1975

FLUID MECHANICS

STUDY GUIDE

FLUID MECHANICS

INTRODUCTION

An invigorating shower in the morning is usually a pleasant experience except for the pesky shower curtain slapping your legs and allowing water to run on the floor. You would think that the downward stream of water would be enough to keep the curtain back even without water striking the curtain. But not so: fast-moving fluids (water spray causing a downdraft of air) contain a low-pressure region. Thus the pressure outside the shower is greater than the pressure inside - with the result that the curtain is blown in and flops against your legs.

More technical applications of fluid mechanics include airplane flight, streamlining of boats and cars, blood circulation, water towers, and weather forecasting. Even such mundane phenomena as the pressure of your water faucet and "curves" thrown by pitchers in baseball illustrate the ideas of fluid mechanics.

In this module conservation of energy will be recast into a form that is more suitable for application to fluids. No new fundamental physical laws will be introduced. The concepts of energy, work, and the conservation of matter will be used to study fluids at rest and in motion (statics and dynamics).

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Relate the resultant force on a particle to all forces of interaction of that particle and distinguish between weight and mass (needed for Objectives 1 and 2 of this module)	Newton's Laws Module
*Relate the work done on a particle to its change in kinetic energy (needed for Objective 3 of this module)	Work and Energy Module
*Define potential energy and apply the law of conservation of energy to mechanical systems (needed for Objective 3 of this module)	Conservation of Energy Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Pressure - State Pascal's principle and use it to solve problems involving the absolute or relative pressure, density, relative density, or depth of a fluid at rest.

2. Buoyancy - State Archimedes' principle and use it to find the pressure, density, relative density (specific gravity), or amount submerged for stationary objects floating in fluids.

3. Fluid flow - Describe various types of fluid flow: steady versus nonsteady, rotational versus irrotational, compressible versus incompressible, or viscous versus nonviscous, and give an example of each.

4. Hydrodynamics - State the equation of continuity and Bernoulli's principle, and use them to solve problems concerning the pressure, velocity, or height above some arbitrary reference level of incompressible, steady, nonviscous, irrotational fluid flow.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

The reading includes all of Chapter 14 except Sections 14.3 and 14.4. Study the Problems with Solutions carefully, and solve the Assigned Problems before attempting the Practice Test. Begin by reading the material relative to Objective 1, that is, General Comment 1 and Sections 14.1, 14.2, and 14.5.

To supplement the text, read General Comment 2, where the characteristics of fluid flow are described more clearly. Pay particular attention to the treatment of fluid particles as in Figures 14.7, 14.10, and 14.12.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text (Illus.*)	Study Guide	Text (Chap. 14)	(Chap. 14)
1	General Comment 1, Secs. 14.1, 14.2, 14.5	A, B	14.2	G, H	10	8, 9, 11
2	Sec. 14.6	C, D	14.3	I, J	18	15, 17, 19, 21
3	Sec. 14.7, General Comment 2					
4	Secs. 14.8 to 14.11, General Comment 3	E, F		K, L	26	23, 24, 25, 27

*Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

The reading includes all of Chapter 15. Study the Problems with Solutions carefully, and solve the Assigned Problems before attempting the Practice Test. Begin by reading the material relative to Objective 1, that is, General Comment 1 and Sections 15-1 through 15-6.

In Sections 15-2, 15-3, and 15-4 the concepts of pressure and density, Pascal's principle, and Archimedes' principle are explained. Before these two principles are introduced, there is a treatment of the variation of pressure in a fluid at rest, which provides a theoretical basis for them. Equation (15-4) is a special case of the pressure variation in a moving fluid that you will encounter in Section 15-8 (Bernoulli's principle).

Pay particular attention to the treatment of fluid particles as in Figures 15-2, 15-11, and 15-13. See Section 15-7, Eqs. (15-5) and (15-6). The phrase "along any tube of flow" must be understood to follow these equations.

In Section 15-8, Bernoulli's equation has been derived for a pipeline, but can equally well be applied along any streamline. The necessity of keeping to a streamline was forgotten in the subsequent diagram of a Pitot tube (Fig. 15-15);

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text (Chap. 15)	Assigned Problems: Study Guide	Assigned Problems: Text (Chap. 15)	Additional Problems (Chap. 15)
1	General Comment 1, Secs. 15-1 to 15-6	A, B	Ex.* 1,	G, H	8	7, 12
2	Sec. 15-4	C, D	Ex. 2	I, J	19	13, 15, 17, 20
3	Sec. 15-6, General Comment 2					
4	Secs. 15-7 to 15-9, General Comment 3	E, F	Sec. 15-9, 1, 2, 3	K, L	29	26, 30, 31, 39, 42

*Ex. - Example(s).

point b should be at the tip of the Pitot tube, so that the same streamline passes through points a and b. If we could neglect this condition, then we could move point a just inside the Pitot tube, where the velocity is zero - and thereupon find no pressure difference at all!

In Section 15-9, 3: Dynamic Lift, what Bernoulli's equation really does is to relate the pressure to the speed along each of two streamlines, one of which goes through point 1 (above the ball or wing), while the other goes through point 2 (below). But this makes no difference to the arithmetic, since p and v become the same for all streamlines if we follow them back sufficiently far in front of the ball or wing. It's important to note that v must be measured in the reference frame in which the streamlines are at rest. (Remember that the pipeline was at rest in the derivation of Bernoulli's equation.) Had we measured v in the rest frame of the ambient air, the ball would seem to be pulled down! Finally, note these are examples - don't try to remember the resultant equations!

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

The reading includes all of Chapter 12 except Section 12-7 and all of Sections 14-1 to 14-4 of Chapter 14, except for the rocket example on p. 201 in Section 14-4. Study the Problems with Solutions carefully, and solve the Assigned Problems before attempting the Practice Test. Begin by reading the material relative to Objective 1, that is, General Comment 1 and Sections 12-1 through 12-5.

In Sections 12-1 to 12-6, the concepts of pressure and density, Pascal's principle, and Archimedes' principle are explained. Before these two principles are introduced, there is a treatment of the variation of pressure in a fluid at rest that provides a theoretical basis for them. Equation (12-4) is a special case of the pressure variation in a moving fluid you will encounter in Section 14-3 (Bernoulli's principle). Pay particular attention to the treatment of fluid particles as in Figures 12-1, 14-3, and 14-4. In Section 14-4, 5 and 6, what Bernoulli's equation really does for us is to relate the pressure to the speed along each of two streamlines, one of which goes through point 1 (above the ball or wing), while the other goes through point 2 (below). But this makes no difference to the arithmetic, since p and v become the same for all streamlines if we follow them back sufficiently far in front of the ball or wing. It's important to note that v must be measured in the reference frame in which the streamlines are at rest. (Remember that the pipeline was at rest in the derivation of Bernoulli's equation.) Had we measured v in the rest frame of the ambient air, the ball would seem to be pulled down! Finally, note that these are examples - don't try to remember the resultant equations!

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 12-1 to 12-5, General Comment 1	A, B	Ex.* (Sec. 12-4)	G, H	12-5	12-1 to 12-4, 12-6
2	Sec. 12-6	C, D	Ex. (Sec. 12-6)	I, J	12-13	12-11, 12-19, 12-21, 12-33
3	Sec. 14-1, General Comment 2					
4	Secs. 14-1 to 14-4, General Comment 3	E, F	Sec. 14-4, 1 to 6	K, L	14-19	14-9, 14-11, 14-17, 14-21

*Ex. = Example(s).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

The reading includes all of Chapter 15 except Sections 15-1 and 15-2. Study the Problems with Solutions carefully, and solve the Assigned Problems before attempting the Practice Test. Note that the text covers the material in reverse order from this study guide. You may want to skim through the sections of the text and then decide whether to follow the table in order as suggested below or start with Objectives 3 and 4 and do Objectives 1 and 2 last. Either way is good.

Pay particular attention to the treatment of fluid particles as in Figures 15-5, 15-7, and 15-8. There will not be any problems like Example 15-4 on the Mastery Tests, where an integral of dF must be done. However, note that at the end of the example the difference in forces acting on the dam is not $(1/2)gLH^2$. What should it be? (Try dimensional analysis.)

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Secs. 15-3, 15-7, General Comment 1	A, B	Ex.* 15-1	G, H	15-14	
2	Sec. 15-8	C, D	Ex. 15-5	I, J	15-17	15-15, 15-9 to 15-21
3	Sec. 15-4, General Comment 2					
4	Secs. 15-5, 15-6, General Comment 3	E, F	Ex. 15-2, 15-3	K. L	15-9	15-10 to 15-12

*Ex. = Example(s).

GENERAL COMMENTS

1. Pascal's Law

A fluid is a substance that does not maintain its shape against external distorting forces, or, more simply, a fluid is something that can flow. In fluid mechanics the basic laws of particle physics apply, but special formulations are needed. Instead of forces we often speak of pressure (force per unit area, in newtons per meter squared or pascals), instead of mass we often use density (mass per unit volume, in kilograms per meter cubed). Keep this in mind when reading your text and doing the problems.

With pressure we sometimes use gauge pressure, i.e., pressure above atmospheric pressure. For example, if the gauge pressure in your car tires is 25.0 lb, this means that the pressure is 25.0 $lb/in.^2$ above the atmospheric pressure of 14.7 $lb/in.^2$. The absolute pressure is 39.7 $lb/in.^2$ in your tires, but the pressure difference between the inside and outside of the tire is 25.0 $lb/in.^2$. Density is often stated relative to water, thus it is called specific gravity. For instance, the specific gravity of mercury is 13.6. This means that the density of mercury is 13.6 times that of water, or 13.6 g/cm^3.

Pascal's law may be stated as follows: "A pressure change in one portion of an incompressible liquid in a closed container is transmitted to all parts of the liquid." The liquid must remain at rest for this to be true. For a definition of incompressible, see General Comment 2. The hydraulic brake system of a car is a good example of Pascal's principle. The small force of the brake pedal on the small piston of the master cylinder produces a pressure in the brake fluid, which produces a large force on the correspondingly large piston at the wheel.

2. Characteristics of Fluid Flow

As is customary in physics, fluid motion will be idealized in order to obtain workable problems. You will recall that "frictionless" surfaces and bearings were much used in your studies of particle motion; for fluid motion, the corresponding adjectives are "steady, irrotational, incompressible, and nonviscous" - truly a frightening list! Yet the resulting idealized fluid motion is close enough to reality to have practical applications. The following summary defines these terms and gives an example of them.

Steady versus nonsteady flow: Fluid motion is steady if the velocity $\vec{v}$ of all particles passing any given point does not change with time. An example of steady flow is a gently flowing stream or flow of fluid in a pipe. Nonsteady motion is more common: the chaotic, turbulent motion of rapids, or the pounding waves at the seashore.

Incompressible versus compressible flow: Even though gases are highly compressible, sometimes the changes in density are unimportant. For example, the air around airplane wings at low speeds is nearly incompressible, and the mathematical analysis is much simpler. Liquids can usually be considered as incompressible, i.e., they do not change density.

Irrotational versus rotational flow: If a fluid element at a point has a net angular velocity about that point, the fluid flow is rotational. It is probably best illustrated by a small paddle wheel. In Figure 1 the paddle wheel does not turn, thus the motion is irrotational (as in a straight pipe). In Figure 2 the paddle does spin, thus the motion is rotational (as in a whirlpool).

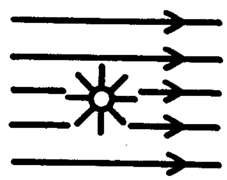


Figure 1

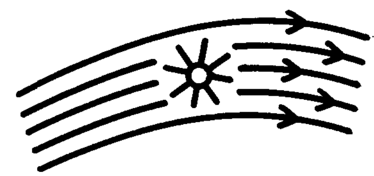


Figure 2

Nonviscous versus viscous flow: Viscosity is the internal friction of a fluid between individual layers of the fluid and between a layer of the fluid and the constraining tube through which it flows. Viscosity introduces retarding forces between layers of fluid in relative motion and dissipates mechanical energy. Air has very little viscosity, whereas thick syrup is very viscous. Nonviscous fluids have no velocity discontinuities in the fluid, but a complete discontinuity between the liquid and the wall of the pipe.

Streamlines: In steady flow a streamline is the path that a particle traces out as time passes. At each point the streamline (or path) is tangent to the velocity $\vec{v}$ of the particle. Thus the streamlines indicate the direction of the fluid's velocity at each point. A high velocity may be indicated by the streamlines being close together. Thus the direction of streamlines and the number of them per unit area give the direction and speed of the fluid's flow.

3. Hydrodynamics

All applications of Bernoulli's equation and the equation of continuity in this module concern nonviscous flow, in which mechanical energy of the fluid is conserved. Notice that the fluid pressure, which has the dimensions of force per area or energy per volume, plays the role of a potential energy per unit volume in Bernoulli's equation. That's a helpful idea when solving problems.

Note how the derivations of pressure dependence on depth in a fluid, the equation of continuity, and Bernoulli's equation employ either the technique of considering a small bit of fluid as though it were a particle or otherwise artificially isolating a region of fluid for purposes of the analysis. This is also a useful technique for solving problems, and I recommend that you keep it in mind to guide your work and to help you check your results in terms of your knowledge of particle mechanics.

PROBLEM SET WITH SOLUTIONS

A(1). Two hemispheres are put together as shown in Figure 3 and then evacuated to 0.100 atmospheric pressure. (1 atm = 1.013×10^5 N/m^2 or Pa.*)

(a) What is the minimum radius of the hemispheres if a force of 1.00×10^5 N cannot pull them apart?

(b) If the hemispheres were moved to the Moon (no atmosphere) what force would be required to hold them together?

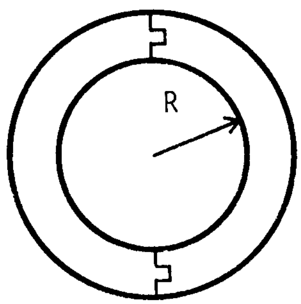


Figure 3

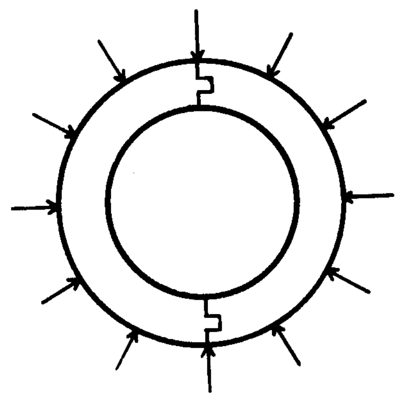


Figure 4

Solution

(a) See Figure 4. The pressure difference between the inside and the outside is (1.00 - 0.100) atm = 0.90 atm = 9.1×10^4 Pa. The atmospheric force on each element of area is directed radially inward. Considering one hemisphere only, we see that the vertical components cancel and the horizontal components add together to yield a total force equal to the pressure difference (Δp) times the area (A) of a circle of radius R. Therefore if we pull on the hemispheres with a total force of $F = \Delta pA$ they will just come apart:

$$F = \Delta pA = \Delta p\pi R^2,$$

$$R = (F/\pi\Delta p)^{1/2} = [10^5/\pi(9.1 \times 10^4)]^{1/2} = 0.59 \text{ m}.$$

(b) In empty space, the smallest external force, now directed inward, that will hold the hemispheres together is $F = \Delta pA = (P_{At}/10)A = 1.100 \times 10^4$ N.

B(1). A simple, uniform U-tube, open on both sides, contains mercury as in Figure 5. If water is poured into the left side until the column of water is 28.0 cm high, how high on the right does the mercury rise above its initial level? The density of mercury is 1.40×10^4 kg/m^3. Water and mercury do not mix (ρ for water equals 10^3 kg/m^3).

*The pascal (Pa) is the SI unit for pressure used in this module. 1 Pa = 1 N/m^2.

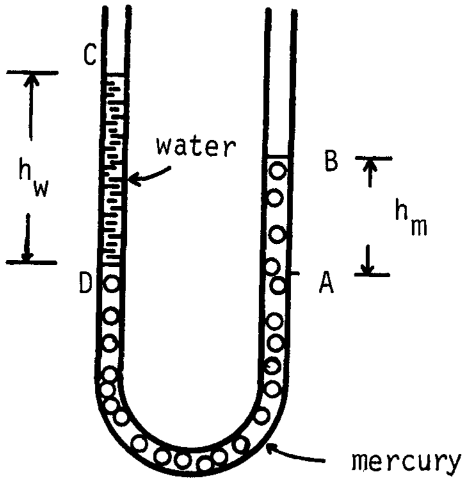


Figure 5

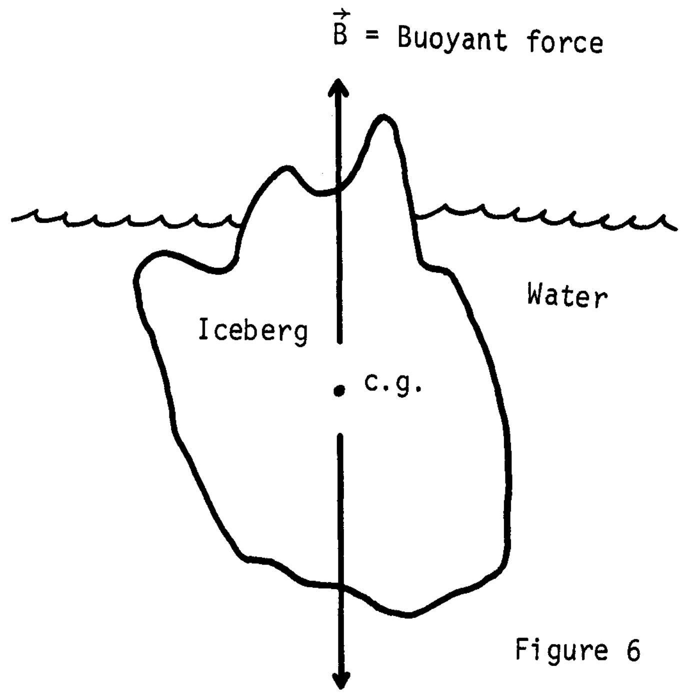


Figure 6

Solution

Consider first the mercury below D and A in the tube. If the pressure at D and A were not equal, the water would flow. Since it does not, we conclude that the pressure at D equals the pressure at A (Pascal's principle). The pressure at a height h below the surface of a liquid is ρgh, thus $P_D = \rho_w gh_w$ and $P_A = \rho_m gh_m$. Equating these, we find

$$\rho_w h_w = \rho_m h_m, \quad h_m = (\rho_w/\rho_m)h_w = [10^3/(1.40 \times 10^4)]28.0 \text{ cm} = 2.00 \text{ cm}.$$

Before the water was added, D and B were at the same height. The left side fell by the same amount the right side rose up, thus the right side has risen 2.00 cm/2 = 1.00 cm above its original position.

C(2). The density of ice is 92% that of fresh water. What fractional volume of an iceberg on a lake floats on top of the water?

Solution

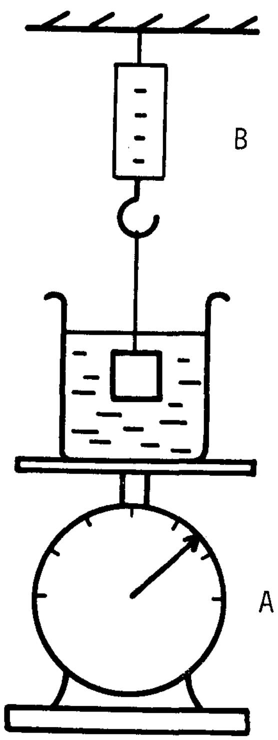
The weight of the ice is $\vec{W} = \rho_i V_i \vec{g}$, where ρ_i is the density and V_i is the volume of the ice. The weight of the volume V_w of water displaced is the buoyant force, $\vec{B} = \rho_w V_w \vec{g}$. Since the iceberg is in equilibrium,

$$|\vec{B}| = |\vec{W}|, \quad \rho_i V_i g = \rho_w V_w g, \quad \text{and} \quad V_w/V_i = \rho_i/\rho_w = 0.92\rho_w/\rho_w = 92\%.$$

The volume of the submerged portion of iceberg is V_w = 92% of the total volume. So only 8.0% of the iceberg is exposed.

D(2). A beaker partly filled with water has a mass of 300 g (see Fig. 7). A piece of metal with density 3.00 g/cm^3 and volume 100 cm^3 is suspended by a spring scale B, so that the metal piece is submerged in water but does not rest on the bottom. What scale readings on A and B will be observed with the metal piece suspended in the water?

Figure 7



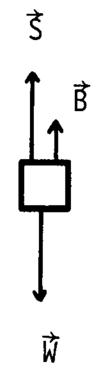


Figure 8

Solution

A free-body diagram of the metal piece is shown in Figure 8. $\vec{S}$ is the tension from the spring scale; $\vec{B}$ is the buoyant force from the displaced water; and $\vec{W}$ is the weight from gravity. Since the metal is in equilibrium,

$$\vec{W} = \vec{B} + \vec{S}.$$

By Newton's third law the buoyant force of the water on the metal is equal and opposite in direction to the force exerted on the water by the metal. This force on the water is the extra amount that the beaker and water weigh owing to the metal. The buoyant force is equal to the weight of the water displaced.

Since the volume of the piece of metal is 100 cm^3, the weight of the water displaced is (0.100 kg)(9.8 m/s^2) = 0.98 N. The weight of the metal piece is

$$mg = \rho Vg = (3000\ \text{kg/m}^3)(10^{-4}\ \text{m}^3)9.8 = 2.94\ \text{N} = \vec{W},$$

$$\vec{S} = \vec{W} - \vec{B} = 2.94\ \text{N} - 0.98\ \text{N} = 1.96\ \text{N},$$

and scale B reads 200 g. The beaker now weighs its original weight plus an amount equal to the buoyant force or (0.300 kg)(9.8 m/s^2) + 0.98 N = 3.94 N, and scale A reads 400 g.

E(3,4). A water pipe having a 2.00-cm inside diameter carries water into the basement of a house at a velocity of 1.00 m/s at a pressure of 2.00×10^5 Pa. If the pipe tapers to 1.00 cm inside diameter and rises to the second floor 10.0 m above the input point, what are the (a) velocity and (b) water pressure there?

Solution

(a) By the equation of continuity $A_1v_1 = A_2v_2$,

$$v_2 = A_1v_1/A_2 = \pi R_1^2 v_1/\pi R_2^2, \tag{1}$$

$$R_1 = 2R_2,\ \text{thus}\ v_2 = (2R_2)^2 v_1/R_2^2 = 4v_1 = 4(1.00\ \text{m/s}) = 4.0\ \text{m/s}.$$

(b) Bernoulli's equation is

$$p_1 + \rho g h_1 + (1/2)\rho v_1^2 = p_2 + \rho g h_2 + (1/2)\rho v_2^2. \tag{2}$$

Since $h_2 - h_1 = 10.0$ m, $p_1 = 2.00 \times 10^5$ Pa, $\rho = 1000$ kg/m^3, $v_1 = 1.00$ m/s, and $v_2 = 4.0$ m/s, we can solve for p_2:

$$p_2 = p_1 - \rho g(h_2 - h_1) - (1/2)\rho(v_2^2 - v_1^2)$$

$$= 2.00 \times 10^5 - 1000(9.8)10.0 - (1/2)(1000)[(4.0)^2 - (1.00)^2]$$

$$= (2.00 \times 10^5) - (0.98 \times 10^5) - (0.075 \times 10^5) = 9.5 \times 10^4\ \text{Pa}.$$

Note the use of the two relationships (1) and (2) in combination.

F(3,4). Each wing of an airplane has an area of 10.0 m^2. At a certain air speed in level flight air flows over the upper wing surface at 50 m/s and over the lower wing surface at 40 m/s. The density of air is 1.29 kg/m^3. Assume that lift effects associated with the fuselage and tail are negligible. What is the weight of the plane?

Solution

The region above the wing is one of higher velocity and lower pressure than that below the wing. It is this pressure difference that gives rise to the lift on the wing. On streamlines connecting the upper and lower wing surfaces and a point far from the wing, Bernoulli's equation tells us that

$$p_u + \rho g h_u + (1/2)\rho v_u^2 = p_L + g h_L + (1/2) v_L^2.$$

The difference in height will contribute a very small amount to the pressure difference. (Try it, assuming a difference of, say, 1.00 m.)

$$p_L - p_u = (1/2)\rho(v_u^2 - v_L^2).$$

The total force for each wing is $F = \Delta pA$. The weight of the plane in level flight is then equal to the total force upon both wings:

$2\Delta pA = 2(1/2)\rho(v_u^2 - v_2^2)A$, weight $= (1.29)(50^2 - 40^2)10.0 = 1.16 \times 10^4$ N (1180 kg mass).

Problems

G(1). In a car brake system the small piston at the brake pedal (or master cylinder) has a cross-sectional area of 2.00 cm^2, and the wheel cyclinder has a cross-sectional area of 40 cm^2. What is the force applied at the wheel cylinder if a force of 400 N is applied at the brake pedal?

H(1). A tube 0.0100 m^2 in cross-sectional area is attached to the top of a container 0.100 m high and of cross-sectional area 1.00 m^2 as in Figure 9. Water is poured into the system filling it to a depth of 1.00 m above the bottom of the vessel.
(a) What is the force exerted against the bottom of the vessel by the water (excluding atmospheric pressure)?
(b) What is the weight of the water in the system?
(c) Explain why (a) and (b) are not equal.

I(2). An iron casting weighs 1000 N in air and 600 N in water. What is the volume of the cavities in the casting? The density of iron is 7.8 times that of water. (Hint: find the volume of the iron in the casting and the volume of the displaced water.)

J(2). What is the minimum mass of wood (density = 0.80 times the density of water) necessary to support a 70-kg man standing on a block of wood floating on water?

Figure 9

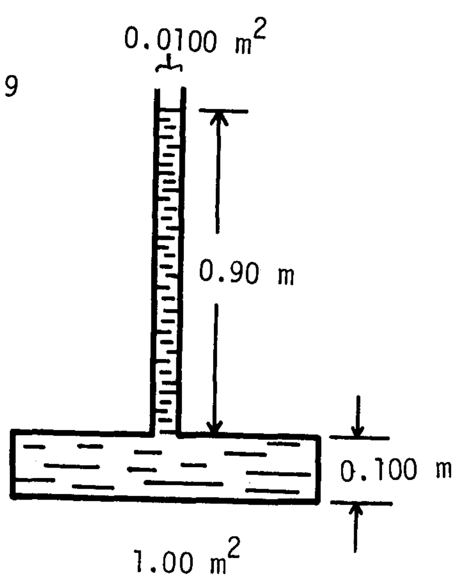
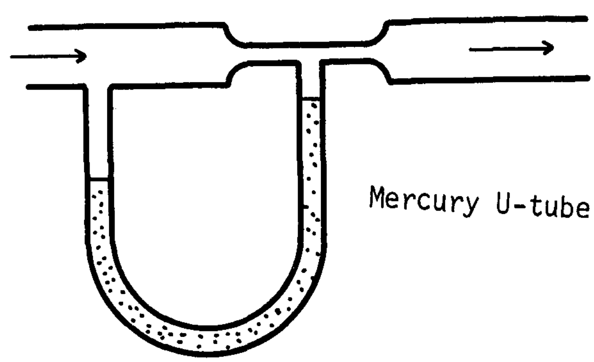


Figure 10



K(3,4). A tank is filled with water to a depth H. A hole is made in one of the vertical walls at a depth h below the water surface.
(a) Apply Bernoulli's equation to a streamline connecting the water surface and the hole to find the horizontal speed of water coming out of the hole. (The area of the hole is much less than the cross-sectional area of the tank.)
(b) If the hole has a bent pipe attached to it so that the water shoots directly upward, how high will the water rise?
(c) If the water is flowing horizontally out of the hole, how far from the foot of the tank will the stream strike the floor? (The bottom of the tank is at the same level as the floor.)

L(3,4). The section of pipe shown in Figure 10 has a cross-sectional area of 30.0 cm^2 at the wider portion and 10.0 cm^2 at the constriction. 3.00 ℓ of water is discharged from the pipe in 1.00 s.
(a) Find the velocities at the wide and the narrow portions.
(b) Find the pressure difference between these portions.
(c) Find the difference in height between the mercury columns in the U-tube (density of mercury = 1.40×10^4 kg/m^3).

Solutions

G(1). $|\vec{F}| = 8.0 \times 10^3$ N.

H(1). (a) $F = 9.8 \times 10^3$ N. (b) $W = 1.07 \times 10^3$ N. (c) There is also an upward force on the top of the 1.00-m^2 container.

I(2). Volume of cavities = 2.8×10^{-2} m^3.

J(2). m_{wood} = 280 kg.

K(3,4). (a) $v = \sqrt{2gh}$. (b) To the top of the water surface in the tank if viscous energy losses are neglected. (c) Horizontal distance = $2\sqrt{h(H - h)}$.

L(3,4). (a) 3.00 m/s; 1.00 m/s. (b) 4.0×10^3 Pa. (c) 2.90 cm.

PRACTICE TEST

1. The large piston of a hydraulic automobile lift is 40 cm in diameter. (a) What pressure is required to lift a car weighing 1000 kg? (b) If the fluid is forced into the main chamber by a small piston with an area of 5.0 cm^2, what force must be applied to this small piston to lift the 1000-kg car?

2. A 1.00-kg block of iron is floating on mercury as in Figure 11. How much lead would have to be placed on top of the iron in order that the iron might float barely below the surface as pictured? (The density of mercury is 13.6 g/cm^3; the density of iron is 7.9 g/cm^3; and the density of lead is 11.3 g/cm^3.)

3. Briefly describe what is meant by steady versus nonsteady flow; illustrate each with an example.

4. The water level in a tank of large cross-sectional area on the top of a building is 40 m above the ground. The tank is open to the atmosphere, and supplies water to the various apartments through pipes of 10.0 cm^2 cross-sectional area. Each faucet through which the water emerges has an orifice of 2.00 cm^2 effective area. (a) What is the gauge pressure in a water pipe at ground level when the faucet is closed? Recall that gauges read the pressure in excess of atmospheric pressure. (b) How long will it take to fill a 20.0-ℓ pail in an apartment 30.0 m above the ground? (c) With what speed is the water flowing in the pipe (of cross-sectional area 10.0 cm^2) leading to this faucet?

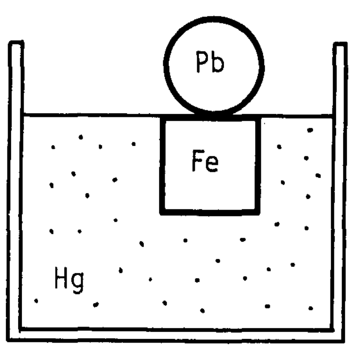


Figure 11

Practice Test Answers

1. (a) 7.8×10^4 Pa.
   (b) 39 N.
2. 720 g (Pb).
3. In steady flow the fluid velocity $\vec{v}$ at every point is constant in time. For examples, steady flow: gentle flowing stream; nonsteady flow: seaside surf.
4. (a) 3.9×10^5 Pa.
   (b) 7.1 s.
   (c) 2.80 m/s.

FLUID MECHANICS | Date ____________

Mastery Test   Form A

pass   recycle

1   2   3   4

Name ____________   Tutor ____________

1. The expansion tank of a household hot-water-heating system is open to air, and the water level is 10.0 m above a pressure gauge attached to the furnace.
(a) What is the gauge pressure at the furnace? Recall that gauges read the pressure in excess of atmospheric pressure. The density of water is 1000 kg/m^3.
(b) If the gauge is removed and the hole is filled with a plug having an area of 1.00 cm^2, what is the resultant force on the plug?

2. A 300 000-kg iceberg is floating on salty water ($\rho_{salt\ water}$ = 1030 kg/m^3). This iceberg has a relative density (or specific gravity) of 0.92.
(a) What volume of the iceberg is below the surface of the water?
(b) What volume of it is above the water surface?

3. Briefly describe what is meant by viscous versus nonviscous fluid flow and illustrate each with an example.

4. Point A in Figure 1 is 20.0 m above ground level; points B and C are 3.00 m above ground level. The cross-sectional areas at points B and C are 0.200 m^2 and 0.100 m^2, respectively. The surface area at point A is much larger than that at B or C.
(a) Compute the discharge rate in cubic meters per second.
(b) Find the gauge pressure at point B.

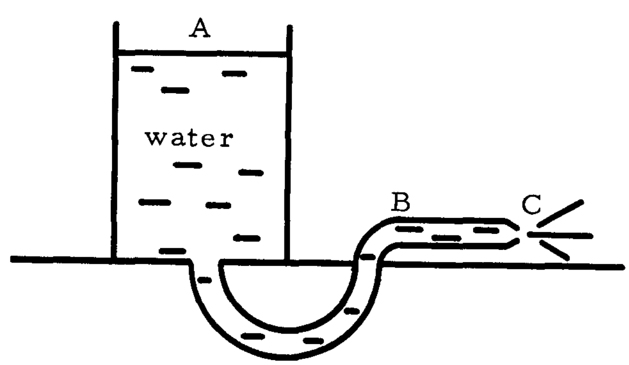


Figure 1

FLUID MECHANICS

Date ______________________

Mastery Test   Form B

pass   recycle

1   2   3   4

Name ______________________________   Tutor ______________________________

1. In a hydraulic press, the small piston has an area of 2.00 cm^2, and the large piston an area of 72 cm^2. What weight can be lifted if a force of 5.0 N is applied to the small piston?

2. A raft is made from four logs, held together by light, thin ropes. Each log has a diameter of 40 cm and a length of 2.00 m. The raft floats in fresh water (ρ = 1000 kg/m^3) with the logs exactly half submerged. When the River Riders climb on board the raft floats with the logs just barely submerged. If there are six River Riders, what is the average weight of each?

3. Briefly describe what is meant by rotational versus irrotational flow and give an example of each.

4. (a) You are designing an airplane to have a lift of 1.00×10^3 N per square meter of wing area. Make the usual assumptions of smooth flow, negligible viscosity, etc. If the air flows under the wing at 100 m/s, how fast must it flow over the top surface? (At the specified altitude, ρ_{air} = 1.25 kg/m^3.)

   (b) A fluid of density 0.60 g/cm^3 flows through a horizontal pipe whose area at the inlet is 25.0 cm^2, inlet speed is 2.00 cm/s, and outlet speed is 5.0 cm/s. Find the area of the outlet.

FLUID MECHANICS

Date ______________

Mastery Test Form C

pass recycle

1 2 3 4

Name ______________ Tutor ______________

1. A simple U-tube manometer contains water as in Figure 1. If 2.50 cm of oil having a relative density (specific gravity) of 0.80 is poured into the tube on the right, how high does the water in the left arm rise above its initial level?

2. A swimmer (volume $\equiv V_s = 0.070$ m^3) is floating with just her nose and chin (volume $\equiv V_{nc} = 140$ cm^3) out of the water. The water in the swimming pool has a density $\rho_w = 1000$ kg/m^3. What is her specific gravity? Express your answer first in terms of the symbols V_s, V_c, and/or ρ_w and then numerically.

3. Briefly describe what is meant by compressible versus incompressible flow, and illustrate each with an example.

4. Oil of specific gravity of 0.80 flows through a pipe as shown in Figure 2, emerging from the lower end of the pipe into the atmosphere.

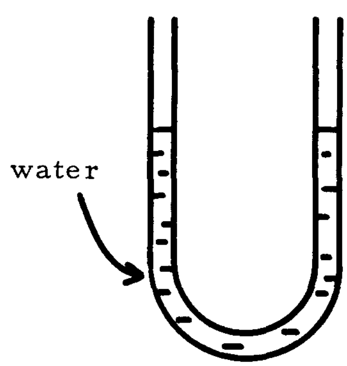
   $A_1 = 8.0$ cm^2, $A_2 = 2.00$ cm^2,

   $V_1 = 6.0$ cm/s, $\Delta h = 50$ cm,

   $p_2 = 1.01 \times 10^5$ Pa.

   (a) Find the speed at which the oil emerges.
   (b) Find the absolute pressure at point 1 in the pipe.

Figure 1



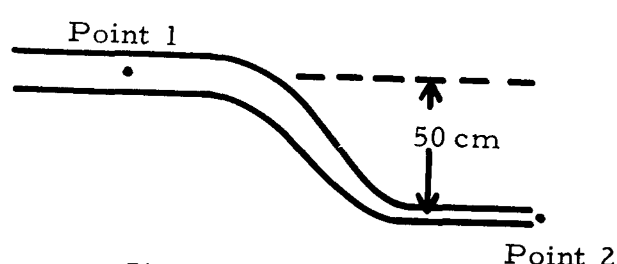


Figure 2

P 10 FLUID MECHANICS

Mastery Test Form D

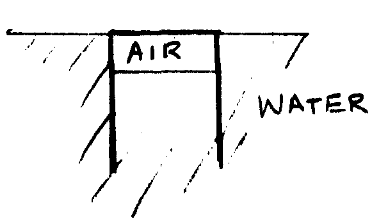
Date ______________

pass recycle

1 2 3 4 5

Name ________________________________ Tutor ____________________

1. An automobile hydraulic jack must lift a 1000 N car when a force of 50 N is applied to the small piston. What is the ratio of the area of the large piston to the area of the small piston?

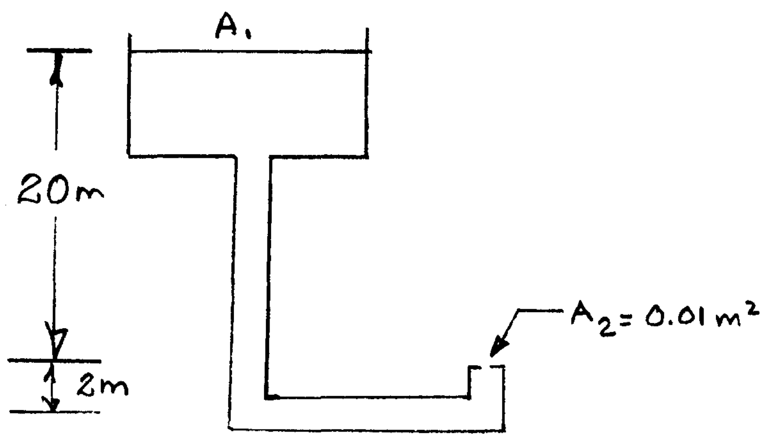
2. 

An inverted beer can weighing 10^{-2} N is found barely to float as shown. Find the volume of air trapped in the can. (You may neglect the mass of the air.)

3. Briefly describe what is meant by rotational versus irrotational flow and give an example of each.

4. For the water tank shown, A_1 is much larger than A_2. (a) Find the velocity of the water as it comes out of the 0.01 m^2 hole.

(b) Neglecting air resistance, how high will the water spray?



Courtesy of University of Missouri-Rolla

P 10 FLUID MECHANICS

Mastery Test Form E

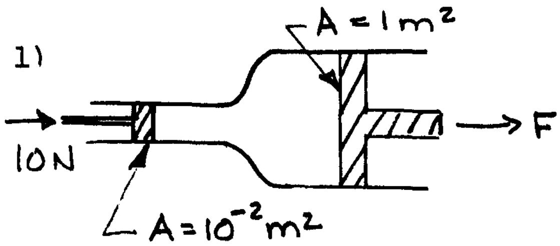
Date ______________

pass recycle

1 2 3 4 5

Name ________________________________ Tutor ______________________

1)

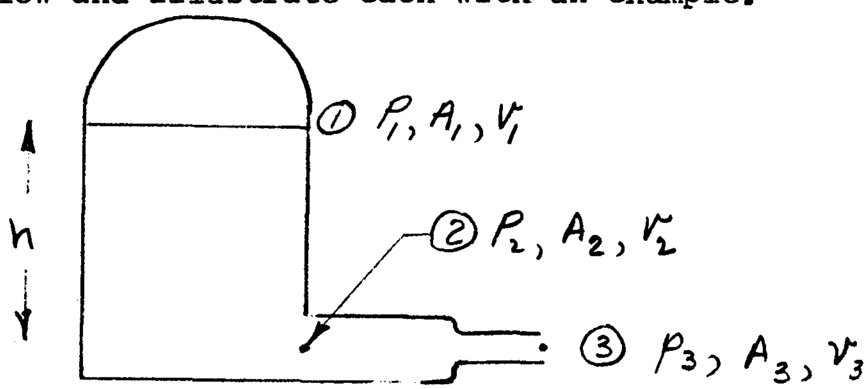


For the hydraulic system shown, find the force on the large piston.

2) A bar of soap floats on the water with 15% of its volume above the surface. What is the specific gravity of the soap? (ρ of water-10^3kg/m^3)

3) Briefly describe what is meant by viscous versus nonviscous fluid flow and illustrate each with an example.

4.



Water in a closed tank flows out a valve at A_3 when the valve is turned on. What absolute pressure P_1 is needed to produce a flow velocity V_2=10 m/s with the valve open?
Data: $A_1 >> A_2$, A_2=.01 m^2, A_3=.002m^2, P_3=$1x10^5 N/m^2$, h=50m.

Courtesy of University of Missouri-Rolla

P 10 FLUID MECHNAICS

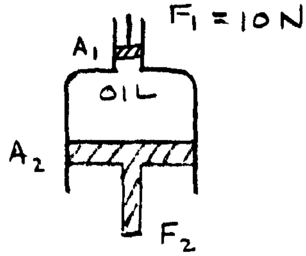
Mastery Test Form F

Date ______________

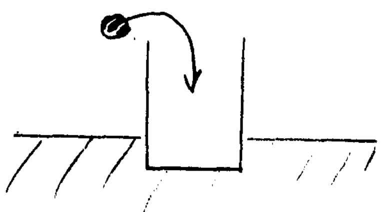
pass recycle

1 2 3 4 5

Name ________________________________ Tutor ____________________

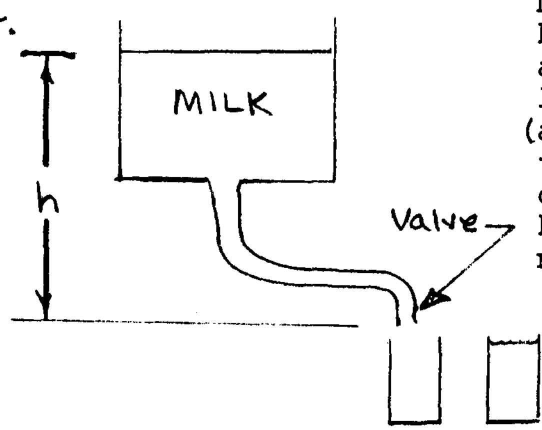
1. 

You are designing a hydraulic punch press that requires a force F_2 on the punch of 10^6N. If the oil pump can deliver a force of 10 N, what will you choose for the ratio of the areas of the large and small pistons?

2. 

A beer can floats in the water. The can is .02 m in radius and 0.1 m long. The empty can has a mass of 0.03 kg. How many 0.01 kg mass stones must be dropped into the can to sink it?

3. Briefly describe what is meant by compressible versus incompressible flow, and illustrate each with an example.

4. 

Milk containers on an assembly line are being filled from a hose connected to a large reservoir which can be raised or lowered. The automatic value (area=$10^{-4}m^2$) on the hose opens for 3 s to fill the milk containers with $10^{-3}m^3$ of milk ($\rho_{milk}=10^3$ kg/m^3). Find the height required so that proper amount of milk is dispensed into each container.

MASTERY TEST GRADING KEY - Form A

1. What To Look For:

   (a) What is Pascal's principle? What does it apply to here? (p_0 = atmospheric pressure is transmitted from top to furnace.) (b) Why should we use gauge pressure?

   Solution:

   (a) $p = p_0 + gh$, gauge pressure $= p - p_0 = \rho gh = (1000)(9.8)(10.0)$

   $= 9.8 \times 10^4$ Pa.

   (b) $p = F/A$, $F = pA = (9.8 \times 10^4 \text{ Pa})(10^{-4} \text{ m}^2) = 9.8$ N.

2. What To Look For: (a) What is ρ_{ice}? $\rho_{ice} = 0.92\rho_{water} = 0.92(1000) = 920 \text{ kg/m}^3$.

   Solution: (a) Weight of iceberg $= W_i = \rho_i V_i g$,

   Buoyant force $= \vec{B} = \rho_w V_w \vec{g}$, where V_w = volume of water displaced = volume of ice below surface.

   $B = W$, $\rho_i V_i g = \rho_w V_w g$, $\dfrac{V_w}{V_i} = \dfrac{\rho_i}{\rho_w} = \dfrac{920 \text{ kg/m}^3}{1030 \text{ kg/m}^3} = 0.89$.

   Thus 89% of iceberg is below water.

   $\rho_{ice} = \dfrac{m}{V}$; Volume $= \dfrac{m}{\rho_{ice}} = \dfrac{300\ 000 \text{ kg}}{920 \text{ kg/m}^3}$, $V_i = 326 \text{ m}^3$,

   $V_{\text{below surface}} = 0.89 \times 326 = 290 \text{ m}^3$.

   (b) $V_{\text{above surface}} = 0.110 \times 326 = 36 \text{ m}^3$.

3. Solution: Viscous flow: friction between layers of the fluid, e.g., thick syrup. Nonviscous flow: no friction between layers, e.g., air.

4. What To Look For: Does $V_A = 0$ sound reasonable?

   Solution: (a) Apply Bernoulli's theorem for points A and C along a streamline:

   $$p_A + \rho g h_A + (1/2)\rho v_A^2 = p_c + \rho g h_c + (1/2)\rho v_C^2.$$

   Now $p_A = p_C$ = atmospheric pressure, $h_A - h_C = \Delta h = 17.0$ m. By the equation of continuity $A_A v_A = A_C v_C$:

   $A_A >> A_C$, $v_A = (A_C/A_A)v_C \simeq 0$, $\rho g(h_A - h_C) = (1/2)\rho v_C^2$,

$v_C = \sqrt{2g\Delta h} = 2(9.8)(17.0) = 18.3$ m/s.

Discharge rate = Av = (0.100 m^2)(18.3 m/s) = 1.83 m^3/s.

What To Look For: (b) Could also use points B and A if they find v_B first. Did they use the equation of continuity to find v_B?

Solution: (b) For a streamline between B and C, Bernoulli's equation states

$$p_B + \rho g h_B + (1/2)\rho v_B^2 = p_C + \rho g h_C + (1/2)\rho v_C^2, \quad h_B = h_C.$$

$A_B v_B = A_C v_C$, so $v_B = (A_C/A_B)v_C = 9.1$ m/s.

$p_B = p_C + (1/2)\rho(V_C^2 - V_B^2) = p_C$ + gauge pressure,

gauge pressure = $(1/2)(1000)(18.3^2 - 9.12^2) = 1.26 \times 10^5$ Pa.

MASTERY TEST GRADING KEY - Form B

1. What To Look For: What is Pascal's principle?

   Solution: Pascal's principle says that the pressure will be the same everywhere, thus on the small piston

   $p = F/A_s = (5.00\text{ N})/(2.00 \times 10^{-4}\text{ m}^2) = 2.50 \times 10^4\text{ N}.$

   On the large piston

   $F = pA_r = (2.5 \times 10^4)(72 \times 10^{-4}) = 180\text{ N}.$

   Thus a weight of 180 N (mass = 18.4 kg) can be lifted.

2. What To Look For: What is Archimedes' principle? Did they multiply mass of water by g to obtain weight?

   Solution: The volume of the logs is $4(\pi r^2)\ell$. With the riders on,

   weight of logs + riders = buoyant force = weight of displaced water = $\rho_w 4(\pi r^2)\ell g$,

   weight of logs = weight of displaced water with no riders = $(1/2)\rho_w(4\pi r^2)\ell g$.

   Thus the weight of one rider is one-sixth the weight of all riders.

   $$W_{rider} = \frac{1}{6}[\rho_w 4\pi r^2\ell - \frac{1}{2}\rho_w 4\pi r^2\ell]g = \frac{\rho_w \pi r^2 \ell g}{3} = \frac{(1000)\pi(0.200)^2 2(9.8)}{3}$$

   $= 820\text{ N}$ (mass = 84 kg).

3. Solution: Irrotational flow: a fluid element at a point has zero net angular velocity about that point (e.g., flow in a straight pipe). Rotational flow: a fluid element has a net angular velocity about a point (e.g., whirlpools).

4. What To Look For: (a) Why is $h_1 \simeq h_2$? ($\rho g\Delta h$ is very small if $\Delta h \simeq 1.00$ m.)

   Solution: (a) Apply Bernoulli's equation:

   $p_2 + \rho g h_2 + \frac{1}{2}\rho V_2^2 = p_1 + \rho g h_1 + \frac{1}{2}\rho V_1^2$, $\quad p_1 - p_2 = 1.00 \times 10^3$ Pa

   (1 = below, 2 = above).

   $h_1 \simeq h_2$, therefore $\quad (1/2)\rho V_2^2 = p_1 - p_2 + (1/2)\rho V_1^2$,

   $V_2 = [2(p_1 - p_2)/\rho + V_1^2]^{1/2} = [(2.00 \times 10^3)/1.25 + 100^2]^{1/2} = 108\text{ m/s}.$

   (b) Equation of continuity:

   $A_0 v_0 = A_i v_i$, $\quad A_0 = A_i(\frac{v_i}{v_0}) = 25.0\text{ cm}^2[\frac{2.00\text{ cm/s}}{5.6\text{ cm/s}}] = 10.0\text{ cm}^2.$

MASTERY TEST GRADING KEY - Form C

1. What To Look For: What is Pascal's principle?

   Solution: See Figure 15. Points A and B are at the same pressure (Pascal's principle).

   $p_A = \rho_{oil} g h_{oil}$, $p_B = \rho_w g h_w$, h_w = height of water above B, $p_A = p_B$.

   Therefore,

   $\rho_o g h_o = \rho_w g h_w$, $h_w = (\rho_o/\rho_w)h_o$, $\rho_o/\rho_w = 0.80$.

   Thus, $h_w = 0.80(2.50) = 2.00$ cm. If the difference in height of the water is 2.00 cm, one side moved down by 1.00 cm, and the other side moved up by 1.00 cm.

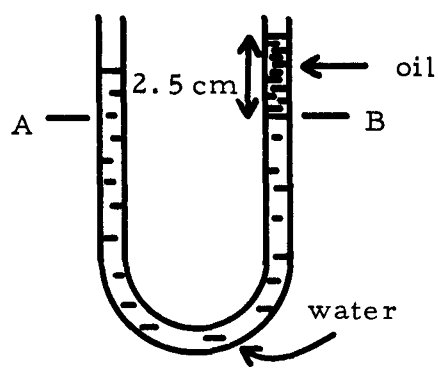


Figure 15

2. What To Look For: What is Archimedes' principle? (buoyant force = weight of displaced water.) Can you draw a free-body diagram of swimmer?

   Solution: Volume of swimmer is $V_s = m_s/\rho_s$.

   Weight of swimmer = $m_s g = \rho_s V_s g$, weight of displaced water = $\rho_w(V_s - V_{nc})g$.

   Weight of swimmer = weight of displaced water, $\rho_s V_s g = \rho_w(V_s - V_c)g$,

   ρ_s/ρ_w = specific gravity = $(V_s - V_c)/V_s = 1 - V_{nc}/V_s = 1 - (1.40 \times 10^{-4})/0.070$
   $= 0.998$.

3. Solution: Incompressible: the fluid cannot change density under ordinary pressures of a few atmospheres (e.g., water). Compressible: the fluid easily changes density under pressure of a few atmospheres (e.g., a gas like air).

4. What To Look For: (a) What is the discharge rate at point 2? (A_2v_2.) (b) Can you show that the units in every term are N/m^2 (Pa)? What is the gauge pressure at point 1? (-3.9×10^3 Pa.)

Solution: (a) From the equation of continuity:

$A_1v_1 = A_2v_2$, $v_2 = (A_1/A_2)v_2 = (8/2)(6)$ cm/s = 24.0 cm/s.

(b) Apply Bernoulli's equation to 1 and 2:

$$p_1 + \rho gh_1 + (1/2)\rho v_1^2 = p_2 + \rho gh_2 + (1/2)\rho v_2^2,$$

$$p_1 = p_2 + \rho g(h_2 - h_1) + (1/2)\rho v_2^2 - v_1^2),$$

$h_2 - h_1 = -0.50$ m, $\rho = 0.80\rho_w = 800$ kg/m^3,

$V_1 = 0.060$ m/s, $V_2 = 0.240$ m/s,

$$p_1 = 1.01 \times 10^5 + 800(9.8)(-0.50) + (1/2)(800)[(0.240) - (0.060)] = 9.7 \times 10^4 \text{ Pa.}$$