1-1-1975

FARADAY'S LAW

Module ______

STUDY GUIDE

FARADAY'S LAW

INTRODUCTION

Consider the electric light you may be using to read this module by and the influence on your life style of the vast amounts of electrical energy produced in the United States. This module treats the fundamental principle that allows for the transformation of mechanical energy into electrical energy. The physical law that governs the production of electric current is named after its discoverer, Michael Faraday.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate scalar products (needed for Objectives 1 through 3 of this module)	Vector Multiplication Module
*Use Ohm's law to determine the current, knowing the resistance and voltage (needed for Objective 3 of this module)	Ohm's Law Module
*Given the current direction, determine the direction of the magnetic field; or given the direction of the magnetic field, determine the direction of the source current (needed for Objectives 2 through 4 of this module)	Ampère's Law Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Faraday's law - Write the equation for Faraday's law in the form $\mathcal{E} = -d\phi/dt$ and define all terms with correct units.

2. Magnetic flux - Determine the magnetic flux or the time rate of change of the magnetic flux for an area in a magnetic field.

3. Application of Faraday's law - Determine, using Faraday's law, the induced current and/or voltage for a situation involving either (a) a stationary circuit in a time-varying magnetic field, or (b) a conductor moving in a magnetic field.

4. Lenz's law - Apply Lenz's law to a situation as listed in Objective 3 to determine the direction of the induced current and explain your reasoning.

GENERAL COMMENTS

1. Faraday's Law

In this module you will be held responsible for the following form of Faraday's law:

$$\mathcal{E} = -d\phi/dt,$$

where the magnetic flux is

$$\phi = \oint \vec{B} \cdot d\vec{A}.$$

The induced emf is produced by the time rate of change of the magnetic flux. This is the fundamental principle of the generation of electrical energy. This form of Faraday's law allows us to calculate the induced emf caused by a time-varying magnetic field or a moving circuit in a magnetic field.

What is the induced emf? The voltage induced in a circuit is produced by an electric field (force per unit charge) acting over the current path (circuit):

$$\mathcal{E} = \oint \vec{E}_n \cdot d\vec{\ell}.$$

This is the work required to move a unit charge around a complete circuit. In this case, the force that produces the electric field is magnetic in origin and therefore produces a nonconservative force or electric field E_n. The integral of a nonconservative force per unit charge over a complete path is not equal to zero but, rather, is equal to the work done per unit charge, which is the induced emf. Mechanical work is required to produce a current in a magnetic field. The operation of the Betatron is based on this equation.

The complete form of Faraday's law is

$$\int \vec{E}_n \cdot d\vec{\ell} = -d\phi/dt,$$

where the left-hand side is the induced emf and the right-hand side tells you how to calculate the induced emf. This form suggested that there is a very intimate relationship between electric and magnetic fields.

Why have we chosen to test you on the first listed form of Faraday's law? In a certain sense, the decision is arbitrary because a relativistic treatment of electric and magnetic fields shows that they are merely different perspectives of the same quantity, which depends on the velocity of the observer. The reason for using the form $\mathcal{E} = -d\phi/dt$ is that it is more useful for a course at this level. A future module Inductance is based on this form of Faraday's law. The complete form of Faraday's law is very powerful, but its usefulness is more readily seen in an advanced course in electricity and magnetism.

2. Lenz's Law

The minus sign in Faraday's law can best be explained by the use of Lenz's law. Lenz's law, or the minus sign in Faraday's law, is merely a consequence of the conservation of energy. It tells us that we must do mechanical work to rotate a loop of wire in a magnetic field to produce electrical energy.

Objective 4 requires that you explain your reasoning for determining the direction of the induced current. It is suggested that you answer this question in terms of the first cause; for example, rotating a coil in a magnetic field requires a torque, and your answer should be in terms of the direction of the induced current that would oppose this rotation.

3. Units

You have experienced the confusion produced by different sets of units, and magnetic-field units are perhaps the worst. Textbooks use a variety of units, but this module (Problem Set, Practice Test, and Mastery Tests) will use webers (Wb) for magnetic flux, and teslas (1 T = 1 Wb/m^2) for magnetic fields. From Faraday's law it is possible to show that one volt = one weber per second. You may wish to prove this for yourself.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

First read General Comments 1 through 3 and then read Sections 25.1 and 25.6 in Chapter 25 of your text. Sections 25.2 through 25.5 will be treated in the module Inductance.

The text's treatment of Faraday's law [Eq. (25.2)] and Lenz's law is brief, but adequate. Study Problems A through E and Illustrations 25.1 and 25.6 (first half only) before working Problems F through I and 1, 3, 8, 9, and 21 in Chapter 25. Take the Practice Test before attempting a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text (Chap. 25)
1	Sec. 25.1	A			
2	Sec. 25.6	B		F	21
3	Secs. 25.1, 25.6	C, D	Illus.[a] 25.1, 25.6 (first half)	G, H	1, 3, 8
4	Sec. 25.1	E		I	9

[a]Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

First read General Comments 1 through 3 and then read Chapter 31, Sections 31-1 through 31-4. Equation (31-1) gives Faraday's law. Section 31-5 is not required but does have a limited discussion of the relationship between time-varying magnetic fields and electric fields as mentioned in General Comment 1.

Study Problems A through E and Examples 1 and 2 in Chapter 31 before working Problems F through I and 3, 9, 16(a), (b), 17, and 20 in Chapter 31. Take the Practice Test before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Assigned Problems: Text (Chap. 31)
1	Secs. 31-1, 31-2	A			
2	Secs. 31-2, 31-4	B	Ex.[a] 1	F	
3	Secs. 31-1, 31-2, 31-4	C, D	Ex. 1, 2(b)	G, H	3, 9(a), 16(a), (b), 17, 20
4	Sec. 31-3	E	Ex. 2	I	9(b)

[a]Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

First read General Comments 1 through 3 and then read Chapter 33, Sections 33-1, 33-2, 33-4, and 33-5. See especially Eqs. (33-3) and (33-6). Your text derives Faraday's law by starting with the expression for the nonelectrostatic force per unit charge. Section 33-2, "The Search Coil," is an application of Faraday's law.

Study Problems A through E before working Problems F through I and the Assigned Problems from the text. Take the Practice Test before attempting a Mastery Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions	Assigned Problems	
		Study Guide	Study Guide	Text
1	Sec. 33-1	A		
2	Secs. 33-1, 33-2	B	F	33-8(a)
3	Secs. 33-1, 33-2, 33-4	C, D	G, H	33-6(b), 33-8(b), 33-9, 33-10, 33-15(c), (d)
4	Sec. 33-5	E	I	33-20, 33-21

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

First read General Comments 1 through 3 and then read Chapter 31, Sections 31-1 through 31-4. See especially Eq. (31-5). Although not required, your text does have a partial discussion of the relationship between time-varying magnetic fields and nonconservative electric fields in Section 31-5 and in particular on p. 632.

Study Example 31-1 and Problems A through E before working Problems F through I and the Assigned Problems from your text. Take the Practice Test before attempting a Mastery Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems	
		Study Guide	Text	Study Guide	Text
1	Sec. 31-1	A			
2	Secs. 31-2, 31-3	B	Ex.[a] 31-1	F	
3	Sec. 31-3	C, D	Ex. 31-1	G, H	31-1(a), 31-2, 31-8, 31-10(a), 31-12
4	Sec. 31-4	E		I	31-5

[a]Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(1). Write the equation for Faraday's law and define all terms with correct units.

Solution

$$\mathcal{E} = -d\phi/dt \quad \text{(Faraday's law)},$$

where $\mathcal{E}$ is the induced voltage (volts), ϕ is the magnetic flux = $\int \vec{B} \cdot d\vec{A}$ (webers), B is the magnetic field (webers per square meter, or teslas), and t is time (seconds).

B(2). A square wire loop (edge d = 0.200 m) is hinged as in Figure 1 so it can rotate about one edge. This loop is then placed in a $\vec{B}$ field of magnitude B_0 = 1.50 T, directed perpendicular to the hinged edge. Express the flux through the loop as a function of the plane of the loop angle θ with the magnetic field direction.

Solution

The flux is

$$\phi = \int \vec{B} \cdot d\vec{A},$$

but $\vec{B} \cdot d\vec{A} = B\,dA \sin\theta$ (careful with the angle), and B and θ are constant; therefore

$$\phi = B \sin\theta \int dA = BA \sin\theta = (1.50\ \text{T})(0.200\ \text{m})^2 \sin\theta = (0.060 \sin\theta)\ \text{Wb}.$$

If θ was 0°, then $\vec{B}$ and the plane of A would be parallel and there would be zero flux because the projection of $\vec{A}$ on $\vec{B}$ would be zero.

C(3). Determine the magnitude of the induced emf as a function of time for Problem F.

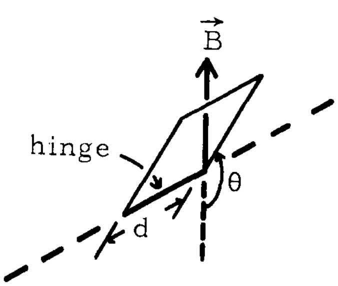


Figure 1

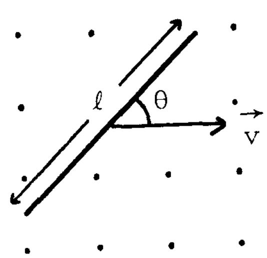


Figure 2

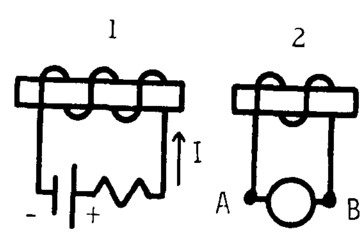


Figure 3

Solution

This is a situation of a time-varying magnetic field:

$$\mathcal{E} = -d\phi/dt.$$

We are interested in the magnitude of $\mathcal{E}$ and will therefore ignore the minus sign:

$$\phi = \int \vec{B} \cdot d\vec{A}.$$

$\vec{B}$ is a function of time but is uniform in space, therefore

$$\phi = \int \vec{B} \cdot d\vec{A} = B_0 L^2 e^{t/c},$$

$$\mathcal{E} = (d/dt)(B_0 L^2 e^{t/c}) = B_0 L^2 (d/dt) e^{t/c} \quad \text{or } \mathcal{E} = (B_0 L^2/c) e^{t/c}.$$

Check the units and remember that $e^{t/c}$ is dimensionless.

D(3). A straight wire, ℓ = 3.00 m long, which is part of a circuit, moves in the plane of Figure 2 as shown with a velocity $\vec{v} = 10.0\hat{i}$ m/s. If the angle θ is 30° and the magnetic field has a magnitude of B = 0.120 T and is directed perpendicularly out of the paper, what is the magnitude of the induced emf?

Solution

This is a situation of a moving conductor in a constant magnetic field. The conductor sweeps out an area per unit time of $\ell v \sin\theta$, therefore (ignoring the negative sign) Faraday's law is $\mathcal{E} = d\phi/dt$, where

$$\frac{d\phi}{dt} = \frac{d}{dt}\int \vec{B} \cdot d\vec{A} = B\ell v \sin\theta$$

and

$$\mathcal{E} = B\ell v \sin\theta = (0.120\ \mathrm{T})(3.00\ \mathrm{m})(10.0\ \mathrm{m/s})(1/2) = 1.80\ \mathrm{V}.$$

For $\theta = 0$, the wire would not sweep out an area. If you wish you may check how reasonable this answer appears in terms of the magnetic force on a moving charge $\vec{F} = q\vec{v} \times \vec{B}$ and determine whether $\theta = 0$ or $\theta = 90°$ would give the greater charge separation.

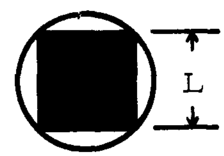
E(4). Use Lenz's law to determine the direction of the induced emf in coil 2 in the following cases and explain your reasoning (see Figure 3):
(a) 2 is moved toward 1.
(b) The current is decreased in 1.

Solution

(a) The cause of the induced current is the movement of coil 2 toward coil 1; therefore, the induced current must be in such a direction as to produce a force that will oppose this motion. Therefore, an induced emf from B to A through the meter will (using the right-hand rule) produce a magnetic field in coil 2 such that the magnetic interaction will be repulsive.

(b) The cause of the induced current is the decreasing current that produces a decreasing field in coil 2. The induced current must be in the direction to compensate for the decreasing current field produced by coil 1. This would be produced by an induced emf from A to B through the meter in coil 2 that would tend to compensate for the decreased field (using right-hand rule).

Figure 4



Problems

F(2). The uniform magnetic field in Figure 4 acts in the square shaded region and varies in time according to $\vec{B} = B_0 e^{t/c}\,\hat{k}$, where $B_0 = 3.00$ T, $c = 0.0100$ s, $L = 0.40$ m, and $B = 0$ elsewhere. Determine the magnetic flux at $t = 0$ within the circular wire.

1.00 sec.

G(3). A magnetic field is uniform in space but varies with time as $\vec{B} = B_0(1 + t/c)\hat{k}$, where $B_0 = 0.300$ T, $c = 1.00$ T and $\hat{k}$ points out of the paper. A wire with a 10.0-Ω resistance in the form of a square with sides $L = 0.50$ m is arranged such that the plane of the wire is perpendicular to $\vec{B}$. Determine the current in the wire at $t = 0$.

H(3). Determine the magnitude of the induced emf in Problem F at $t = c$.

I(4). Use Lenz's law to determine the direction of the induced emf in coil 2 for the same situation as Problem E in the following cases and explain your reasoning:
(a) Coil 1 is moved toward 2.
(b) Current is increased in 1.

Solutions

F(2). $\phi = B_0 L^2 = 0.48$ Wb (at $t = 0$). (Be careful: the magnetic field exists only in the area L^2.

G(3). $I = 0.0075$ A.

H(3). 131 V.

I(4). (a) From B to A through the meter.
(b) From B to A through the meter.

PRACTICE TEST

1. The magnetic field in a certain region of space is given by

   $\vec{B}(t) = 0.300 \sin(100t)\hat{k}$ T.

   A 50-turn loop of cross-sectional area 0.60 m^2 lies in the xy plane.

   (a) Write Faraday's law and define all terms with correct units.

   (b) Calculate the flux through the coil at t = 0.

   (c) Use Faraday's law to calculate the magnitude of the induced emf in the coil at t = 0.

2. A current-carrying solenoid is moved toward a conducting loop as in Figure 5. Use Lenz's law to determine the direction of the induced current and explain your reason.

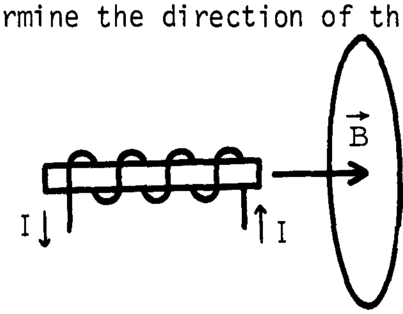


Figure 5

Practice Test Answers

1. (a) Faraday's law is $\mathcal{E} = -d\phi/dt$. The induced emf = $\mathcal{E}$(volts). The flux is $\phi = N\int \vec{B} \cdot d\vec{A}$(webers) = the magnetic field $\vec{B}$ (teslas) integrated over area A (square meters).

   (b) $\phi = N\int \vec{B} \cdot d\vec{A} = NB\int dA = NBA = N(0.300) \sin(0)(0.60) = 0$.

   (c) $\mathcal{E} = d\phi/dt = N(d/dt)(BA) = NA(dB/dt) = NA\,[d(0.300)\sin(100t)/dt]$
   $= NA(0.300(100 \cos 100t)$. At t = 0, $\mathcal{E} = (50)(0.60\ m^2)(0.300\ T)(100\ s^{-1})$
   $= 900$ Wb/s $= 900$ V.

2. The induced current will be counterclockwise. The cause of the induced current is the movement toward the ring. Therefore the current in the loop must be in such a direction as to oppose this motion. A counterclockwise current would produce a magnetic field that would repel the solenoid.

FARADAY'S LAW

Date ____________________

Mastery Test   Form A

pass   recycle

1 2 3 4

Name ____ Tutor ____________________

A plane circular wire loop in Figure 1 of radius = 0.050 m and resistance = 2.00 Ω lies in the plane y = 0 in a spatially uniform magnetic field $\vec{B} = 0.100 \cos(2\pi t/8)\hat{j}$.

1. Write Faraday's law and define all terms with correct units.

2. Calculate the magnetic flux through the loop at t = 1/6 s.

3. Use Faraday's law to determine the induced current in the loop.

4. Use Lenz's law to determine the direction of the induced current in the loop at t = 1/6 s and explain your reason.

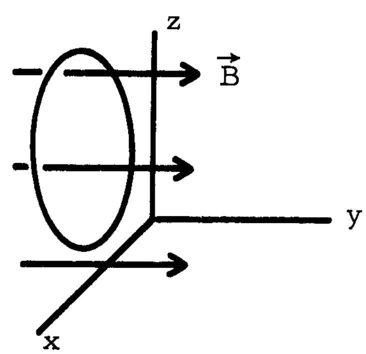


Figure 1

FARADAY'S LAW

Date ____________________

pass recycle

Mastery Test Form B

1 2 3 4

Name ____

Tutor ____________________

A conducting rod AB of length ℓ makes contact with the metal rails CA and DB in Figure 1. The apparatus is in a uniform magnetic field $\vec{B} = 0.50\hat{k}$ T(into the paper). The rod moves with a velocity $\vec{v} = 4.0\hat{i}$ m/s.

1. Write Faraday's law and define all terms with correct units.

2. Calculate the time rate of change of magnetic flux swept out by the wire.

3. Use Faraday's law to calculate the magnitude of the induced emf.

4. Use Lenz's law to determine the direction of the induced current and explain your reason.

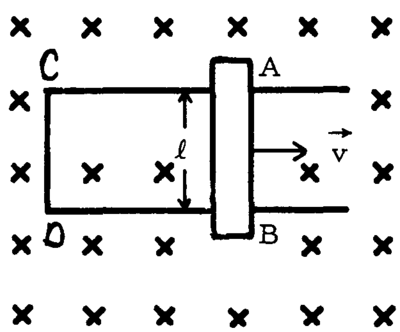


Figure 1

FARADAY'S LAW

Master Test   Form C

Date ____________________

pass   recycle

1   2   3   4

Name ____

Tutor ____________________

A circular wire coil of radius b in the plane of this page and resistance R is in a uniform magnetic field that varies with time as $B = B_0 e^{-t/t_0}$, where B_0 and t_0 are constant and the magnetic field $\vec{B}$ makes an angle of $\pi/3$ with the plane of the coil.

1. Write Faraday's law and define all terms with correct units.

2. Calculate the magnetic flux through the circular wire at $t = t_0$.

3. Use Faraday's law to determine the magnitude of the induced current in the coil.

4. Use Lenz's law to determine the direction of the induced current and explain your reason.

MASTERY TEST GRADING KEY - Form A

1. Solution: $\mathcal{E} = -d\phi/dt$. $\mathcal{E}$ is the induced voltage (in volts), ϕ is the magnetic flux and equals $\int \vec{B} \cdot d\vec{A}$ (in webers), and $\vec{B}$ is the magnetic field (in teslas).

2. What To Look For: $\vec{B}$ is perpendicular to $\vec{A}$.

   Solution: $\phi = \int \vec{B} \cdot d\vec{A}$. Since $\vec{B}$ is perpendicular to $\vec{A}$,

   $\phi = BA = (0.100\ \text{T})(\cos \underset{24}{\pi/6})(25\pi \times 10^{-4}\ \text{m}^2) = 1.25 \times 10^{-4}\ \text{Wb}.$ 7.8×10^{-4} Wb

3. What To Look For: Differentiate $\vec{B}$.

   Solution: $\mathcal{E} = -d\phi/dt = -A(dB/dt) = -A[d(0.100 \cos 2\pi t)/dt]$

   $= -A(0.100)(\underset{8}{\underline{2\pi}} \sin \underset{8}{\underline{2\pi t}}) = 5\pi^2 \times 10^{-4} \sin(\underset{8}{\underline{2\pi t}})$ Wb/s.

4. What To Look For: Cause is decreasing magnetic field.

   Solution: Magnetic field is decreasing in $\hat{j}$ direction; therefore the induced current must be counterclockwise (looking into the +y axis) to produce a magnetic field in the $+\hat{j}$ direction to compensate for decreasing the magnetic field.

MASTERY TEST GRADING KEY - Form B

1. Solution: $\mathcal{E} = -d\phi/dt$, where $\mathcal{E}$ is the induced voltage (in volts), ϕ is the magnetic flux = $\int \vec{B} \cdot d\vec{A}$ (in webers), and $\vec{B}$ is the magnetic field (in teslas).

2. What To Look For: Area swept out is ℓv.

   Solution: $d\phi/dt = B\ell v = (0.50\ T)(4.0\ m/s)\ell = 2\ell\ Wb/s$.

3. Solution: $\mathcal{E} = d\phi/dt = 2\ell\ V$.

4. What To Look For: Cause is moving wire. The force on the current-carrying wire in a magnetic field.

   Solution: The cause of the induced current is the motion of the wire. The induced current must be in such a direction as to produce a force that will oppose this motion. The induced current will be counterclockwise and will produce a force toward the left.

MASTERY TEST GRADING KEY - Form C

1. Solution: $\mathcal{E} = -d\phi/dt$, where $\mathcal{E}$ is the induced voltage (in volts), ϕ is the magnetic flux $= \int \vec{B} \cdot d\vec{A}$ (in webers), and $\vec{B}$ is the magnetic field (in teslas).

2. What To Look For: $\vec{B}$ makes angle of $\pi/6$ with $\vec{A}$

   Solution: $\phi = \int \vec{B} \cdot d\vec{A} = BA \cancel{=} (B_0/e)\pi b^2$. $\int \vec{B} \cdot d\vec{A} = \cos\frac{\pi}{6} = \pi b^2 \frac{B_0}{e}\frac{\sqrt{3}}{2}$

3. What To Look For: Differentiate $\vec{B}$.

   Solution: $\mathcal{E} = -d\phi/dt = -A(dB_\perp/dt) = -A(\frac{dB_0}{dt} e^{-t/t_0})\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}\frac{AB_0}{t_0} e^{-t/t_0}$

   $I = \mathcal{E}/R = \sqrt{3}/2(AB_0/t_0R)e^{-t/t_0}$.

4. What To Look For: Decreasing magnetic field is the cause of the induced current.

   Solution: The cause of the induced current is the decreasing magnetic field. The induced current must be counterclockwise, which will produce a magnetic field out of the page that will compensate for the decreasing magnetic field.