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Habits-of-Mind Problems,
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Abstract

The focus of my action research is to study the impact of student self-assessment and reflection on my 6th grade math students' abilities to solve deep-thinking math problems, herein referred to as habits-of-mind, or HOM, problems. I investigated the way informal self-assessments and self-reflections impact student learning and motivation. I discovered that students are seldom stimulated to think about their own learning. When they are encouraged to do so, student self-assessments give students ownership of their own learning and provide them with a means for evaluating their growth. Students' assessments of their work also gives teachers a meaningful indication of what they have learned and provide information for improving instruction and for assigning grades. As a result of this research, I plan to focus on a systematic approach to study the impact the reflections and self-assessment devices have on the students' metacognitive skills and their problem-solving abilities using HOM as the vehicle for evaluation.

Problem of Practice

One of my greatest concerns in teaching math has always been how to ascertain the depth of understanding of math skills among all my students. From checking homework and classroom work it often appears that, because all the students' scores are high, they must understand the skills. Or, after a quiz that has examples from each objective, each student passes with high marks and has, seemingly, high levels of understanding. During review sessions for the unit test, students rehearse each skill and have the opportunity to ask questions. It appears that every student is proficient on every objective.

Appearances can be deceiving. Although most students score well on the test, some fail several objectives. Looking at student work on the tests frequently reveals some serious deficiencies. These errors in thinking did not manifest during the review but waited until testing to show. Upon questioning individual students about missed objectives, I almost always find the student had major misconceptions. Their reasons are varied. Sometimes they remark that they just cannot recall which formula to use. Or they say something like: they understood it on their homework, and after giving them just one or two clues, they can remember now, after the test, how to solve the problem. Maybe the students were just afraid to admit not knowing how to complete the problem. And quite often, they did not even recognize that they did not know how to solve the work.

The really gifted students always shine forth. Students who love math always seem to be the most vociferous whether answering questions or asking for clarification. Troublemakers stand out for their own reasons, typically masking their math anxieties or math inadequacies with disruptive behaviors. Teachers usually can identify their math deficiencies right off.

Those students who appear to understand the skills, who appear to be on task, who appear to make the connections, but lack the confidence to risk being called upon are the ones I think of as the truly ‘at risk.’ They may not say anything. They will not appear to be lost. They will not volunteer. They will just sit quietly, hoping that eventually all of this will make sense to them, or, at the very least, hoping that no one will notice they do not understand. They will strive to understand and may even master some skills long enough to do well on the test. But, depth of understanding is not there. True math competency is, for them, a temporary façade. On mid-terms or semester exams or on achievement tests, they will not be able to recall the problem solving skills necessary to be successful. They will continue to be frustrated. They will doubt themselves more and more. In all likelihood, they will quit taking advanced math classes just as soon as they can.

In my ideal classroom I would be able to identify the deficiencies in students’ thinking before they become detriments, before testing. I would have students who are excited about their learning. I want students to be enthusiastic about solving the challenges of difficult mathematical problems.

Problem Statement:

Teachers need a means of student-teacher-student communication which allows them to assess how students are thinking and learning math skills, and how students are assessing themselves. Many studies show that reflective, self-assessment activities provide students with opportunities for exercising metacognitive skills, thinking about their own thinking and learning, and reflecting on how they learn. Students who evaluate their own work are also more motivated to perform well. Requiring students to complete reflective paragraphs and self-assessment activities will provide mechanisms for fostering deeper, long-term understanding.

All mathematics educators would be interested in this research because the NCTM Assessment Standards for School Mathematics (NCTM, 1995) recommend **student self-assessment** as part of a total assessment plan to foster student confidence and independence in learning math. Students who understand how they learn are more confident and more motivated to continue learning. Being able to communicate their knowledge, they also become more adept at problem solving and achieve higher levels of competency. **Communication** is another one of the NCTM Standards for pre-kindergarten through grade twelve. In order for students to organize and consolidate their mathematical thinking through communication, students must consistently justify their solutions. For students to communicate their mathematical thinking coherently and clearly to peers, teachers, and others, they must consistently justify their solutions. Students must look at their own justifications so they will be able to analyze and evaluate the mathematical thinking and strategies of others. As they gain experience in justifying answers, students will use the language of mathematics to express mathematical ideas precisely. If we as mathematics educators want our students to communicate mathematically, it is important that students consistently justify their solutions. Reflective, self-assessment activities are ways in which teachers can ensure student competence, foster confidence, and promote independence in learning math.

Literature Review

This past December, Nebraska Commissioner of Education, Doug Christianson released the annual state report card. Results of 4th, 8th, and 11th grade students from every school in Lincoln as well as many other schools from Nebraska were published in the Lincoln Journal Star. The ‘No Child Left Behind’ Act requires schools to report student achievement in those grades for Reading, Writing, and Mathematics. The stakes are high. Federal funding is both the carrot and the stick for those schools which perform well and for those which do not. Published results impact school reputations and, per force, school morale. To ensure that NCLB goals are reached, school districts often impose their own intermediate assessments. Not only do students take the annual achievement exams; they might now also take Criterion Reference Tests (CRT), Norm Exams, or State Standards Exams, Reading Inventories, State Writing Assessments, Cognitive Tests, Aptitude Tests, or Ability Tests. These are in addition to the weekly Spelling tests, chapter tests in English, Math, Science, and Social Studies. With all of these assessments, it would seem that all students would know exactly how they learn and how well they learn. New forms of assessment must be created that are sensitive to students’ backgrounds, knowledge, and attitudes so that students are motivated to do their best (Blatchford, 1997).

Formal assessments give teachers ‘snapshots’ of students’ abilities. They provide teachers with information that should guide instruction toward more effective means of teaching. Typically, student evaluation is regarded as the last feature of an instructional plan and is conducted by the teacher. Occasionally students are somewhat involved in the evaluation process when they are asked to correct each others’ papers. Seldom is the student stimulated to think about his or her own learning (Block, 1999).

The focus of my action research is to study the impact of student self-assessment and reflection on their abilities to solve deep-thinking math problems, herein referred to as habits-of-mind, or HOM, problems. Ainsworth and Christianson (1998) wrote that it is important to involve students in the assessment process for three key reasons.

- *Student Motivation*: Students who evaluate their own work and the work of others using assessment criteria…are more motivated to perform well.
- *Understanding Assessment Criteria*: …a standard for acceptable performance is established. Students internalize this standard and understand what the resulting grade means.
- *Reinforcement of Criteria*: …students are reviewing content related to the unit focus questions. Students are searching their performance-task projects to verify they have the content required…which continually reinforces their own understanding of the material presented. (p. 39)

Student self-assessments give students ownership of their own learning and provide them with a means for evaluating their growth (Wells, 1998). Students' assessments of their work also give teachers a meaningful indication of what they have learned and provide information for improving instruction and for assigning grades. Reflection and evaluation can encourage understanding of what is expected, improve motivation, and lead to pride in positive achievement.

Self-assessment is found in reflection, which itself has focused on three topics within Kathleen Blake Yancey's (1998) Language Arts classroom:

- The questions we might ask of students.

- The ways we read the responses to the questions and how student perceptions influence our judgment of the quality of their work.
- The need to integrate reflection into the classroom if we expect students to produce reflective texts. (p. 2 of 6)

Within the past two decades, numerous studies have been conducted with regard to the role of prior knowledge and knowledge structures in the performance of complex mathematical tasks. Student self-assessment provides opportunity for exercising metacognitive skills-thinking about their own thinking and leads to reflecting on what they learn and how they learn (Rose, 1999).. Tobias (1992) noted that the way complex problems are perceived allows students to develop representations that trigger appropriate problem solving procedures and solutions in memory.

I intend to have a variety of formats to measure student assessment. Rubrics are scoring devices that have recently received high marks from teachers around the country. Student rubrics used for self-assessment encourage learners to participate in the evaluation process (Rose, 1999). According to Jacobs (2005), student descriptors of classes that fostered achievement were:

- **Fun** (intellectual challenge and advanced studies).
- Student **discussion** as part of instruction.
- Class assignments were real or **relevant**.
- **Process** of learning equally important to final **product** in assessing achievement. (p. 14)

Kagan and Kagan (1998) wrote about a four-step process for student self-evaluation which can be applied to any content. First, students are involved in defining criteria. Next they are instructed in how to apply the criteria. Students engage in self-evaluation and receive feedback on their self-evaluations. Finally, students use their self-evaluations as a basis for developing action plans for improvement.

Having used HOM problems last year and this year, I have seen how students enjoy the challenge of attempting to solve them. Until this year I focused grading these problems on two parts: the **process**, step by step attempts to solve; and on the final **product/answer**. I began using the HOM problems this year as a means to have students reflect on how they solved the problems. They were asked to explain how they solved the problems. However, I had not really organized their results into any actionable evaluation. My action research project focuses on a more systematic approach to study the impact the reflections and self-assessment devices have on the students' metacognitive skills and their problem-solving abilities.

Purpose Statement

The purpose of my study is to determine the impact of self-assessment/reflection activities on developing problem-solving skills among middle school math students. The following questions have surfaced in thinking about this study:

1) How do teachers know when they have reached all their students? How do they know how well they have reached the students?
2) What tools are available for developing self-assessment activities and student reflection activities? Which tools are appropriate for a middle school math classroom?
3) How often should self-assessment/reflection activities be conducted?

4) When they assess themselves, do students assess themselves against a set standard or against their peers?
5) What is the impact on student motivation?
6) How does the student see him/herself as a math student?

To help me answer these questions I collected five different assessments, aside from regular homework and general observations. All data was collected in my sixth grade classroom during the spring 2006 semester. The first assessment was a Mathematics Attitude Survey (Appendix 1), consisting of 20 questions, which was given once at the beginning and once at the end of this project. Secondly, at least once each week I completed a HOM Rubric (Appendix 2) which I frequently used to discuss with each student or group about how a solution was found. For the third assessment, I asked each of the students to complete a HOM Guide Rubric/Data Sheet/Reflection Form (Appendices 3A and 3B) for most of the HOM problems. To evaluate the students' progress in developing math skills, I required them to complete a 50 minute-timed Math Skills Competency Exam (Appendix 4) once at the beginning and once at the end of the project. Finally, I invited each student to complete a Student Interview Questions form (Appendix 5).

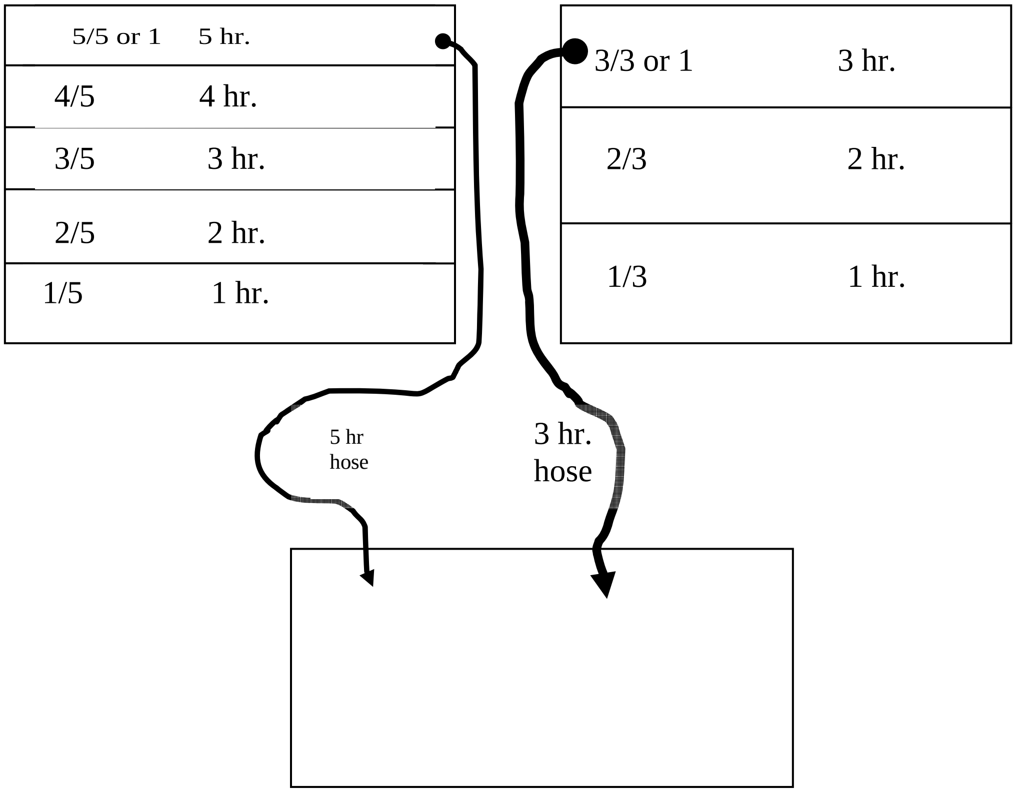
Analysis

In early February 2006, I began this action research project. In order to see how well I was reaching each of my students, I began assigning a series of self-assessment/self-reflection activities. I provided my students with folders in which to keep their Habits-of-Mind homework, their self-assessments, and their self-reflections. Folders were handed in following each HOM activity given two or three times a week. In the beginning, the self-assessment activities focused on the work each student used to solve, or attempt to solve, the HOM. Less emphasis was placed on the getting the correct solution as was placed on showing how the student got a solution.

The self-reflections were intended for me to catch glimpses into how students were thinking, to have them voice their metacognition. One student wrote that she came to realize she tends to make simple things much harder than necessary. Another student wrote that some problems which appear simple are really difficult. One student was so frustrated with HOM problems that initially he had not handed in any reflection or assessment forms. He could not find a solution to the problems that made sense to him. Rather than hand in an incorrect solution, he chose to not hand in anything at all. His mother told me during Parent-Teacher conferences in late February that her son was feeling overwhelmed by the difficulty of these problems. He felt he was in over his head, but did not want the other students to know he was struggling. I explained to her, and the next day I reminded him, that I was more interested in learning about his thinking processes than if he could solve the problem. I needed to remind him it was just as important to communicate with me his feelings of frustration. Once he began reflecting on his thinking processes, he began to understand himself better, and he began finding ways to improve himself. By the end of the project he made considerable improvements. Several other parents stated at the February Parent-Teacher conferences that they were pleased their children were being challenged in this way.

Giving just the answers to problems provides little insight into a student’s thought processes. The following excerpt from a student’s folder illustrates this point. I assigned a HOM called The Hot Tub Problem. In it students are asked to help Maria fill her hot tub.

Maria wants to fill her hot tub for the party tonight. With one hose it will take 5 hours to fill the tub. With a larger hose, it takes only 3 hours. If she uses both hoses at the same time, how long will it take her? Student #10 provided this solution:



Self-Assessment

“This problem is about filling a hot tub with two hoses. I’m supposed to find how ling it will take to fill the tub with two hoses, one that would fill the tub by itself in five hours and the other in 3 hours.

$$1 \text{ hr} = \frac{1}{5} + \frac{1}{3} = \frac{32}{60} \qquad 2 \text{ hr} = \frac{32}{60} + \frac{32}{60} = \frac{64}{60}$$

$$\left(\frac{1}{3} = \frac{20}{60}, \quad \frac{1}{5} = \frac{12}{60}\right)$$

“So, I know that the answer is less than two hours.

"With the 5 hr. hose it takes 300 min. to fill. It will fill $\frac{1}{300}$ in 1 min. The 3 hr. hose will take $\frac{1}{180}$ in 1 min.

Z = min. to fill $\qquad$ 1Full tub = $(\frac{1}{300}+\frac{1}{180})$ x Z min.

$$1/300 + 1/180 = 8/900$$

(900) x 1 Full tub = $8/900$ x Z x (900)

$$900 = 8Z$$

$$900 \div 8 = 8Z \div 8$$

112.5=Z $\qquad$ "112.5 min = 1 hr. 52.5 min to fill the tub with both hoses.

Self-Reflection

"First I found fractions to represent how much of the tub would be filled in one hour and in one minute. Then I found common denominators for these fractions.

"Next I found an expression to represent the rate that both hoses fill the tub together. After that I solved the equation and got the time it took to fill the tub with both hoses in minutes.

"Finally I converted the minutes to hours. I think my answer makes sense. At first I thought it was easy, 4 hours, I averaged 3 and 5 hours. Then I thought, that doesn't make sense because one of the hoses only takes 3 hours by itself. The time has to be less than 3 hours and my answer does."

There are levels of math comprehension here that I would never have been able to glean without her written explanations. Her problem solving skills were outstanding. Her sketch allowed her to find a unit-rate for each hose. I could see she had a solid understanding of fractions, not only how to add them, but also how they represent time in minutes and hours, and how to manipulate them.

Do HOM problems have a positive impact on student motivation? In early February, after we had discussed and solved our second HOM problem, one of my students wanted to know if we could do more of these kinds of problems. This led to a good class discussion. The class generally felt the daily homework from the text book was too easy. They wanted to know if they could do more HOM. During the course of this project I frequently observed some of my students, while waiting for school to begin, ask their classmates if they had solved the HOMs yet. On February 24, one student wrote on her self-reflection form, “When I got it I thought how easy it was and I thought it was really cool. I was so happy I solved it I screamed!” Once she had solved it, another student wrote, “I felt super-duper relieved. Looking back, I learned to think out of the ordinary.”

Several students admitted that one HOM, called the T-puzzle, was frustrating and that they needed hints from those who had already solved it. They were feeling somewhat guilty because they had “cheated by getting help from others.” This led to some great discussion about how mathematicians really work together. It is called collaboration. I explained to them how during collaboration, often the sum of their collective knowledge is greater than the sum of their individual knowledge. They were able to comprehend how a suggestion from one partner can often steer their own thinking toward the correct path. There were very few times after this discussion when any of the students chose to work by themselves. They understood the power of collaboration.

On March 14, I interviewed 4 students, chosen at random by drawing names from a hat. Each of them said they enjoy doing the HOM problems much more than just doing regular math skills. One student said the HOM helped her become a better problem solver because they “forced her to use new strategies that normal problems never would have needed.” Another one said, “The habits of mind problems use what we are learning in a book and puts it in a real life scenario. I really think that HOM problems increase understanding of mathematical thinking.”

During this semester I was fortunate to be assigned a student teacher. He began to take over teaching duties for most of the regular math assignments in March. However, I reserved 15 minutes from each math class to either assign or to discuss HOM problems. Several students expressed concern that they would not get to do any more ‘fun’ problems. On March 13, I was absent and my student teacher substituted for me. He said he had to look for my HOM notebook because the class had nearly demanded they have a HOM problem to work.

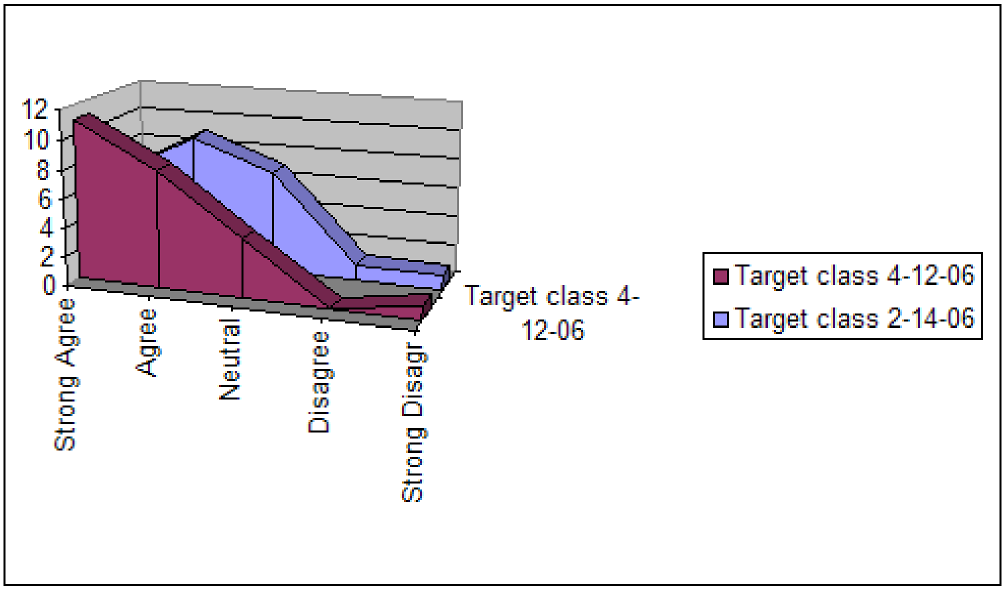
I had my students complete a Mathematics Attitude Survey twice, once in early February, and the other in mid-April of this year. The surveys were identical. A comparison of the two sets of answers/data reveals that most of my students were more positively impacted by the HOM.

In February, 15 of 24 respondents replied positively that they ‘Look forward to math class every day: six strongly agreed, nine agreed, seven responded neutral, and two disagreed. In April, 19 of 24 respondents replied positively that they 'Look forward to math class every day: 11 strongly agreed, eight agreed, four neutral and one disagreed.'

Mathematics Attitude Survey

Query: #1 ***I look forward to math class every day***.

	Strong Agree	Agree	Neutral	Disagree	Strong Disagree
Target class 2-14-06	6	9	7	1	1
Target class 4-12-06	11	8	4	0	1

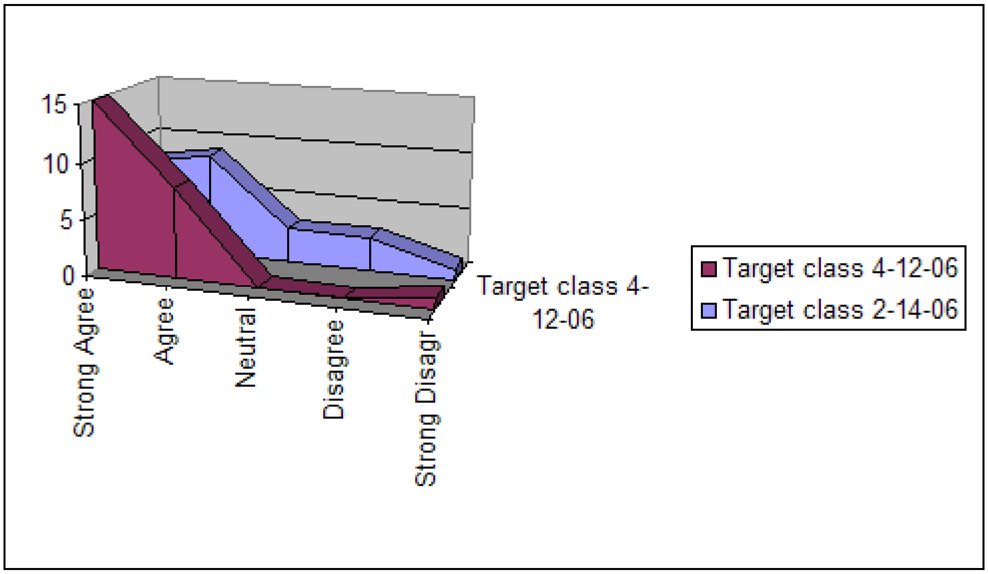


In February, 17 of 24 respondents answered positively that they feel they 'Are getting better at solving math problems: eight strongly agreed, nine agreed, and seven replied either neutral or disagreed. In April, 23 of the 24 responded positively about 'Getting better at solving math problems: 15 strongly agreed, eight agreed, and only one disagreed.

Mathematics Attitude Survey

Query: #8 I ***am getting better at solving math problems.***

	Strong Agree	Agree	Neutral	Disagree	Strong Disagree
Target class 2-14-06	8	9	3	3	1
Target class 4-12-06	15	8	0	0	1

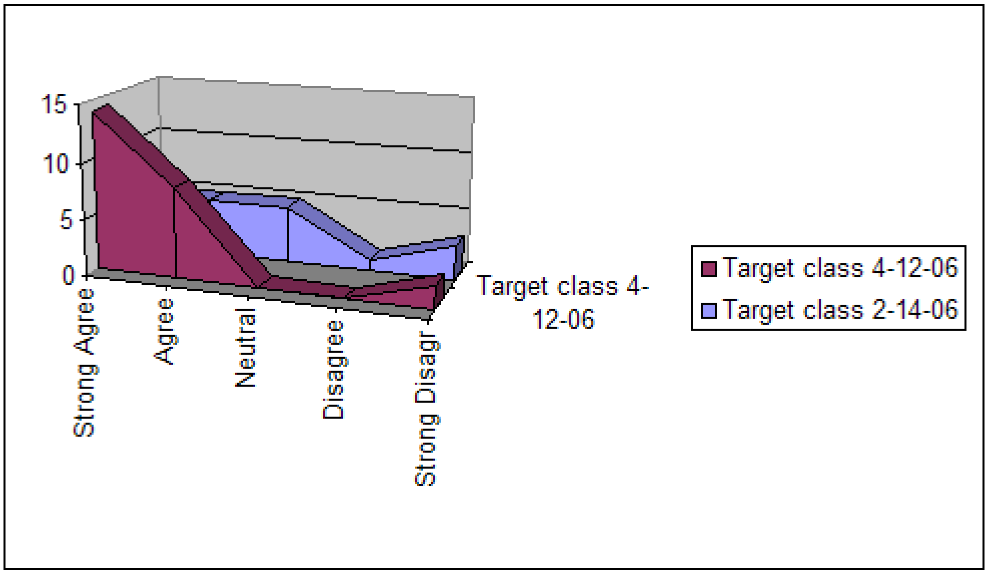


Where only five students in February responded with strongly agree and five agreed that they '<u>Planned to continue taking challenging math classes </u>(five were neutral and four disagreed). April's responses had 15 students strongly agreeing and eight agreeing to '<u>Continue taking challenging math classes</u>. Only two students disagreed and do not plan to take higher level math courses.

Mathematics Attitude Survey

Query: #20 ***I plan to continue taking challenging math classes.***

	Strong Agree	Agree	Neutral	Disagree	Strong Disagree
Target class 2-14-06	5	5	5	1	3
Target class 4-12-06	14	8	0	0	2



The general trend for all 20 questions showed an overwhelming increase in student confidence and motivation. More students felt increased confidence when solving problems. They felt that problem solving was a more realistic way to learn math concepts. I found it interesting that the only question which did not show improvement was question #15, which asked if students learned best when working with others. There were several students who wrote caveats to their answers that said things like one girl's response, "I want to learn it for myself and when I work with friends, sometimes they solve it before I do, and I don't really learn how it was solved. But, I do like getting the chance to ask a friend how to get over a tough spot when I get stuck so usually I like working together best" (See Appendix 6).

Not all of my students were positively impacted. One student in particular had been a negative force in every way possible: failing to complete homework, refusing to participate in groups, denigrating the teacher and student-teacher. In spite of numerous conversations with this student's parents and academic interventions the student had only completed 3 out of 24 HOM assignments. The student's class average dropped from a 91% in early January to a 74% by late April. This student's parents had recently divorced and he was living three days each week with mom and four days with dad and new wife.

My research showed me that when they assess themselves, students assess themselves in multi-faceted ways: against a set of standards, against their peers, and against a preconceived notion of what they believe their teacher wants from them. In the beginning one group of three or four students wanted just to have correct answers. There was seldom any work shown. Having done a few HOM problems, a couple of them began to see the value of writing all of their work down. They learned that being organized on paper had some merit. There was a format for answering HOMs that I required them to follow. Those who followed the format were almost always ready to discuss theirs or post their solutions on the board. Those who did not follow the format were not as comfortable discussing their solutions or their work in front of their peers. One student said he was glad I did not evaluate his answers solely on the basis of being correct or not. He thought he contributed a lot of work to his answer but he made a simple arithmetic error. He was glad that he still received a good score because of the rubrics I used to assess their work. He said he knows now why I want them to write all of their work down, even if it’s incorrect.

No matter what I tried, I had four students who never completely embraced the idea of HOM or of self-reflections. The only times they did any HOM work was when they were able to see the answer in their heads without doing much pencil and paper work. Of the 25 HOM tasks, one of the students only completed 11 tasks, five of the self-assessments, and two of the self-reflections. Another completed most of the tasks but only completed eight of the self-assessments and eight self-reflections. One of those reflections stated simply that doing reflections “was boring.” The other teachers on our team in other subjects responded that these four students were having the same attitudes in Language Arts, Science, and Social Studies classes.

The majority of the rest of the students seemed to feel that their math skills greatly improved. Some said this was their favorite class because it really challenged them to think ‘outside the box.’ One girl wrote that she liked how I made math fun, that I made it like a game where they got to try to beat the teacher.

I selected these seven examples of student work from the final HOM assignment to show the differences in their math logic. The problem was, “If 20% of a number is 12, find 30% of that number.” All the students who completed the task arrived at the same solution. I allowed them to compare notes before making their presentations to the class. There was a general murmur of success because they each had the same answers, but they solved them in such varying ways. I attempted to type them as close to the way they wrote them as possible, complete with misspellings and grammar errors.

Nearly half the class solved this problem similarly to the way student #10 did. Most people who have an understanding of basic algebra would solve it this way, and frankly, it’s the way I expected most students to solve it because we had studied Hands-On-Equations Algebra just 2 months prior to this assignment.

Student #10

Data Sheet

If 20% of a # is 12, then what is 30% of that same #?

20% of x = 12

$$20\% = \frac{20}{100} = 0.2$$

$$0.2\ X = 12$$

$$X = \frac{12.0}{0.2} = \frac{120}{2.0} \text{ so } \begin{array}{r} 60 \\ 2\overline{)120} \end{array}$$

$$X = 60$$

$$30\% = \frac{30}{100} = 0.3$$

30% of 60 = 0.3 x 60 = 18.04

Since 20% of 60 is 12, 10% of 60 is 6. So 12 + 6 = 18 and 10% + 20% = 30%

30% of 60 is 18

Write Up

“This problem asks if 12 is 20% of a number what 30% of that same number is. To solve this problem I had to write an equation and solve it.

First I wrote an equation and solved it for the unknown number. Next I multiplied the unknown number by 30% or 0.3. Finally I found the answer.

The answer is that the unknown number is 60 and 30% of 60 is 18. I know this because I used the definition of percent to write an equation and I solved it two different ways that both got the same answer.”

Student #12

Write Up

“If 20% of a number is 12 what is 30% of the same number?

In this problem I was supposed to find 30% of a number, with given the clue that 20% of the same number is 12.

First I divided 12 by 2, or 20%. Which is 6, then I realized that for every 10% it is equal to 6. After that I, multiplied 6 times 3 or 30%, which is 18. Next, to double check my answer I divided 18 by 3 which is 6. I concluded that once again for every 10% it is equal to 6 and the total would be 60 because 6 times 10 equals 60.

My answer is 18, because 30% of 60 equals 18, also 6 times 3 or 30% is 18.”

Data:

20% = 12

30% = 18 $6\times 10 = 60$

$\begin{array}{r} 6 \\ 2\overline{)12} \end{array}$ $6\times 3 = 18$ $\begin{array}{r} 6 \\ 3\overline{)18} \end{array}$

Student #12 solved this in the same manner that several other students in the class did. She simplified the problem by recognizing that if 20% is 12, 10% must be 6, or half of the original amounts.

The following example is from student #21. She is identified as highly gifted in math. Her solutions always amaze me with the multiple-levels of understanding she displays. Her depth of understanding is just incredible for a 6th grader. I singled out this solution to save in my files because I never have considered using any of these steps to solve this problem.

Student #21

Problem: If 20% of a # is 12, what is 30% of that same number?

1) 60 is the number 2) 42 off of 60

$60 \times 70 = 4200$ $\begin{array}{r} 42 \\ 100\overline{)4200} \end{array}$ 60-42=18 is what 30% of 60 is

Problem Statement

“This problem is about percentages of a certain number. In this case 60. I’m supposed to find what 20% of a number is 12, and what 30% of that number is.”

Work Write Up

“First I found out that 20% is 1/5 of 100%. So I multiplied 12 by 5, and that’s 60. Then I took 60 multiplied by 70 because I made the problem like a shopping sale. 70% off 60, so that’s why I took 60 multiplied by 70, and that’s 4200. Then you do 4200 divided by 100 because of the 100%. And since it’s 70% off then you take 42 off of 60 which is 18. So after I got that I realized another way to solve the problem. It was to take 18 multiply it by 3 because 30% goes into 100% three and a third times. So I did 18 multiplied by three and 1/3 of 18 is 6 so add that to 54 and it’s 60.”

Answer

“My answer for the first part is 60 and for the second question is 18. I think my answer makes sense because 20% is 1/5 for 100% and 12 is 20% of 60. And for the second question makes sense because if 20% of 60 is 12 then 30% shouldn’t be much higher so 18 is pretty close to 12.”

Student #6

If 20 percent equals 12 then what is 30%?

20% =12

100% = 60 (20 x 5 = 100; 12 x 5 = 60)

100% ÷ 10 = 10%

10% =6

10 x 3 6 x 3

30 18 30% = 18

“This problem is about percentages and numbers and we have to do is if 10% equals 5 then what is 20% and find the answer.

To find my answer what I did was: First what I did was I thought if 20% equals 12 than what is 100% and I found out that it is 60. Then with that new data I thought that what would 10% be well 100% by 10 is ten so 60% of 10 would be 6. Next what I did with that was if 10 x 3 is 30%, then 6 x 3 would be 18.

So my answer is 30% = 18. I think it is reasonable because when I was doing my work I found and checked it and it was right each time.”

Student #6 solved it with a thorough understanding of what percents are and how to manipulate them. His solution shows a working knowledge of pre-algebra.

In the following example, even with the errors in student #2’s sketch, she was able to talk her way through it with the write up. I had her orally explain how her sketch connected to her work. That was when she picked up on the errors--it should not be 20%

and 30% under the circled dots. But the fact that she recognized the errors showed me that she just used the sketch as a springboard to get started solving the problem

Student #2

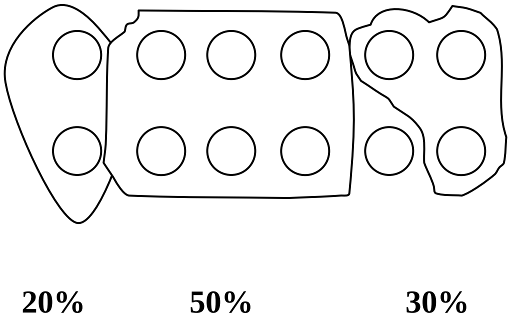
<u>Work Write-Up</u>

“Well, I forgot how to do percentages. My mom helped explain to me. First she gave me many examples and I remembered of means multiply. I first took 12 divided by .20 or 20%. I got 60. I made an equation which is 20% x n = 12. I got n = 60. Then I simply took 30% x 60 because n = 60. My answer is 18.”

<u>Answer</u>

“My answer is 18. I think my answer is correct because 20% of 60 is 12, and 0.30 x 60 = 18. My answer also makes sense because I drew a picture. I used multiplication and division to solve the problem.”

Example A:



Student #14

12 + 12 = 24

20% + 20% = 40% $\cancel{12}, \cancel{13}, \cancel{14}, \cancel{15}, \cancel{16}, \cancel{17}, \boxed{18}, \cancel{19}, \cancel{20}, \cancel{21}, \cancel{22}, \cancel{23}, \cancel{24}$

“1st I figured out what 40% was; 24. Then I found the midpoint between 12 and 24 and got 18 as my answer. I think my answer (18) makes sense because I showed my work, I double checked, and I used the right math.”

I found student #14’s solution to be one of the most unique and simplest ways to solve this problem. It was so simple that when he put his answer on the board for the class, there was an audible groan and several, “Why didn’t I see that? And “Why did I do it the hard way?”

The skills in this next solution showed that student #20 also had a deep understanding of percents and how to use them.

Student #20

“I am supposed to find a number knowing that 20% of that number is 12. Then I had to figure out what 30% of that same number was.

First I figured out that if 12 is 20% of the number, there are 5 20’s in 100. That made me take 5 x 12 which equals 60. So the number was 60. Then I needed to find 30% of 60. I found that 10% of 60 is 6, and I did that because you need to add another 10% on to 20% to make 30%. Finally I added those 2 percentages together and got my answer, 18.”

Had I only required an answer from these students, and not required them to show their work, all of these students would have arrived at the same answer, and all would have been correct. Even if they had only shown their work without adding a written explanation, I would have seen that they each knew how to get the answer and I would have surmised that basically they had all done their work in similar or nearly-similar ways. It was only through reading their explanations, what they entitled ‘Write-Up’, that I was able to see just how diverse were their thoughts and solutions.

Interpretation

The written explanations and reflections have given me more insight into students' thinking and problems solving tactics than any other means I have used before. It escapes me why I never tried to do this before now. I was able to pinpoint what difficulties each student was having and was able to find ways to overcome those obstacles. Even with those students who exhibited little interest in problem solving, I was able to identify the reasons for their lack of interest. Most importantly, I was able to see the many thought processes and problem solving skills my students used.

Student motivation has never been higher. Time and again I had students tell me how much they enjoyed the challenges of the HOM. The survey results bear this out, too (see Appendix 6). One student's survey comments included, "…you really influenced me to work and try me best in my math class and other things that involve math. I have almost always loved math but thanks to you, you have helped me realize I want a career with math." Several parents contacted me through emails, through Parent-to-Staff communiqués, and through phone calls telling me how pleased they were that their child was challenged this year in Math; that they feel their students are well-prepared for the more rigorous pre-Algebra classes next year.

These students have a much higher awareness of their own learning styles. The reflections and self-assessments promoted a routine of analyzing their work, checking the reasonableness of their answers, and adjusting for accuracy. Student assessments fostered ownership of their learning in a way that provided them with a means for evaluating their growth. They know what they learned, and they know how they learned it. Even those students who balked at the HOM problems and reflections learned more about themselves through this project.

Next year I am slated to teach two classes of regular 6th grade math, one of differentiated 6th grade math, and two classes of science. I am excited to be able to better understand my students by using the pre- and post-surveys, the self-assessments, the self-reflections, and the Habits of Mind problems. I anticipate even greater successes in guiding my math students toward higher order thinking skills next year.

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Appendix 1: Mathematics Attitude Survey

Please answer the following questions to the best of your ability to accurately describe your attitudes and opinions regarding Mathematical problem solving. Do **NOT** put your name on the survey. All results will be kept strictly confidential. Thank you for sharing your thoughts with me.

Scale:

- **1 = Strongly Disagree**
- **2 = Disagree**
- **3 = Neutral**
- **4 = Agree**
- **5 = Strongly Agree**

Circle **ONE** number for each question.

1. I look forward to Math class every day.	1	2	3	4	5
2. I like trying to solve new problems in math.	1	2	3	4	5
3. I feel confident when solving problems.	1	2	3	4	5
4. Solving problems is easier when working in groups or pairs.	1	2	3	4	5
5. I keep working at a problem even when I feel frustrated.	1	2	3	4	5
6. There is more than one way to solve a problem.	1	2	3	4	5
7. I enjoy presenting my solution to the class.	1	2	3	4	5
8. I am getting better at solving problems.	1	2	3	4	5
9. My teacher takes time to make sure I understand math concepts.	1	2	3	4	5
10. Most people in my class are better at solving problems than me.	1	2	3	4	5
11. Problem solving is a better way to practice and understand math concepts.	1	2	3	4	5
12. Mathematics is NOT important in everyday lives.	1	2	3	4	5
13. I learn best when working by myself.	1	2	3	4	5
14. I learn best when working with others.	1	2	3	4	5
15. I use different strategies when solving problems.	1	2	3	4	5
16. Learning how to solve problems has helped me in my other classes.	1	2	3	4	5
17. I don’t usually know where to start when solving problems.	1	2	3	4	5
18. I am learning more math than students in other classes.	1	2	3	4	5
19. This level of math is too challenging for me.	1	2	3	4	5
20. I plan to continue taking challenging math classes.	1	2	3	4	5

Appendix 2

Math Self-Assessment Form

Name: Date:

Lesson Title:

Rate yourself based on how well you think you understood today’s lesson. Feel free to add comments below your grade.

A = I WAS AWESOME! I WAS LEADING THE PACK!

B = RUNNING WITH IT!

C = I GOT MOST OF IT.

D = I THOUGHT I GOT IT AT FIRST, BUT I GOT LOST AND CONFUSED.

Explain what you learned about today’s lesson and how well you finished it.

Explain in detail the steps necessary to complete today’s lesson. (Think: Write an explanation to help a student who was absent yesterday understand the lesson and get caught up?)

Appendix 3A

Habits of Mind Rubric

For a grade of:

A	**B**	**C**	**D**
Follows the Habits of Mind Guide completely Data sheet is complete; no further explanation is needed. Paragraph 1 of the write-up includes all the information give in the problem. Paragraph 2 includes specific math steps and math vocabulary. Paragraph 3 includes correct answer and verification.	Answer is correct Follows the Habits of Mind Guide. Data sheet shows how problem is solved, but the solution needs more explanation. Specific math steps are missing in the write-up. Verification doesn't prove the answer.	Answer is incorrect or answer is correct, but write-up doesn't show how the problem was solved. Habits of Mind Guide wasn't followed completely.	Answer is incorrect. One or more sections required on Habits of Mind Guide are missing.

Appendix 3B

Habits of Mind Guide

Follow these directions exactly to create a data sheet.
- Write your name and the date at the top of a sheet of paper.
- Write the title of the Habits of Mind with the words *Data Sheet*.
- Show all the work you did to solve the problem using words, pictures, and numbers. Don't erase. It is important to show your errors and how you overcome them.
- Write a concluding sentence at the end of your data sheet that clearly states the answer to the problem.

On a separate page, record your write-up.
- Write your name and the date at the top of a sheet of paper.
- Write the title of the Habits of Mind with the words *Write-up*.
- Copy the paragraph titles shown below, and use the sentence starters provided to complete your write-up. Everything you write must refer to the math content, the procedures you followed, and the strategies you used to solve the problem. You can print, type, or use cursive writing. Write in paragraph form.

Paragraph 1
Problem Statement

- This problem is about …
- I am supposed to find …

Paragraph 2

Work Write-up

Explain step by step and in detail everything you did to complete your Data Sheet and arrive at your answer. Think of it as a recipe for someone to follow or as directions to your house. You may include strategies that you used that didn't work.

- First I …
- Then I …
- Next I …
- After that I …
- Finally I …

Paragraph 3
Answer

Verify, or prove, you answer by referring to the math you did. Don't write that you checked it on a calculator, you did it twice, or a friend or family member told you.

- My answer is …
- I think my answer makes sense because …

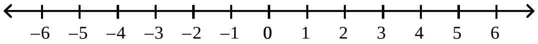
Appendix 4

6th Math Competency Exam

Short Answer

1. A book contains 77 pages. If a printer plans on printing 8 copies of the book, estimate how many sheets of paper the printer will need. Show your work.
2. Evaluate the following expression. Show your work.

   $41 + 4^3 \bullet 4 - (6 \bullet 2)$
3. Determine which of the given values of the variable is a solution. Show your work.
   $m + 4 = 19$; $m = 23, 15$, or 16
4. Solve the inequality $5 \ ^3 \ x + 1$, and graph the solution on a number line. Show your work.



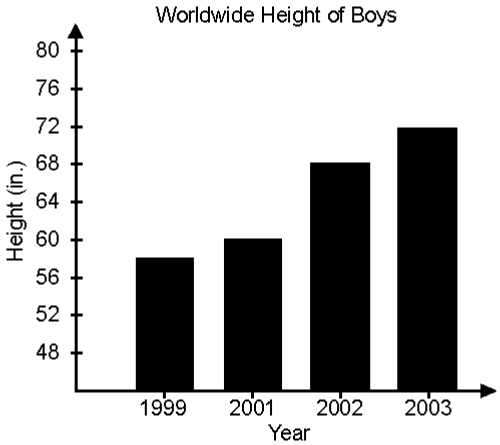
5. A log weighing 4.56 pounds is split into two pieces. If one of the pieces weighs 2.79 pounds, what does the other one weigh? Show your work.
6. A box of cereal weighs 450 grams.
   a. How many kilograms does it weigh? Show your work.
   b. How many milligrams does it weigh? Show your work.
7. A small box of donuts weighs 8.5 oz. A large box of donuts weighs 1.6 times as much. How much does a large box weigh? Show your work.
8. Write the prime factorization of the number 72. Show your work.
9. This winter has seen higher than average snowfall. In December 7.3 inches of snow fell, in January seven and one fourth inches of snow fell, and in February $7\frac{1}{2}$ inches of snow fell. Order the months from least to greatest amount of snowfall. Show your work, or explain in words how you determined your answer.
10. Ethan’s parents sent him outside to the garden to pick $\frac{5}{6}$ pound of green beans for dinner. He came back with $\frac{3}{4}$ pound. Did he get enough beans? Show your work.
11. List all of the factors for each of the following numbers. Tell whether the number is prime or composite, and explain how you determined your answer.
    a. 28
    b. 29
12. Below is a table of scores on the latest biology test in Mrs. Smith’s class.

Student	Score
Chris	98
Arturo	86
Gael	94
Rachelle	92
Antonio	83

Juan	98
Kim	61
Alice	81

a. Find the mean, median, and mode of the data without Kim's score. Round to the nearest hundredth.
b. Find the mean, median, and mode of the data with Kim's score. Round to the nearest hundredth.
c. How does Kim's score affect the mean, median, and mode?

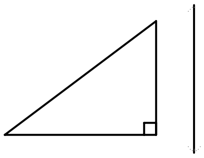
13. Explain why the bar graph below is misleading.



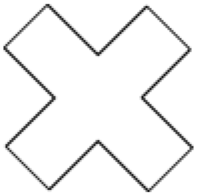
14. Use the data in the table below to make a steam-and-leaf plot.

Hourly Pay Rate of High School Students with Part-time Jobs							
7	9	7	8	6	6	9	8
11	10	12	9	12	10	7	10

15. Draw a horizontal reflection of the figure.



16. Jordan cut the following pattern out of a piece of construction paper. How many lines of symmetry does the pattern have? Draw a diagram to illustrate all of the lines of symmetry of the figure.



17. A man is standing next to a giraffe. The man is 6 feet tall and casts a shadow that is 9 feet long. The man measures the shadow of the giraffe to be 27 feet long. Use a proportion to approximate how tall the giraffe is.

18. A flower store sells carnations for $18.50 per dozen. The sales tax is 6%.
    a. How much is the sales tax on a dozen carnations?
    b. What will be the total cost of a dozen carnations?

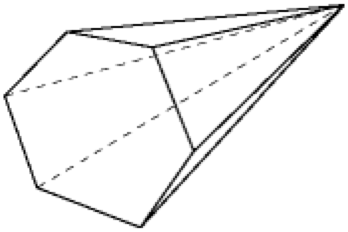
19. After two rounds of golf, Emma's score was –11. Her first round score was –5. What was her second round score? Show all your work.

20. Consider the following integers: 7, –4, 0, –1, and 3.
    a. Order the numbers from least to greatest.
    b. Graph the integers on the same number line.

21. The Flores family has a rectangular oriental rug on their dining room floor that has a length of 10.5 ft and a width of 8 ft. Find the area of the floor covered by the rug. Show your work.

22. Jerry bought a rectangular trunk that has a length of 4.5 feet, a width of 3.5 feet, and a height of 2 feet. Find the volume of Jerry's trunk. Show your work.

23. Maria's math book has the following shape on the cover of the book.



    a. Name the solid figure represented by the shape.
    b. Identify the number of faces, edges, and vertices the figure has.

24. Ramon flipped a button 20 times and recorded which side of the button landed up. Based on Ramon's experiment, which side of the button is more likely to land up? Show all of your work.

Outcome	Front	Back
Frequency	12	8

Appendix 5

Student Interview Questions **Number: __________**

1. Please react by telling me how much each statement describes you and your beliefs about math, Habits-of-Mind problems, self-assessments, and reflections. Your answers may be as brief or as explicit as you desire.
    a. I learn math concepts pretty easily

    b. Math problems have only one correct answer.

    c. There is only one way to get the right answer to math problems.

    d. Working in pairs or in groups is the best way to solve math problems.

2. Please describe strategies that you usually use to help you solve math word problems.
3. What do you usually do if you get stuck on a problem?
4. How do you know when you get a problem right?
5. What helps you learn math the best?
6. Have the Habits-of-Mind problems helped you become a better math problem solver? If so, in what ways?
7. How do you feel about the reflection and self-assessment tasks?
8. Do you think I should continue to assign Habits-of-Mind problems to next year's students? Why or why not?
9. Do you think I should have my students do self-assessing and reflecting next year? Why or why not?
10. Would you want your math teacher next year to have you do Habits-of-Mind problems? Why or why not?
11. Please feel free to add any other comments.