1975

Inductance

INDUCTANCE

INTRODUCTION

Anyone who has ever grabbed an automobile spark-plug wire at the wrong place, with the engine running, has an appreciation of the ability of a changing current in (part of) a coil of wire to induce an emf in the coil. What happens is that the breaker contacts open, suddenly interrupting the current, and causing a sudden large change in the magnetic field through the coil; according to Faraday's law, this results in a (large) induced emf. In general, the production of an emf in a coil by a changing magnetic field due to a current in that same coil is called self-induction; and the ability of a coil to produce an emf in this way is commonly measured by its self-inductance L, usually called more briefly its inductance. A coil used in this way is more formally called an inductor.

The transmission of electric signals by television, radio, and telephone depends on time-varying currents and fields to represent the appearance of pictures and the sound of voices; and so, as you can well imagine, capacitors and inductors play an important role in the circuits of such devices. You already know that a capacitor can store energy; so can an inductor. If an inductor carrying a current is connected to a resistor, its energy is dissipated as heat in the resistor, much as for a charged capacitor. But now suppose you connect it instead to a capacitor; the inductor will try to give its energy to the capacitor - and vice versa - but the initial energy is not quickly dissipated from the electrical circuit. What do you suppose happens? If you do not already know, can you guess, before studying this module?

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Use Ampère's law to calculate $\vec{B}$ inside toroids and long solenoids (needed for Objective 1 of this module)	Ampère's Law Module
*Relate the emf induced in such toroids and solenoids to the time rate of change of $\vec{B}$ or Φ_B (needed for Objective 1 of this module)	Faraday's Law Module
*Find the power dissipated by a resistor (needed for Objective 2 of this module)	Ohm's Law Module
*Add voltages around an RC circuit to verify the exponential time dependences of the current and voltage (needed for Objective 2 of this module)	Direct-Current Circuits Module
*Find the energy stored in a capacitor (needed for Objective 3 of this module)	Capacitors Module
*Relate the motion of a mechanical oscillator to the mathematical expression for its displacement (needed for Objective 3 of this module)	Simple Harmonic Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Inductance - Apply the definition of inductance, Ampère's law, and Faraday's law to toroids and long solenoids to (a) find the inductance L; and (b) relate the induced emf to the rate of change of current or flux.

2. LR circuits - Determine currents, voltages, stored energies, and power dissipations in simple LR circuits. (This includes adding up voltages around the circuit to find a differential equation and determine the time dependence.)

3. LC circuits - Determine charges, voltages, currents, and stored energies in simple LC circuits. (This includes using the principle of energy conservation to find maximum values, as well as to obtain a differential equation and determine the time dependence.)

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read the General Comments on the following pages of this study guide, along with Sections 25.3 through 25.5 and 27.8 in your text; but the part of Section 25.5 after Eq. (25.7), and Cases 2 and 3 of Section 27.8 (including Illustration 27.8) are optional, for the purposes of this module. Optional: Read Section 25.2. Recommended: Read Sections 34-1 through 34-3 in Halliday and Resnick (HR)* or Section 34-1 in Weidner and Sells (WS),* for further discussion of electromagnetic oscillations.

Although this is not explicit in the text, Bueche assumes the self-inductance L to be defined as the proportionality factor between the current and the flux:

$$N\phi = Li. \qquad \text{(B1)}$$

When there is no ferromagnetic material near the coil, ϕ is proportional to i, and L is thus constant. Differentiation then yields L(di/dt) = N(d /dt), which by Faraday's law is just the induced emf $-\mathcal{E}$. Therefore, as in Eq. (25.2),

$$\mathcal{E} = -L(di/dt). \qquad \text{(B2)}$$

Equations (B1) and (B2) are equivalent for coils that do not have ferromagnetic cores, as will be the case in the problems of this module. You will find that Eq. (B1) is sometimes more convenient to use than Eq. (B2).

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 25.3	A	Illus.[a] 25.2	F, G	Chap. 25: Quest.[a] 2, 11
2	General Comment 1; Secs. 25.4, 25.5 thru Eq. (25.7)	B, C		H, I	Chap. 25: Probs. 12, 13
3	General Comment 2; Sec. 27.8; HR*: Secs. 34-1 thru 34-3; or WS*: Sec. 34-1	D, E		J, K	L

[a]Illus. = Illustration(s). Quest. = Question(s).

*HR = David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974).

WS = Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2.

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read the General Comments on the following pages of this study guide, along with Sections 32-1 through 32-4, and 34-1 through 34-3 in your text.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	Secs. 32-1, 32-2	A	Chap. 32, Ex.[a] 1	F, G	Chap. 32: Probs. 1, 2, 5, 9, 10(a), 11(b), Quest.[a] 4
2	General Comment 1; Secs. 32-3, 32-4	B, C	Chap. 32, Ex. 2, 3, 4	H, I	Chap. 32: Probs. 11(a), 12 thru 25, Quest. 7 thru 11
3	General Comment 2; Secs. 34-1, 34-2, 34-3	D, E	Chap. 34, Ex. 1, 2, 3	J, K	L; Chap. 34: Probs. 1 thru 17

[a]Ex. = Example(s). Quest. = Question(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read the General Comments on the following pages of this study guide, along with Sections 33-9 through 33-12 in your text; but the latter part of Section 33-10, on p. 477, is optional for the purposes of this module. Optional: Read Section 33-8. Recommended: Read Sections 34-1 through 34-3 in Halliday and Resnick (HR)* or Section 34-1 in Weidner and Sells (WS),* for further discussion of electromagnetic oscillations.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text	Assigned Problems: Study Guide	Additional Problems
1	Sec. 33-9	A	Example in Sec. 33-9	F, G	33-12, 33-20, 33-21
2	General Comment 1; Secs. 33-10, 33-11	B, C		H, I	33-30 thru 33-34
3	General Comment 2; Sec. 33-12; HR*: Secs. 34-1, 34-2, 34-3; or WS*: Sec. 34-1	D, E		J, K	L; 33-35

*HR = David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974).

WS = Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

Read the General Comments on the following pages of this study guide, along with Sections 32-1 through 32-3 and 34-1 in your text.

There is a typographical error in Eq. (34-7); the second-to-last term should read $(Q_m \cos \omega t)^2/2C$.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems	Additional Problems
		Study Guide	Text	Study Guide	
1	Sec. 32-1	A	Ex.[a] 32-1	F, G	32-12(a)
2	General Comment 1; Secs. 32-2, 32-3	B, C	Ex. 32-2	H, I	32-2, 32-5 thru 32-11, 32-12(b), 32-13
3	General Comment 2; Sec. 34-1	D, E		J, K	L; 34-1

[a]Ex. = Example(s).

GENERAL COMMENTS

1. LR Circuits

Suppose you are given the circuit shown in Figure 1. Before the switch is closed, the current is zero. When the switch is closed, the current starts to rise - but only at a finite rate, since the inductor will not allow any sudden change in the current. [The induced emf $-L(di/dt)$ would be infinite!] Furthermore, the current will not rise indefinitely, because of the opposing voltage $V_R = -Ri$ across the resistor. Therefore, the behavior of the current has the appearance of Figure 2. Since the emf induced in L vanishes as the current approaches its final, unchanging, value i_0, we see that

$$i_0 = V_B/R. \qquad (1)$$

Next, adding voltages around the circuit, much as you did in the module Direct-Current Circuits for circuits containing only resistors and batteries, leads to the equation

$$V_B - Ri - L(di/dt) = 0. \qquad (2)$$

This "differential equation" gives a mathematical description of the behavior of the current i after the switch is closed.

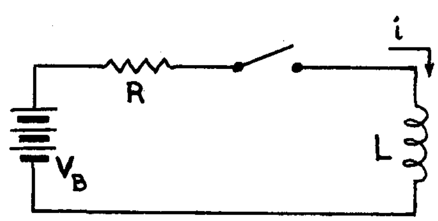


Figure 1

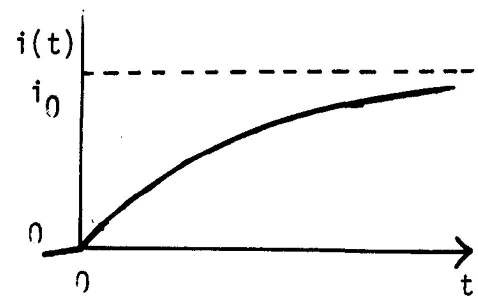


Figure 2

Another possibility is the circuit shown in Figure 3. Initially, the switch is closed; and we imagine that the current i has reached its steady-state value, so that the emf induced in L is zero. Therefore, the voltage difference across R is zero, and the current through the inductor is

$$i_0 = V_B/R_1 \qquad (3)$$

while the switch is closed. When the switch is opened, the emf induced in the inductor again prevents any instantaneous change in i; its initial value is therefore just i_0. The current now flows through the only path open to it, namely, through R; as a result, the resistor R heats up. Evidently, this supply of heat is not unlimited (or we would use it to heat houses!); the current must fall toward zero, as in Figure 4. Incidentally, since the resistor does heat up, we have seen that an inductor stores energy when a current is flowing through it.

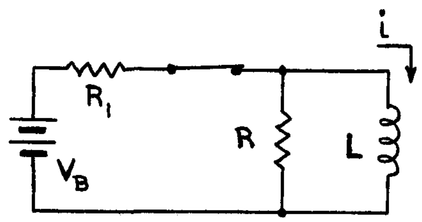


Figure 3

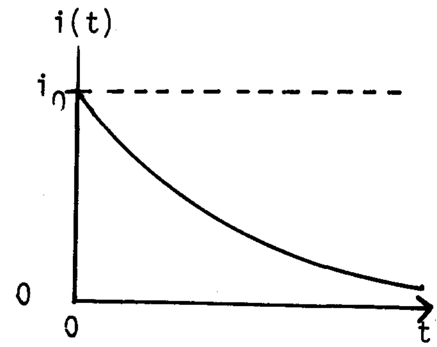
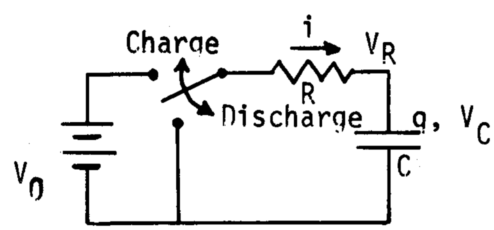


Figure 4

Figures 2 and 4 should recall to mind the behavior of the charge and current in the RC circuits that you studied in the module Direct-Current Circuits. A synopsis of the results for such circuits is presented in Figure 5. If you look back, you will find, for example, that the voltage across the capacitor obeys the

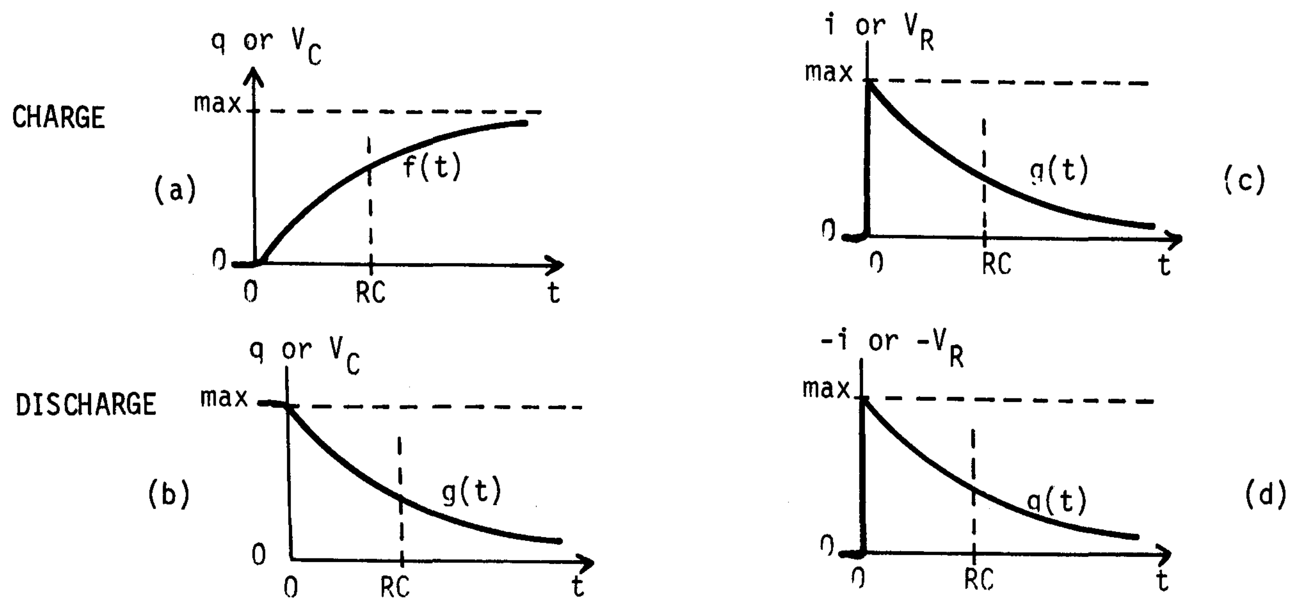
Figure 5: RC Circuits (τ = RC)



Note proportionalities:

$V_C = q/C$; $V_R = Ri$.

Switch is moved at t = 0.



equation

$$V_C = V_B[1 - e^{-t/\tau}] \qquad \text{(where } \tau = RC\text{)} \tag{4}$$

when the switch is moved to the "charge" position, and

$$V_C = V_B e^{-t/\tau} \tag{5}$$

when the switch is moved back to "discharge" after the capacitor has become fully charged. In fact, all the quantities indicated in Figure 5 can be expressed as

$$y = \text{const} \times f(t) \quad \text{or} \quad y = \text{const} \times g(t), \tag{6}$$

$$\text{where} \quad f(t) = 1 - e^{-t/\tau} \text{ and } g(t) = e^{-t/\tau}. \tag{7}$$

Note that $f(0) = 0$ and $f(\infty) = 1$, $g(0) = 1$ and $g(\infty) = 0$.

In each case, "const" is just the maximum value of the quantity in question, which is either the limiting value for large times, or the initial value at $t = 0$.

Of course, we are not really interested in re-solving RC circuits in this module! The point of all this is that we need to find the solution of the differential equation (2). The similarity between Figures 2 and 5(a) - both curves rise from zero to an asymptotic value - suggests that we "try" a solution similar to Eq. (4). (Actually, if you checked the differential equations, you would find that they are similar, too.) That is, we set

$$i(t) = Af(t) = A(1 - e^{-t/\tau}), \tag{8}$$

where the constant A is determined by the condition, from Eq. (1), that

$$i(\infty) = i_0 = V_B/R = A(1 - 0) = A. \tag{9}$$

Substitution of the expression (8) into the differential equation (2) leads to

$$0 = V_B - RA(1 - e^{-t/\tau}) - L(A/\tau)e^{-t/\tau} = V_B - V_B + (V_B - LV_B/R\tau)e^{-t/\tau}, \tag{10}$$

when the value $A = V_B/R$ is inserted, from Eq. (9). This equation is satisfied if and only if

$$\tau = R/L. \tag{11}$$

With these particular values, Eq. (8) becomes

$$i(t) = (V_B/R)(1 - e^{-t/\tau}); \tag{12}$$

we have found the needed solution to Eq. (1), for the current $i(t)$ in the circuit of Figure 1! (It can also be shown that this solution is unique.)

Other LR circuit problems can be analyzed in this same way; as above, the steps are:

(a) Determine the qualitative behavior of the current as a function of time, including its maximum value.

(b) Add voltages around the circuit to find the appropriate differential equation.

(c) "Try" a solution to (b) of the form $Af(t)$ or $Ag(t)$, where $f(t)$ and $g(t)$ are defined in Eq. (7), depending on whether (a) was increasing or decreasing. [The constant A is equal to the maximum value found in part (a).] The resulting equation gives the correct value of τ.

Once you know $i(t)$, it is a relatively simple matter to find any other quantity, such as the voltage across the resistor, $V_R = Ri$, or the induced emf $\mathcal{E}_L = -di/dt$. A summary of the results for typical LR circuits is presented in Figure 6.

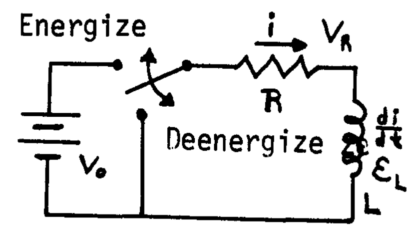
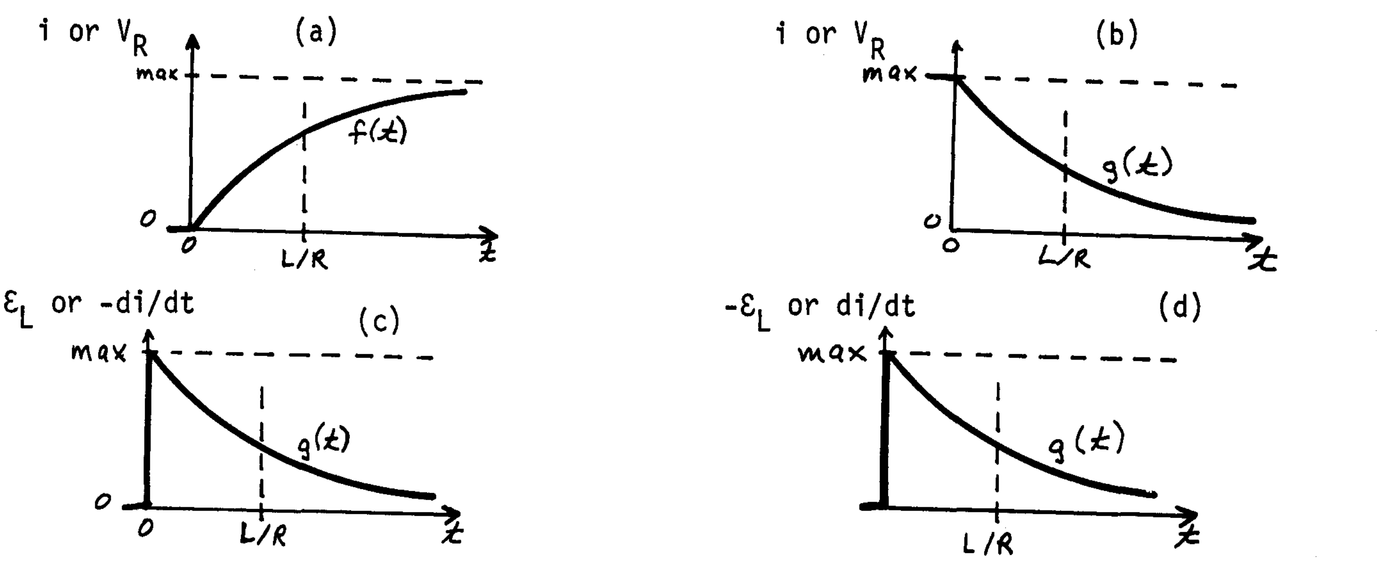


Figure 6: LR Circuits ($\tau = L/R$)

Note proportionalities: $\mathcal{E}_L = -L(di/dt)$; $V_R = Ri$. Switch is moved at $t = 0$.

ENERGIZE (Corresponds to "Charge")

DEENERGIZE (Corresponds to "Discharge")



2. LC Circuits

As the term "LC" implies, we are assuming idealized inductors and capacitors, with negligible resistive or other dissipative effects; the circuit shown in Figure 7 is constructed from such idealized components. With the switch in position a, the capacitor acquires the charge

$$q_0 = CV_B. \tag{13}$$

As you learned in the module Capacitors, this implies that an amount of energy

$$U_C = (1/2)(q_0^2/C) = U_0 \tag{14}$$

is stored in the capacitor.

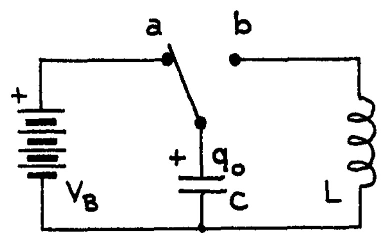


Figure 7

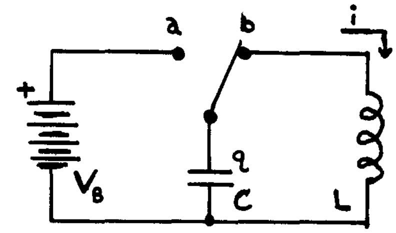


Figure 8

When the switch is moved to position b as in Figure 8, positive charge starts to flow from the upper plate of C through L. Eventually, the capacitor becomes discharged (q = 0), at which time

$$U_C = 0. \tag{15}$$

But the energy U_0 that was originally stored in the capacitor must have gone somewhere. It could not have been converted to internal ("heat") energy, since there are no resistors in the circuit; therefore it must have gone into the inductor. That is, the current must have the value $i = i_0$ such that the energy stored in the inductor is

$$U_L = (1/2)Li_0^2 = U_0. \tag{16}$$

Of course, the inductor will not let the current stop abruptly; the capacitor thus proceeds to charge up again, but with <u>negative</u> charge on the top plate. The current <u>does</u> stop, however, when all the energy has been transferred back to the capacitor, i.e., when

$$U_C = U_0 \quad \text{and} \quad U_L = 0. \tag{17}$$

Next, the potential difference across the charged capacitor plates again starts a current flowing through the inductor, but in the opposite direction from before - and so on.

Thus, there is a continual transfer of energy back and forth between the capacitor and the inductor, in such a way that the total energy is constant:

$$U_C(t) + U_L(t) = U_0. \tag{18}$$

See Figure 9. This transfer of energy back and forth is very nicely portrayed by the upper circular diagram on p. 10 [Fig. 11(a)].

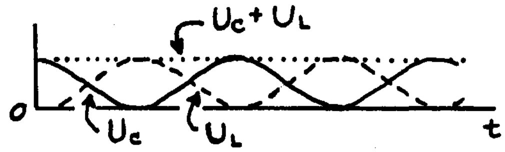


Figure 9

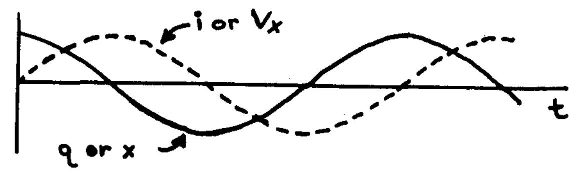


Figure 10

The lower circular diagram [Fig. 11(b)] shows the analogous situation in a mechanical oscillator; the spring potential energy is the analog of U_C, and the kinetic energy of the moving mass is the analog of U_L. The oscillation of the positive charge between the upper and lower plates of C is very similar to the back and forth motion of the mass in the mechanical case; in fact, we can represent the charge q and the displacement x (or the current i and the velocity v_x) by the same graph, as in Figure 10, provided we match up the amplitudes, frequencies, and phases.

The curves in Figure 10 were drawn to be sinusoidal; how can we check this claimed behavior? Simple enough:

(a) Write the equation of energy conservation:

$$(1/2)q^2/C + (1/2)Li^2 = U_0.$$

(b) Differentiate with respect to time:

$$(1/C)q(dq/dt) + Li(di/dt) = 0.$$

(c) Use i = dq/dt, cancel a factor, and rearrange:

$$LC(d^2q/dt^2) + q = 0. \quad (19)$$

Another differential equation! But do not despair; we are merely going to check the claimed sinusoidal behavior - thus we set

$$q(t) = q_m \cos(\omega t + \phi), \quad (20)$$

(where q_m, ω, and ϕ are constants to be determined) and substitute this expression into Eq. (19) to see whether or not it is a solution. The substitution leads to

$$-LC\omega^2 q_m \cos(\omega t + \phi) + q_m \cos(\omega t + \phi) = 0, \quad (21)$$

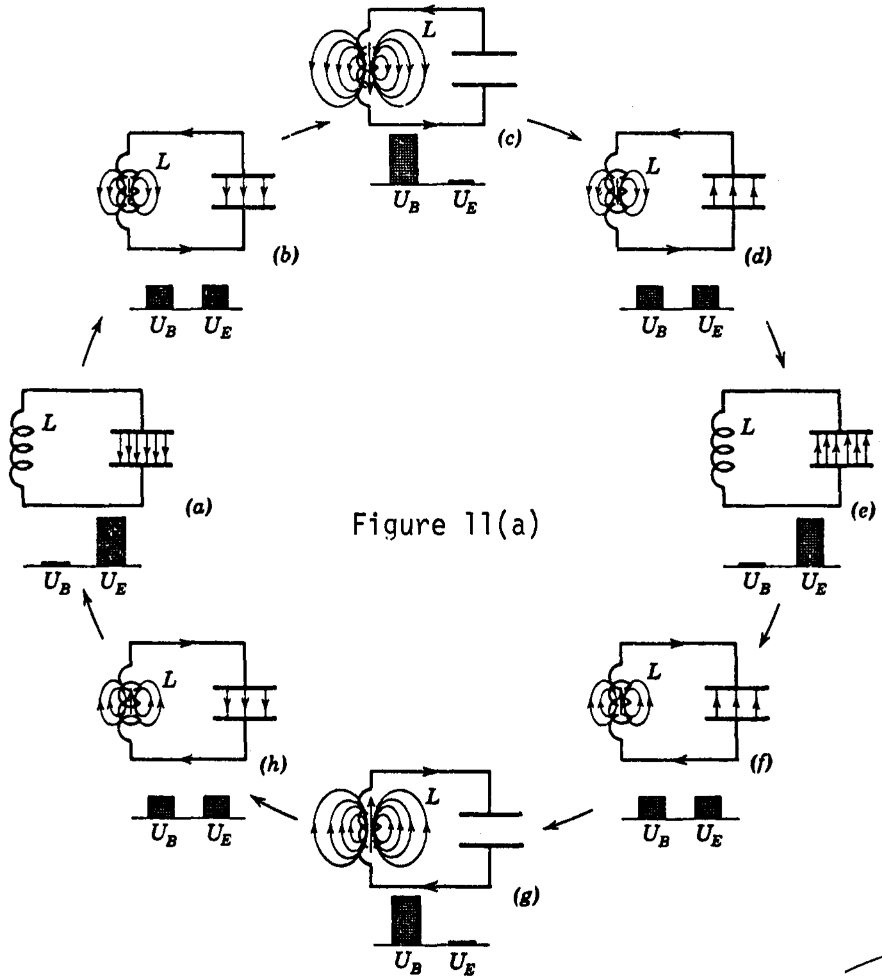


Figure 11(a)

Figures 11(a) and (b) visualize the energy transfer that occurs during one cycle of an electrical oscillator [Figure 11(a)] and of a mechanical oscillator [Figure 11(b)]. Note the amazingly similar behavior of these apparently dissimilar devices.

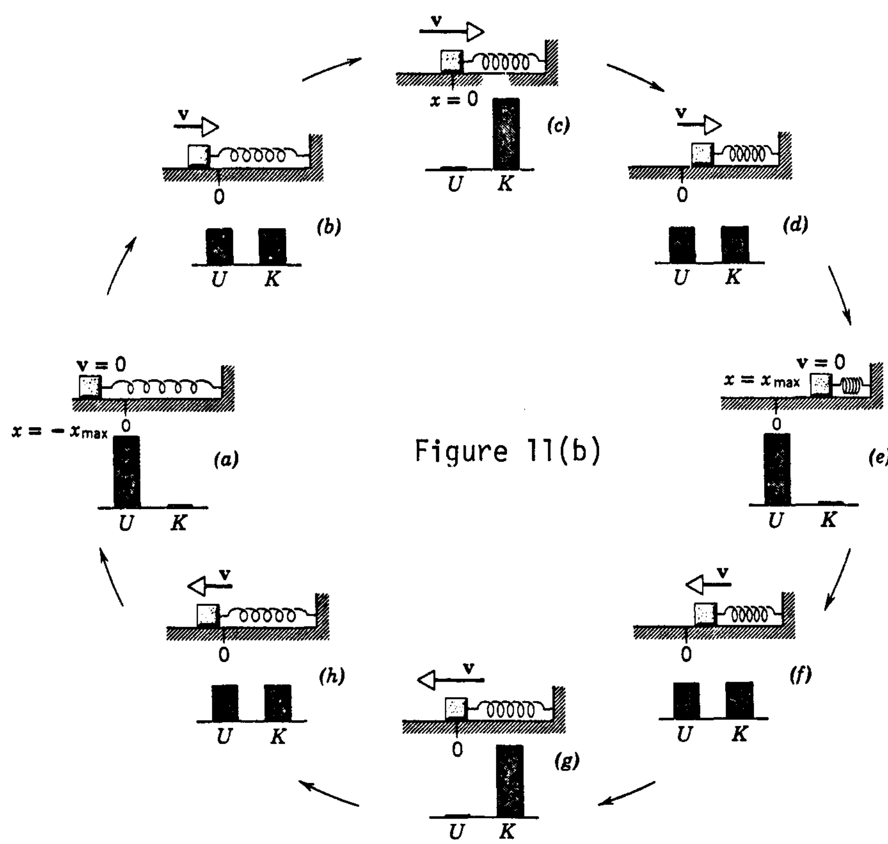


Figure 11(b)

The diagrams on this page [Figures 11(a) and 11(b)] have been reprinted from Fundamentals of Physics, by David Halliday and Robert Resnick (Wiley, New York, 1970; revised printing, 1974), with permission of the publisher. In the text they are Figures 34-1 and 7-4, respectively.

which is satisfied provided

$$\omega = \sqrt{1/LC}. \tag{22}$$

The sinusoidal behavior (20) is thus verified, and we have found the frequency

$$f = \omega/2\pi = (1/2\pi)\sqrt{1/LC}. \tag{23}$$

As in the mechanical case, the values of q_0 and ϕ are determined by initial conditions. For example, if the switch in Figure 7 is moved to b at t = 0, then

$$q(t = 0) = CV_B \quad \text{and} \quad dq/dt(t = 0) = 0; \tag{24}$$

applying these conditions to Eq. (20) yields

$$q_m = CV_B \quad \text{and} \quad \phi = 0. \tag{25}$$

ADDITIONAL LEARNING MATERIALS

Auxilliary Reading

Stanley Williams, Kenneth Brownstein, and Robert Gray, Student Study Guide with Programmed Problems to Accompany Fundamentals of Physics and Physics, Parts I and II, by David Halliday and Robert Resnick (Wiley, New York, 1970).
Objective 1: Sections 31-1 and 31-2;
Objective 2: Sections 31-4 and 31-7 through 31-9;
Objective 3: Section 33-1.

Various Texts

Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition: Sections 25.3 through 25.5 and 27.8.

David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974): Sections 32-1 through 32-4 and 34-1 through 34-3.

Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition: Sections 33-9 through 33-12.

Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2: Sections 32-1 through 32-3 and 34-1.

PROBLEM SET WITH SOLUTIONS

A(1). An inductor in Figure 12 has been wound on a long cylindrical form with a square cross section measuring 1.00 cm by 1.00 cm. The winding has been painted over, so that it is impossible to count the turns; however, you are able to determine that the flux through the center is 1.00 μT when the current is 4.0 A.

(a) Apply Ampère's law (indicate your path) to find the number of turns per meter.
(b) If the inductance is 200 μH, about how long is the winding? Neglect the spreading of the magnetic lines of force at the ends.
(c) If a potential difference of 1.00 mV is applied between the ends of the inductor, at what rate does the current increase, as long as the resistance of the inductor can be neglected?

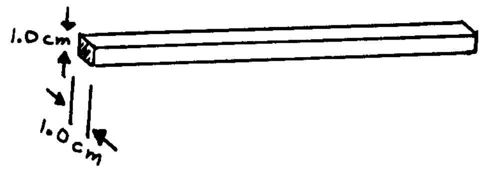


Figure 12

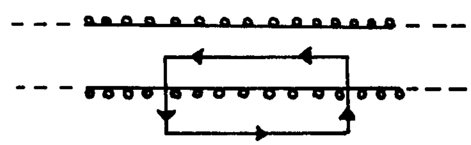


Figure 13

Solution

(a) Applying Ampère's law to the rectangular path in Figure 13 yields

$$\ell B = \int \vec{B} \cdot d\vec{\ell} = \mu_0 N' i,$$

where ℓ is the length of the rectangle, B is the strength of the magnetic field, N' is the number of turns enclosed by the rectangle, and i is the current in each. Since B is the quotient of flux and area, the number of turns per unit length is

$$n = \frac{N'}{\ell} = \frac{B}{\mu_0 i} = \frac{1.00 \times 10^{-6}\ \text{turns}}{(4\pi \times 10^{-7})(4.0)(1.00 \times 10^{-4})\ \text{m}} = 1.99 \times 10^3\ \text{turns/m}.$$

(b) Since the total number of turns is $N = Li/\Phi_B$, the length must be

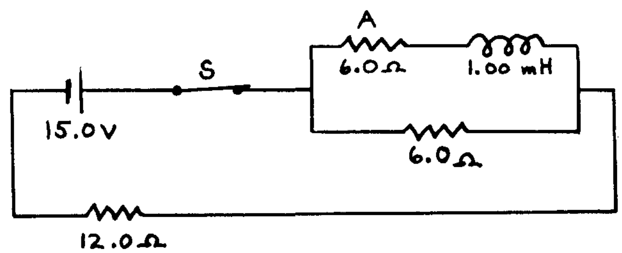
$$\frac{N}{n} = \frac{Li}{\Phi_B n} = \frac{(2.00 \times 10^{-4})(4.0)}{(1.00 \times 10^{-6})(1.99 \times 10^3)}\ \text{m} = 0.40\ \text{m}.$$

(c) Since $|\mathcal{E}| = L(di/dt)$,

$$\frac{di}{dt} = \frac{|\mathcal{E}|}{L} = \frac{1.00 \times 10^{-3}\ \text{A}}{2.00 \times 10^{-4}\ \text{s}} = 5.0\ \text{A/s}.$$

B(2). (a) After switch S in Figure 14 has been closed for a long time, what is the current through the inductor?
(b) The switch is then opened at time t = 0. What is the voltage across resistor A at a later time t? Verify your answer.
(c) How much energy is dissipated in resistor A between time t = 0 and t = ∞?

Figure 14



Solution

(a) The presence of the inductor can be neglected, since $di/dt = 0$. The series-parallel combination of resistors has an effective resistance of 15.0 Ω; the total current is thus 1.00 A, and the current through the inductor is 0.50 A.

(b) Adding voltages around the small loop yields

$$-Ri - Ri - L(di/dt) = 0 \quad \text{or} \quad di/dt = -(2R/L)i.$$

This differential equation is satisfied by $i = i_0 e^{-2Rt/L}$; and we must have $i_0 = 0.50$ A to get $i(0) = 0.50$ A, as required by part (a). Therefore

$$i = (0.50\ \text{A})e^{-1.20 \times 10^4 t\ \text{s}}.$$

(c) At $t = 0$, the energy stored in the inductor is $(1/2)Li_0^2$. This energy will be dissipated in the two resistors; and since the resistances are equal, half must clearly go to each. Thus $(1/4)Li_0^2 = 6.2 \times 10^{-5}$ J is dissipated in resistor A.

C(2). Find the voltage $V_R(t)$ across the resistor R in the circuit of Figure 3 (General Comments) when the switch is opened after having been closed a long time.

Solution

Step (a), finding the qualitative behavior, has already been done in Figure 4 above: $i(t)$ is a decreasing function, and must therefore be proportional to $g(t)$. Also, we found that its maximum value, which occurs at $t = 0$, is just

$$i(0) = i_0 = V_B/R_1.$$

[See Eq. (3) and the text immediately following.] Step (b): Adding up voltages around the circuit (with the switch open) yields the differential equation

$$-Ri - L(di/dt) = 0. \qquad (*)$$

Step (c): Setting

$$i(t) = Ag(t) = Ae^{-t/\tau}, \qquad \text{with } A = V_B/R_1$$

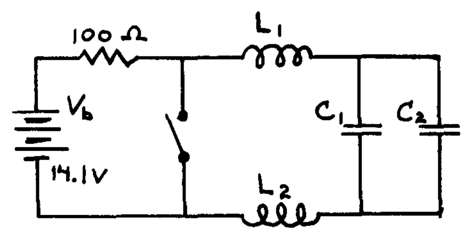
to satisfy $i(0) = V_B/R_1$, and substituting these expressions into (*) yields

$$-RAe^{-t/\tau} + (LA/\tau)e^{-t/\tau} = 0.$$

This is satisfied by the choice $\tau = L/R$, so that

$$i(t) = (V_B/R_1)e^{-Rt/L} \quad \text{and} \quad V_R(t) = Ri(t) = (RV_B/R_1)e^{-Rt/L}.$$

Figure 15



D(3). In the circuit shown in Figure 15, $C_1 = C_2 = 50\ \mu F$, $L_1 = L_2 = 1.80$ mH, and the resistance of the inductors is negligible.

(a) Find the energies stored in each of the capacitors and inductors, given that the switch has been open a long time.

(b) The switch is now closed; write down the energy-conservation equation that applies now. Do this directly; do not combine capacitors and inductors.

(c) Use (b) to derive a "differential equation" and show that the current in the circuit can be written in the form $i = i_0 \sin \omega t$. Derive the value of ω.

Solution

(a) The current is zero, and the potential difference V_b appears across each capacitor. Thus the energies stored in C_1, C_2, L_1, and L_2 are $(1/2)C_1V_b^2 = 5.0$ mJ, the same, 0, and 0, respectively.

(b) $$(1/2)L_1i^2 + (1/2)L_2i^2 + \frac{(1/2)q_1^2}{C_1} + \frac{(1/2)q_2^2}{C_2} = U_0 \text{ (= sum of energies above).}$$

(c) First, let us set $L_1 = L_2 \equiv L$, and $C_1 = C_2 \equiv C$. We note that i is the derivative of the total charge $q = q_1 + q_2$; and $q_1 = q_2$. It will thus be convenient to replace q_1 and q_2 by $(1/2)q$. The energy equation is now

$$Li^2 + q^2/4C = U_0.$$

Differentiating this and replacing i by dq/dt yields

$$2L \frac{dq}{dt} \frac{d^2q}{dt^2} + \frac{1}{2C} q \frac{dq}{dt} = 0 \quad \text{or} \quad 4LC \frac{d^2q}{dt^2} + q = 0.$$

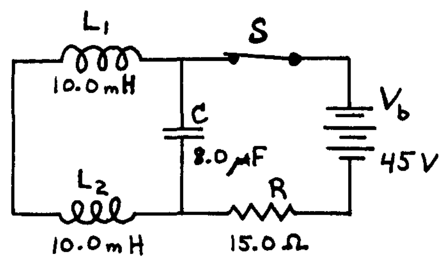
This is the desired differential equation. The expression $q = q_0 \cos(\omega t + \phi)$ satisfies this, provided $\omega = \sqrt{1/4LC}$ - just try it and see. Differentiating that expression yields

$$i = -\omega q_0 \sin(\omega t + \phi) = i_0 \sin(\omega t + \phi),$$

where $i_0 = -\omega q_0$. Finally, we must have $\phi = 0$ to get $i = 0$ at $t = 0$.

E(3). The inductors in the circuit shown in Figure 16 have negligible resistance. The switch has been closed a long time.
(a) Find the energies stored in the inductors and in the capacitor.
(b) The switch is now opened, at t = 0. Use energy conservation to determine the differential equation for the charge in the capacitor.
(c) Write an expression for the charge as a function of time for t > 0, and show that it satisfies the differential equation. Evaluate all parameters (such as ω) that occur in this expression.
(d) Find the voltage across the capacitor at $t = \pi\sqrt{2CL_1/6}$.

Figure 16



Solution

(a) The current through L_1 is V_b/R; thus the energy stored in it is $(1/2)L_1 i_2^2 = L_1 V_b^2/2R^2 = 45$ mJ. The same is true of L_2. There is no voltage across C; there is thus no energy stored in it.

(b) The current is the same in L_1 and L_2; call it i. Thus the total energy is

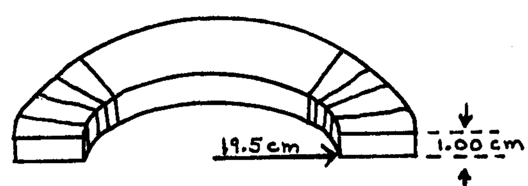
$$(1/2)L_1 i^2 + (1/2)L_2 i^2 + \frac{(1/2)q^2}{C} = Li^2 + \frac{(1/2)q^2}{C} = U_0,$$

where $L = L_1 = L_2$ and $(1/2)U_0$ is the energy calculated in (a). Differentiating with respect to time and replacing i by dq/dt yields

$$2L\left(\frac{dq}{dt}\right)\left(\frac{d^2q}{dt^2}\right) + \left(\frac{q}{C}\right)\left(\frac{dq}{dt}\right) = 0 \quad \text{or} \quad 2LC\left(\frac{d^2q}{dq^2}\right) + q = 0.$$

(c) $q = q_0 \sin \omega t$. (We have already arranged the phase to make q = 0 at t = 0.) The second derivative of this is $-\omega^2 q_0 \sin \omega t$; thus the differential equation is satisfied provided $\omega = 1/\sqrt{2LC} = 2.50 \times 10^3$ rad/s. The derivative of the expression for q is $i = \omega q_0 \cos \omega t$, which becomes ωq_0 at t = 0. But this must be the same as the current before the switch was opened, which was found in part (a) to be V_b/R; we therefore have $q_0 = V_b/R\omega = 1.20$ mC.
Alternate evaluation of q_0: When i = 0, $q = q_0$; at this time all the energy resides in the capacitor. Therefore by energy conservation, $U_0 = q_0^2/2C$, or $q_0 = \sqrt{2U_0C}$, where from part (a), $U_0 = 90$ mJ.
(d) $V = q/C = (q_0/C) \sin \omega t \rightarrow q_0/2C = 75$ V at the given time.

Figure 17



Problems

F(1). A toroidal inductor as in Figure 17 having 1000 turns is wound on a form with an inner radius $r_0 = 19.5$ cm and square cross section of base and height equal to 1.00 cm.
(a) Find the $\vec{B}$ field inside the inductor as a function of the radius r from the toroid's center and the current i. Indicate the path you used when applying Ampère's law.
(b) Find the inductance of the inductor. You may neglect the variation of $\vec{B}$ with radius inside the windings.
(c) What is the induced emf when a current through the winding is increasing at 10.0 A/s?

G(1). You are winding an inductor on a cylindrical form with a rectangular cross section measuring 1.00 cm by 2.00 cm, as in Figure 18. The wire you are using allows you to get 10 turns per centimeter. Assume that you will wind an inductor long enough that the spreading of the lines of force at the ends can be neglected, as a good enough approximation.
(a) Find the flux Φ through a turn near the center, as a function of the current i. Use Ampère's law, and indicate the path you have chosen.
(b) Approximately how long a winding do you need for an inductance of 150 μH?
(c) At what rate $d\Phi/dt$ will the flux increase when a potential difference of 3.00 mV is applied across the inductor?

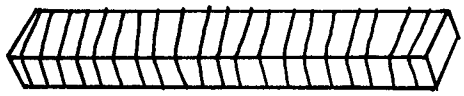


Figure 18

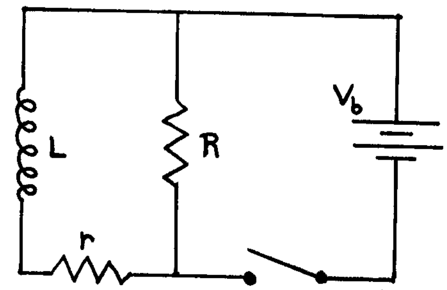


Figure 19

H(2). In Figure 19 an inductor (with inductance L and resistance r) and a resistor R in parallel are connected to a battery until the currents reach steady values. Then the switch leading to the battery is suddenly opened.
(a) Express the following four currents in terms of V_b, L, R, and r:

(i) through the inductor before the switch is opened;
(ii) through the inductor immediately after the switch is opened;
(iii) through the resistor R before the switch is opened;
(iv) through the resistor R immediately after the switch is opened.

(b) What emf appears across the resistor R immediately after the battery is disconnected?
(c) What is the exponential time dependence of the current after the switch is opened? Derive the differential equation for the current, and use it to verify your answer. Sketch this behavior as a function of time.
(d) Compute the total energy dissipated.

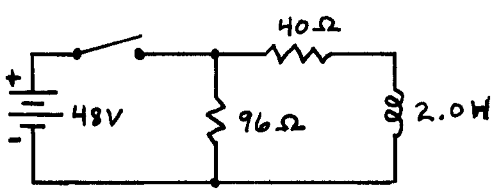


Figure 20

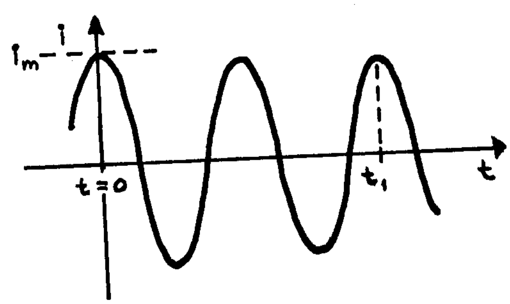


Figure 21

I(2). The switch in Figure 20 has been open a long time; it is now closed. The resistance of the inductor is negligible.
(a) Sketch the time dependence of the current through the inductor.
(b) What is the current through the 40-Ω resistor 0.0200 s later?
(c) How much energy is stored in the inductor at this instant?
(d) If you come back the next day, how much power do you find being dissipated by the 40-Ω resistor?

J(3). An oscilloscope connected to plot the current through the inductor in an LC circuit as a function of time produces the trace shown in Figure 21. Note that the current is a maximum at $t = 0$; you are able to determine that this maximum current is $i_m = 10.0$ mA. Also, you find that the time t_1 is 4.0 ms. The capacitor in this circuit has a capacitance of 0.200 μF.
(a) Neglecting the slow decrease of amplitude of the oscillations caused by energy losses, how long does it take for the current to drop from i_m to $(1/2)i_m$?
(b) How large is L?
(c) Again neglecting energy losses, use energy conservation to find the maximum value V_m of the potential across the capacitor.

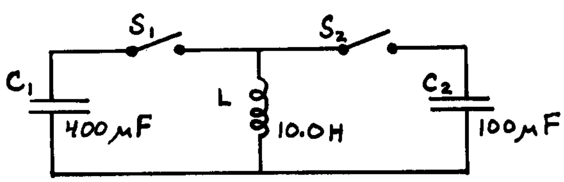


Figure 22

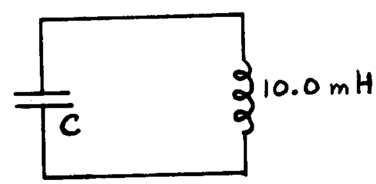


Figure 23

K(3). Initially, C_1 in Figure 22 is charged to a potential of 100 V.
(a) Describe how you can manipulate switches S_1 and S_2 to charge C_2 to a potential larger than 100 V. (If you act fast enough!)
(b) What is the largest potential you can obtain this way?
(c) How long after you close S_2 should you open it again?

L(3). The circuit shown in Figure 23 is oscillating at the frequency $f = 5.0 \times 10^4$ Hz. At the instant $t = 0$, there is a current $i = 0.0100$ A flowing through the inductor, but no charge on the capacitor plates.
(a) Find the capacitance C.
(b) Use energy conservation to find q_{max} and V_{max} for the capacitor.
(c) Find the current in the inductor at $t = 2.50 \times 10^{-6}$ s.
(d) Use energy conservation to derive a differential equation involving q, and from that derive the expression for f that you used in part (a).

Figure 24

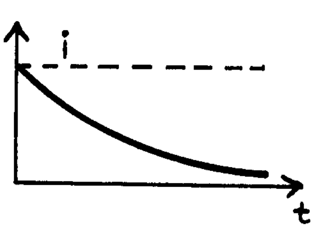
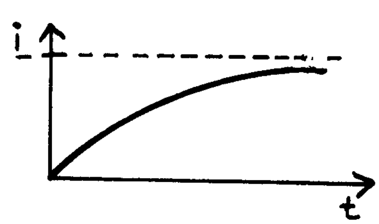


Figure 25



Solutions

F(1). (a) $(2.00 \times 10^{-4}$ Wb/A m$)i/r$. (b) 1.00×10^{-4} H. (c) 1.00 mV.

G(1). (a) $(2.50 \times 10^{-7}$ Wb/A$)i$. (b) 0.60 m. (c) 5.0 μWb/s.

H(2). (a)(i) V_b/r; (ii) V_b/r; (iii) V_b/R; (iv) V_b/r. (b) V_bR/r.
(c) $i = i_0e^{-(R + r)t/L}$. Adding emfs around the circuit yields $-L(di/dt) = -(R + r)i = 0$; substituting the expression for i yields $[L(R + r)/L - (R + r)]i_0e^{-(R + r)t/L} = 0$. See Figure 24. (d) $LV_b^2/2r^2$.

I(2). (a) See Figure 25. (b) 0.40 A. (c) 0.160 J. (d) 58 W.

J(3). (a) 0.330 ms. (b) 0.51 H. (c) 15.9 V.

K(3). (a) Close S_1; wait until all the energy is in L; close S_2 and open S_1; wait until all the energy is in C_2; open S_2. (b) 200 V. (c) 1/4 cycle = 0.050 s.

L(3). (a) 1.01×10^{-9} F. (b) 3.2×10^{-8} C, 31 V. (c) 7.1 mA.
(d) $\frac{1}{2}(q^2/C) + \frac{1}{2}Li^2 = 0$.
Differentiate and set $i = dq/dt$ to get $q + LC(d^2q/dt^2) = 0$; $q = q_0 \sin \omega t$ satisfies this provided $\omega^2 = LC$, or $f = 1/2\pi\sqrt{LC}$.

PRACTICE TEST

1. The toroidal inductor in Figure 26 consists of 2000 turns wound on a form with an average (principal) radius $r_0 = 20.0$ cm. The cross section of the winding is somewhat unusual: an isosceles triangle with base b = 1.00 cm and height h = 1.00 cm.
   (a) Choose an appropriate path (indicate what it is), and use Ampère's law to find $\vec{B}$ for points inside the winding as a function of the distance r from the center C and of the current i in the windings.
   (b) Find the inductance of this toroid, neglecting the variation of $\vec{B}$ with r for points inside the winding.
   (c) What is the induced emf when the flux through the toroid is increasing at the rate $d\phi/dt = 3.00$ μWb/s?

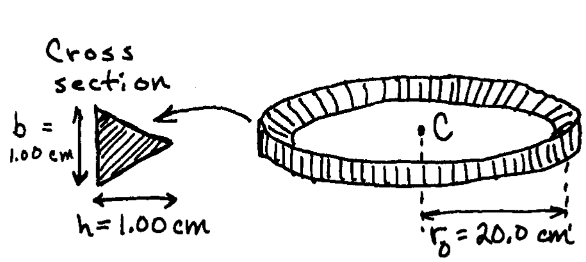


Figure 26

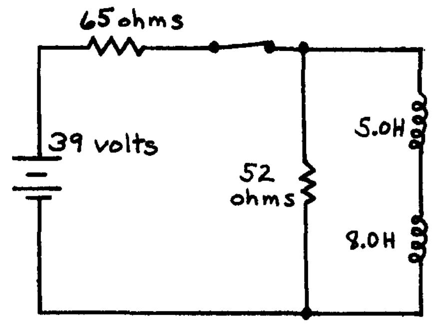


Figure 27

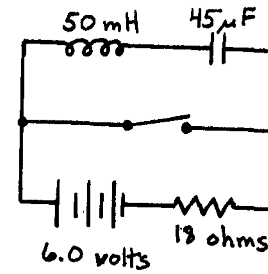


Figure 28

2. (a) The switch in Figure 27 has been closed a long time. What is the power dissipated by the 65-Ω resistor? The resistance of the inductors is negligible.
(b) The switch is now opened. Add voltages to determine the time dependence of the current.
(c) What will the current through the inductors be after 0.150 s?
(d) How much energy will be stored in the two inductors together at that instant?

3. The switch in Figure 28 has been open a long time; it is now closed.
(a) What is the frequency of oscillation?
(b) What is the maximum current in the inductor?
(c) Use energy conservation to derive a differential equation for the charge on the capacitor.
(d) Use your result (c) to show that the charge can be expressed in the form $q = q_0 \cos \omega t$, assuming the switch was closed at $t = 0$.

Practice Test Answers

1. (a) Use a path with radius r, inside the toroid; $B = (4.0 \times 10^{-4}$ Wb/A m)(i/r). (b) 200 μH. (c) 6.0 mV.

2. (a) 23.4 W. (b) The differential equation is $di/dt + (4.0\ s)i = 0$, which is satisfied by $i = i_0 e^{-4.0t\ s}$. (c) 0.33 A. (d) 0.71 J.

3. (a) 106 Hz. (b) 0.180 A. (c) Energy conservation reads $(1/2)(q^2/C) + (1/2)Li^2 = U_0$; upon differentiation and simplification, this becomes $q + LC(di^2/dt^2) = 0$. (d) The expression given has its maximum at $t = 0$, as it should; substituting it into the differential equation yields $q_0 \cos \omega t - \omega^2 LC \cos \omega t$, which is satisfied provided $\omega = \sqrt{1/LC}$.

INDUCTANCE

Date ____________________

Mastery Test   Form A

pass		recycle
1	2	3

Name ________________________________   Tutor ________________________

1. The inductor in Figure 1 is wound on a toroidal form with average (principal) radius r_0 = 20.0 cm. The cross section of the toroid is an ellipse of height 2a = 1.00 cm and width 2b = 2.00 cm. There are 400 turns around the toroid. Recall that the area of an ellipse is equal to πab, where a and b are the large and small radii, respectively.

   (a) Use Ampère's law to express $\vec{B}$ inside the toroid as a function of the current i in each turn and of the radius r from the center C of the toroid. Indicate the path you used.
   (b) Find the inductance of this toroid. (Neglect the variation of $\vec{B}$ with r inside the toroid.)
   (c) At a certain instant of time, the current through this inductor is increasing at the rate di/dt = 2.00 A/s. What is the induced emf?

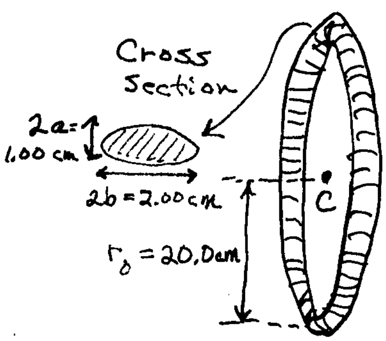


Figure 1

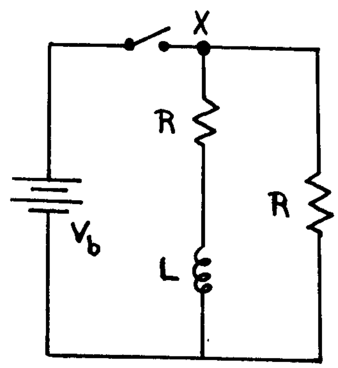


Figure 2

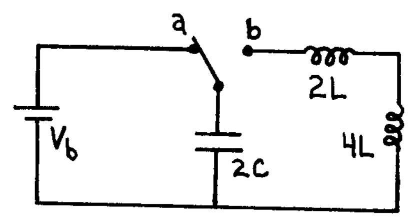


Figure 3

2. For the circuit shown in Figure 2:

   (a) State the current at point X immediately after the switch is closed at t = 0 s.
   (b) Find the energy stored after a very long time of operation.
   (c) Find the current as a function of time at point X after the switch is opened at $t = t_0$, when the circuits have been operating for a long time. Show that your answer is correct by adding voltages around the appropriate path.

3. The inductors in Figure 3 have negligible resistance. The switch is moved from a to b at time t = 0 s. Answer the following in terms of L, C, V_b, t, and constants:

   (a) Use energy conservation to determine the differential equation for the charge on the capacitor.
   (b) Write an expression for the charge as a function of time for $t \geq 0$ s.
   (c) Show that this expression satisfies the differential equation.
   (d) Evaluate all the parameters (such as ω) that occur in your expression for part (b).
   (e) Find the voltage across the capacitor at $t = (1/2)\pi\sqrt{3LC}$.
   (f) How much energy is stored in the "4L" inductor at that time?

INDUCTANCE

Date ____________

Mastery Test   Form B

pass   recycle

1   2   3

Name ____________   Tutor ____________

1. A solenoid 1.00 m long is wound with a double layer of thin copper wire allowing 1000 turns/m in each layer. The radius of the solenoid is 3.00 cm.
(a) Find the inductance. Indicate the path you use when applying Ampère's law.
(b) What voltage must be applied to cause the current to increase at the rate of 5.0 A/s?

2. The switch in Figure 1 has been closed a long time. It is now opened. Assume the inductors have negligible resistance.
(a) Add voltages to find a differential equation for the current through the 51-Ω resistor, and sketch it. [Hint: Since the voltages across the two inductors are the same, $L_1(di_1/dt) = -\mathcal{E}_1 = -\mathcal{E}_2 = L_2(di_2/dt)$, which can be integrated to yield $L_1 i_1 = L_2 i_2$ at all instants of time.]
(b) What is the current through the 51-Ω resistor 0.75 s after the switch is opened?
(c) At what rate is energy being dissipated in the 51-Ω resistor at this instant?

3. In Figure 2, t = 0, q = 2.00 μC and i = 0.
(a) What is q_{max}? What is V_{max} across the capacitor? Use energy conservation to find i_{max}.
(b) At what frequency f does this circuit oscillate?
(c) Write down expressions for q and i at any instant of time. (Give numerical values for all constants that appear in these equations.)
(d) Use energy conservation to obtain a differential equation involving q; and use this to verify the expression for q that you wrote in part (c).

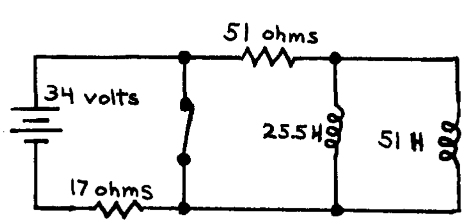


Figure 1

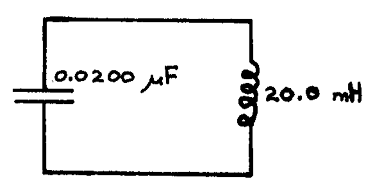


Figure 2

INDUCTANCE

Date ______________________

Mastery Test   Form C

pass   recycle

1   2   3

Name ______________________________   Tutor ______________________________

1. An inductor is wound on a toroidal form with circular cross section of radius $r = 1.50$ cm. The radius of the toroid to the center of the cross section is 30.0 cm; it has 2000 turns of wire.
(a) Find the inductance. (Be sure to indicate the path you use when applying Ampère's law.)
(b) If there is an induced emf of 10.0 μV, at what rate is the flux increasing through the toroid?

2. The switch in Figure 1 is closed at $t = 0$ s, after having been open a long time. Assume the inductors have negligible resistance.
(a) Add voltages around the appropriate loop to determine the exponential time dependence of the current.
(b) What is the current flowing through the inductors at $t = 0.50$ s?
(c) How much energy is dissipated in the 35-Ω resistor from the time the switch is closed until the next day?

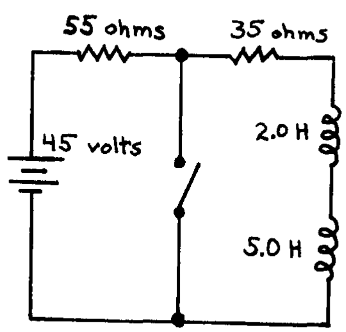


Figure 1

3. After the circuit in Figure 2 has reached the steady state, switch S is closed. Calculate the following for the LC circuit:
(a) the frequency of oscillation;
(b) the energy in the circuit; and
(c) the maximum current in the inductor.
(d) Wite the appropriate energy-conservation equation, differentiate it, and simplify to obtain a "differential equation" for the charge. Use this equation to verify the expression for frequency that you used in part (a).

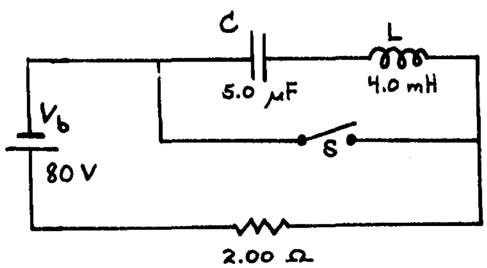


Figure 2

MASTERY TEST GRADING KEY - Form A

1. What To Look For: (a) Make sure student indicates path used, and really applies Ampère's law.

S Solution: (a) The easiest path is a circle of radius r, inside the toroid. Ampère's law gives $2\ rB = \mu_0 Ni$, so

$$B = \frac{\mu_0 N}{2\pi}\left(\frac{i}{r}\right) = \frac{(4\pi \times 10^{-7})(400)}{2\pi} \text{ W/A m}\left(\frac{i}{r}\right) = (8.0 \times 10^{-5} \text{ W/A m})\left(\frac{i}{r}\right).$$

(b) $L = \frac{N\Phi}{i} = \frac{NBA}{i} = \frac{(400)(8.0 \times 10^{-5})(0.50 \times 10^{-4})\pi}{(0.20)} \text{H} = 2.5 \times 10^{-5}$ H. $\times \pi = 2.5 \times 10^{-5}$ H

(c) $|\mathcal{E}| = L(di/dt) = (2.5 \times 10^{-5})(2.00)$ V $= 50.0$ μV.

2. What To Look For: (c) Check that student derives the differential equation.

Solution: (a) $2V_b/R$ (no current through L).

(b) The current in the inductor is V_b/R, thus the energy is $(\frac{1}{2})(LV_b^2/R^2)$.

(c) The current will have a decreasing exponential dependence on t:

$$i = i_0 e^{-2R(t - t_0)/L},$$

Adding voltages: $-Ri - Ri - L(di/dt) = 0$, or $di/dt + (2R/L)i = 0$. Substituting the expression for i:

$$(-2R/L + 2R/L)i_0 e^{-2R(t - t_0)/L} = 0.$$

3. Solution: (a) $(\frac{1}{2})\frac{q^2}{2C} + \frac{1}{2}(2L)i^2 + \frac{1}{2}(4L)i^2 = U_0 (= \frac{1}{2}V_b^2 C)$ or $\frac{q^2}{2C} + 6Li^2 = U_0$. Differentiate:

$$(q/C)(dq/dt) + 12Li(di/dt) = 0 \quad \text{or} \quad d^2q/dt^2 + (1/12LC)q = 0.$$

(b) $q = q_0 \cos \omega t$ where $q_0 = 2CV_b$ and $\omega = \sqrt{1/12LC}$.

(c) Substituting (b) into the D.E. yields $-\omega^2 q_0 \cos \omega t + (1/12LC)q = 0$.

(d) Already done above; note $q(0) = 2CV_b$.

(e) $T = 2\pi/\omega = 2\omega\sqrt{12LC}$, so $\frac{t}{T} = 1/8$ period; $V = \frac{q}{2C} = \frac{q_0 \cos(\pi/4)}{2C} = \frac{V_b}{\sqrt{2}}$.

(f) $|i| = |dq/dt| = \omega q_0 \sin \omega t = \frac{\frac{1}{2}\sqrt{1/3LC}(2CV_b)}{\sqrt{2}} = V_b\sqrt{C/6L}$. So Energy $= \frac{1}{2}(4L)i^2 = \frac{1}{3}CV_b^2$.

MASTERY TEST GRADING KEY - Form B

1. What To Look For: (a) Check path and actual use of Ampère's law.

Solution: (a) See Figure 36. Only the "top" of the rectangle gives a contribution to the integral. $\ell B = \mu_0 n \ell i$, where n = turns/m.

$$L = \frac{N\Phi}{i} = \frac{N\pi r^2 B}{i} = (n\ell)(\pi r^2)\mu_0 n = \pi\mu_0 r^2\, n^2 \ell$$

$$= \pi(4\pi \times 10^{-7})(3.00 \times 10^{-2})^2(1.00)(2.00 \times 10^3)^2 = 14.2\text{ mH}.$$

(b) $|\mathcal{E}| = L(di/dt) = 71$ mV.

2. Solution: (a) $L_1 i_1 = L_2 i_2$, $i_1 = (\frac{L_2}{L_1 + L_2})i$ and $i_2 = (\frac{L_1}{L_1 + L_2})i$, where $i = i_1 + i_2$. Adding emfs around one loop yields

$$-R_1 i - R_2 i - \frac{L_1 L_2}{L_1 + L_2}\left(\frac{di}{dt}\right) + V_b = 0,$$

which reduces to $+(4.0/A)i + (1.00\text{ s/A})(di/dt) = 2.00$.

(b) Since we need a rising exponential, we write $i = i_0(1 - e^{-at})$ and substitute this into the D.E. (see Figure 37):

$$4i_0(1 - e^{-at}) + ai_0 e^{-at} = 2.$$

In the limit $t \to \infty$, this requires $i_0 = 0.50$ A; this leaves us with $-2e^{-at} + (1/2)ae^{-at} = 0$, which requires that $a = 4.0$/s. Thus $i = (0.50\text{ A})(1 - e^{-4.0t\text{ s}})$. When $t = 0.75$ s, $i = (0.50\text{ A})(1 - e^{-3.00}) = 0.48$ A.

(c) $P = i^2 R = 11.8$ W (11.4 W if a more accurate value of i is used).

Figure 36

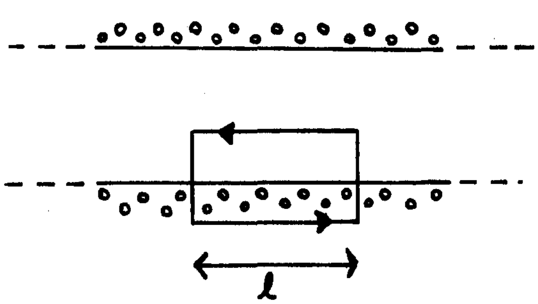
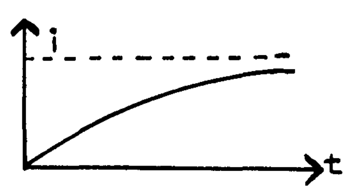


Figure 37



INDUCTANCE

3. Solution: (a) $q_{max} = 2.00\ \mu C$; $V_{max} = q_{max}/C = 100$ V. $(1/2)Li_{max}^2 = (1/2)(q_{max}^2/C)$, or

$$i_{max} = q_{max}/\sqrt{LC} = \frac{2.00 \times 10^{-6}}{[(2.00 \times 10^{-8})(0.0200)]^{1/2}} = 0.100 \text{ A.}$$

(b) $f = \omega/2\pi = (1/2\pi)\sqrt{LC} = (1/4\pi) \times 10^{-5} = 8.0 \times 10^3$ Hz.

(c) $q = q_{max} \cos \omega t$, where $\omega = 5.0 \times 10^4/s$;

$$i = \frac{dq}{dt} = -\omega q_{max} \sin \omega t = -i_{max} \sin \omega t.$$

q_{max} and i_{max} were evaluated in (a).

(d) $(1/2)Li^2 + (1/2)(q^2/C) = V_0$ (Energy conservation). Differentiate: $Li(di/dt) = (q/C)(dq/dt) = 0$. Simplify: $LC(d^2q/dt^2) + q = 0$. Substitute expression for q, from (c): $(-\omega^2 LCq_{max})(\cos \omega t) = 0$, provided $\omega = \sqrt{1/LC}$.

MASTERY TEST GRADING KEY - Form C

1. What To Look For: (a) Check path and actual use of Ampère's law.

Solution: (a) The best path is a circle of radius R = 30.0 cm, inside the toroid. Then Ampère's law gives us

$$2\pi RB = \mu_0 Ni, \quad \text{or} \quad B = \mu_0 Ni/2\pi R.$$

$$L = \frac{N\Phi}{i} = \frac{N\pi r^2 B}{i} = \frac{\mu_0 N^2 r^2}{2R} = \frac{(4\pi \times 10^{-7})(2000)^2(1.5 \times 10^{-2})^2}{2(0.30)} \text{ H} = 1.88 \text{ mH}.$$

(b) $10.0\ \mu V = |\mathcal{E}| = N(d\Phi/dt)$; therefore,

$$\frac{d\Phi}{dt} = \frac{|\mathcal{E}|}{N} = \frac{(10.0 \times 10^{-6})}{2000} = 5.0 \times 10^{-9} \text{ W/s}.$$

2. Solution: (a) $-iR - L_1(dt/dt) - L_2(di/dt) = 0$ or $(5.0/s)i + di/dt = 0$; thus $i = i^0 e^{-5.0t/s}$.

(b) $i_0 = V_b/(R + R') = 0.50$ A; thus at t = 0.50 s, $i = 0.50e^{-2.50}$ A = 0.041 A.

(c) $(1/2)Li_0^2 = 3.5(0.50)^2 = 0.88$ J.

3. Solution: (a) $f = 1/2\pi\sqrt{LC} = 1.13 \times 10^3$ H_2.

(b) $(1/2)V_b^2 C = 16.0$ mJ($=V_0$).

(c) $(1/2)Li_{max}^2 = V_0$, or

$$i_{max} = \sqrt{2V_0/L} = \sqrt{32 \text{ mJ}/4.0 \text{ mH}} = \sqrt{8.0} = 2.80 \text{ A}.$$

(d) $(1/2)Li^2 + (1/2)(q^2/C) = U_0$ by energy conservation.

Differentiate: $1i(di/dt) + (q/C)(dq/dt) = 0$.

Simplify : $LC(d^2q/dt^2) + q = 0$. The expression $q = q_0 \cos \omega t$ satisfies

$$-LC\omega^2 q_0 \cos \omega t + q_0 \cos \omega t = 0,$$

i.e., if $\omega = \sqrt{1/LC}$. And, of course, $f = \omega/2\pi$.