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Correction to "The Theory of Quaternion Orthogonal Designs"

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Corrections

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Seberry *et al.* claimed that even though the dual-polarized transmission channel cannot be considered as described by means of a single quaternionic gain, the maximum-likelihood (ML) decoding rule can be decoupled for orthogonal space–time-polarization block codes (OSTPBCs) derived from quaternion orthogonal designs (QODs) [1, Sec. IV]. Regretfully, a correction is necessary, and we will show that decoupled decoding using the method presented therein is only optimal for codes derived from certain QODs, not from arbitrary QODs as previously suggested.

Previously, we have utilized the representation of a quaternion variable $s = z_1 + z_2\mathbf{j}$ as $s = [z_1, z_2]$, so that a quaternion matrix $\mathbf{Q}$ can be converted into a complex matrix with twice as many columns; we have abused notation and referred to the complex representation of $\mathbf{Q}$ again as $\mathbf{Q}$. It was possible to use the context (e.g., the implied size or domain) to determine which representation of $\mathbf{Q}$ was being utilized. In this note, we will formalize the notation, thus illuminating a problem with the decoding discussion in [1, Sec. IV].

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Let us define an operator $\mathbb{C}$ from the quaternion to the complex domain such that

$$\mathbb{C}\{z_1 + z_2\mathbf{j}\} = [z_1 \;\; z_2] \text{ and } \mathbb{C}^{-1}\{[z_1 \;\; z_2]\} = z_1 + z_2\mathbf{j}. \tag{1}$$

Then, using the background in Section IV of [1], the ML decoding rule for any OSTPBC is equivalent to finding a set of signal symbols that minimizes the following quaternion analog of the standard complex Frobenious norm

$$\|\mathbf{R} - \mathbb{C}^{-1}\{\mathbb{C}\{\mathbf{Q}\}\mathbf{H}\}\| \tag{2}$$

where $\mathbf{R}$ is the received signal vector, $\mathbf{Q}$ is the code matrix, and $\mathbf{H}$ is the matrix of complex channel coefficients. However, due to an irresponsible abuse of notation, we assumed in [1] that this is equivalent to finding a set of signal symbols minimizing the squared norm $\|\mathbf{R} - \mathbf{QH}\|^2$. This abuse of notation and the subsequent expansion of the norm implicitly—and incorrectly—assume that the operator $\mathbb{C}$ and its inverse are commutative with the quaternion and Hermitian transposes. Although the last line in the incorrect expansion of $\|\mathbf{R} - \mathbf{QH}\|^2$ on [1, p. 263] is now irrelevant, we note for further clarification that the usage there of the orthogonality of $\mathbf{Q}$ would be incorrect, even if it were possible to get to that line: the matrix should be formally written as $\mathbb{C}\{\mathbf{Q}\}$, which is not orthogonal when viewed correctly as a complex matrix. Therefore, in the general case, the decoupled decoding statistics derived using method presented in [1, Sec. IV] do not lead to ML decoding. To achieve the ML decoding rule, one needs to minimize the norm given by (2), which can only lead to decoupled decoding in special cases, like the case of an example considered in [1].

References

[1] J. Seberry, K. Finlayson, S. Spence Adams, T. A. Wysocki, T. Xia, and B. J. Wysocki, "The theory of quaternion orthogonal designs," *IEEE Signal Trans. Signal Process.*, vol. 56, no. 1, pp. 256–265, Jan. 2008.