of doing homework presentations, hearing homework presentations, and writing about your thinking and solutions.
- Is there anything you would like to ask me about increasing your mathematical understanding through oral and written communication?

Appendix F

Personal Journal Prompts

I will complete a form weekly for my personal journal.

Kim Cotton
Personal Journal
Date: _____

1. What are two or three observations you made this week regarding students' attitude and responses toward daily presentations and journal writing?

2. What are two or three observations you made this week regarding students' preparedness in presentations and journaling?

3. What are two or three observations you made this week regarding students' attitudes while working in class?

4. How has this week's presentations and journaling influenced your lesson plans (if at all)?

5. What is one question you have after this week?

6. Other comments or observations:

Appendix G

Student Mathematical Attitude Survey Results

This table shows the frequency of student responses to each of the fourteen statements for the pre- and post-survey.

Statement	Strongly Agree		Agree		Undecided		Disagree		Strongly Disagree	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1. Mathematics is enjoyable and stimulating to me.	1	3	7	5	4	5	1	0	0	0
2. Communicating with other students helps me have a better attitude towards mathematics.	5	7	5	4	2	2	1	0	0	0
3. I am interested and willing to acquire further knowledge of mathematics.	3	4	6	7	3	1	1	1	0	0
4. The skills I learn in this class will help me in other classes.	2	5	10	3	1	3	0	0	0	1
5. I learn mathematics well from lectures.	1	1	5	5	3	4	2	1	2	2
6. I am sure I can learn mathematics.	8	7	4	5	1	1	0	0	0	0
7. Knowing mathematics will help me earn a living.	8	8	4	5	1	0	0	0	0	0
8. Math is a worthwhile, necessary subject.	5	8	6	4	1	1	1	0	0	0
9. Math is a fun subject.	2	4	3	4	3	1	6	4	1	0
10. My teachers give me examples of how math is important in life.	1	8	4	1	6	3	2	0	0	1
11. I believe math homework is helpful.	1	3	4	5	6	2	2	3	0	0
12. I believe teachers assign math homework to help us learn better.	3	2	10	9	0	2	0	0	0	0
13. I am good at completing my math homework assignments.	2	4	7	4	2	2	2	2	0	1
14. I would be better at math if I understood how math relates to my life.	5	4	2	5	3	1	3	3	0	0