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# **Improving Communication about Mathematics through Vocabulary and Writing**

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Math in the Middle Institute Partnership  
Action Research Project Report

in partial fulfillment of the MA Degree  
Department of Teaching, Learning, and Teacher Education  
University of Nebraska-Lincoln  
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## **Improving Communication about Mathematics through Vocabulary and Writing**

### **Abstract**

In this action research study, where the subjects were my 6<sup>th</sup> grade mathematics students, I investigated the impact of direct vocabulary instruction on their communication and achievement. I strategically implemented the addition of vocabulary study into each lesson over a four-month time period. The students practiced using vocabulary in verbal discussions, review activities, and in mathematical problem explanations. I discovered that a majority of students improved their overall understanding of mathematical concepts based on an analysis of the data I collected. I also found that in general, students felt that knowing the definition of mathematical words was important and that it increased their achievement when they understood the words. In addition, students were more exact in their communication after receiving vocabulary instruction. As a result of this research, I plan to continue to implement vocabulary into daily lessons and keep vocabulary and communication as a focus of my 6<sup>th</sup> grade mathematics class.

## INTRODUCTION

My goal in teaching was for all students to achieve to their fullest potential. When beginning this research project, I looked for ways that I could foster higher achievement in my classroom. My thoughts turned to what my teaching may be lacking; what could I be doing better as a teacher? I decided that what makes any subject interesting and exciting to the learner is the communication that takes place not only between the teacher and the students, but also among the students themselves. I had already changed my teaching techniques with respect to communication, since starting Math in the Middle. I used cooperative learning grouping and student problem presentations, so I tried to think of other ways that communication could improve achievement.

I realized that I did not focus my lessons at all on using precise mathematical vocabulary or written communication. I explained words that I thought were necessary to know in order to understand the mathematical concept, and even more often, used simpler terms to explain. I was worried that using sophisticated mathematical vocabulary might make the actual mathematical work more confusing for the students. At the same time, I was worried that the words that I was skipping from the textbook were words that students would need to know in the future. So I chose this as the area that I wanted to focus on. I wanted to see how specific instruction focused on vocabulary would impact student communication and achievement.

In addition, my students already regularly used verbal communication during class activities, but written communication was an area I had yet to take advantage of. During my formative education, I was never exposed to the idea of writing about mathematics. However, through my higher education, I have learned the importance of

written work and being able to be a fluent mathematical writer, but this just had never translated into my mathematics teaching. I definitely wanted to explore this idea in my research and implement it as a part of my regular classroom routine.

Next, I asked myself questions that I felt I needed to find the answer to in my research. I wanted to be able to find a connection between vocabulary instruction and written communication. One major area that I wanted to focus on was how the two areas could influence each other. The relationship between having a large vocabulary and being a good writer is a well-accepted fact in the area of language arts. I wanted to know if that fact carried over to mathematical vocabulary and writing. Also, I wanted to know to what extent students would use exact mathematical vocabulary in their writing assignments, after receiving vocabulary instruction. I also wanted to find out to what extent student use of exact mathematical vocabulary would correlate with student mathematical understanding.

Then for my growth as a professional, I wanted to look at what my teaching looked like when I challenged and supported students in vocabulary instruction. My overall goal in teaching mathematics was not only for my students to learn mathematical skills, but to develop a mathematical way of thinking, or mathematical literacy. If students learn to think more like mathematicians, they will be better problem solvers, now and in the future.

Before my research, vocabulary instruction was a minimal part of my lesson plans. Sometimes I mentioned the vocabulary words from the textbook or other times, I skipped over them. I felt that as long as students could do the mathematical work, then I was doing my job. Each mathematical concept seemed to be separate from the others; if

students could perform each individual task, then they mastered that concept. I did not necessarily combine concepts to increase the idea of spiral reviewing. When the curriculum is spiraled, students see the same concepts year after year. I wondered if knowing the vocabulary well would increase retention of the material when it was taught in the following years. I hoped that focusing on vocabulary would allow me to use words repeatedly and connect multiple mathematical ideas for students. I also hoped that the more often the students heard the words being used correctly, the more likely they would be to use the words themselves when communicating, both orally and written.

Since my students were already communicating within small groups, as well as in front of a large group, I wanted to focus on another form of communication, writing. I had done many written explanations in my Math in the Middle classes. I felt that written explanations helped me understand the concepts better because I really had to think through my work to write about my solution. In fact, when I wrote about a problem, I solved it first, then went back to explain. In a way, this became part of my revision process. I wanted to see how this worked for my students. I wanted them to work the problem, then go back and explain it, which would give them an opportunity to evaluate their own work by reviewing the procedures they performed. I felt this would help students eliminate small mistakes in their work, as well as procedural errors.

This was a big change in my classroom because students really only showed their work and gave an answer in numeric digits. Typically, the only letters that they wrote were labels for their numeric answers. I knew that it would be a long process to get the kind of written explanation that I hoped for. By the end of my project, I wanted students

to be able to produce a cohesive written explanation with the correct answer that would earn a four on the rubric.

### **PROBLEM STATEMENT**

Any teaching method that can increase student achievement was worth studying and knowing about. Focusing on vocabulary and written expression were two ways that I felt I could change my teaching, and I wanted to see what would happen with student learning when I did. If I found that these strategies increased achievement, then I felt that any teacher could easily work with vocabulary and/or written communication in addition to teaching techniques they already use. It does not take any specialized skills, just time to fit it into the curriculum. It would not be difficult for any teacher, at any grade level, to include written communication into their lessons. Mini lessons can be included during class time, but students could do their writing at home, as to not take up more class time. I felt that adding a vocabulary focus was not such a dramatic change that my students were thrown off or needed time to adjust. I took the time to explain to the students why I was changing some of our routines. They seemed to be comfortable with the change, and excited to try something new. A teacher could easily use some of the vocabulary strategies that I used to increase achievement.

One obstacle was that extra time would be necessary to teach writing in math class, but I felt that it would benefit my students, so the time was well worth it. My class was made up of differentiated, or advanced, students. Most of my math students were also in the differentiated Language Arts class, and therefore already fairly good writers. Having students that were smart and motivated made the time that I used to teach writing less than it may be in other classes, with students of differing abilities.

Many other teachers and researchers have been interested in either vocabulary and writing in mathematics. These have not been linked together in many other studies, which is why mine is unique. Effective teachers are always seeking strategies to increase learning, so if I can learn one more strategy for my repertoire, then my research was a success.

## **LITERATURE REVIEW**

When I looked at my research questions about written expression and vocabulary, a few themes emerged and I focused on reading more about them. Those themes were communication, writing theories, writing strategies, vocabulary theories, and vocabulary strategies. I wanted to look at the prior research of others to see if they found vocabulary and written communication to be a worthwhile use of time in the mathematical classroom. Then I wanted to explore multiple vocabulary and writing strategies to use in my classroom. I will provide information based on recent research concerning the importance of direct vocabulary instruction and written communication within the mathematics classroom along with effective strategies for doing each with students.

Two important ideas about communication that the articles I read made me think deeper about are risk taking and higher-level thinking. I know that my students already take risks in communication because they present homework problems daily and higher-level mathematics solutions biweekly. At the beginning of the school year, giving presentations was quite difficult for most students. Most students struggled with not only having the confidence to present their work in front of peers, but also the knowledge to explain why and how they found a solution. I hoped writing explanations would assist those who are still struggling by giving them a form to work from.



Since my students did not typically use written communication as much as oral communication, they would need to take a risk when asked to write. Writing about a mathematical solution is a personal expression of knowledge that scares some students. Many of them had never been asked to use writing in math class and were apprehensive about it. Meyer, Turner, and Spencer (1997) studied why 14 fifth and sixth graders are and are not risk takers, as defined by their study. They found that a risk taker prefers tasks that are difficult, can tolerate failure, and use flexible strategies. This means that teachers need to anticipate how students will react in the face of risk and plan for it. Teachers need to know which students will easily take risks and which ones need more support to take a risk. The first obstacle that I needed to overcome was aiding students in taking risks. I needed to help students get rid of the initial fright of sharing their thought process.

At times, students' risk taking is attempting to answer questions that they think are out of their realm of knowledge. Writing about mathematics often requires higher-level thinking skills. Being able to communicate their thoughts while problem solving can facilitate much deeper learning for students than just isolating concepts. When students learn math through thinking and reasoning, they are more likely to understand conceptually (Wood, Williams, & McNeal, 2006). I want my students to take the vocabulary words that they learn and apply them to their communication, both orally and written.

The National Council of Teachers of Mathematics (NCTM) considers communication to be an important part of mathematical education. NCTM reported, "when students engage in speaking, writing, reading, and listening in mathematics class,

they not only communicate to learn mathematics, but they learn to communicate about mathematics” (2000, p. 60). NCTM also noted that in grades six through eight, students need to communicate their mathematical thinking coherently and clearly to peers, teachers, and others, as well as using the language of mathematics to express mathematical ideas precisely. “When students are given the opportunity to communicate about mathematics, they engage thinking skills and processes that are crucial in developing mathematical literacy” (Pugalee, 2001, p. 296).

Pugalee studied the incorporation of the communication standard into a middle school classroom. He found that it was best for students if teachers communicate about math problems with multiple representations and manners of problem solving. He concluded that communication should be a part of a well-balanced mathematics program. This enables students to be comfortable communicating mathematical ideas orally and written, as well as evaluating their own thinking and the thinking of others.

I planned to incorporate many ways of collecting data. I think it is important to show what happened in my research by both quantitative and qualitative data. I used methods from Meyer, Turner, and Spencer’s (1997) research and Zaslavsky and Shir’s research, which both included questionnaires or surveys. However, Meyer, Turner, and Spencer followed up by interviewing students. Wood, Williams, and McNeal’s (2006) research differed from the others because it was based on observations and experiences.

No matter the differences in the students that were studied, all findings seemed to overlap. Meyer, Turner, and Spencer (1997) as well as Zaslavsky and Shir (2005) found that communication within mathematics class promotes higher-level thinking skills. Pugalee (2001), along with Meyer, Turner, and Spencer concluded that in order to have a

well-rounded mathematical curriculum, communication is a necessity. In fact, Meyer, Turner, and Spencer noted that if in fact teachers want to improve communication, then current curriculum might need to be altered. In order to have good communication in the classroom teachers should be working by collaborating with students. They stress that education should be project based and that the curriculum should reflect personal histories and beliefs. By incorporating vocabulary and written expression into my teaching, I hope to create a more well-rounded curriculum and allow students to bring their own personal knowledge and thoughts into the classroom.

The important ideas that I focused on when researching theories of the use of writing in mathematics classrooms are using multiple methods of delivery, student ownership of knowledge, justification of a solution, and student engagement. These themes were common amongst many of the articles that I read.

By using multiple modes to deliver instruction, teachers and students work together to create learning as a classroom. “Teachers and students need to develop a mathematical-linguistic vocabulary that equips them with tools of thinking and speaking about the various forms of language available to them so they can make informed, critical judgments about which forms to use” (Burton & Morgan, 2000, p. 451). Burton and Morgan found that students engage in successful mathematical work when teachers explicitly address the correct mathematics.

When students and teachers are able to read and write about mathematics critically, they are able to gain authority and ownership over the concept. When students feel that they own a concept they are able to use it in different contexts and apply it to

different problems. When students write about what they know, they can show the reader that they know what they are writing about and their solution is believable.

The writer needs to know how to write in ways that are likely to convince such readers that she or he has the authority to write on this topic, that the subject matter is important enough to be interesting, and that paying attention to what is being said is worthwhile (p. 451).

The idea of writing in math class has many formats and uses, just as in a literacy class. Most importantly, writing about math cannot solely consist of stating what math operations have been done. A more in depth explanation of why specific operations have been used is necessary. “Justifying an answer is actually telling the reader what you did to solve the problem and providing a rationale for why the steps you took were appropriate” (Anderson & Little, 2004, p. 470). Anderson and Little found this manner of writing was most beneficial for their students in their research on reasoning and communication. They also applied problem-solving skills across their curriculum in hopes of students effectively communicating in all areas. Whitin and Whitin (2002) researched students who were asked to use a general form of the scientific method with a representation of their method to express their knowledge. The researchers reported that when students are justifying their answers, they have the opportunity to formulate questions about the concept or their thinking process.

The use of expository writing, which is intended to describe and explain, in the mathematics classroom, was the focus of a study done by Shield and Galbraith (1998). They found that comprehension was improved when the concepts were meaningful to students. Linking new concepts to known concepts proved to be especially beneficial.

This is an area that can be researched even deeper to find specific strategies that prove to be most effective.

An interesting study done by Porter and Masinglia (2000) showed results indicating no significant difference between students who did and did not write in math class. The most significant finding in respect to my research was that students must be engaged intellectually with mathematical tasks that require them to justify their ideas. This is a finding that is particularly useful when working with middle-school children. In middle school, students have a tendency to be easily distracted. Especially in sixth grade because it is their first year of middle school and they are interested in getting to know new people. They also noted that student achievement was dependent upon individual learning styles. Students need to make a connection with the concept to promote long-term retention.

Another study that indicated students need to connect to the curriculum in order to achieve retention of the concepts that were taught was done by Clarke, Waywood, and Stephens (1993). It showed how using writing allowed students to put math in personal terms and therefore allow students to connect to it. “The learning of mathematics is fundamentally a matter of constructing meaning” (p. 235). Their finding showed that students used their writing to explain concepts that helped students add meaning to the curriculum.

Writing is very important to the learning process and can be used in any subject to show what students know. “Teachers and researchers are seeing that the process of writing mirrors the process of learning and can be seen as supportive of it” (Clarke, Waywood, & Stephens, 1993, p. 236). The writing process involves procedures that are

essential to the learning process was well. Writing supports learning by allowing students to use their own words to express an idea.

Writing allows all students to write at their own pace as well to receive personal feedback, which engages students in the curriculum. Borasi and Rose (1989) found that writing enabled the students not only to learn the concept but also clarify the process in their own mind. “Students are too often content with externally manipulating symbols and doing routine problems, without ever reaching a deep and personal understanding of the material” (p. 347). It is just not enough for students to be able to do the mathematical work. They need to have an understanding of why and how a concept is used to really own it.

Interestingly, Borasi and Rose (1989), then Porter and Masingila (2000) both concluded from their studies that writing done in a math class was not of significant importance. Instead, what was important was that students were engaged in the material being taught. Borasi and Rose stated that writing is an alternative to typical lecture teaching methods.

I think of writing as a way for my students connect themselves to mathematics. They can solidify their thought process as well as the mathematical concepts that were involved in the solution. It forced them to be engaged in mathematics, which as Porter and Masinglia (2000) found, is most important. If students are actively engaged in the concepts being taught, they are more likely to retain the information.

When deciding to incorporate writing into mathematics, many factors must be considered, such as physical setup, the amount of time spent writing, writing instruction, and the use of a rubric. When I decided to use writing in my classroom, I had to decide

what outcome I wanted students to achieve and plan accordingly. Many articles suggest ways to teach writing in a mathematics or literacy class. There is no method that stands out as being most effective. There are many important strategies that have been successful in many classrooms, which I fit into my own classroom.

One area that has been questioned by researchers is the set up of the physical paper for writing in mathematics class, such as math on the top and writing on the bottom of a piece of paper or math on the left and writing on the right. Both Thompson and Rubenstein (2000) as well as Price (1989) suggest the using the latter. Studies have not been done to investigate whether one setup is more effective than the other.

It is also important to consider when and how long students will be writing. Williams and Wynne (2000) suggest allowing students to write in class for five to ten minutes. Williams and Wynne experimented with writing in their classrooms, finding that it can be time-consuming to implement. I knew this as an obstacle that I must overcome in order for students to produce well written explanations.

One idea shared by Price (1989), as well as Shield and Galbraith (1998), is having students explain the concept to an absent peer. They both suggested that students need to explain the situation to someone who needs help, instead of the instructor who already knows the information. The ultimate test of what a student writes is whether someone else can learn from his or her paper. This forces students to be accurate and reader-friendly.

Another area of importance is the grading of writing. The use of a rubric is suggested by Anderson and Little (2004), as well as by Thompson and Rubenstein (2000). Anderson and Little suggested giving the rubric to students so that students knew

what was expected of them. This allows students were able to evaluate their own work, just as the teachers would. Allowing students to see examples of exemplary and non-exemplary work is suggested by Thompson and Rubenstein (2000). Both allowing students to see how teachers will grade them and examples of how their teacher has graded other work will emphasize student awareness. Williams and Wynne (2000), who worked with high school students, and Anderson and Little (2004), who worked with fourth grade students both suggested the use of rubrics, means that rubrics are useful for a large age and ability range of students.

Not much research exists concerning writing strategies in middle school mathematics classrooms. I think this is an area that has not yet been thoroughly researched. I gave students time in class to write, to increase the completion rate. I also used the idea of students explaining to an absent peer. For sixth graders, I think this writing strategy is appropriate. I also used a rubric that I created to grade student writing. As Anderson and Little (2004) and Thompson and Rubenstein (2000) suggested, I gave students the rubric and shared examples of weak and strong written explanations.

The three most important theories of teaching vocabulary in a mathematics classroom, with respect to my research, were a connection to prior knowledge and multiple exposures to essential words. These two areas Chard (2003) mentions that teaching vocabulary in a math class should be similar to teaching vocabulary in a language arts class. Many studies have been done about effective strategies to teach vocabulary in language arts class, which carry over to mathematics class.

Teachers must introduce vocabulary words and connect them to prior knowledge. Most importantly, the vocabulary words learned in class must support the mathematical



concepts that the students are currently learning. Vocabulary knowledge is as essential to learning mathematics as it is to learning how to read (Chard, 2003). He also noted that vocabulary is essential to education and therefore improves achievement.

“Definitions are considered fundamental in mathematics and mathematics education” (Zaslavsky & Shir, 2005, p. 317). They found that definitions are typically formed over a school career by the spiraling of mathematical concepts. They also noted that it was necessary to know definitions in order to problem solve. Since problem solving is more than just a mathematical skill and is required throughout a lifetime, it makes vocabulary knowledge of the utmost importance. Students cannot accurately solve a problem that they do not understand. Students need to be taught words that are important to mathematics that are also commonly used in real life. This will not only help them in the classroom, but in their everyday life as well.

One study that focused on the implementation of a vocabulary-based curriculum found that those students who had more exposures to the vocabulary words were more successful (Zimmerman, 1997). The more opportunities to use the words in a communication setting, the more benefit the students can gain. Students in this study reported that they liked vocabulary instruction that was interactive. Students also believed that they gained more from these activities, rather than definition-based assignments.

Each time a student encounters a specific vocabulary word, they are more likely to use the word when they communicate. “Students must encounter words in context more than once to learn them” (Marzano, Pickering, & Pollock, 2001, p. 124). In fact the chances of a medium-ability student learning new words in context without instruction is 12 percent. The chances decrease to eight percent for a low-ability student and increase to

19 percent for a high ability student. Their research shows that, especially for low-ability students, direct vocabulary instruction is crucial. They noted that in order for students to remember a word and be able to use it correctly, students need to have encountered it at least six times. In my research, I tried to use the vocabulary words at least six times so that my students could achieve high retention rates.

In addition to multiple exposures, it is important for students to see words used in different applications. Stahl and Fairbanks (1986) also suggested that the use of vocabulary in different contexts was more beneficial to students than the same number of exposures in the same context. They suggest that this method is how words are acquired as children and teachers can replicate that learning process. With these ideas in place, teachers need to identify the words that are essential to each concept and intentionally teach those words in multiple contexts.

Zimmerman (1997), Stahl and Fairbanks (1986), and Marzano, Pickering, and Pollock (2001) had some similar findings. All found that vocabulary words need to be covered multiple times in multiple contexts. They also concluded that the more exposures a student has to the word in a meaningful context, using varying teaching methods, the more likely a student is to add that word to their own vocabulary. I used many different strategies to allow students to connect with the words they were learning. I also made sure to organize my teaching and the curriculum so that I allowed for multiple exposures to mathematical vocabulary words.

Just as other researchers, Stahl and Fairbanks (1986) concluded that the most effective way to teach vocabulary is to use multiple methods. Stahl and Fairbanks studied 80 different vocabulary strategies through research that was already done. Their goal was

to identify the most effective teaching strategy. From the research they reviewed, no specific strategy was noted as most effective, but that a combination of interactive strategies works best. The strategies that I read about and wanted to use in my classroom were graphic organizers that create relationships, a word wall, and other activities that are creative. Giving students the opportunity to feel ownership over new vocabulary was a common notation within the strategies.

One strategy that is a good start to working with vocabulary is to use a graphic organizer. Chard (2003) writes that graphic organizers can help children understand terms in relation to each other. Graphic organizers mentioned by Chard are word splashes and scaffolding. Venn diagrams are suggested by Marzano, Pickering, and Pollock (2001). These could be kept in a personal glossary or dictionary, as suggested by Thompson and Rubenstein (2000).

Another way to organize the vocabulary words is to categorize them, suggested by two different articles of research. Classifying is defined by organizing elements into groups based on similarities (Marzano, Pickering, & Pollock, 2001). Students can put the words into categories given by the teacher or kids can assign titles to categories of words that they are given. Readence and Searfoss (1980) believe that categorization helps students organize the words and make sense of them. Students also are able to use the words in their own experiences and that can help improve problem-solving skills.

Another relationship-based strategy suggested by Marzano, Pickering, and Pollock (2001) is the use of analogies. Analogies are probably the most complex format for identifying similarities and differences in that they deal with “relationships between

relationships” (p. 26). This strategy allows students to see how words may be similar and different which will increase understanding.

An important common notation was that of building relationships between words. Making some sort of association of new vocabulary words to known concepts was suggested in nearly all of the articles I read. Although many specific strategies were suggested, it is obvious that this is a significant determinant to effective vocabulary instruction.

A word wall is a strategy of teaching vocabulary in which words are posted in the classroom for easy reference. Rubenstein and Thompson (2002) suggest that word walls may also include definitions, pictures, and uses that are created by the students. Word walls give students a visual representation to remind students of the definitions. Since students create the word wall, it increases retention of the information.

Rubenstein and Thompson also suggested using more creative modes of learning vocabulary, such as stories, cartoons, bumper stickers, skits, raps, songs, and poetry. These strategies would be particularly helpful for kinesthetic learners. Creativity is an important aspect of effective strategies. Students need to use their creative talents to boost their knowledge of vocabulary words.

St. Clair Otten (2003) as well as Rubenstein and Thompson (2002) found that students enjoyed activities and learned when the teacher was creative with the assignment and when students were able to be creative too. One creative strategy created by St. Clair Otten is a matching game of sorts for her students. She described the process in which students were asked to cut and paste definitions onto a list of words. She noted that she made the activity a competition to increase motivation, which she reported worked. I

used this as a review activity at the end of every chapter. This strategy was useful to reinforce learning, especially for kinesthetic learners.

My research was similar to those articles that I read because I am using some aspect of each. However, it was different because it will include many more strategies. I had students use graphic organizers, as suggested by Chard (2003), which were organized in a personal glossary of vocabulary terms, suggested by Thompson and Rubenstein (2000), on a daily basis. I also did activities that will allow students to create relationships between the words, such as a word wall, matching terms and definitions, and other creative methods, as suggested by Marzano, Pickering, and Pollock (2001) and Rubenstein and Thompson (2002).

After reading research studies and professional opinions, the importance of vocabulary and written communication in the mathematics classroom seems obvious to me. It is essential to mathematics understanding that students communicate ideas. Communication occurs best when the vocabulary used is accurate. Written communication is one form of communication where students can internalize their understanding and see the process in which they solve problems. My biggest question emerged as to what degree does written communication and direct vocabulary instruction improve communication and achievement?

My research was different from those I read because it encompassed writing and vocabulary. I did not only study the impact that written communication and direct vocabulary instruction had on mathematics achievement, but also the effectiveness of direct vocabulary instruction on written communication. My research can be carried over to other subject areas as well. For example, Science teachers could incorporate direct

vocabulary instruction into their lessons. Since I teach Language Arts and Science in addition to Math, I hoped to be able to use what I learned from this study in those classes as well. I hoped to learn more effective teaching strategies.

### **PURPOSE STATEMENT**

One purpose of my study was to see how much students used mathematical vocabulary after receiving direct instruction. I wanted to see how much the vocabulary instruction influenced their written problem explanations. Another purpose of the study was to see the effect of direct vocabulary instruction on mathematical understanding and achievement. I wanted to explore the idea that when students hear the correct vocabulary being used repeatedly and vocabulary is the major focus of instruction, then a greater understanding of mathematics concepts might take place. The last purpose of my study was to see how my teaching changed when I put such a major focus on vocabulary and written communication. Obviously, I knew there would be changes in the timing and organization of the lessons, but I was not sure what further impacts this study would have on my delivery of the material.

At the end of this project, I wanted to be able to understand if there was a correlation between vocabulary, written expression, and achievement. I wanted to decide if the strategies that I implemented in my classroom were worthwhile. The questions that I wanted to find the answers to were, to what extent will students use exact mathematical vocabulary in their writing assignments after receiving vocabulary instruction, to what extent does student use of exact mathematical vocabulary correlate with student mathematical understanding, and what did my teaching look like when I challenge and support students in vocabulary instruction?

## METHOD

My data came in a variety of different forms and was collected in multiple ways. The first part of my data collection was to record achievement information, before the focus on vocabulary and written expression began. I used data based on classwork, quizzes, chapter test, and criterion-referenced tests. I wanted to see how much students achieved using an individual-concept focus, as I did at the beginning of the school year. I organized this information in a spreadsheet so that I could easily see progress. I labeled each student individually, as well as found the mean across students for each type of data.

The next kind of data that I collected was a writing sample. I gave the students a problem over a concept that we had already covered. I asked them to solve the problem and explain their process. This allowed me to see where each student was in the writing process and gave me a great starting point for my teaching. I graded student writing on a scale of one to four, with one being weakest and four being strongest, using a rubric that I made, (Appendix 1). I included this information in my spreadsheet as well.

My focus on vocabulary instruction started with a Vocabulary Book. I used the vocabulary words that were included in each chapter of my text. Each student had his or her own bound book that was kept in the classroom and used each day. In the book, the students and I recorded each vocabulary word from the textbook on one page. Each page was set up as a vocabulary square, which has a space for the word, definition, examples, a memory trick, and a word problem example, (Appendix 2). The students helped decide what to write for examples, the memory trick, and the word problem. For most vocabulary words, the students and I drew pictures to show examples of the word. For the word problem examples, the students made up a word problem that went along with the

concept, and then we solved it together. The memory trick is a way to link the vocabulary word to the definition to help students connect the two together and gain long-term retention.

At the end of each chapter, I gave each student a list of the vocabulary words for that chapter and a separate list of their definitions. Each student was required to cut out the definition and paste it to the correct vocabulary word. Students did this assignment two days before the test to help retain the meanings of the vocabulary words.

The day before each chapter test, the students played a review game called, “I am... I have...” This is where each student got a slip of paper with an answer and a question on it. However, the answer did not match the question. One student started by reading, “I have...” and the question. Then the student with the correct answer read, “I am...” and the answer, then “I have...” and their question. The activity continued until it came back to the first person. This was a fresh, new way to practice vocabulary words as well as mathematical concepts.

At the beginning of each chapter, I gave students a pretest of the vocabulary words. Some words were review words and some were completely new to the students. This gave me a great chance to see what they already knew and what words I needed to emphasize. Then at the end of the chapter, I retested the students to see what words they retained. I organized the test scores in a spreadsheet with individual student scores, as well as mean scores for each test.

On each quiz, which was approximately weekly, students were required to write about how they solved a problem. I chose the problem that they wrote about, which was typically a story problem. I hoped this would allow them to elaborate more. I graded all



writing based on the same four-point rubric as the pre and post writing tests. I gave the students the rubric so they could see how they would be graded and also see what it would take for them to get a four. Students put the rubric in their Vocabulary Book so they could use it often. On the quizzes I also counted how many vocabulary words they used. I was looking for a correlation between the number of words used and the scores. Again, I organized these in a spreadsheet. I tried to look at this information from multiple angles, such as mean number of words used, how many students got each score, and of the students that got each score, how many vocabulary words they used.

I also made a poster that hung in the classroom that helped students know what to write (Appendix 5). It is a scaffolding method of writing that helps remind the students that they need to explain what they did, why they did it, and what their solution means. I used it as a way to begin teaching writing within a mathematics classroom. It also helped students who needed guidance in their writing.

During the middle of the project and at the end of it, I surveyed the students. The surveys were exactly the same both times (Appendix 3). I wanted to see how students were feeling about the changes and if their opinions changed over time. I used a likert scale, from one to five, on each question. I organized the answers in a spreadsheet with mean scores and individual scores. I also interviewed the students in the middle of the project. I asked students similar questions as the survey, but gave them an opportunity to expand on their opinions. I audio recorded the interviews, then played them back and took notes, (Appendix 4). For the end-of-project interviews, open-ended questions were asked again, however, this time the students wrote the answers down instead of expressing them verbally.

The last piece of data I used was my teacher journal. I wrote a weekly journal in order to remember my thoughts during the research. Although this was hard to find time to do, I know that it helped my research.

## **FINDINGS**

Overall, this research project was successful in raising achievement and incorporating vocabulary into written communication. I became a more effective teacher and learned new teaching strategies. The research that I read, the data I collected, and the conversations I had with students helped to facilitate my learning throughout this research process.

## **TEACHING**

When answering the question, what does my teaching look like when I challenge and support students in vocabulary instruction, I found that my teaching is more effective and more focused when I concentrate on vocabulary instruction. Two of the statements on the student survey given on March 13, 2008, stood out to me as a connection to this question. When reading the statement, “I hope my math teacher next year uses the vocabulary square,” seven out of fifteen students responded in agreement. Four more students responded that they were not sure. This means that only four out of fifteen students, indicated they would not like to use the vocabulary square again. This supports the fact that students feel it helped them learn, which means that these new strategies are effective and therefore makes my teaching more effective.

In addition, when asked in the oral interviews what students liked best about math, about half of the students who responded mentioned that I was the reason they liked math, while the reason they disliked math mostly dealt with homework. On the final

written interview, seven out of sixteen students noted that I was a great teacher, when asked what I needed to change to become a better teacher.

Part of what makes a teacher effective is that students enjoy class and enjoying learning the concepts in that class. I wanted to be able to note whether students enjoyed the class more or less than they did before my study. A piece of data to support the likability and therefore effectiveness of my teaching came when I was interviewing students. When asked, “Why do you like or dislike the vocabulary square?” all students responded with a positive answer. Since the vocabulary square is an integral part of my teaching now, I feel that positive comments about it reflect on my teaching as well. Also, when asked, “What do you hope your math teacher does next year with vocabulary?” Brooke responded, “I hope that she is just like you because it’s fun in this class because we do games and we don’t even know that we are learning, but we actually are learning.” The games she is talking about are the "I am... I have..." review, which all of the students enjoy.

The students also enjoy making up word problems for the vocabulary book, which I have noted multiple times in my teacher journal. In my first journal, February 4-8 2008, I noted that as soon as I gave the books out to students, they took ownership of them. They were excited from the beginning about trying a new learning strategy. Even when I felt that there might have been too much emphasis on vocabulary and students might be bored, they were completely engaged in the books, as I noted in journal six, March 10-14, 2008. Students seemed to really understand the importance of vocabulary, as noted by Holly in the oral interviews. She said, “I hope they care about vocabulary or else we

wouldn't understand," when asked what she hoped her 7<sup>th</sup> grade teacher would do in regards to vocabulary.

Also, in my teacher journal, I have made several mentions of student changes since implementing the vocabulary book. I think this relates directly to the changes I made in my teaching because it shows that since students are learning more, my teaching is better. In my very first journal, February 4-8, 2008, I had already noticed a change was taking place. I said, "So far, the book is a great device that allows both my students and myself to focus on the concept and the vocabulary." Specifically, one student, Paul, has improved his behavior and overall grade. At the beginning of the year, he was sent to think time repeatedly and assigned to the after-school room for not completing his work. Think time is a technique used by many schools to deescalate students who are in a problem situation. When a student is sent to think time, they exit the classroom in the attempt to move them out of an emotional mindset and get them into a thinking mode to fix the situation. Just in my classroom, he was sent to think time approximately once per week, before the project. Since I have begun this research project, he has only been sent to think time approximately once per month.

Paul's behavior was an ongoing battle that has now seemed to diminish in my classroom. His test grades have also increased from an 79.77% mean to an 82.3% mean. I realize that there may be other factors involved, such as maturity and mathematical content covered, but I do feel that this is one of them. I think he has a focus during class that he did not have before, vocabulary, specifically the vocabulary book. I think Paul needed a focus during class because he was easily distracted by his own behavior and the behavior of others. He is a very impulsive student who tends to act on his feelings at the

moment, rather than thinking of the consequences. The vocabulary book seemed to give him an outlet to really focus his energy on.

I noted in my first journal, from the week of February 4 – 8, 2008, that the vocabulary book worked best as a summarization technique. During the first week of using the vocabulary book, I experimented with the most effective way to use it. I found that it worked best to talk about the words during class while working on examples of conceptual problems, and then fill it out at the end of the lesson, as a review, before the students started on individual work. Again in my third journal, February 18-22, 2008, I noted that the time I was spending on vocabulary instruction was worth it. Again, I said that I found it effective to introduce the words at the beginning and then summarize them at the end. I said, “This seems like a good way to introduce the words and use them to summarize the lesson.”

I think the new techniques that I used to teach the vocabulary were mostly effective. Unfortunately, I know that the use of the word wall was not one of those effective strategies. I had good intentions of using it to refer to words, but I did not find it to be as useful as I wanted it to. I mentioned in my fifth journal, March 3-7, 2008, that I was not using it often in my teaching and that I would like to incorporate it more into my lessons, but that just did not happen. I did use it a few times to remind students to use correct terms, but not nearly as much as I thought I would.

My biggest struggle during this study was the time aspect. I had a hard time fitting all of the vocabulary words into some lessons. For example, during chapter 10, on geometry, when we talked about triangles, the vocabulary included all classifications of angles and triangles. Having to cover ten words or more in one period proved to be too

much. It was difficult for both the students and myself. I noted in journal 10, April 7-11, 2008, that this was my biggest struggle between teaching and researching. This was also evident in the oral interviews. When I asked for the students to give me advice about using the vocabulary square, about half of the students said that sometimes I needed to slow down. I know they were feeling the crunch when there were a lot of words, just like I was.

### **VOCABULARY USAGE IN WRITING**

When researching the question, to what extent will students use exact mathematical vocabulary in their writing after receiving vocabulary instruction, I noticed that students typically use exact mathematical vocabulary when communicating with others, especially writing about their solution to a problem. In fact, I also noticed that generally, the more vocabulary words students in their writing, the better grade they earned on the writing. When students used zero or one vocabulary word, 82% of the time, they received a score of a one, 44% received a two, and 17% received a three, while only 2% earned a four. That last percentage means that only one student quiz out of 149 quizzes total, used zero or one vocabulary words and still earned a grade of a four. On the opposite end of that spectrum, students who used six or more vocabulary words earned a three or a four 70% of the time.

I assigned ten quizzes throughout this research project. On each of the quizzes, I assigned one problem for the students to solve and then write about how they solved it. In the table below you can see that the scores are generally rising. The number of words used by students in writing is not continuously rising, but I think this is due to the problem I chose and number of words that are associated with it. It was important to

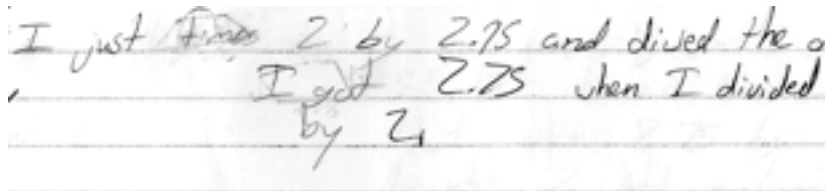
choose problems that incorporate several concepts so they can be explained using multiple vocabulary words. I mentioned this issue in journal 5, March 3-7, 2008, by saying, “I need to seek out the problems that will elicit the most words.” Students also felt that this was important, as noted by Adam in the interview, “If you know the words, you’ll be able to explain it a lot better.” I also mentioned this in my fourth journal, February 25-29, 2008. It is much easier to follow a student’s thoughts when they are very precise. This was best achieved when the wording of the explanation includes vocabulary words that we have discussed.

Table 1: Mean Rubric Scores and Mean Vocabulary Usage on Quizzes		
	Mean rubric score	Mean of vocab words used
Chapter 8, Quiz 1	2.07	1.5
Chapter 8, Quiz 2	1.93	.87
Chapter 9, Quiz 1	2.4	1.4
Chapter 9, Quiz 2	2.15	1.13
Chapter 10, Quiz 1	2.67	4.2
Chapter 10, Quiz 2	1.4	4.27
Chapter 11, Quiz 1	2.6	1.67
Chapter 11, Quiz 2	2.33	3.2
Chapter 11, Quiz 3	2.27	2.47
Chapter 12, Quiz 1	2.73	4.6

I found individual qualitative data that represents the above table of quantitative data. The pre and post writing samples from those two students is shown below. Both are students of color who have participated in the differentiated program in the past. This means that both students are considered bright students who can be successful when challenged, however, the results in my research were very different for these two students. It is important to mention that, Greg, the student who had the lowest average

writing score, 1, also had the lowest vocabulary word usage on the quizzes, 0.7 words per problem. This shows that on an individual level, that a student who does not use exact vocabulary words to explain their work, they will not do as well.

Student Work 1: Greg, scored 1



The image shows a student's handwritten work on lined paper. The text is written in cursive and is difficult to read. It appears to be a student's attempt to explain a calculation. The first line says "I just find 2 by 2.75 and divided the a". The second line says "I got 2.75 when I divided". The third line says "by 2". The handwriting is messy and lacks proper punctuation and capitalization.

Notice that Greg did not use any vocabulary words. His writing is also difficult to follow and gives no explanation as to why he performed the operations that he did. He did not get the problem correct and earned a rubric score of a one.

On the other hand, Pennie, the student who had the highest average writing score, 3.89, also had the highest vocabulary word usage on the quizzes, 6.89. Notice that she uses multiple vocabulary words and her explanation is much easier to follow.

Student Work 2: Pennie, scored 4



$A = b \times h$  For problem  
 $A = 2.75 \text{ cm} \times 2 \text{ cm}$  number four, my  
 $A = 5.5 \text{ cm}^2$  answer for the  
area of the parallelogram  
is  $5.5 \text{ cm}^2$ . I received this  
answer by using the formula  
 $A = b \times h$ . So, I multiplied  
the ~~base~~ of the parallelogram  
and multiplied it by the ~~height~~  
of the parallelogram, which is  
 $2.75 \times 2$ . Since area equals  
~~base of a parallelogram~~ multiplied  
by the height of a parallelogram,  
the area is  $5.5 \text{ cm}^2$ .

In Pennie's writing, she used nine specific vocabulary words, which are highlighted. Her explanation is thorough and correct, which earned her a score of a four. She also labeled her answer correctly so that her solution made sense.

For these two specific students, I think personality played a big part in their work. Pennie tends to rise to the occasion while Greg tends to take less of a risk so that he does not fail. Both pieces of writing are the same assignment. Pennie's writing incorporates multiple vocabulary words from the textbook. She explains her process very well and earned a rubric score of a four.

I also grouped the students by the rubric score they earned, and then looked to see how many words those students used. I found this to be a very important piece of data. I really think it shows that the more vocabulary words they incorporate into their writing, the more exact their explanations are, therefore, they earn a higher score on the rubric. The following table shows the number of words used on each quiz, according to the

students' rubric score on that quiz. Notice that in general, the higher the rubric score, the more words that were used. On any quiz, the scores generally get higher from left to right.

Table 2: Number of Vocabulary Words Used with respect to Rubric Score				
	Number of vocab words used by students who scored a 1	Number of vocab words used by students who scored a 2	Number of vocab words used by students who scored a 3	Number of vocab words used by students who scored a 4
Chapter 8, Quiz 1	1	1.64	5	n/a
Chapter 8, Quiz 2	0	.83	0	4
Chapter 9, Quiz 1	0.5	1.25	1	3.33
Chapter 9, Quiz 2	.6	1	1.33	3
Chapter 10, Quiz 1	5	3.25	4.17	5
Chapter 10, Quiz 2	3.25	6.2	7	n/a
Chapter 11, Quiz 1	1	.5	1.25	3.75
Chapter 11, Quiz 2	1.4	2	6	5.25
Chapter 11, Quiz 3	1.5	1.67	2.5	7
Chapter 12, Quiz 1	2	2	5.1	10

At the beginning and end of every chapter, I gave the students a quiz over the definitions of the vocabulary terms from that chapter. Their knowledge of the terms increased dramatically. I was a bit disappointed that the post-test scores were not as high as I would have hoped. I would have liked to see more perfect scores on the posttests, but since this is a new form of assessment, I have to give them time to get used to it. I also think that since the text I teach from is one grade level higher than the students, some words are difficult for them to understand. They have had prior exposure to words such as circle, center, and equilateral triangle. However, words such as polygon and quadrilateral were very new to them. The following table shows the average pre and posttest scores for

the five chapters I taught throughout the project. Notice that even though the posttest scores were not perfect, they were much better than the pretests. This means that there was knowledge of the textbook vocabulary words being attained by the students.

Chapter number and concepts	Pretest mean	Post test mean
Chapter 8 Ratios, proportions	3%	59%
Chapter 9 Percents	21%	67%
Chapter 10, Part 1 Geometric Figures	23%	80%
Chapter 10, Part 2 Geometric Figures	21%	76%
Chapter 11 Measurement, Area	18%	81%
Chapter 12 Surface Area, Volume	16%	65%

After looking at chapters individually, I looked at the mean of all of the chapters. The pretest mean for all chapters was 17% correct, while the post test mean for all chapters was 72%. This shows a 55% increase in students' knowledge of vocabulary terms. I think the general trend of the data shows great improvement. It is important to mention where the data does not agree with the trend. I think that by the posttest of chapter 12, the students were mentally exhausted from the school year. They began putting less importance on school as the countdown to the end of they year got smaller. This was a problem for me as a teacher because the beginning of the school year when students seem to put the most effort into their work, the curriculum is mostly a review of concepts. At the end of the school year, the students motivation seems to have tapered off, however, the concepts have grown harder and require more effort by the students, (Appendix 6).

Throughout the research project, I have also seen improvements by individual students. I have one student, Paul, who generally does little homework and pays minimal attention during class. However, since beginning a focus on vocabulary, he has participated in class activities and used his vocabulary book. He improved his vocabulary posttest from chapter 8 to chapter 9 to chapter 10. In fact, from the vocabulary pretest to the posttests, he improved an average of 46%. He also improved his writing scores from 1 to 4. I have also noticed him, as well as other students, using the vocabulary words more when discussing problems or asking questions about problems.

Below are examples of his writing from the beginning of the project, middle of the project, and end of the project. Notice that his pre writing sample is one sentence and earned a rubric score of a one. He did not explain his thinking process or his mathematical work.

Student Work 3: Paul, scored 1

(Pre-writing sample)

At a market, a pound of salmon costs \$6.00 and a pound of shrimp costs \$12.00. Mr. Hart wants to buy  $2\frac{1}{3}$  pounds of salmon and  $3\frac{1}{2}$  pounds of shrimp. If he has \$55.00, can he buy the amounts of salmon and shrimp that he wants? Explain your answer.

$\begin{array}{r} 12SA \\ \hline \$14 \end{array}$	$\begin{array}{r} 9H \\ \hline \$42 \end{array}$	$\begin{array}{r} \$42 \\ +\$14 \\ \hline 56 \end{array}$
--	--	---

NO he cant buy the amount because hes short 1 dollar.

Notice that he did very little mathematical work on paper and had very little writing to explain his work. Although he did the problem correctly, there was no flow of his thought process to follow.

His writing sample from the middle of the study shows progress. He earned a rubric score of a four and did a much better job explaining what he did to solve the problem. He used several vocabulary words and his explanation flowed much better than in the beginning.

Student Work 4: Paul, scored 4

①

$$\begin{array}{r} 4 \\ 180 \\ \times 6 \\ \hline 720 \end{array}$$

1080°

The way I solved is by making an polygon and started at the top left corner. Then I went to every corner and made triangles. And each triangle equals 180° so 4 times 180° by 6 and got 1080° and that was my answer.

Notice that his writing after only two months has definitely improved. He has more complete sentences, as well as more sentences. His thought process is more apparent due to his drawing.

His post-writing sample shows even more progress. He explained the mathematical work he did and why. His work was done correctly and he was better at explaining his solution.

Student Work 5: Paul, scored 4

Mindy can type 248 words in 4 minutes. Find the unit rate. Then find how long it would take Mindy to type 558 words. Explain your answer.

$$\begin{array}{r}
 62 \\
 4 \overline{) 248} \\
 \underline{24} \phantom{8} \\
 8
 \end{array}$$

Unit rate = 62 words per minute

$$\begin{array}{r}
 9 \\
 62 \overline{) 558} \\
 \underline{55} \phantom{8} \\
 8
 \end{array}$$

$$\begin{array}{r}
 62 \\
 \times 9 \\
 \hline
 558
 \end{array}$$

$$\begin{array}{r}
 62 \\
 \times 9 \\
 \hline
 434
 \end{array}$$

$$\begin{array}{r}
 62 \\
 \times 9 \\
 \hline
 496
 \end{array}$$

To get the unit rate I did 248 divided by 4 and got 62. So the unit rate equals 62. Then I divided 558 by 62 because 558 is the number of words and 62 is the unit rate. And I got 9 minutes. So she can type 558 words in 9 minutes.

Notice that his writing is more explanatory and well written. He shows more work and that makes it easier to follow his process. There was a dramatic increase in his level of written communication in only four months.

I made mention of this student in three of my ten journals as well. In journal six, March 10-14, 2008, I mentioned that he had been gone and did not have knowledge of the new vocabulary words, yet he participated and tried his best. I said, "It was very exciting to see him actively participating in the review game. He was urging other students to give their answer or even shouting out the correct word to help out. WOW!"

This happened when I challenged the class to play the review game with a time limit.

Adding a competition element to the game seemed to increase his motivation.

In journal seven, March 17-21, 2008, the following week, I noticed again that the same student had worked hard on his vocabulary book, participated in reviews, and paid more attention in class. At this point, I began to really consider the improved attitude of this student and what the causes of it were. I can never really know what that student was thinking, but I believe that a new class focus was part of the improvement.

A very striking piece of evidence is that fifteen out of fifteen students have incorporated vocabulary into their writing. Every student has used vocabulary in his or her writing at some point. I have given ten quizzes and required students to write on all of them. The following chart shows how many times students have incorporated mathematical vocabulary into their written explanation. For example, three students used vocabulary on their quizzes five out of ten times. The two students who did not complete all ten quizzes used vocabulary words on nine out of nine and five out of nine.

On how many quizzes have the students used vocabulary?	How many students?
1	0
2	0
3	0
4	0
5	3
6	0
7	2
8	3
9	4
10	1

In addition to the data from my point of view, I wanted to add data from the students' point of view. I wanted to know how they felt about the use of mathematical vocabulary. When surveyed on March 13, 2008, eight out of fifteen students said that the vocabulary square helps them with writing. Another five out of fifteen students said that they were not sure. That means that only two students felt that the vocabulary square was not beneficial to them when writing. Then on the survey given on May 13, 2008, at the end of the project, seven out of fifteen students said that the vocabulary square helped them on their writing. Only six out of nineteen said their were not sure if it helped or not. The last two students did not feel that the vocabulary square helped them when writing. To me this shows that students value the use of the vocabulary square and the learning of the mathematical vocabulary itself.

During the interviews in March of 2008, students made several comments supporting the fact that vocabulary helps them write more descriptively. When asked, "Does the vocabulary square help you with writing?" Ryan said, "Yes because if you explain something, there is a word you can use that you didn't know before." Kaden agreed and stated, "You get to remember the definition and instead of using shorter and other words, you can use smarter words." In another groups, Pennie said, "It helps you understand it so that you can write it in a sentence." Also, when asked, "How does your knowledge of vocabulary help your writing in math?" Adam said, "If you know the words, you'll be able to explain it a lot better." Information gathered at the interview doesn't necessarily mean that students are using the words, but it shows that they understand the importance of using correct mathematical words and are trying to incorporate them into their writing.



At the end of the project, instead of interviewing the students verbally, I asked them open-ended questions on paper. When I asked if they liked writing about math problems, seven out of fifteen students responded no, which seems like it would be discouraging, but it is not. When they answered why they do not like writing, four of the nine students mentioned that it took too much time, while the other three said that they did not like writing in general. To me, these reasons are not based on mathematics or on my research. The reasons that students gave for not liking the vocabulary book were rooted in their educational background. The four students who said they thought the vocabulary books took too much time are students who typically rush through their work and want to be done as quickly as possible, regardless of the outcome. I think that as I used the vocabulary books again, I could resolve the time issue. In addition, the three students who do not like writing are students who do not do well when writing in their Language Arts class. Specifically, one student is gifted in math and has a learning disability in writing.

I even mentioned this issue in my eighth journal, March 24-28, 2008. I said, “I think a lot of students just don’t like to write, which deters them from being able to fully explain a problem in math.” On the positive side, seven out of fifteen students said they liked writing about math. Of those seven, six said it was because it helped them understand better, while the other one said it was fun. The last student answered both yes and no to the questions. Nathan agreed with other students that it did take a lot of time, but again, it helped with understanding.

On the same written interview, I asked what connection students saw between writing about math and being good at math. Nine out of the fifteen students said that it

helped them understand the concepts better when they wrote about it. Three additional students said that it helped them learn to describe the mathematical process better. In fact, Paul said, “To show you know about math, you have to be able to write about it.” This supports my belief that in order to really understand an idea, you need to be able to explain it to someone else. This is basically what my students practiced doing throughout this study.

On the written expression prewriting, that I used to gauge their prior skills, the mean score of the students was 1.73, on a scale of one through four. This shows that they did not have a good concept of how to explain a solution in writing. I looked at this a great opportunity to expand their knowledge and written fluency. I feel that I was successful in this endeavor because the post writing yielded a mean score of 3.13. This growth of 1.4 points may not seem like much, but with a high score of a four, that does mean a lot. In four months, thirteen out of fourteen students improved their writing. Considering students only had ten writing assignments in those four months, I feel that is a real accomplishment. The following chart shows the number of students with each score on the beginning and end writing samples.

	Scored a one	Scored a two	Scored a three	Scored a four
Pre writing sample	4	8	2	0
Post writing sample	0	3	7	5

The following are pre and post writing samples done by two students. The first student is Nathan who said that he did not like writing in math class. It is important to note that even though he said he did not particularly enjoy it, he still grew to be a better

mathematical writer. Notice that in his pre writing sample, he only described, very minimally, the mathematical process.

Student Work 6: Nathan, scored 2

(Pre-writing sample)

At a fish market, a pound of salmon costs \$6.00 and a pound of shrimp costs \$12.00. Mr. Hart wants to buy  $2\frac{1}{3}$  pounds of salmon and  $3\frac{1}{2}$  pounds of shrimp. If he has \$55.00, can he buy the amounts of salmon and shrimp that he wants? Explain your answer.

$$\frac{6}{1} \cdot \frac{7}{3} = \frac{42}{3} = 14 \quad \frac{12}{1} \cdot \frac{7}{2} = \frac{84}{2} = 42 \quad \begin{array}{r} \$14 \\ +42 \\ \hline \$56 \end{array}$$

No because  $\$6.00 \cdot 2\frac{1}{3} = 14$  and  $\$12.00 \cdot 3\frac{1}{2} = 42$  and  $\$14 + \$42 = \$56$  which is over \$55.00.

Although he does show some mathematical work, he does not explain what process he used to get to his solution. Nathan just gives his solution and feels that is enough. Then in his post-writing sample, he explains his thoughts during the mathematical process.

Student Work 7: Nathan, scored 4

(Post-writing sample)

Mindy can type 248 words in 4 minutes. Find the unit rate. Then find how long it would take Mindy to type 558 words. Explain your answer.

$$\begin{array}{r} 62 \\ 4 \overline{)248} \\ \underline{24} \phantom{0} \\ 88 \\ \underline{88} \\ 0 \end{array}$$

62 words per min

$$\begin{array}{r} 9 \\ 62 \overline{)558} \\ \underline{558} \\ 0 \end{array}$$

558 words in 9 min

In the problem it told me to find the unit rate of how she can type. So I set up the problem 248 divided by 4 because 248 is the amount of words and 4 is the time. After dividing 248 and 4 I ended up with 62 words per min. Next I had to find how long it would take her to type 558 words. So I then divided 558 by 62 because 558 is the amount and 62 is how

fast she types, and I got 9 mins. So my two answers were 62 words and 9 min.

Notice what a dramatic difference there is between his pre and post writing samples. For the post-writing sample, he gave a very thorough explanation that shows his

thought process and is fairly easy to follow. He uses the correct terminology and labels his work.

The second student, Brooke, did explain her mathematical process in the pre-writing sample, but it was not mathematically correct. Although she uses complete sentences, her work is not easy to follow.

Student Work 8: Brooke, scored 1

(Pre-writing sample)

At a fish market, a pound of salmon costs \$6.00 and a pound of shrimp costs \$12.00. Mr. Hart wants to buy  $2\frac{1}{3}$  pounds of salmon and  $3\frac{1}{2}$  pounds of shrimp. If he has \$55.00, can he buy the amounts of salmon and shrimp that he wants? Explain your answer.

what I did was make  $2\frac{1}{3}$  into a mixed fraction. and then I turned the  $3\frac{1}{2}$  into a mixed number. then I multiplied by  $\frac{7}{3} \cdot \frac{10}{2} = \frac{70}{6}$   
 $\frac{7}{3} \cdot \frac{6}{1} = 14.00$ .  $\frac{10}{2} \cdot \frac{12}{1} = \frac{120}{2} = 60.00$   
 $60.00 + 14.00 = 74.00$  so no he will not have enough money for both

By looking at her eraser markings, she obviously was unsure of the correctness of her mathematical process. She attempted multiple ways of solving this problem and was still unsuccessful.

Student Work 9: Brooke, scored 4

(Post-writing sample)

Mindy can type 248 words in 4 minutes. Find the unit rate. Then find how long it would take Mindy to type 558 words. Explain your answer.

What you would do to find the unit rate which means how many words Mindy can type in one minute. To solve for the unit rate you would do the 248 words divided by the 4 min.

$$\begin{array}{r} 62 \\ 4 \overline{)248} \\ \underline{24} \phantom{0} \\ 8 \phantom{0} \\ \underline{8} \phantom{0} \\ 0 \phantom{0} \end{array}$$

$248 \div 4 = 62 \text{ wpm}$ . She can type 62 wpm. Now to see how long it takes for her to type 558 words you would do  $558 \text{ w} \div 62 \text{ w} = \underline{\quad} \text{ min}$ . I did this because if she can type 62 wpm that's our unit rate. When you need to find minutes or hours you either multiply or divide with whichever is easiest. In this case dividing. So when I do  $558 \dots \div 62 \text{ w}$ , ~~since~~ I didn't have a calculator

I had to estimate I started off by doing  
 $\frac{62}{5}$  and got 300 so I knew it had  
 to be a higher next I tried 7.  
 $\frac{62}{7}$  and got 434. It was still too low.  
 So I tried  $9 \times 9 = 81$  and got 558 mins. then  
 I put 9 into my division problem  $\frac{558}{9}$ . She  
 can type 558 words in 9 mins. She can also  
 type 62 words in 1 min.

Her post-writing sample does a much better job of explaining the mathematical process and her own thoughts. Notice that just by the length, it is obvious that she was trying hard to thoroughly explain her thought process. She includes her mathematical work in her writing, which helps make her work easier to follow.

Overall, I saw a great amount of growth in my students writing abilities during second semester. They used vocabulary words when writing to make their work more precise. This was the outcome I tried to achieve in this study.

### **EFFECTS ON ACHIEVEMENT**

When researching the question, to what extent does student use of exact mathematical vocabulary correlate with student mathematical understanding, I noticed that students understand concepts better when they learn and use correct vocabulary. In

fact, I found that in general, students' grades have gone up since beginning this study. I found evidence in multiple forms of data to support this assertion.

On the survey given on March 13, 2008, ten out of fifteen students said that knowing vocabulary words helps them understand the mathematics more. Two more students said they were not sure if the vocabulary helped them. Then on the survey given on May 13, 2008, again, ten out of fifteen students said that knowing the vocabulary words helps them understand more, while four more said they were not sure. This means that two out of the fifteen students who did not feel that the vocabulary square helped them, now think that it might.

When interviewing students in March 2008, most students agreed that knowing the vocabulary helped them understand how to solve mathematical problems. When asked why the students think I feel knowing vocabulary is important, Seth said, "It's a lot easier to understand words problems when you know what the words mean." In a separate interview, Paul said, "It will help us understand what we are doing." Pennie gave an example of how vocabulary helps, by stating, "On the test they ask you to find the adjacent and supplementary angle and if you don't know what that is, you'll probably get the problem wrong." Another thoughtful comment made by Pennie was, "The vocabulary helps you solve the problem." All of these comments help me to know that learning vocabulary was important to the students.

In my fourth journal, February 25-29, 2008, I said, "I think understanding of mathematical concepts is increased when the vocabulary associated with the concept is directly taught and focused on. This idea is strengthened everyday when my students are achieving more than they did before. I really do think the more we use the words, the



more they understand them, and the more they understand the relationships between them.” I noted this because I saw improvement in conceptual understanding based on quiz scores. In the first quiz for chapter 9, fourteen out of fifteen students passed the quiz. In fact, eight out of fifteen students got 100%. The material covered on this quiz was proportions, which is generally difficult for 6<sup>th</sup> graders to set up and solve. The students would not have been able to pass if they did not know the vocabulary words at all. I mentioned the proportion lesson earlier and noted that the concept was made easier because students were able to connect the concept to equivalent ratios, which is a concept they already knew.

In journal five, March 3-7, 2008, I wrote that the post vocabulary quizzes were done well. I have a few students who invest very little effort into class and even they did better than I would have thought. I think that students who put little effort into school are used to just sitting and daydreaming, whereas during my research, they were expected to write in their vocabulary books, which they did. Also, in my journal, I have noticed that students understand the material better when they see how the concepts relate. Students are able to correlate new words to words they already know because this is a literature skill that they are taught early on in life. However, math is often taught in isolation, so it is hard for students to use multiple concepts in one problem.

One example of connecting vocabulary words is when I taught cross products during chapter 8. Right away, students saw that you just multiplied across, but they did not see how this really worked. I related cross products back to equivalent ratios, which they already knew. This gave them the opportunity to connect new knowledge to prior knowledge. I noted in journal two, February 11-15, 2008, that cross products is always a

little confusing to students. They typically understand how to solve, but really never know why. “Since we had discussed in more depth what a ratio was and how that related to a proportion, they quickly and more deeply understood cross products.” This made the entire chapter have more cohesion than before, due to the depth at which students learned the vocabulary. Since the vocabulary words were connected, so to were students’ thoughts, and therefore so were the mathematical concepts. Then in chapter 9, when I taught percents, fractions, and decimals. This chapter was much easier for the students because they already understood why cross products works. They quickly saw that you could use a proportion to convert between percents and fractions.

When I have spoken with students informally, helping with a problem and such, I have noticed them using more of the vocabulary words. Instead of saying, “I don’t get it,” or “How do I solve this?” they have begun asking, “Should I use a proportion?” or “Will you help me set up this proportion?” This gives me a better idea of what the student is not understanding so I can better help them. I was not looking to elicit this outcome in my research, so I was pleasantly surprised to notice it.

Another question I asked students on during the post interview was what connection they saw between knowing mathematical vocabulary and being good at solving words problems in math class. Eleven of the fifteen students responded by saying that it helped them understand the problem better or know how to solve it. Two more students said that it made the problem easier. These results were encouraging because I knew that the focus on vocabulary was having a positive impact on my students.

The last important piece of quantitative data to show that students increased mathematical knowledge when instruction is focused on vocabulary is the improvement

in grades. Thirteen out of fifteen students have improved their classwork grade since beginning this study. In fact, eight out of fifteen students have improved by 10% or more, which is an entire letter grade; for example, from a B to an A. On quizzes, twelve out of fifteen students improved their grades. Surprisingly, seven of the fifteen students have improved by more than 5%, which is half of a letter grade; for example, from a B to a B+. On tests, only nine out of fifteen students have improved their test scores. Lastly, ten out of fifteen students improved their criterion-referenced test. This improvement is important because the most difficult concepts come at the end of the year.

If students increased their grade from the beginning of the year to the end of they year, that shows much more understanding than if their grade decrease throughout the school year. I also noted in my fifth journal, March 3-7, 2008, that the students who were making the least progress were students who had attendance problems. “When they are absent, it doesn’t give them opportunity to interact with the curriculum.” The following table shows the mean of each of the criteria that I used to evaluate, before in study began and after.

	Before study	After study
Classwork	72.73%	82.74%
Quizzes	83.39%	86.88%
Tests	85.23%	85.38%
CRT	88.93%	91.11%

Another piece of qualitative data to show is that I asked students what connection they saw between knowing mathematical vocabulary and being good at math in general. Thirteen out of fifteen of them said that knowing what the words mean helps them understand math concepts better. Paul said, “Knowing vocabulary is everything in math.”

Coming from a student who is very talented, but does not often achieve it, this really means a lot to me. Another student, Peter, said, “If your good at the math, you probably know the vocabulary.” I thought this statement illustrated the same idea, but in reverse. Lastly, Holly said, “To know math, you need to know the vocab.” I felt like these students in particularly really got to the heart of my research.

Finally, when I asked students how they felt about the changes in class from first semester to second semester, fifteen out of fifteen students had positive comments. Six of them said it made class more fun, while three more said the changes were good. I had three students say that it was harder second semester, however, they were referring to the concepts in the curriculum, which is definitely true. One student noted that they became a better student and a better writer, while the other two students said they remember the concepts better and longer than before. Ryan noted that, “I’ve been learning more, better than I have before,” which describes how I hoped the students would feel about this study.

## **CONCLUSIONS**

After looking at all of the information from my study, I believe that when students receive direct vocabulary instruction, they are able to use those terms when communicating with others in writing. I believe students need to interact with the words in order to put them in their personal vocabulary. I was able to give them this opportunity through the use of vocabulary books. Since they write each word, its definition, examples of problems, and memory tricks to help them remember each word, they have multiple opportunities to use and interact with the word. I also have the students help decide what will go into the book. They have enjoyed using their own names when making up story

problems. I think this allows the students to relate the words to themselves and their interests, so they remember the words better. I had always thought that by serving up knowledge on a silver spoon, I was giving my students what they needed to know, but this gave me the opportunity to turn the thinking over to my students. This gave the students the chance to discover the mathematics that they were learning, hopefully resulting in long-term retention.

I also believe that when students have multiple modes in which to interact with the vocabulary, they are more likely to remember the words. Students use visual, kinesthetic, verbal, mathematical, interpersonal, and intrapersonal intelligences when learning the words. I think this has helped all students be able to retain the words. It seems as though no matter what type of learner they are, they were able to acquire the words because so many of the intelligences were used.

Students are able to achieve more when they understand the vocabulary associated with the mathematical concept. In the same way that we connect new words to known words, students are connecting new mathematical concepts to known concepts. This enables students to use multiple concepts on one problem. This is such an important skill as the students get older. With each year of school, students are expected to incorporate multiple concepts together to solve problems. The sooner students see the connections between mathematical concepts, the better off they will be.

After having direct vocabulary instruction, students have more confidence going into a summative assessment. Students realize the importance of knowing words and feel that if they know the words, the work will be easier because they will know more about

the mathematical concept that they are working on. If students feel that they can do better, they probably can.

### **IMPLICATIONS**

I cannot promise that I will continue to use the vocabulary books in the future because they were very time consuming. I feel that at times, I spent much more time giving direct vocabulary instruction rather than doing guided practice or helping students individually. However, I know that vocabulary will remain a major focus in my classroom. I will continue to use the “I am... I have...” review activity, as well as having students write about their work. I have found that the student thoroughly enjoyed the review activity and although some did not want to admit it, their written solutions helped them understand the mathematical process better and in turn improved their achievement.

Overall, this has been such a great learning experience for myself, as well as my students. They have enjoyed trying a new educational strategy and have obviously grown from it. I have not only learned about my study, but have learned the action research process, which I will be able to carry on with me onto new ideas.

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**4**

- Complete correct response with strong supporting details
- Reasoning is connected to mathematical knowledge
- Precise mathematical language and notation used
- Uses details that make sense, signals words, and paragraphing if necessary
- May include accurate visual aid, examples or nonexamples

**3**

- Correct or incorrect response with some supporting details
- Reasoning is somewhat connected to mathematical knowledge
- Some mathematical language and notation used
- Uses some details and signal words, but may contain minor gaps
- May include inaccurate visual aid

**2**

- Correct or incorrect response with details which may not support
- Reasoning is mathematical, but not connected to topic
- Little mathematical language and notation used
- Few details which may not help explain topic
- Visual aid is not related to topic

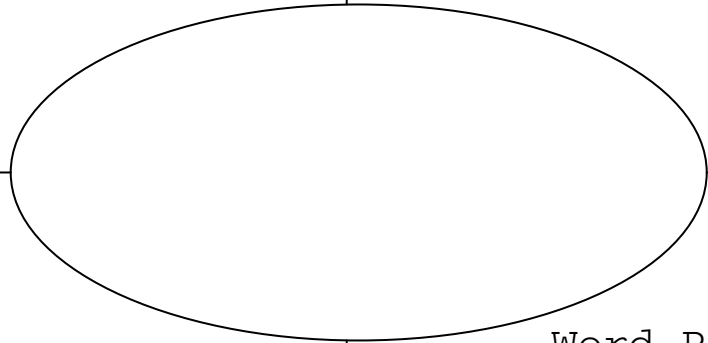
**1**

- Incorrect response with minor details which do not support
- Little or no connection to mathematics
- Familiar, everyday language used
- Confusing or inaccurate details
- Little or no visual aid

**Appendix 1**

Definition:

Example:



Memory Trick:

Word Problem Example:

**Appendix 2**

Name \_\_\_\_\_

## Student Survey

Circle your answer.

1. I like math.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
----------------------------------	----------------------------------	----------------------	-------------------------------	----------------------------

2. I am good at math.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
----------------------------------	----------------------------------	----------------------	-------------------------------	----------------------------

3. I understand vocabulary words in math.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
----------------------------------	----------------------------------	----------------------	-------------------------------	----------------------------

4. Knowing vocabulary words helps me understand the math work more.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
----------------------------------	----------------------------------	----------------------	-------------------------------	----------------------------

5. I like using the vocabulary square.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
----------------------------------	----------------------------------	----------------------	-------------------------------	----------------------------

6. The vocabulary square helps me remember what the words mean.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
----------------------------------	----------------------------------	----------------------	-------------------------------	----------------------------

7. I hope my math teacher next year uses the vocabulary square.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
----------------------------------	----------------------------------	----------------------	-------------------------------	----------------------------

8. All of next year Mrs. Georgius should use the vocabulary square with her math class.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
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9. The vocabulary square helps me with my writing.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
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10. I like writing in math.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
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11. The writing we do in math helps me understand the math more.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
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12. Knowing more math vocabulary helps me solve word problems.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
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13. It is important to know the meaning of math vocabulary words.

<b>1</b> Strongly disagree	<b>2</b> Somewhat disagree	<b>3</b> Not sure	<b>4</b> Somewhat agree	<b>5</b> Strongly agree
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### Appendix 3

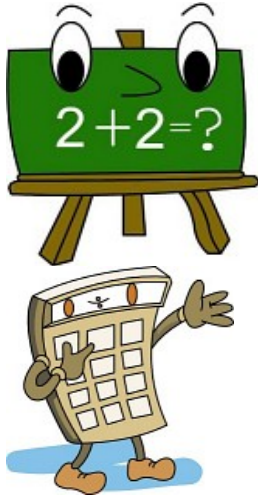
Names \_\_\_\_\_

Date \_\_\_\_\_ Time \_\_\_\_\_

## Student Interview

1. What do you like best about math?
2. What do you like least about math?
3. Why do you think that I tell you it is important to know the meanings of vocabulary words in math?
4. Do you like working word problems? Why or why not?
5. Do you like writing about math problems? Why or why not?
6. Why do you think I have been asking you to write about math problems?
7. Why do you think that I tell you it is important to write and explain your answers in math class?
8. Why do you like or dislike the vocabulary square?
9. How does the vocabulary square help you with writing?
10. How does your knowledge of vocabulary help your writing in math?
11. What connection do you see between knowing math vocabulary and being good at solving word problems in math?
12. What advice would you give me about using vocabulary squares and writing next year in my math class?
13. What do you hope your math teacher next year does with vocabulary and writing in math class?
14. Is there anything else I should know about you to better understand your problem solving in math or your general math experience?

### Appendix 4



What did you do?

Why did you do  
it?

What does your  
solution mean?



## **Appendix 5**

### Chapter 8

ratio  
equivalent ratio  
rate  
unit rate  
slope  
proportion  
cross products  
scale drawing  
scale  
scale model

### Chapter 9

percent  
circle graph  
interest  
principal  
simple interest  
annual interest rate  
balance  
ray  
angle  
vertex  
degrees

## **Appendix 6**

### Chapter 10

acute angle  
right angle  
obtuse angle  
straight angle  
complementary  
supplementary  
adjacent angles  
vertical angles  
congruent angles  
plane  
parallel lines  
intersecting lines  
perpendicular lines  
corresponding angles  
acute triangle  
right triangle  
obtuse triangle  
congruent sides  
equilateral triangle  
isosceles triangle  
scalene triangle  
quadrilateral

trapezoid  
parallelogram  
rhombus  
polygon  
pentagon  
hexagon  
heptagon  
octagon  
regular polygon  
similar polygons  
congruent polygons  
transformation  
image  
translation  
reflection  
line of reflection  
rotation  
center of rotation  
angle of rotation  
line symmetry  
line of symmetry  
rotational symmetry

### Chapter 11

square root  
perfect square  
radical expression  
hypotenuse  
legs  
base of a parallelogram  
height of a  
parallelogram  
base of a triangle  
height of a triangle  
bases of a trapezoid

height of a trapezoid  
circle  
center  
radius  
diameter  
circumference

Chapter 12

solid  
prism  
pyramid  
cylinder  
cone  
sphere  
faces  
edges  
vertex  
surface area  
net  
volume