Photographic Interpretation Handbook, United States Forces: Section 10 Height Finding and Direction of North from Shadow

Robert L. Bolin Depositor

University of Nebraska-Lincoln, rbolin2@unl.edu

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PROCEDURE IN DETERMINING HEIGHT OF OBJECTS FROM SHADOW LENGTHS ON AERIAL PHOTOGRAPHS

For all practical purposes this procedure makes but two simple assumptions: (a), that the shadow falls on level ground and that (b), the earth is a sphere.

To apply the formula for height from shadow length, the following information is needed:
1. The date and time of photography.
2. The latitude and longitude of photography.

From this information we may calculate and list the following factors utilized in the formula:

\[ a = \text{Sun's declination or latitude. This may be determined from graph (curve 1) or Solar Ephemeris for the date of photography.} \]
\[ b = \text{Latitude of photography (known).} \]
\[ c = \text{Difference in longitude between position of sun and position of photography. This may be calculated as follows:} \]

1. From time of photography (and referring to a time zone map) determine G.C.T. (Greenwich Civil Time).
3. Multiply this interval by 15 (remembering that minutes are not decimal parts of hours), giving the longitude of the sun at the time of photography. (The sun's longitude will be East if G.C.T. is A.M. and West if G.C.T. is P.M.).
4. A correction should be applied to the above calculated longitude of the sun for relatively accurate measurements. This correction involves adding or subtracting (as indicated) the small angle derived from graph (curve 2), if the longitude of the sun is West. If the longitude of the sun is East, the signs indicated in graph (curve 2) are reversed.
5. From the calculated longitude of the sun and the known longitude of the photography determine the difference in longitude between these two positions. = c.

From Tables of Natural Functions substitute the sine and cosine values in the following formula:

\[ \sin x = (\cos a)(\cos b)(\cos c) \pm (\sin a)(\sin b) \]

The sign between the two parts of this equation will be plus if the sun and photography are the same side of the Equator. If they are on opposite sides the sign will be minus.

From the value of '\( \sin x \)' calculated above determine from Tables of Natural Functions the value of '\( \tan x \)'. Height in the photograph will be shadow length multiplied by '\( \tan x \)'.

EXAMPLE:

It is desired to determine the height of a radio tower in Dakar which casts a shadow 77 feet long. The photograph was taken at 9 A.M. on the 4th of August. The latitude of Dakar is 14°40' North and the longitude is 17°20' West.

\[ a = \text{Sun's declination (or latitude) from graph (curve 1, page 10.03) } \approx 17°20' \]
\[ \cos 17°20' \approx .9546 \quad \sin 17°20' \approx .2979 \]
**HEIGHT SHADOW**

**MATHEMATICAL METHOD (CONT.)**

\[ b = \text{Latitude of photograph} = 14^\circ 40' \]
\[ \cos 14^\circ 40' = 0.9674 \quad \sin 14^\circ 40' = 0.2532 \]

\[ c = \text{Difference in longitude between sun and photograph} \]

1. From time zone map Dakar is seen to be 1 hour west of Greenwich, therefore G.C.T. = 10 A.M.
2. Interval between G.C.T. and Greenwich noon = 2 hours.
3. \( 2 \times 15 = 30 \) or \( 30^0 \) East is longitude of sun.
4. Correction from Table I on August 4th is seen to be \(-1^0 31'\), but longitude of sun is \( 1^0 \) East, so sign is changed to \(+1^0 31'\) making corrected longitude of sun \( 31^0 31' \) East.
5. Longitude of photograph = \( 17^0 20' \) West. Difference in longitude between sun and photograph = \( c = 48^0 51' \cos 48^0 51' = 0.6581 \).

\[ \sin x = (\cos a)(\cos b)(\cos c) \pm (\sin a)(\sin b) \]
\[ \sin x = (0.9546)(0.9674)(0.6581) \pm (0.2979)(0.2532) \]

The sign in the equation is plus because sun and photograph are on same side of Equator.

\[ \sin x = 0.6080 + 0.0754 = 0.6834 \]
\[ x = 43^0 7' \]
\[ \tan x = 0.9347 \]

Length of shadow = 77 feet.
\[ 0.9347 \times 77 = 72 \text{ feet, height of radio tower.} \]

In the foregoing procedure it is assumed that the time of day for the photography is known. If this information is lacking, the angle of elevation of the sun may still be determined if the direction of true north on the photograph is known. The accuracy of this method depends largely on the accuracy with which the angle between the direction of the shadow and true north may be measured on the photograph.

The following formula applies (substituting values from tables of natural functions):

\[ \sin x = \frac{(\cos b)(\cos N) \sqrt{(\cos^2 a) - (\cos^2 b)(\sin^2 N)} + (\sin a)(\sin b)}{1 - (\cos^2 b)(\sin^2 N)} \]

The sign in this equation is plus if the latitude of the sun and of the photography are on the same side of the equator. The sign is minus if they are on opposite sides.

\[ a = \text{Latitude of sun (as before)} \]
\[ b = \text{Latitude of photography (as before)} \]
\[ x = \text{Angle of elevation of sun (as before)} \]
\[ N = \text{Angle between direction of shadow and true north measured on photograph.} \]

Height of object = length of shadow multiplied by 'tan x', (converting sin x to tan x by tables of Natural Functions).
GRAPHIC METHOD OF DETERMINING HEIGHT OF OBJECTS FROM SHADOW LENGTHS ON AERIAL PHOTOGRAPHS

The following procedure requires that the date and time of photography are known as well as the geographic coordinates. The sun's longitude is determined in graph I from the time of day and time zone involved, and a small correction derived from graph II. Graphs III to V solve the relation: \( \sin \alpha = \cos \beta \cdot \cos \gamma \cdot \sin \phi \), \( \sin \beta \), \( \sin \gamma \), without the use of trigonometric tables or calculations. Graph VI gives the height of the object from the shadow length and the numerical value (\( \sin x \)) obtained from Graphs III to V. The various steps in the procedure are outlined below:

1. Referring to graph I showing relationship between time of day and longitude of sun, select point along lower margin of graph corresponding to indicated time of photography. Follow vertical line corresponding to time to proper oblique line representing zone to which indicated time is referred. From point of intersection follow horizontal line to right margin for longitude of sun.

2. Apply small correction to sun's longitude for the particular day of the year as indicated in graph II. This correction is added or subtracted as indicated by the graph if the longitude of the sun is west. If the longitude of the sun is east, the signs indicated for addition or subtraction are reversed.

3. Difference in longitude between sun and locality photographed may be determined by subtracting one from the other if both are east or if both are west. The two longitudes are added if one is east and the other west. If the result of this addition is greater than 360°, subtract 360°.

4. From date of photography indicated on left half of lower margin, below Graph V, follow vertical line for day concerned to curve in Graph III representing latitude of locality photographed. From this point in Graph III follow horizontal line into Graph IV to oblique line representing difference in longitude as determined in above paragraph. From point of contact in Graph IV follow vertical line down to lower margin of Graph IV to determine value of \( \gamma \).

5. Again, from left lower margin indicating date of photography follow same vertical line to curve in Graph V which represents latitude of locality photographed. From point of contact in Graph V follow horizontal line to right margin of Graph V to determine value of \( \gamma' \).

6. The values of \( \gamma \) and \( \gamma' \) are added together if the sun's latitude (or declination) and that of the photograph are on the same side of the equator. If they are on opposite sides of the equator, \( \gamma' \) is subtracted from \( \gamma \). Note: Sun is north of the equator from Mar. 21 to Sept. 21.

7. In Graph VI select point along lower margin representing length of shadow. Follow vertical line for value of shadow length to oblique line representing the value of \( \lambda \) or \( \lambda' \), as determined in paragraph above.

NOTE:

1. Referring to graph I showing relationship between time of day and longitude of sun, select point along lower margin of graph corresponding to indicated time of photography. Follow vertical line corresponding to time to proper oblique line representing zone to which indicated time is referred. From point of intersection follow horizontal line to right margin for longitude of sun.

2. Apply small correction to sun's longitude for the particular day of the year as indicated in graph II. This correction is added or subtracted as indicated by the graph if the longitude of the sun is west. If the longitude of the sun is east, the signs indicated for addition or subtraction are reversed.

3. Difference in longitude between sun and locality photographed may be determined by subtracting one from the other if both are east or if both are west. The two longitudes are added if one is east and the other west. If the result of this addition is greater than 360°, subtract 360°.

4. Vertical line A - B representing Nov. 7th in the left lower margin is followed to point B in Graph III representing latitude 22°48' for Okayama. From point B a horizontal direction is followed to point C representing \( \gamma \) difference in longitude in Graph IV. From point C a vertical line is followed to point D at lower margin of Graph IV, giving a value of .875 for \( \gamma' \).

5. Returning to line A - B representing November 7th, point E on the date line represents a position in Graph V corresponding to the latitude of Okayama. From point E a horizontal direction is followed to the right margin of Graph V where the value of \( \gamma \) is found to be .108.

6. \( \gamma' \) is subtracted from \( \gamma' \) inasmuch as the sun and locality are not on same side of equator. .875 minus .108 equals .767.

7. Referring to lower margin of Graph VI locate point G representing 82°4 feet for shadow length. Follow horizontal line representing 82°5 to position H between oblique lines .76 and .77, representing .767. A vertical line from point H to point I at top margin shows height of radio tower to be 986 feet high.
GRAPH TO BE USED IN DETERMINING HEIGHT OF OBJECT FROM LENGTH OF SHADOW ON AERIAL PHOTOGRAPHS

Y ± Z
(SINE OF ANGLE OF ELEVATION OF S)

LENGTH OF SHADOW

HEIGHT OF OBJECT

0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
The following procedure requires, as in determining height of objects from shadow length, that the date and time of photography are known, as well as the geographic coordinates. The graphs prepared specifically for determining the angle between a north-south direction and shadow direction are a solution of the equation: \( \sin N = \cos a \cdot \sin c - \cos x \).

The procedure involves use of the curves designed to give height of objects from shadow length so far as obtaining the value of \( Y \pm Z \) (sine of the angle of elevation of the sun). The various steps in the procedure are outlined below:

1. On graph VII showing angle between north (or south) and shadow direction, select point along left half of lower margin representing value of \( Y = Z \) and follow vertical line to the curve representing difference in longitude between sun and locality as indicated along left margin. From point of intersection with proper curve, or interpolated position, follow horizontal line to curve or position between curves on right representing date of photography. Note: The curves shown represent the 21st day of the month indicated and approximate positions for intervening dates will be found by interpolation. From point where horizontal line intersects curve of photography follow vertical line to lower margin of graph to read angle between shadow and a north-south line. At this point the interpreter is required to visualize the approximate direction of the shadow in order to determine in which quadrant the shadow falls, and hence in which direction from the shadow that the angle determined above is to be plotted.

**EXAMPLE:** Same data as in height vs. shadow length problem:
Photos taken at 1115 hours, Nov. 7th. Latitude and Longitude: 22°48' N. and 120°15' E.

2. Selecting point .767 on left half of lower margin of graph VII follow vertical line to curve representing 8° difference in longitude. From point of intersection follow horizontal line to interpolated position for Nov. 7th in curves representing dates of photography to right. From latter point of intersection follow vertical line to lower margin to find angle from north to be 12°. Since time of photography was 1115 hours, presumably referred to an air base in China at least an hour earlier than sun time in Formosa, it is actually early afternoon in Formosa, hence the shadow is directed to the northeast and the angle of 12° would be plotted counter clockwise from the shadow in the photograph for the direction of north.
Graphs for determining angle between shadow direction and north from value of $\sqrt{\sin^2 \theta}$ (sine of angle of elevation of sun), difference in longitude between sun and locality, and date of photography.

Office of the A.G.A.S , Intelligence
Washington, D.C.

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