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Ultrasonic and Stereo-Optical Characterization Techniques for Applications in Mechanical Testing

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ULTRASONIC AND STEREO-OPTICAL CHARACTERIZATION TECHNIQUES
FOR APPLICATIONS IN MECHANICAL TESTING

By

Jonathan M. Hein

A THESIS

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Ultrasonic and Stereo-Optical Characterization Techniques for Applications in Mechanical Testing

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This thesis considers methods developed based on ultrasonic reflections from interfaces and stereo-optical surface strain measurements to study the mechanical characterization of materials and bodies. The ultrasonic method is used to characterize wave speed and attenuation for highly attenuating samples where other methods that require traveling of the wave through the material might not provide any information. The stereo-optical strain measurements are used to characterize the effects of genes on the mechanical properties of bone, and the dynamic characterization of blast waves in the UNL Shock Wave Trauma Mechanics facility. In each case the stereo-optical system provides a capability to do measurements that would not otherwise be possible.

A method is described and developed for characterizing the wave speed and attenuation spectrums from reflected waves from the contact surface with an unknown material. The method is demonstrated on samples of PDMS (polydimethylsiloxane), PC (polycarbonate), Sillyputty® (silicone rubber material), and bovine liver. This method has potential in its ability to characterize soft materials in the dynamic range where other methods cannot do so due to high wave attenuation.

In a study of the effects of genotypes on bone rigidity that was conducted in collaboration with Creighton University, the mechanical properties of mouse tibia were
characterized. An important consideration in this study is that the mouse tibia is small (in the order of 10 mm) and of irregular shape. With the stereo-optical system, a special method was developed to construct a 360 degree view of the deformation of each tibia as the sample was being subjected to axial compression. These were then used to calculate the effective modulus of the bone based on geometrical and strain data from the tests. This study was done using an ARAMIS 3-D optical system.

The ARAMIS system is also capable of performing high speed measurements such as is needed in shock loading. A series of experiments and analysis were performed to characterize the nine inch shock tube in the Shock Wave Trauma Mechanics facility at UNL.
To God,

May all glory, honor, and praise be forever his!
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Chapter 1 – Introduction

There is a great need to understand the properties of complex materials under a variety of different conditions. Both static and dynamic tests using two different experimental methods are studied in this thesis. One of the main areas of research is in the characterization of materials for the study of traumatic brain injury. Hundreds of millions of dollars have been spent by the military on the study of traumatic brain injury over the last several years [1] and to understand the mechanisms that cause traumatic brain injuries. For this the properties of the brain, skin, and skull must be well characterized. Biological specimens are traditionally difficult to perform experiments on, especially in the dynamic range. It is critical to develop new experimental methods that can perform measurements that are repeatable and accurate.

In my research I have worked on two experimental techniques that provide new and unique measurement capabilities. The first technique is a stereo optical displacement and strain measurement instrument called ARAMIS. Through a digital image correlation algorithm it can precisely measure full field displacement and strains from areas of mm$^2$ to m$^2$. It is an extremely robust and flexible system that allows the user to have an exceptional visualization of the deformation as it progresses in time. The second method is an ultrasonic technique which has the ability to perform measurements on soft materials without needing a uniform geometry or known dimensions. The technique can determine the sample wave speed, attenuation coefficient, and the linear viscoelastic relaxation modulus (if both the wave speed an attenuation coefficient is known). The wave speed was measured, but it was not possible to measure the attenuation of samples
with the current test setup. The reflected wave ultrasonic method provides dynamic information about the material properties that could not normally be gathered with through transmission methods.

Three experimental case studies are presented in this thesis: Ultra-High Speed ARAMIS Tests, Mapping Surface Strains on Mice Tibia, and Reflected Wave Ultrasonics. The ultra-high speed ARAMIS system was used extensively in the characterization of the 9-inch shock tube in the UNL Trauma Mechanics Shock Wave Laboratory. Chapter 3 presents four experiments involving the shock tube. Chapter 4 presents static compression tests of mice tibia. The tests are conducted for a set of samples representing three genotypes and ARAMIS is used to measure the full field deformation. A method to link multiple tests together to obtain a 360 degree view is also presented. Finally, Chapter 5 presents the theory, method, and results using the reflected wave ultrasonic method.

The high speed ARAMIS system provides the exceptional ability to visualize full field strains and displacements in the dynamic characterization of materials. This robust system can perform measurements on samples that are mm$^2$ or m$^2$. It is easily configurable and the full field measurements give good understanding of how the deformation progresses with time for the entire sample surface within the field of view of the cameras. The ARAMIS system is described in detail in Chapter 2 and the theory used in performing the measurements is also presented.

The characterization of mouse bones is necessary since rodent models are used extensively in the study of bone disease. Currently strain gages are used in an attempt to
characterize the deformation of the bone under compressive loading, but due to the highly non-uniform geometry, it does a poor job. ARAMIS cannot only provide the full field surface displacements and strains, but can stitch multiple views of the same test together to create a full 360 degree view. From this 360 degree view, geometrical information can be obtained for cross sections at different elevations. Strains can also be obtained for points on specific cross sections and based on this information the Elastic Modulus can be calculated using elastic beam theory. This analysis is performed for one bone from each genotype and the results are presented in Chapter 4.

The reflected wave ultrasonic method presented in Chapter 5 allows for the characterization of soft materials with non-uniform geometries. The method works for both longitudinal and shear ultrasonic waves and results for both modes are presented. Using traditional contact transducer techniques, soft materials cannot be tested due to deformation during measurement and their usually large attenuations which make passing a wave through the sample difficult. This reflected wave method accounts for both of these short comings and allows for a rather simple characterization of soft materials. One difficulty in the method is in the measurement of the attenuations; phase shifts on the order of 0.004 radians need to be measured between the reference configuration and the test configuration. Experimental results for PDMS, PC (polycarbonate), Sillyputty®, and bovine liver are presented along with ways in which the experimental setup can be improved in an attempt to measure attenuation values.
Chapter 2 – Introduction to ARAMIS

2.1 – System Overview

ARAMIS is a high performance optical system which performs measurements on complex surfaces and obtains their deformations and strain during loading. It is robust and has full-field non-contact displacement and strain measuring capabilities. ARAMIS performs unique non-contact measurements of 3D deformations and analyzes the displacement and strain fields through 3D digital image correlation methods (DIC), using high-resolution stereo CCD (charge coupled device) cameras. It is capable of performing both static and dynamic measurements based on the cameras speeds selected for test measurements.

During testing the test object can be viewed using one camera to perform 2D measurements and two cameras for 3D measurements. Images of the sample or test object are taken as the deformation progresses under the given loading conditions (separate from ARAMIS). The results are calculated using the progression of the deformation captured by the images and evaluated through digital image processing algorithms. This provides 3D coordinates for all of the points on the sample that are within view of both cameras. The 3D surface displacements and strain can be resolved from how these points move. Full 3D measurements can only be made with stereo cameras. Using one camera provides a two dimensional deformation view and therefore no out of plane measurements can be calculated. One limitation of the system is that it
can only provide surface measurements on the samples and no information about the deformation; the interior of the sample can be obtained.

aramis is highly versatile and can measure test samples from large bridge trusses under static loading, to dynamic tests under shockwave impacts, to extremely small mice tibia under compressive loading. The system can easily be adjusted and calibrated to make measurements on very different size scales from approximately 4mm x 4mm (with a special fixture) up to very large samples. Strains from 0.05% up to 500% can be measured for any size test object (as long as it remains in view). ARAMIS is extremely flexible in that it can be configured to perform measurements on almost any sample. The method in preparing the samples and calibrating the system can easily be scaled and changed based on a given set of test parameters.

The system is comprised of a pair of high resolution monochrome cameras, a trigger mechanism, a computer with ARAMIS software, and accessories. Different calibration panels are used based on the size of the test objects. Larger calibration objects can be created by the user for large test specimens. Triggering the cameras is performed by a device which sends simultaneous signals to both cameras at the same instant in time, ensuring measurements are taken at the exact same time. Our low speed system uses dual, two megapixel CCD cameras with a maximum frame rate of 12 images per second (Figure 1) and the high speed system uses dual one megapixel Photron SA1.1 cameras (Figure 2) with a maximum frame rate of up to 675,000 images per second (at reduced resolution).
Figure 1: Low speed ARAMIS cameras with 50 mm lenses.

Figure 2: High speed cameras with 24mm lens (left) and 105mm lens (right).
2.2 – Principles of Operation

The application of digital image correlation (DIC) techniques to experimental mechanics has increased dramatically over the last 15 years. This increase is due to the improvement of digital cameras, computers, and the development of commercially available systems. Image processing and correlation algorithms have been developed to provide robust displacement and strain measurement capabilities. These systems have sub pixel measurement accuracy which leads to very accurate measurements of displacement and strain. The typical measurement sensitivity for the ARAMIS system is 1/30,000 the field of view for displacement and 100 με for strain. The theories for two-dimensional and three-dimensional image correlation are similar, however only the two-dimensional theory will be presented in detail because it simpler and a brief overview of the three-dimensional theory will be presented with a source for further information provided.

A number of techniques have been developed to perform two-dimensional displacement and strain measurements. For the two-dimensional measurements, the out of plane displacements and strains are assumed to be zero. One of the methods developed was the FFT method which uses the discrete Fourier transforms of the image intensity pattern in two sub-regions (deformed and undeformed). The sub-regions are at the same location on the sample, but different locations on the image. The displacements of the points (located in the sub-region) are then estimated by measuring the peak to peak distance of the function based on the FFT results. This method proved to be useful and accurate for displacement measurements under rigid body motion, but under any strain a
loss of accuracy was encountered [2]. The second method used is the DIC-2D technique and it is still used today. It operates on the principal that when an image is taken with a digital camera it is digitized into digital data through an analog to digital (A/D) converter. The light intensity is converted into a digital value, where black is low intensity and white is high intensity [2]. Figure 3 shows an intensity plot of a sample 10x10 pixel image.

![10x10 pixel intensity plot from image](image)

**Figure 3:** 10x10 pixel intensity plot from image [2].

At this point two images could be compared to each other and correlated through the discrete intensity plots, where the intensity values are a function of the X and Y pixel locations. This however would not allow for accurate measurements if the intensity plots for the two images did not match exactly since the images could only be correlated at best to ±1 pixel accuracy. To allow the image correlation to have sub pixel accuracy the discrete intensity plot must be converted back to a continuous function. For an eight bit A/D converter the intensity levels are discretized into 256 values. These intensity values each have a specific spatial location on the image sensor (measured by pixel location).
For the 10x10 pixel intensity image from Figure 3, there are 100 values that the intensity level takes on over the 100 pixel squared area. Interpolating over this discrete system, a continuous function for the intensity level as a function of spacial location can be calculated. This continuous function now provides values for the intensity over the entire area and not just discrete locations. Figure 4 shows the original intensity plot with the assigned pixel locations and gray (intensity) levels, and three different interpolation methods: bi-linear interpolation, bi-cubic interpolation and bi-cubic spline interpolation.

![Interpolation Methods](image)

**Figure 4:** Interpolation of raw image data using different numerical schemes [2].
All of these methods were tested and shown to work by Sutton [2]. To increase the accuracy of the image correlation, the camera resolution could be increased. This would lead to better spatial resolution and the intensity interpolation functions would be more accurate to the original image since more points are available to fit for the same area. Likewise, a higher resolution A/D converter will allow the interpolation functions to more precisely resemble the original, continuous intensities. A 12-bit A/D converter would give 4096 values for the intensity levels during discretization where as an 8-bit converter only gives 256. This would allow the discretized intensity values to more accurately represent the true values. Sutton achieved typical errors of $\pm 0.02$ pixels using an 8-bit digitizer with bilinear interpolation, yielding displacement accuracies on the order of $\pm 200 \times 10^{-6}$ mm and strain accuracies on the order $\pm 200 \mu$e (0.02%).

The theory for three-dimensional image correlation is much more complicated since a pair of cameras (stereo setup) must be used and the calibration procedure is also much more complex. A total of 11 parameters are needed to describe the imaging process and global orientation of each camera. There are five intrinsic properties which are unique to the cameras and lenses and 6 extrinsic parameters which describe the orientation and position of the cameras. The calibration process determines the global coordinate system and the relative orientation of each camera to the global coordinates. Figure 5 shows the five coordinate systems that must be related through the calibration procedure.
Numerical algorithms using the relationship of the coordinate systems and the intrinsic parameters are used to create the three-dimensional image. The details of this procedure are outlined in *Advances in Two-Dimensional and Three-Dimensional Computer Vision* [2]. The calibration of the measurement system in both the two-dimensional and three-dimensional setups is the most important part in being able to accurately measure displacements and strains. The calibration methods for ARAMIS are presented in section 2.5.1. There is a specific theory on the calibration of the system and the sensor, but it is beyond the scope of this introduction. Again, more specific information can be found in *Advances in Two-Dimensional and Three-Dimensional Computer Vision* [2].
2.3 – Strain Formulation in ARAMIS

ARAMIS uses simple principles from Continuum Mechanics for the displacement and strain formulation. The theory used to measure strain will be examined for the 3D case since this is the most general and applicable to both measurement types. The following section references The Mechanical and Thermodynamical Theory of Plasticity [3]. The two most common measures of strain at a point are the change in length and change in angle for line elements. The change in length will be looked at in further detail below.

*Reference Configuration (RC)*

*Current Configuration (CC)*

Figure 6: Strain measurements for a line element via change in length.

Figure 6 shows the change of length of the line element with differential length \( \text{d}X \) in the reference configuration and length \( \text{d}x \) in the current configuration. To understand the extension of line elements, the length of the line elements must first be calculated. Letting \( \text{d}s \) denoted the length of the line element \( \text{d}x \), then \( \text{d}s^2 = \text{d}x \circ \text{d}x \). The deformation gradient \( \textbf{F} \) maps points from the reference configuration (RC) to the current configuration (CC). This is done through the relation: \( \text{d}x = \textbf{F}\text{d}X \), where \( \textbf{F} \) is the gradient of \( \text{x} \) with respect to changes in \( \text{X} \). 
Therefore, the strain can now be written as

\[ ds^2 = dx \odot dx \] (1)

\[ ds^2 = (FdX) \odot (FdX) \] (2)

\[ = (FdX)^T \odot (FdX) \]

\[ = (dXF^T) \odot (FdX) \]

\[ = dX \odot [(F^TF)dX], \] (3)

where \( C = F^TF \) is the right Cauchy stretch tensor. This gives

\[ ds^2 = dX \odot (CdX) \] (4)

Therefore, the strain can now be written as

\[ \varepsilon = \frac{ds - ds_0}{ds_0} = \frac{\sqrt{dX \odot (CdX)} - \sqrt{dX \odot dX}}{\sqrt{dX \odot dX}}. \] (5)

The deformation is fully characterized by the deformation gradient \( F \). Through the polar decomposition theorem, the deformation gradient can be decomposed into a rotation tensor \( R \) and stretch tensor \( U \) and \( V \). The relationship is

\[ F = RU = VR, \] (6)
where $R$ is an orthogonal second order rotation tensor and completely accounts for any rigid body rotation of the sample (see Figure 7) and the tensors $U$ and $V$ are symmetric second order tensors representing pure triaxial extension, with no rotation. $U$ is the right factor in the polar decomposition of $F$ and $V$ is the left factor; see Figure 7 for a description.

![Polar decomposition diagram]

**Figure 7:** Polar decomposition of the deformation gradient. The two symmetric factors ($U$, $V$) have identical eigenvalues and their eigenvectors are rotated by the value of the orthogonal factor ($R$)

The right Cauchy strain tensor $C = F^T F$, by the polar decomposition, can be related directly to the stretch tensor $U$ through
The definition for the Green Strain is \( E = \frac{1}{2} (C - I) \), or based on the above relation can be written as \( E = \frac{1}{2} (U^2 - I) \). The ARAMIS manual provides the relationship for the Green Strain as \( \varepsilon^G = \frac{1}{2} (\lambda^2 - 1) \), which is the scalar counterpart of this for the stretch \( \lambda \) along the line \( \hat{N} \) in the reference configuration given by \( \lambda^2 = \left( \frac{ds^2}{ds_0^2} \right) = \hat{N} \cdot (C \hat{N}) = \hat{N} \cdot (U^2 \hat{N}) \).

It is important to know the strain values and how to obtain them in ARAMIS, but it is necessary to be able to apply ones understanding of continuum mechanics theory to understand what the values truly mean. The deformation gradient maps points from the undeformed configuration into the deformed shape. This second order tensor allows mapping for all points on the body. Since ARAMIS can only perform surface measurements, only points on the surface of the body can be tracked and thus the strain and displacement measurements are surface measurements only. As such, complex deformations may occur inside of a sample, but ARAMIS cannot measure this. Most other deformation measurement devices also only measure surface strains, such as strain gages and LVDTs (linear variable differential transformer). ARAMIS provides full field measurements over the entire surface where as these other techniques give only one value and must be in contact with the test sample.

ARAMIS has the ability to transform the coordinate system so that the user has complete control in defining the local coordinate system directions. Currently, only Cartesian coordinates are available within the software. The global coordinate system is used to map the points on the surface of the sample. They are defined as three
dimensional vectors and all points are mapped from this coordinate system. The local coordinate frame takes on the same orientation as the global coordinate system, but the origin is at the local point of interest. Normally for each test the user defines the local coordinate system based on the geometry of the sample. This is related back to the global coordinate system to measure the displacements and strains. This proves very useful in tests with non-uniform geometries.
2.4 – Advantages of ARAMIS

There are many advantages of the ARAMIS system when compared to traditional deformation measurement tools. Table 1 provides a list of the advantages and disadvantages.

**Table 1:** Advantages and disadvantages of the ARAMIS system

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-contact measurements</td>
<td>Long setup time</td>
</tr>
<tr>
<td>Full field measurements</td>
<td>Long computation time</td>
</tr>
<tr>
<td>Perform full three-dimensional tests</td>
<td>Need to be able to pattern surface</td>
</tr>
<tr>
<td>High Accuracy</td>
<td>Lighting requirements</td>
</tr>
<tr>
<td>Ability to visualize deformation</td>
<td>Large physical size</td>
</tr>
<tr>
<td>Ability to determine high strain regions</td>
<td></td>
</tr>
<tr>
<td>Measure virtually any sample size</td>
<td></td>
</tr>
<tr>
<td>Can perform dynamic measurements</td>
<td></td>
</tr>
<tr>
<td>Generate test images and movies</td>
<td></td>
</tr>
<tr>
<td>Can perform test through windows</td>
<td></td>
</tr>
<tr>
<td>(ovens, shock tube, etc.)</td>
<td></td>
</tr>
<tr>
<td>Easy sample preparation</td>
<td></td>
</tr>
</tbody>
</table>

The advantages of ARAMIS far outweigh its disadvantages. Two of the most notable benefits over traditional gages is its contact free measurement capability and its ability to capture full field displacements and strains. ARAMIS requires the application of a high contrast stochastic pattern in order to track points. A very thin speckle pattern is applied to the sample using white and black paint which allows the software to correlate the two images as discussed in section 2.2. It is very important to apply only a thin layer of paint so the measurements are not affected.
Techniques using strain gages and LVDTs for measuring displacements and strain have been used for many decades. They are reliable and there are many applications where they are still the best option. They are much less expensive and require less time to perform tests than ARAMIS. Their main disadvantage is that they can only provide one data point per gage. The non-contact, full field measurement from ARAMIS provides an answer to this problem. It produces thousands of measurements over a sample’s surface. The non-contact measurement taken by ARAMIS removes the errors due to mounting gages on the surface. Both strain gages and LVTDs must be rigidly fixed to the sample and although slight, may introduce errors in the measurement. Due to the requirement to fix the gages to the sample, strain gages and LVDTs are not applicable when testing very small or soft samples. Two examples where ARAMIS provides measurements where strain gages and LVDTs can’t are: (a) studying the fracture in PDMS (Polydimethylsiloxane) which is a soft silicone rubber and (b) creating a 360 degree deformation view for mice tibia compression tests. The high speed ARAMIS system has the ability to measure the strains over the entire surface of a PDMS sheet both at the crack location and in the far field region. Strain gages were originally used to try to determine the complex deformation process for the mouse bones; ARAMIS however provides data which allows the user to see the deformation of the entire bone and not just one point.

Full field measurements of displacement and strain can be taken with ARAMIS, allowing for a better understanding of the deformation of the test specimen. The system achieves this by its ability to measure deformations for any region that is captured in both of the images. Full field measurements are not possible with traditional strain gages.
which only give data point per gage. There are however, other techniques such as interferometry, or Moiré fringe techniques that can provide full field measurements but these systems are not flexible in their ability to measure objects of many sizes. The advantage of ARAMIS is that full field measurements can be taken for a variety of sample sizes and geometries. These full field measurements allow regions of high strains or shear bands to easily be seen and recognized after the analysis is performed. Figure 8 exemplifies how ARAMIS can show areas of larger strain. A cylindrical sample of polycarbonate was compressed and one can see that there are larger compressive strains near the platen location as opposed to the middle of the sample where there is a strain band that is approximately 10% less than the top and bottom. This information would not be available if an extensometer or LVDT was used to perform the strain measurements.
Static and dynamic measurements are easily performed with the ARAMIS system just as they can be with strain gages. The limiting factor for ARAMIS is the speed and resolution of the current high speed CCD camera technology. The high speed cameras at UNL are the Photron Fastcam SA1.1 cameras which can take 5400 images per second at 1 Megapixel resolution (maximum camera resolution) and 675,000 images per second at 64x16 pixels. As technology progresses, the speed at which measurements can be taken will increase; already, the newest Photron SA5 can take 7,500 images per second at full 1 Megapixel resolution. Since the image acquisition and image analysis can occur independently, the speed at which measurements can be taken is only limited by current

**Figure 8:** Aged polycarbonate compression sample with small shear and compression band.
camera technology. Having the acquisition and analysis be independent creates a highly adaptable system that can improve and adapt with new technologies and the software can be used with nearly any camera setup.

ARAMIS provides a user friendly interface that allows for the easy operation of the system. It has simple features to import and analyze images and data. Once the data has been computed, there are many tools which can be used to transform the coordinate system, measure distances or strains at certain locations, create best fit cylinders or spheres to the test object and much more. Finally, when the analysis is completed, images and videos of the deformation can be exported along with data and user created reports. This allows for easy integration of the results into presentations or other programs to analyze data from ARAMIS.
2.5 – Sample preparation and testing procedures

Performing tests with ARAMIS must follow the proper pre-test calibration and sample preparation procedures to ensure the reliability and accuracy of the measurements. Calibration is the most important part of this procedure and is performed using special calibration panels. The samples must also be prepared with a high contrast stochastic pattern. There are several calibration panel types and users can create their own for large sample sizes. This is what allows for the versatility of ARAMIS with respect to sample size. Calibration provides the true spatial resolution and proper coordinate distances based on the camera positions and other factors such as lens distortion. The quality of the calibration performed directly affects the quality and accuracy of the measurements taken.

2.5.1 Calibration Procedures

The calibration object is selected based on the size of the sample to be tested. The size of the sample should be approximately the same size as the dimensions given on the calibration panel. When the proper calibration panel is selected it must also be selected in the software so the proper point to point distance can be applied. This is necessary because all of the lengths between points on the calibration panels are precisely measured and this information is stored in the software where it is used for calibrating the system. A series of 13 images is taken at different distances, rotations and angles of the calibration panel. Based on these images a calibration file is created if the calibration meets quality standards so that it can be used for testing. The quality of the calibration is measured based on the calibration deviation and is given in terms of the deviation in pixels. As discussed in section 2.2, the images are correlated through fitting the images
to each other based on the intensity fields. The calibration deviation is a measure of how
successful this matching process is. GOM, the manufacturer of ARAMIS advises that the
maximum calibration deviation that should be used is 0.04 pixels. The following
example will show how the calibration deviation relates to errors in the displacement
fields:

Unknown:

Error (standard deviation) for displacement measurements

Known:

Horizontal Field of view (distance) = 200mm

Horizontal Field of view (resolution) = 1600 pixels

Calibration deviation = 0.03 pixels

Solution:

\[ \text{Distance per pixel} = \frac{200 \text{ mm}}{1600 \text{ pixels}} = 0.125 \frac{\text{mm}}{\text{pixel}} \]

\[ \text{Displacement Error} = \text{Calibration deviation} \times \text{Distance per pixel} \]

\[ = 0.03 \text{ pixels} \times 0.125 \frac{\text{mm}}{\text{pixel}} = 0.00375 \text{ mm.} \]

Therefore, with a calibration deviation of 0.03 pixels and a field of view that is
200mm/1600 pixels, the standard deviation in the measurement is 0.00375 mm or
0.000148 inches. This error is very small and gives ARAMIS the ability to precisely
measure the deformation. It is very important to ensure that the calibration is completed
with precession otherwise the calibration will have to be performed again. If the
calibration cannot be calculated a message appears to the user saying the calibration
calculated was divergent.
Figure 9: 55x44mm ceramic calibration panel manufactured by GOM.

Figure 9 shows one of the standard calibration panels provided with ARAMIS. For samples larger than the provided calibration panels a large area calibration panel has to be made by the user. A large area calibration panel can be seen on the following pages in Figure 10 and Figure 11. Individual points are printed out and arranged on a rigid object such as a metal plate so the points don’t move with respect to each other. When the calibration panel is made the same series of images is taken as in the regular calibration, however, extra images of the calibration panel are normally taken with the panel in the corner and edges of the image to ensure proper calibration is achieved. This is not necessary for the normal calibration objects because there are prescribed distance parameters in which each panel can be used in. Therefore, for the traditional object only the thirteen prescribed images are necessary. The images for the large area calibration
must be analyzed in a program called Tritop which generates the large area calibration file which is imported into ARAMIS along with test images.

Figure 10: Large area calibration panel made on aluminum plate, there are twenty calibration points on the panel.

Figure 11: Large area calibration points with number identifiers (each point is unique and recognized by the calibration software).

The calibration methods described, although different for the regular and large area, create the same calibration file which is necessary for analysis of the images. It is important that precision is used in placing the calibration panel for different calibration
shots. The more attention to detail, the better the calibration will be and therefore distances can be more accurately measured. This will produce more reliable data.

2.5.2 Sample Preparation

The samples must be prepared by applying a special surface pattern. It must be a high contrast black and white stochastic pattern of approximately ‘ellipses’. The size of the ellipses needs to be approximately seven pixels in diameter when viewed through the cameras. Therefore the user can determine the actual size that the dots need to be by measuring the field of view and dividing the distance in a certain direction (vertical or horizontal) by the number of pixels in that direction. For small dot patterns spray paint is used and for large dot patterns white spray paint and permanent markers are used. It is not important whether the pattern is black on white or white on black, as long it is a high contrast pattern. The pattern must be as thin as possible so that it does not influence the test outcome. Since ARAMIS only performs surface measurements, if there is a thick layer of paint, the properties of the paint and not the sample will be measured.

The sample patterns shown in Figure 12, Figure 13 and Figure 14 are examples of common spray patterns. Only Figure 12 exhibits an acceptable pattern. Figure 13 has a good, high contrast pattern, but the large black dots will cause issues. Figure 14 has a very low contrast pattern and the software will have difficulties analyzing the test with this pattern.
Figure 12: High contrast stochastic pattern (good).

Figure 13: High contrast pattern with large dots (bad).

Figure 14: Low contrast pattern (bad).
2.5.3 Test Procedures

Once the system has been calibrated and the sample prepared with a high contrast stochastic pattern, a pretest must be performed to measure the noise of the system. A series of images is taken of the sample in its reference, or unloaded configuration. These few pictures are then analyzed and the noise of the system can thus be determined. The noise of the system is generally between 50-100 microstrain (the strain error is generally the same for all sample sizes since it is normalized by the length measurements). This is the uncertainty of the measurement, and therefore, the strains that are being measured should be approximately 10 times this, or 0.05%-0.075%. If the noise floor is too high the system must be recalibrated and then the pretest performed again.

The first image is arguably the most important in that all successive images are measured based upon this image. Therefore this image must be taken of the undeformed configuration so that the entire deformation is measured. Generally several images are taken of the reference configuration so that a good baseline measurement can be taken.

The frame rate, shutter speed and aperture must be determined and set for each test. The values for these parameters are determined based on several factors including: test duration, measurement type (2D or 3D) and the number of images ARAMIS can analyze. ARAMIS can process thousands of images, however the time it takes to perform this analysis becomes very long, and normally the maximum number of images used is 1500. An average test however will typically have 200-300 images. Based on this limiting factor, the desired number of frames can be divided by the approximate test time to determine the necessary frame rate. After this parameter is set, the shutter time is then constrained to a value less than the inverse of the frame rate. By reducing the
shutter time more light is needed to properly see the test sample, but this reduces the risk of having blurred images. This is more critical for high speed applications. The aperture can also be adjusted, and in doing so the depth of field is also changed. If the aperture is open all the way, more light is let into the sensor, but the depth of field is very small. This means that the area in front of and behind the focal point of the camera will be out of focus and thus cannot be analyzed by ARAMIS. The smaller the aperture setting, the larger the depth of field, and the more light needed. A balance of all of these parameters must be met and adjusted for each test to ensure optimal conditions so the images are as clear and crisp as possible. With the camera parameters adjusted and set, the test can be started. After all of the images have been taken and imported into ARAMIS the analysis can begin.

2.5.4 Test Analysis

The analysis of the images is critical in obtaining relevant results and there are many parameters that can be adjusted to obtain the most accurate results. All of the images are referenced to the very first image taken and therefore all of the displacement and strain values are relative to this image (called ‘stage’ in ARAMIS). There are several parameters that can be adjusted within ARAMIS to optimize the analysis. First and foremost, the facet size and facet step must be adjusted, along with the area to analyze (see Figure 15 for description of facet size and facet step). It is important to reduce computation time by selecting only the area comprised by the sample in the image to analyze and not the entire image. The facet size and step adjust how the image is analyzed and how many data points will be created. Facet size is a parameter that sets the size of an analysis square and is measured in pixels. For each square one point is created.
The facet step is the parameter that sets the distance between the centers of the squares and must always be less than the facet size so that there is some overlap.

Displacement and strains are then based on the movement of the point field created from the facets. It is necessary for the user to define an initial start point (see Figure 16); this identifies the exact same location in both the left and the right image and is needed to perform the digital image correlation. After these parameters are adjusted and the start point it created, the test data can then be computed.
After completion of the computation, the displacement and strain values can all be viewed and reports and data can be exported. Data can be created for specific points or averaged over areas using the statistics feature and then graphs can be created. Images, graphs and data values can be combined into reports and movies generated from these. The ability to create reports using images and graphs allows the user to simply and precisely present the test and results in an easy to understand way. Figure 17 shows a report created for a 3-point bending and fracture test for polycarbonate. The strain field is seen in the top right image and the quantity displayed is the horizontal strain ($\varepsilon_{xx}$) and a zoomed in image of the crack location is shown in the top left image. A graph has also been included, showing the strain at a location above the crack. Reports such as these are useful in the explanation of the experiment and a lot can be learned from them.

**Figure 16:** Start point selection for left and right images (3D measurement).
Figure 17: Example report for horizontal strain showing three-point-bending of polycarbonate, with the high strain region enlarged and shown on the left.
Chapter 3 – Ultra-High Speed ARAMIS Tests

3.1 – Introduction to High Speed ARAMIS Tests

ARAMIS is capable of performing high speed, dynamic measurements by using the high speed cameras in Figure 2 of the previous chapter. A pair of Photron SA1.1 cameras were purchased to conduct high speed experiments such as shock loading, compression and torsional Kolsky bar measurements under the Army Research Office traumatic brain injury project. The cameras have a top speed of over six hundred thousand frames per second. The cameras have eight gigabytes of onboard memory and can record just over one second of video at maximum resolution and frame rate. For high speed tests this is more than enough time to capture the events. Table 2 provides seven settings showing the resolution setting with the maximum frame rate for that setting.

Table 2: Photron Fastcam SA1.1 high speed video system settings.

<table>
<thead>
<tr>
<th>Resolution</th>
<th>Frames Per Second (fps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,024 x 1,024 pixels</td>
<td>5,400</td>
</tr>
<tr>
<td>832 x 608 pixels</td>
<td>10,800</td>
</tr>
<tr>
<td>640 x 480 pixels</td>
<td>18,000</td>
</tr>
<tr>
<td>512 x 512 pixels</td>
<td>20,000</td>
</tr>
<tr>
<td>256 x 256 pixels</td>
<td>67,500</td>
</tr>
<tr>
<td>256 x 16 pixels</td>
<td>500,000</td>
</tr>
<tr>
<td>64 x 16 pixels</td>
<td>675,000</td>
</tr>
</tbody>
</table>

Triggering the cameras is a crucial part of timing in high speed tests. The cameras can be controlled through the computer (software trigger), a user controlled hardware trigger switch, or a TTL trigger can be attached which allows for exact timing with other electronic measurement devices. The TTL triggering is the preferred method
and is used for the shock impact experiments in UNL’s Shockwave Mechanics Facility. The cameras are triggered to start taking images at the same time as the other data acquisition devices (i.e. pressure sensors). This allows the data from both instruments to be aligned in time and gives a clear understanding of when events occur. There are many trigger modes and several are described in Table 3. These are the most often used ones, but the cameras have the ability to take images at one speed for a certain amount of time and then upon a trigger take images at a different speed. The minimum frame rate that the cameras can take is 60 frames per second. This can be adjusted by using a square wave form generator to trigger the cameras off the leading edge of the pulse.

**Table 3: Trigger mode list and description.**

<table>
<thead>
<tr>
<th>Trigger Mode</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Start</strong></td>
<td>The cameras begin taking images when the trigger is received</td>
</tr>
<tr>
<td><strong>End</strong></td>
<td>The cameras continuously take images until the trigger is received and the pictures currently in memory are kept</td>
</tr>
<tr>
<td><strong>Center</strong></td>
<td>The cameras continuously take images and when the trigger is received they take pictures which will fill half of the memory (so half of the pictures are before and half are after the trigger)</td>
</tr>
<tr>
<td><strong>Random</strong></td>
<td>The cameras will take the user defined number of images each time a trigger is received.</td>
</tr>
</tbody>
</table>

Many different experiments have been conducted with the high speed ARAMIS system including, shock loading experiments, kolsky bar measurements, fracture of asphalt, fracture of PDMS, surface wetting experiments, dynamic inflation of PDMS and more. The following sections will focus on the characterization and experimentation related to the 9 inch shock tube. First, polycarbonate and tempered glass windows, which are used to view the experimental chamber in the shock tube, were analyzed to determine their deflection and strains. This was done to determine if the window distortion would have any effect on the optical measurements. Second, measurements that looked through
the shock tube (looking through two windows) were performed to see if the distortion caused by the shock front would affect measurements. Finally, two experiments are analyzed: (a) a polycarbonate tube fixed to the top and bottom of the shock tube was studied to characterize its deformation and compare to finite element models, and (b) the RED Head (realistic explosive dummy head) was subjected to shock loading outside of the shock tube to study deformation and the wave propagation through the skull (wave propagation through the skull measurements not taken with ARAMIS).
3.2 – Shock Tube Window Deflection Experiments

3.2.1 Introduction

ARAMIS can be used in high rate and high energy experiments such as shock loading, impacts, or explosions, however, certain safety precautions to protect the system and user must be taken. This generally is done by using a high strength, high toughness window. The windows are necessary, but can cause measurement errors since as the window deforms, it causes a lens effect and distorts the images. Under most circumstances, this distortion is small, but under shock loading inside a shock tube the deflections from the shock over pressure are not known. The window deflection needs to be characterized to ensure the lens effect and window deflection is not too large. Although this problem could be modeled and analyzed, it is simpler to perform direct measurements to quantify the deflection and deformation.

Two types of windows were designed and manufactured to be used in the shock tube. One was made of polycarbonate and the other was a tempered glass. The in and out of plane displacements, and horizontal and vertical strain values needed to be quantified for both windows. Tempered glass is a stiffer material than polycarbonate, but is not as tough or fracture resistant [4]. Each window was subjected to similar shock loadings of approximately 20 psi. Below are images of the polycarbonate window and the tempered glass window. The windows were painted white and patterned using a black permanent marker.
Polycarbonate:

Figure 18: Polycarbonate window for 9" square shock tube.

Tempered Glass:

Figure 19: Tempered glass window for 9" square shock tube.
The windows were designed to be easily removed and interchangeable with each other, the sensor plates and test plates. Figure 20 shows one of the window openings on the 9 inch square shock tube. The window fixtures are clamped into place using the four bolted connections which can swivel to allow for easy removal. The shock tube has three ports that the windows can be installed in. When ARAMIS is used, imaging from only one side is possible due to lighting constraints.

**Figure 20:** Window opening in 9 inch square shock tube with bolted clamps (to hold window in place).
3.2.2 Experimental Setup and Hypothesis

The experiment was prepared by following these steps:

1. Place the high speed cameras in their approximate position.
2. Make the appropriate connections for the full three-dimensional measurement.
3. Turn on the cameras and determine the size of the field of view and frame rate needed and adjust the camera position until both of this variables are satisfied.
4. Make a large area calibration panel (10mm radius objects) as seen in Chapter 2, Figure 10.
5. Take the calibration images (at full camera resolution) and process them in Tritop to obtain a calibration file.
6. Based on the size of the field of view in pixels and mm, determine the size that the pattern must be (in mm), i.e. determine the equivalent distance of 7 pixels.
7. Set up the lighting by setting the cameras to the test resolution and frame rate and viewing the sample under these conditions. Close the aperture as far as possible to obtain the largest depth of field.
8. Perform a pretest to ensure that the noise floor is acceptable.
9. Ensure that there is a way to trigger the cameras and know when the shock wave passes. (For this test an electronic trigger had not been made yet, so the cameras were manually triggered and a piece of paper was used to show the approximate time the shock wave passed the window).
10. Perform the test.
11. Analyze the data.
3.2.3 Experimental Results & Conclusions

Glass

Figure 21: Tempered glass window subjected to shock loading. Visualization is out-of-plane deflection in millimeters.
Polycarbonate

**Figure 22:** Polycarbonate window subjected to shock loading. Visualization is out-of-plane deflection in millimeters.
The above images give a great visualization of the displacement of the glass and PC windows subjected to a shockwave. The corresponding graphs for out of plane displacement and the horizontal (EpXX) and vertical (EpYY) strains are shown in the following pages. Figure 23 shows the data point location that is employed in all of the graphs. The strains in the polycarbonate window are an order of magnitude larger than those in the glass window and the out of plane displacements are approximately three times larger than the glass. This clearly means the lens effect is increased by using the polycarbonate verses the glass. The one clear advantage of polycarbonate is that it is a tougher material. If the glass fails, it will fail catastrophically, likely with no prior indicators.

![Data point locations for graphs](image)

**Figure 23:** Data point locations for graphs (shock wave travels from the left to right)

Although it is important to know what the displacements and strains of the window are, the final check comes from using ARAMIS. If ARAMIS can analyze the specimen and the deviation parameter is less than the predefined limit then the errors are small enough that the measurement is not affected. If the deviation parameter in
ARAMIS is exceeded, it could mean that the window deflection is too large, or the cameras relative position has changed.

The following graphs show the strain components for the center point of the glass window and the out of plane displacement for the three points as shown in Figure 23. The out of plane displacement of the glass window is largest at the center at approximately 0.6 mm. The displacements for the leading edge and trailing edge are less at peak values of 0.3 mm and 0.38 mm. These values should be approximately the same and the difference could be accounted for by rigid body motion of the entire tube. It appears that the value that the displacements trend to are not zero which means that there is most likely some rigid body motion of the shock tube itself. The strain components (Figure 24) are at the minimum measuring capabilities of ARAMIS (<0.05%). The horizontal strain (EpXX) direction is larger than the vertical direction.
Figure 24: X and Y strain components for glass window center point.

Figure 25: Out of plane displacement (positive) for Leading Edge, Center and Trailing Edge data points for glass window.
Figure 26 and Figure 27 show the displacements and strains for the polycarbonate window which are much larger than those for the glass window. The center point displacement is nearly three times larger at a maximum amplitude of 1.85 mm than that for the glass window. The displacements for the leading edge and trailing edge are approximately the same at a maximum of 0.7 mm which is what is expected to happen. The strains also exhibit the expected pattern with the vertical strain being larger than the horizontal strain. The maximum strain is approximately 0.37% which is nearly 10 fold larger than the maximum strain on the glass window. No permanent deformation was seen in the window at these strain values. The frequency of oscillation for the displacements and strain is approximately 500 Hz which is approximately the same as for the glass window.

Figure 26: Out of plane displacement (positive) for Leading Edge, Center and Trailing Edge data points for polycarbonate window.
The out of plane displacements are shown in Figure 28 for both window materials. From the data and test analysis, it was determined that the glass windows provide more optimal test conditions for the cameras. The lens effect and distortion caused by the glass windows will be less than the PC window. Therefore the errors will be less due to the distortion for the glass windows. The errors due to the polycarbonate window will also be small, but the glass reduces these errors even more. The glass also has an advantage because it is much more difficult to scratch the surface whereas polycarbonate is very easily scratched. The edges of the glass window easily chip when they are installed or removed from the shock tube. This could reduce the strength of the windows or cause injuries to the user.

**Figure 27:** X and Y strain components for polycarbonate center point.
The hypothesis that the tempered glass window would deform less under shock loading was proved correct through experimental testing. Both the out of plane displacements (of three points) and the horizontal and vertical strains were less for the glass window than those of the polycarbonate window. The maximum displacement of the center point was approximately one third of the out of plane displacement of the PC window and the maximum strain was nearly ten times smaller than the maximum strain in the PC window. The glass window while exhibiting less deformation has a problem in that the edges chip easily. It is however a better material due to the fact that the deflection is smaller and that it does not scratch as easy as polycarbonate.
3.3 – Shock Tube Through Window Experiments

3.3.1 Introduction

The shock front is a location in a compressible fluid where there is nearly a discontinuous jump between the thermodynamic and fluid dynamic properties. The shock front region can be seen and is caused by a change in density across the front [5]. Errors generated by the distortion in light due to the change in density at the shockwave front must be quantified so its effect can be removed from the ARAMIS measurements. This disturbance essentially looks like a blurred region in the high speed photographs. All of the windows were subjected to approximately a 35 psi overpressure wave (0.24 MPa).

3.3.2 Experimental Setup

To understand the effect of the shock front disturbance a sample that would not deform under shock loading was needed. Finding a sample that exhibited no deformation and could easily be fixed in the shock tube was not worthwhile so instead, a patterned plate was placed outside the shock tube where it would not exhibit any deformation. The cameras were placed on the opposite side of the shock tube and looked through both windows of the tube to see the sample on the other side. One of the complications to this setup was that part of the measurement could be in error due to window deflection, but as this was previously measured to be small, it was not a concern. By placing the sample outside of the shock tube, there is theoretically no deformation since it is not subjected to any loading, and one could conclude that all of the deformation seen was due to the shock
front. Figure 29 shows the experimental setup. The shock moves from the right to left and is generated by a diaphragm bursting between the driver and driven section.

![Diagram of shock tube experimental setup](image)

**Figure 29:** Through window shock tube experimental setup, looking from the view of the cameras.

Figure 31 and Figure 30 are photos from the actual test setup and show the high speed cameras positioned in front of the shock tube with the sample behind two windows. Two high intensity halogen lights were used to provide enough light on the sample. Both window types were tested and the results are very similar. Results for both window types are presented.
Figure 31: Front on image of through window experimental setup.

Figure 30: Top view of through window tests. The test panel is clamped behind both windows.
3.3.3 Experimental Results

The following image overlays and graphs show the measured strain and displacement for both the glass and polycarbonate windows. The distortion from the shock front can clearly be seen by the high strain/high displacement region. The shock front as viewed by the high strain/high displacement region is several centimeters wide. The large width may be in part due to the fact that the shock front is not in focus since the cameras are focused on the sample. There are two peaks on the graphs also due to the fact that the cameras are focused on a plane behind the shock tube. This causes the shock front to pass the cameras at different times. The rest of the sample (away from the shock front) shows virtually no deformation which is expected. All of the graphs show data taken from the black dot labeled ‘Data Point’. The four graphs for strain and displacement for both windows are presented and followed by a discussion of the results and a conclusion of the test.
Glass – Major Strain Visualization

Figure 32: Through window experiment with glass window and Major Strain visualization.
Figure 33: Through window experiment with glass window and displacement magnitude visualization.
Figure 34: Through window experiment with polycarbonate window and Major Strain visualization.
Figure 35: Through window experiment with polycarbonate window and displacement magnitude visualization.
3.3.4 Discussion and Conclusions

This was one of the most important tests that was conducted on the shock tube using ARAMIS. When the shock front was seen with the high speed cameras in some of the first test shots of the shock tube, it was conjectured that this optical disturbance may cause errors in the ARAMIS measurement. After performing the measurements reported above, this hypothesis was proved to be correct. The strain error is almost 3% and the displacement error is approximately 3mm. These are large errors and strains of 2.5% normally need large deformation theory to accurately model. This is a significant error in the measurement and is not easily removed. The error most likely changes with the shock wave over pressure (this was not validated). There is no way to characterize the error during the test measurement. Almost all of the experiments conducted with ARAMIS in the shock tube were performed at lower pressures and the shock wave could not be visually seen as in these tests. Although the shock front at lower over pressures is not easily distinguished with the naked eye, ARAMIS with its sub pixel accuracy most like measures something in these cases.

The timing of the shock front arriving at the sample location and the initiation of loading (and deformation) occur simultaneously. There is no way to decouple these events, and the strain measured in ARAMIS is most likely a combination of the error due to the shock front and the actual deformation of the sample. Therefore the measurement and error cannot be differentiated. One possible solution would be to perform a pretest where these errors could be estimated (a rigid sample approximately the same size and shape as the test sample would have to be used). Once the errors are known they could be subtracted from the test with the real sample. This is not necessarily the best solution
since the pretest sample will have some deformation and the exact burst pressure cannot be precisely controlled. It also adds a fair amount of computation time and increases the number of tests that must be performed. It is important to understand that these errors exist and measurements should be performed with caution. It should be noted that the only errors arise from the shock front and any measurements after the front has passed should not be affected. This means that special attention has to be paid to the few data points where the shock wave interacts with the sample.

With that said, ARAMIS is a great measurement tool and the high speed cameras provide an excellent visual understanding of what is happening on such a short time scale. It still provides a unique ability to measure the full field displacements and strain. The errors at lower pressures (less distinguished shock front) should be small enough to measure the gross motion of the RED Head (Realistic Explosive Dummy Head) and other test specimens as well as estimate the surface strains on the specimens. As stated above, the only problem area in the measurement is at the shock front and all other areas should have very accurate measurements.
3.4 – Polycarbonate Cylinder Shock Loading Experiments

3.4.1 Introduction

Modeling is an important area of engineering and experiments have been performed in the shock tube in order to validate these models. One set of experiments used for this purpose was the shock loading of rigidly mounted polycarbonate cylinders (2 thicknesses) filled with silicone brain simulant in the shock tube test section. The cylinders were filled with brain simulant so that they could be a crude head model with simple geometry. The strains on the thick walled cylinder (1.5 inch diameter, 1/8 inch wall thickness) are expected to be less than the strains on the thin walled cylinder (1.5 inch diameter, 1/16 inch wall thickness). The radius of curvature of the side wall of the cylinder is also measured to determine how it changes during loading. All measurements presented with the exception of the shock wave profile upstream of the test section were measured with ARAMIS. All shock loadings had approximately the same overpressure of approximately 20 psi. This was estimated by the equation:

\[ P_o = -0.0019 \times (P_B)^2 + 0.0426 \times P_B + 0.0336, \]  

(8)

where \( P_o \) is the over pressure in MPa and \( P_B \) is the burst pressure in MPa. The above equation provides the relation for the overpressure in the test section of the 9-inch shock tube with a 4-inch driver section. There is a very strong correlation between the burst pressure and the overpressure in the test section [6]. All of the test shots burst at a pressure of 391 +/- 5 psi. There is sensor data for the over pressure closer to the test cylinder, however the wave interaction becomes complex as the shockwave deflects off of the cylinder and the side walls. That is why the overpressure is being estimated by
using the burst pressure. A schematic of the test setup is given in Figure 36. The figure does not give the exact locations of the pressure sensors and the strain gage was not used on this test, but it gives a general schematic for the experimental setup.

**Figure 36:** Experimental setup for polycarbonate cylinder shock loading tests.

Figure 37 shows the typical shockwave profile that the polycarbonate cylinders were subjected to. This pressure profile shown was taken approximately 40 inches upstream of the cylinder. The maximum pressure is larger here because as the rarefaction wave approaches the shockwave it eats away at the pressure. Therefore, as the shockwave progresses it reduces in amplitude [5] The red profile has been filtered with a second order low pass Butterworth filter with cut off frequency of 10,000 Hz in Labview. This helps to reduce the noise in the signal and provides a more accurate measure of the over pressure.
3.4.2 Experimental Results and Analysis

A total of six experiments were performed, three on the thin walled cylinder, and three on the thick walled cylinder all with nearly identical overpressures of approximately 20 psi. ARAMIS was used to measure the vertical strain at a location approximately 45 degrees to the side of the loading direction. Figure 40 shows the area that was averaged to give the vertical strain measure (black rectangle). Figure 38 shows a screen shot from ARAMIS with a best fit cylinder created for the middle section of the tube. This cylinder is then used to measure the radius of the cylinder and see how it changes during the experiment.

![Graph](image)

**Figure 37:** Shock wave profile that the brain simulant filled polycarbonate cylinders were subjected to.
Figure 38: Best fit cylinder around the middle of the cylinder. This type of cylinder was fit for all of the proceeding tests.

Figure 39 and Figure 40 show the reports generated in ARAMIS for the vertical strain plots for both the thick and thin walled cylinders. The boxed region is the areas in which the strains were averaged over. This area was selected because it is the closest measurement ARAMIS can make to the middle height, head on location. Also, since the tube goes through a bending deformation, the middle line as seen in the strain overlay would be the neutral axis. The graphs following the ARAMIS reports show the vertical strains for all three tests and the standard deviations.
Figure 39: Thick wall cylinder vertical strain visualization with strain history graph.
Figure 40: Thin wall cylinder vertical strain visualization with strain history graph.
Figure 41 shows the vertical strains for all three tests performed on the thin walled cylinder and Figure 42 shows the standard deviations for the thin walled tests. The maximum strain values reach nearly 0.25% and the frequency in which the strains oscillate is approximately 200 Hz. Figure 44 shows the vertical strains for the thick walled cylinder with Figure 43 showing the standard deviations for the three experiments performed. The maximum strains for the thick walled cylinder approach 0.15% which is 40% less than the thin walled cylinder. The thick walled tube strain measurement oscillates at a higher frequency of around 300 Hz. It is predicted that the oscillation frequency is related to the natural frequency for the cylinder. The measured frequency is expected to be less however, due to damping from the silicone gel inside of the test cylinder. The natural frequency for a beam is given as,

$$\omega_n = \lambda_p^2 \sqrt{\frac{EI}{m}}$$

(9)

$\omega_n$…Natural frequency,
$\lambda_p$…Modal parameter,
$E$…Elastic Modulus,
$I$…Moment of Inertia,
$m$…Mass per length [7].

Inputting the parameters for both cylinders, it was calculated that the natural frequency for the thick walled cylinder was 431 Hz, and the thin walled cylinder was 311 Hz. As predicted, the theoretical oscillation frequency for a beam is larger. When comparing the ratio of the thin walled cylinder and thick walled cylinder, the experimental results are approximately 0.67 and the theoretical ratio is 0.72. Therefore, the oscillations seen in
the strain plots are directly related to the first mode of vibration and natural frequency of the cylinder.

Figure 41: Vertical strain in the thin walled tube, shots 87-89.
Figure 42: Vertical strain standard deviation for the thin walled tube.

Figure 43: Vertical strain standard deviation for the thick walled tube.
The strains, as predicted are larger in the thin walled cylinder. For the region selected the maximum strain amplitude is approximately 0.22% and for the thick walled tube, they are approximately 0.12%. The standard deviations for the vertical strain plots are also shown and they are approximately 30 percent of the measured values. One would like the standard deviations to be less than this, but for the given setup with variability in the shock loading pressure and other parameters, it is quite good.

Figure 45 shows how the cylinder changes shape during and after the shock loading. Figure 38 presented how a best fit cylinder was created to obtain the radius of curvature from the side on view in order to help characterize the change of shape of the cylinder. As the tube is compressed by the shock wave, the radius of the best fit cylinder (as seen in Figure 45 A) becomes smaller. The tube then oscillates between the two modes. Therefore, a negative radius change corresponds to Figure 45 A and a positive change in radius corresponds to Figure 45 B.

**Figure 44:** Vertical strain in the thick walled tube, shots 90-92.
Figure 46 shows the change in radius for both the thick and thin walled cylinders. When comparing the maximum amplitudes for both cylinders the thin walled cylinder has an absolute change of 2.5mm whereas the thick walled cylinder has a change of 0.5mm, this is a five-fold difference. The reason that this parameter measurement was created is because it gives some sense of the dilatational motion of the tube. When the radius change parameter is negative, that means that the cylinder is being compressed in the direction of loading (direction that the shock wave is traveling). It takes on the shape

![Diagram showing two modes of dilatational motion of a tube.](image)

**Figure 45**: Top view of two modes of dilatational motion of the tube. The blue circle represents the best fit cylinder created in ARAMIS. The radius of curvatures are given by $\rho_A$ and $\rho_B$. 

A

Decrease in Radius of Curvature

B

Increase in Radius of Curvature

Loading Direction

$\rho_B$
shown in Figure 45 A. For a positive change in radius, the tube takes on the shape in Figure 45 B.

![Graph of radius change over time for thin and thick tubes](image)

**Figure 46:** Tube radius change for 20 psi overpressure blast (plots are average of three tests)

The difference in the frequency of oscillation between the thin tube and thick tube is very apparent in the radius change plot. The thick tube cylinder oscillates at a frequency that is approximately three times larger than the thin tube. The frequencies are approximately 1200 Hz for the thick tube and 400 Hz for the thin tube. Theoretical vibration frequencies for the cylinder are not presented here as they are more difficult to obtain since the second mode oscillation [8]. Further tests need to be performed to obtain a deeper understanding of the results and how the tube reacts. In these tests most of the deformation comes from the bending motion of the tube and not the shock wave itself.
3.5 – Realistic Dummy Head Shock Loading Experiments

3.5.1 Introduction

The Realistic Dummy Head (RED Head) was developed by Dr. Carl Nelson’s research group at the University of Nebraska Lincoln. The instrumented head was created to perform blast measurements and to obtain an accurate understanding of the deformation and wave propagation in the head. A balance between computational modeling and experimental results is necessary, but the two go hand in hand. Since all of the desired experiments cannot be done due to time and monetary constraints, it is important to develop models. These experiments will be used to verify and refine finite element models. The most important parameters are the gross motion of the skull which will be able to provide information about the velocity and acceleration of the head as well.

For the initial experiments a PDMS (Polydimethylsiloxane) skin was fitted over the skull. This proved to be an issue as the skin was not securely attached to the skull and delamination occurred. Figure 47 shows the RED Head with the skin removed. The head and neck assembly attached to a plate which can be fixed to the shock tube. In the 9-inch shock tube these tests had to be performed outside of the tube, but future tests will be performed inside the larger 28 inch shock tube. As the shockwave exits the tube, it becomes extremely non-linear and changes dramatically as it deflects around the end of the barrel [5].
3.5.1 Experimental setup and results

The RED head was subjected to shock loading outside of the tube. According to experiments performed by Nick Kleinschmit, the shockwave behavior becomes very complex and exhibits different behavior based on the distance from the shock tube and the angle made with the shock tube. The shock wave deflects around the end of the tube and the length of the shockwave duration decreases. Therefore the precise loading conditions for this experiment are not known. It was necessary for the tests to be performed outside the shock tube because the RED Head would not fit inside. Although these are limitation of the test and should be modified, for this thesis the ability of ARAMIS to perform the measurements and accurately analyze the data is the part that will be investigated below.
Figure 48: Major strain visualization and graph for front section of PDMS skin (data taken from black dot on right side of head).

Figure 48 – Figure 49 show the results obtained in ARAMIS for the blast loading of the RED Head. The major strain (maximum principal strain) is visualized for both the unlaminated skin (Figure 48) and laminated skin (Figure 49). The strain for the unlaminated section is nearly an order of magnitude larger than those for the laminated section. This happens because the unlaminated section flaps around because it is not glued down. This causes a much larger deformation and the separation between the PDMS skin and skull can clearly be seen in Figure 49 and Figure 51. If the PDMS skin is to be used in the future it will need to be fixed to the head. Rubber cement or another elastic, pliable adhesive would work well since the skin on the human head can move.
relative to the skull. In Figure 49 the strain has a noticeable peak as the shock wave passes by the data point location (black dot). If all of the skin had been fixed to the skull, I believe that this same type of strain profile would be seen everywhere. Figure 50 shows the total displacement magnitude (in 3D) of the point located on the back, laminated section of skin, represented by the black dot in Figure 49. From the graphs, the shockwave arrives around 2 milliseconds into image acquisition. The shock front could not be seen on the high speed cameras, so the arrival time at each point is assumed to be when the slope of the data changes.

**Figure 49:** Major strain visualization and graph for back section of PDMS skin (data taken from black dot on left side of head)
Figure 50: Total displacement for data point shown in Figure 49.

Figure 51: PDMS skin delamination view with major strain overlay. Note the large gap between the front and back pieces of PDMS.
3.5.3 Conclusions and Discussion

From the RED Head experiments many valuable pieces of information were learned. First, it was discovered that by not gluing the PDMS skin to the skull it delaminated as the shock wave interacted with it. This caused large strains and large displacements that were not representative of the actual blast loading of the human head. Secondly, information about the gross motion of the head was obtained and could be used to validate and help improve finite element models. Further tests need to be conducted inside of the shock tube where the overpressures and loading conditions are more precisely known. Performing the experiments outside of the 9 inch shock tube was necessary since the large tube had not been manufactured at the time and still provided valuable information about how the head moves and deforms under shock loading (although the precise loading was not known).

Although ARAMIS is useful in many tests, its capabilities have limited use in this experiment. The reason is that the surface strains are not the critical parameter that needs to be characterized. How stress waves move through the skull and more specifically the brain is the most important factor. The stress waves traveling in the brain are a likely cause for blast traumatic brain injury. The head has been instrumented with small fiber optic sensors (FISO) and the pressure wave transmission has been studied. This will lead to a much better understanding of what is causing injury to the brain. ARAMIS and the high speed cameras work great for seeing the high speed footage and understanding what is happening during the experiment. It helps to visualize and understand certain factors and determine why certain data and outcomes occur.
Chapter 4 – Mapping surface strains on mice tibia

4.1 – Introduction

Quasi-static mouse tibia compression tests were performed using ARAMIS to map the surface strains on mice tibias as part as an osteoporosis related study with Creighton University. A procedure was developed to create a full field 360 degree displacement and strain view based on the results from four tests performed on the same bone. This provided a means to obtain geometrical information for multiple cross sections of the bone. Mouse bone properties were calculated through a technique that uses the geometrical and strain data from ARAMIS and the load from the BOSE mechanical test system. The experimental method developed provides an improvement over traditional techniques which cannot provide full field and three dimensional measurements like ARAMIS.

The goal of this research is to develop a new experimental technique that can measure the full field surface strains using a contact free measurement. This new method is desired because it will be able to better characterize the mechanical response of the bones. It will be able to provide thousands of data points over the entire surface of the bone as opposed to only one data point provided by the strain gages. There are several criteria that the proposed method must meet in order to be considered a success. First it needs to be able to provide a full 360 degree view of the deformation. ARAMIS has the capability to do this by linking multiple projects together, but the procedure in ARAMIS must be modified and adjusted for use in this application. By obtaining a full 360 degree measurement, the user will have an excellent visualization of the deformation.
and it will be easier to compare the experimental results to FEM results. Secondly, as stated earlier, it must be a contact free measurement so that no gages influence the measurement. Optical methods have been used in animal research previously to determine strain during the compression of mouse tibias [9]. This advance was still only able to provide one strain value and not a full field or 360 degree measurement. Finally, the test setup must be able to create full field displacement and strain measurements. This will allow data at any location on the bone surface to be analyzed and compared with other tests. This will enhance the usability and usefulness of the results.

There are several challenges in performing these measurements including creating a high contrast, stochastic pattern on the extremely small mouse bones, creating a set of grips with a unique reference point pattern, and linking multiple tests together to generate a 360 degree view. To create the pattern on the bone, a fine spray paint was used and a 10X magnifying glass helped to ensure that the pattern was even and the proper size. The pattern was ultimately verified during the pre-test measurement by seeing if ARAMIS could find the points and create the displacement and strain grid. The reference point pattern on the grips was crucial to being able to create the 360 degree view. At least three common points had to be seen between each test so that the corresponding planes could be fit to each other. The 360 measurements were also facilitated by placing the BOSE machine on a turntable so that it could be rotated through four different positions. A total of four tests were performed on each sample, one at each position (approximately 90 degrees apart). Then, using the reference point pattern on the grips, individual tests were transformed relative to each other based on the coordinate position of the points. This allowed the four tests to be combined into one, thus creating the 360 degree view.
The experiments were performed on a set of samples which included a total of 14 mouse bones exhibiting three different genotypes as seen in Table 4.

**Table 4:** Mouse bone samples used for full field 360 degree view strain measurements.

<table>
<thead>
<tr>
<th>Genotype</th>
<th>Number of Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wild Type</td>
<td>5</td>
</tr>
<tr>
<td>HBM (High bone mass)</td>
<td>5</td>
</tr>
<tr>
<td>KO (Knock out)</td>
<td>4</td>
</tr>
</tbody>
</table>

All of the test samples were prepared at Creighton University and kept in saline solution and stored in a freezer. A previous set of tests were performed on all of the samples using strain gages placed three millimeters proximal to the tibia-fibula junction (TFJ). The samples were quasi-statically loaded from 0 to 10 Newtons.

Figure 53 shows the separation of tasks for the joint research project between Creighton University and the University of Nebraska-Lincoln. The part of the research that UNL is involved in is experimentation and analysis of the results using ARAMIS. More in depth analysis will be performed using the results from ARAMIS to gain a better understanding of the deformation of the mouse bones during loading. The moment of inertia is calculated for the cross section at varying heights so that the elastic modulus can be estimated based on the strains computed in ARAMIS. This method was selected over using FEM software because the analysis is totally based on the results obtained from ARAMIS.
The following sections of this chapter will present the background to this research, a motivation for this research based on creating a better understanding of the effects of osteoporosis and then previous tests and methods are presented. Next the experimental and test development, procedure, and results are presented.
4.2 – Background

Osteoporosis researches at Creighton University Medical Center were led to the discovery of a genetic mutation that causes abnormally large bone mineral densities (BMD) based on DXA (Dual-energy x-ray absorptiometry) scans of a middle aged female patient. Testing of her family members also showed that several of them had increased bone density as well. After several years of searching for the specific gene that caused this abnormality, the LRP5 protein (low-density lipoprotein receptor-related protein) was discovered. Now researchers are able to genetically modify mice to exhibit the same bone characteristics as the patient they scanned previously [10].

The effect of mechanical stimuli (exercise) on the development and growth of bones can now be characterized for these different mice genotypes. Enhanced bone growth due to mechanical stimulus of varying rates has been widely verified in mouse models [11] [12]. Dr. Akhter’s group at Creighton University Medical Center in Omaha, NE seeks to understand the effect of the specific genetic mutations on bone growth due to mechanical stimuli. Optimal loading patterns exist in which bones (in vivo) exhibit the largest gains in growth. Most of this growth takes place on the outer wall of the bone, thus giving the largest increase in the second moment of inertia therefore increasing the bending strength of the bone [13]. Researchers want to understand the effect of exercise (mechanical stimulus) on genetically different specimens and see whether the genetic mutation causes a difference in bone formation between genotypes.

Through the development of animal models to study the genetic effects of mechanical stimuli, pharmacological treatments and therapies can be developed to help strengthen bones and increase bone mineral density. The ability to precisely measure the
mechanical stimuli and the difference in mechanical properties becomes very important in characterizing how the bones act under different loading conditions. Bone researchers must have an effective way to characterize the mechanical properties of the bones they are studying. The characterization is traditionally performed ex-vivo using strain gages to determine the force-strain response of the bones. The data from the force-strain response curves is then used to load the bones in-vivo to achieve a known deformation [14] [15]. Creating a new technique to more precisely measure the deformation of the bones under loading will provide global deformation information and better analysis of the mechanical differences between genotypes.

The following section provides a motivation for the research. Osteoporosis affects millions of people in the United States alone and advances in the understanding of the disease and how to treat it could improve the lives of many.
4.3 – Osteoporosis Literature Review

Osteoporosis is one of the most common bone diseases and predominately affects older women. It is a disease characterized by low bone mass, low bone mineral density and the deterioration of the trabecular (cancellous) bone structure. The reduction in strength of the bone puts patients at risk for fracture and higher mortality rates. Nearly ten million Americans are affected by osteoporosis and 34 million have low bone mass, putting them at increased risk for osteoporosis. Osteoporosis is a large problem, because it causes high incidences of fracture due to reduced bone strength. Complications from these fractures can lead to limited mobility, disability, surgery and death. Women are at a higher risk for osteoporosis than men, accounting for approximately 80% of patients. Risk for having osteoporosis increases with age, and people 65 and over have much higher chances for fracture. Nearly 75% of hip, spine and forearm fractures occurred in patients over the age of 65. As America’s population ages, the number of people at risk for this disease increases. [16]

![Life expectancy 1960 to 2008 for men, women and combined.](image)

**Figure 54:** Life expectancy 1960 to 2008 for men, women and combined.
Figure 54 shows the average life expectancy for Americans from 1960 to 2008, where the life expectancy in the US has increased by more than eight years. As the population ages and people continue to live longer, more and more people will be at risk for this disease and it is critical that awareness increases and the effectiveness of treatments improves. America’s population in the age group 50 and over grew from 77 million (27.3%) to 91 million (30.4%) from the years 2000 to 2008. Not only is America’s total population increasing, but the percentage of those at high risk for the disease is increasing with respect to the rest of the population [17].

**Figure 55:** Number of births per year from 1940 to 1994 (Highlighted region constitutes baby boom generation (1945-1964)).
America’s population is becoming on average older due to the baby boomer generation as shown in Figure 55. The highlighted region shows the birth years for those considered in the baby boom. The first of those in this generation are starting to turn 65 years old this year (2011), thus putting them in the higher risk group. There was a large increase in the number of births between 1945 and 1964 and most of this population falls in the at risk group for osteoporosis [18]. Osteoporosis research is becoming more and more critical, and a better understanding of the disease is needed to create new and innovative treatments.

Osteoporosis is affecting more and more people worldwide as life expectancy and populations grow. Through bone research into the conditions of deteriorating bone structure and mass, better treatments and therapies can be found. It is of foremost importance to be able to characterize the bones mechanical properties so researchers can properly understand the difference between healthy bones and osteoporotic bones. Once the properties are well characterized the effect of the treatment on the bone health can be quantified and the best therapies and drugs can be selected for further trials. It is absolutely necessary for researchers to be able to use animal models in their research because they allow the research to progress more quickly. All new treatments must first be tested on animals and without this no progress could be made. With that said the animals should be treated humanely and their pain and suffering should be kept to a minimum [19].
4.4 – Previous Tests

4.4.1 Strain gage based measurements

To quantify the amount of deformation (strain) that mouse bones experience during exercise/mechanical stimulus, the overall deformation must be understood. The current method is to measure strain using a strain gage glued onto a specific region on the surface of the bone. Strain measurements are taken ex vivo and the strains are quantified at different loads, which provides a load-strain curve. This information is then applied to in vivo tests (mechanical stimulation of live mice). Specific loading patterns can then be applied to the mouse to determine the amount of strain mouse bones see during a specific loading pattern [14] [15]. See Figure 56 for in vivo loading image. The strains data for this method is only known at one point and not over the entire bone therefore the in vivo loading deformations are not well characterized. There is no way to measure the true strains during the test due to the size and shape of the bones. The strain gages provide measurements with low noise and are reproducible.

Figure 56: In vivo loading of mice tibia.
Strain gages placed in mouse bones are large in comparison to the bone. They also only allow measurements to be taken at one location in an attempt to characterize an extremely complex deformation. The strain measured by the gage varies widely with placement and therefore has large precision errors between samples. On the following pages, Figure 57 to Figure 59 show the size of the mouse tibia tested and the size and placement of the strain gage.

Figure 57: Mouse tibia size comparison.
The bones are not only extremely small, but also have a complex geometry and are inhomogeneous and anisotropic. Therefore, when they are compressed the deformation is extremely complex not only due to the bone geometry, but the mechanical properties of the bone itself. Figure 57 and Figure 58 give a size comparison for the

**Figure 58:** Mouse tibia size comparison.
mouse bones; each bone is approximately 15 mm tall. The strain gage is large compared to the bone. Images of the mounted gage can be seen in Figure 59. The gage which has to be fixed to the surface with epoxy increases the stiffness of the bone where it is applied. Clearly, one strain measurement on the bone surface cannot give a complete description of the deformation. This however is the currently accepted technique to characterize the bone deformation for mechanical loadings. A non-contact, full field measurement would be able to provide a much more complete understanding of the deformation without the influence of having a gage mounted to the specimen.
Figure 59: Creighton University's strain gage placement 3 mm proximal to the tibia-fibula junction (TFJ).
4.4.2 Contact Free Fluorescence Based Strain Measurements

Researchers at the University of Delaware published a paper [9] on a fluorescence-based optical extensometry system for imaging mechanically loaded bone. In their experimental procedure fluorescent spheres were adhered to the surface of the bone and two pattern groups were selected (as seen by the gray boxes in Figure 60) and strain is measured based on the change of length of a line element with endpoints being the center of the boxes. This technique allows for a contact free strain measurement but can only be performed in 2 dimensions since only one camera was used. Although their setup ensured that the camera was focused on a flat side of the bone and calibration tests were performed to determine the error due to out of plane motion, ARAMIS automatically handles out of plane motions by using a full three-dimensional measurement.

Figure 60: Strain measurement technique [9].
There are still several problems with the technique used by Price. It has achieved one of the goals, which is contact free measurement, but there is still only one measure of the strain. A series of different patterns could be selected, and many strain measurements made. Although different patterns can be selected and the strain measured, it is not an automated process and does not give a true full field strain measure. A development of a full field strain measurement tool would provide much more information about the specimen and a more reliable measure of the strain. The entire surface of the bone can be

Figure 61: Delaware University contact free strain measuring apparatus
analyzed and more useful load strain curves can be obtained. Several aspects of the Delaware test setup allowed for better test conditions than the tests performed at UNL. They were able to perform their tests in a saline solution, thus reducing the dehydration of the bones in air. Secondly, they held the bone at a 2 N load for 10 seconds to ensure that the bone was properly held in the grips before the test was started. The load was then brought down to 0.2 N before testing began. This helped ensure that the bone did not move in the grips when testing began. This paper and setup were discovered after the tests at UNL were performed and for future test setups, these experimental techniques will also be employed.
The equipment used for testing of the mouse bones is shown in Table 5. The Bose mechanical test system was used to perform the quasi-static loading and a 5 pound load cell was used to acquire load measurements. The compression grips were made from acrylic cylinders with a small cup machined to ensure that the mouse bone could properly be mounted. The reference point pattern was applied to the bottom compression grip and the turntable was used to rotate the Bose through the four different test views needed to create the 360 degree view.

**Table 5: Mouse bone compression test equipment list.**

<table>
<thead>
<tr>
<th>Test Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARAMIS 3D Stereo-Optical System</td>
</tr>
<tr>
<td>Bose Electroforce Test System</td>
</tr>
<tr>
<td>Compression Grips</td>
</tr>
<tr>
<td>Specialized reference point pattern</td>
</tr>
<tr>
<td>Turntable for Bose Machine</td>
</tr>
</tbody>
</table>

Force control was not used in loading the specimens because the displacement would have continued to increase even when the loads were held constant; this is because the bone is viscoelastic. It was decided that displacement control with a constant ramp would be a better way to load the bones. The bones were displaced until the desired load was reached and then the displacement was held constant while the images could be taken for ARAMIS. The process was repeated for loads of 0, 1, 2, 4, and 6 Newtons. It should be noted that the loads were less than those for the experiments at Creighton
University because the bones had been loaded previously, they were approximately one year old at time of testing at UNL, and damage initiation in the bone was to be avoided.

The above procedure had to be done four times from four different views in order to create the 360 degree view (see Figure 62 for the needed views). The specialized reference point pattern was used to stitch the four views together. Extreme care was taken when rotating the Bose machine on the turntable to ensure that the ARAMIS cameras were not bumped and that the mouse bone did not move in the grips. Had the bone moved in the grips, we would not have been able to accurately create the 360 degree view.

Figure 62: Four views needed in order to compile the 360 degree view.
The specially designed reference point pattern is shown on the compression grip in the test setup as seen in Figure 63. This figure shows both the left and right images with the reference point pattern and the bone fixed in the grips. When the Bose machine is rotated approximately 90 degrees to the left, the reference pattern in the yellow circle will be on the left side. The reference point pattern is created in ARAMIS and its exact coordinates defined. In this same way all of the reference points in all of the views are defined and they are later matched to each other. The four views can then be exported as one combined project.

**Figure 63:** Reference point pattern as seen in view 1.
The following is the procedure used in testing the mouse bones:

**Mouse Bone Testing Procedure:**

1. Remove one mouse bone from the freezer and let thaw (~1 hour). See Figure 67 for an image of the storage vial.

![Mouse bone storage vial](image)

**Figure 64:** Mouse bone storage vial.

2. While the bone is thawing, the ARAMIS system must be setup next to the Bose machine.

3. The ARAMIS system is calibrated.

4. Once the mouse bone is thawed it is painted with a high contrast black and white speckle pattern.
5. The mouse bone is checked with a magnifying glass to ensure the pattern is good.

**Figure 65**: High Contrast black and white stochastic pattern applied to the mouse bone with black and white spray paint
6. The mouse bone is then mounted in the grips and position so proper contact is made.

![Figure 66: Mouse bone mounted in the grips with several reference point pattern groups visible](image)

7. The lighting is checked to ensure that there is no glare on the sample and that there is sufficient lighting.

8. A pretest is performed to ensure that ARAMIS can identify the pattern and measure displacement and strain data.

9. After the pretest and in between each test, the lights are turned off so as to not heat up or dehydrate the sample.

10. If the pretest is okay, the test parameters are defined in ARAMIS including the frame rate, shutter speed and number of images to take.
11. Ten images are then taken for the zero load condition.

12. Once the ten images have been taken, the load is increased (via displacement control) to 1 Newton.

13. Ten images are again taken while the displacement is held constant.

14. Steps 12 & 13 are repeated until the load reaches 6 Newtons.

15. Once the load reaches 6 Newtons and images have been taken the sample is unloaded.

16. The Bose machine is then carefully rotated approximately 90 degrees and steps 11-15 are repeated (Note, there should be one ARAMIS project per view). The Bose machine and turntable can be seen in Figure 67 and Figure 68.

17. This procedure is followed for all views.

18. When the tests for all four views have been completed, the mouse tibia is removed and placed back in the saline solution and returned to the freezer.
Figure 67: Bose electroforce 3200 test frame.

Figure 68: Turntable used to place BOSE on to allow for easy rotation of views.
4.6 – Method for Obtaining Results

The results provided are based on a technique that uses the geometrical and strain data from ARAMIS and the loads from the BOSE to calculate the elastic modulus of the mouse bones. This technique provides a different way to characterize the mechanical properties of the bone. Micro CT scans and finite element analysis has been performed in the past but ARAMIS provides a new technique to obtain results. The micro CT scans can provide more accurate geometrical data than ARAMIS can. However, it is very difficult to match corresponding points from the scan to ARAMIS. It is quite easy to obtain geometrical data for cross-sections and then obtain strain or displacement data from specific points on the cross-section.

The method used to calculate the mechanical properties (elastic modulus) uses statics and elastic beam theory. Figure 69 provides the setup for the problem. The load is applied by the Bose machine along the load axis, which has been estimated as the centroid of the bottom most cross section as obtained from ARAMIS. The force applied at the bottom is balanced by the force at the centroid of the cross section and a moment. The moment is determined by the distance from the cross sectional centroid to the load axis and the force applied. The moment has components along both principle axes, X’ and Y’. Further detail about the specific procedure and quantities needed to solve for the elastic modulus are provided later.
Figure 70 shows the location that cross sectional data was obtained from the mouse tibia and the compiled 360 degree view of the bone from ARAMIS. There are some gaps where ARAMIS could not compute data due to the bone geometry, or the view was obstructed by the fibula. The area and moment of inertia calculations were only performed based on the data points provided by ARAMIS. An interpolation function could be used to fit the data so more points could be used in solving for these values, but the end results would not improve significantly. The load axis was estimated...
to pass through the centroid of the cross section 1mm above the base of the bone as captured by ARAMIS (approximately 3-4 mm from base of bone). The location of the load axis is crucial to the final estimation of the elastic modulus and if it is estimated poorly, the results will not be accurate.

As described earlier, four tests were performed on each bone in order to create a 360 degree view. Although tested at low loads (max 6N), multiple tests on each bone most likely had a negative influence on the tests. Repeated tests of the same bone increases the likelihood of damaging the bone and therefore the different views may not align well. To avoid having to perform several tests on each bone, the 360 degree view can be compiled at zero load. Therefore, the geometrical information (cross section plots, area, moment of inertias etc.) is all obtained for the unloaded sample. Then, the test can

**Figure 70:** Position of cross section data from bottom of mouse bone
be performed from only one view. All of the strain information can then be gathered from this one view and the risk of damaging the bone will be less, while still providing all of the needed values. This method would reduce the number of tests performed and still be able to provide meaningful results.

The remaining portion of this section is devoted to the specifics of the method used to obtain information about the elastic modulus. Based on simple elastic beam theory, the elastic modulus of the bone can be calculated if certain parameters are known. The parameters are the cross sectional area, centroid to load axis vector, and centroid to data point location vector is known. A diagram of the problem is presented in Figure 71 where C is the centroid location, A is the load axis, and P is a data point location on the surface. To solve for the elastic modulus, the principle directions must be known. The principle axes can be obtained by solving the eigenvalue problem for the known moment of inertias (I_{xx}, I_{yy}, and I_{xy}). Once the principal axes are determined, then the vectors (\textbf{CP} and \textbf{CA}) can be transformed into the appropriate coordinate system and the elastic moduli solved for.
Figure 71 provides the details for the problem that is being solved. The X-Y coordinates with corresponding unit vectors i and j, define the coordinates as obtained from ARAMIS. The X’-Y’ coordinate system with corresponding unit vectors $e_1$ and $e_2$ defines the principle axes. The point locations are: point C (centroid of area), point A (in this case, load axis), and point P (data point location). These points have the corresponding point locations in the X-Y coordinate system:

Point $C: (x_c, y_c)$,
Point $A: (x_A, y_A)$,
Point $P: (x_p, y_p)$.
The vector $\mathbf{CA}$ goes from point C (centroid of area) to point A (load axis). $\mathbf{CA}_1$ and $\mathbf{CA}_2$ are the components in the principal coordinate system. Knowing the components of $\mathbf{CA}$ will allow for the moments to be solved for. The vector $\mathbf{CA}$ however, is only defined in the X-Y coordinate system (coordinates from ARAMIS) and we must define it in the principle directions. $\mathbf{CA}$ is defined in the X-Y system as:

$$\mathbf{CA} = (x_A - x_c)i + (y_A - y_c)j.$$  \hspace{1cm} (10)

$\mathbf{CA}$ can be transformed to the principle coordinate system by using the eigenvectors solved for from the eigenvalue problem. The eigenvectors are normal to each other and it is important that they are ordered to define a right handed system so the results are not negative. In this case, since the problem is solved in two dimensions, the eigenvectors have components in the X and Y directions. The vectors are defined as $\mathbf{V}_1$ and $\mathbf{V}_2$ where the cross product of $\mathbf{V}_1$ into $\mathbf{V}_2$ will give the out of plane direction as positive, thus having a right handed coordinate system. The principal axes ($\mathbf{\hat{e}}_1$ and $\mathbf{\hat{e}}_2$) can then be defined in terms of the eigenvectors

$$\mathbf{\hat{e}}_1 = V_{1x} \hat{i} + V_{1y} \hat{j},$$  \hspace{1cm} (11)

$$\mathbf{\hat{e}}_2 = V_{2x} \hat{i} + V_{2y} \hat{j}.$$  \hspace{1cm} (12)

Now $\mathbf{CA}$ can be transformed to the principal axes coordinates ($\mathbf{\hat{e}}_1$ and $\mathbf{\hat{e}}_2$):

$$\mathbf{CA} = (\mathbf{CA} \cdot \mathbf{\hat{e}}_1) \mathbf{\hat{e}}_1 + (\mathbf{CA} \cdot \mathbf{\hat{e}}_2) \mathbf{\hat{e}}_2.$$  \hspace{1cm} (13)

Since the vector $\mathbf{CA}$ is now known in the necessary coordinate system, the moments about $\mathbf{\hat{e}}_1$ and $\mathbf{\hat{e}}_2$ can be solved for. From the way the problem is presented, the moment about $\mathbf{\hat{e}}_1$ will be negative and the moment about $\mathbf{\hat{e}}_2$ will be positive. They are as follows:
where the force is the external compressive force. Using elastic beam theory, the stress can be calculated for the data point P. The point P will experience three stress components, two from bending and one from compression, they are

\[ \sigma_1 = \frac{M_1 CP_2}{I_1}, \]  
\[ \sigma_2 = -\frac{M_2 CP_1}{I_2}, \]  
\[ \sigma_3 = \frac{F}{A}. \]  

And the total stress is the sum of all three and given by

\[ \sigma = \sigma_1 + \sigma_2 + \sigma_3. \]  

Since the strain is known, the stress and strain can be related through Hooke’s Law,

\[ \sigma = E \varepsilon \quad \rightarrow \quad E = \frac{\sigma}{\varepsilon}. \]
4.7 – Experimental Results

Three tests (one of each genotype) were analyzed for this thesis, these were A2100 (HBM), A2132 (WT), and A2107 (KX). Recalling that HBM has a mutation of the LRP5 gene causing high bone mass (and larger bone), WT is the wild type (no genetic mutation), and KX has the LRP5 gene removed causing weaker, smaller bones. The goal was to characterize and understand the differences between each bone. The cross sectional area, second moments of area, and elastic modulus are presented for all three. First, the area as a function of the height of the mouse bone is shown in Figure 72. As expected the HBM has a significantly larger area than both the wild type and knockout. Averaged over the 7 cross sections, it is 63% larger than the wild type and 121% larger than the knock out. The knock out is approximately 25% smaller than the wild type bone, when averaged over all data points.

![Figure 72: Cross sectional area for mouse bone results group.](image)
The area moment of inertia was calculated for the cross sectional areas and then as described in the analysis method above, the principal axes were found by solving the eigenvalue problem. The method used to solve for the principle area moment of inertias is presented below for bone A2100 (HBM)

\[
\begin{bmatrix}
l_{xx} & l_{xy} \\
l_{xy} & l_{yy}
\end{bmatrix} \mathbf{v} = \lambda \mathbf{v},
\]

\[
\begin{bmatrix}
0.09304 & -0.00024 \\
-0.00024 & 0.11890
\end{bmatrix} \mathbf{v} = \lambda \mathbf{v},
\]

Where \( \mathbf{v} \) is the corresponding eigenvector to eigenvalue \( \lambda \). To solve for the eigenvalues, the following problem must be solved:

\[
\text{det} \begin{bmatrix}
0.09304 - \lambda & -0.00024 \\
-0.00024 & 0.11890 - \lambda
\end{bmatrix} = 0
\]

\[(0.09304 - \lambda)(0.11890 - \lambda) + (0.00024)(0.00024) = 0\]

\[\lambda^2 - 0.21194\lambda + 0.011063 = 0\]

Solving the quadratic equation, the two roots 0.118879 and 0.093061 are found.

The eigenvalues can now be used to find their corresponding eigenvectors through solving the equations

\[
\begin{bmatrix}
0.09304 - 0.118879 & -0.00024 \\
-0.00024 & 0.11890 - 0.118879
\end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} = 0,
\]

Which yields

\[
\begin{bmatrix}
-0.02584 & -0.00024 \\
-0.00024 & 0.000021
\end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} = 0.
\]
Solving the above system in terms of $x'$ yields

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix} = \begin{bmatrix}
x' \\
-107.667x'
\end{bmatrix}.
\]

Normalizing the above vector yields

\[
\mathbf{v}_1 = \begin{bmatrix} x'_1 \\ y'_1 \end{bmatrix} = \begin{bmatrix} -0.009287 \\ 0.99995 \end{bmatrix}.
\]

Likewise, the eigenvector corresponding to $\lambda = 0.093061$ will be:

\[
\mathbf{v}_2 = \begin{bmatrix} x'_2 \\ y'_2 \end{bmatrix} = \begin{bmatrix} -0.99995 \\ -0.009287 \end{bmatrix}.
\]

To make sure a right handed base was maintained, we can calculate the cross product, $\mathbf{v}_1 \times \mathbf{v}_2$.

\[
det\begin{bmatrix}
\hat{e}_1 & \hat{e}_2 & \hat{k} \\
-0.009287 & -0.99995 & 0 \\
0.99995 & -0.009287 & 0
\end{bmatrix} = \hat{k}.
\]

Therefore, a right handed coordinate system has been maintained and the eigenvalues and eigenvectors represent the principal area moment of inertias and principal axes respectively.

The eigenvalues and eigenvectors were all solved in Matlab using the built in function: $[V,D] = \text{eig}(A)$, where $A$ is the matrix (in this case the area moment of inertia matrix) and $V$ gives the Eigenvectors corresponding to the Eigenvalues in diagonal matrix $D$. This sped up the calculations and the results were verified as shown above.

Now, the results for the area moment of inertias for the principal axes for all three samples at heights of 7, 9, 11, and 13mm are presented in Figure 73 and Figure 74.
The area moment of inertias show a similar trend to the area, as would be expected. The high bone mass sample is on average 125% and 231% larger averaged over the four cross sections (7mm, 9mm, 11mm, 13mm) when compared to the wild type and knockout samples respectively. The wild type sample is 49% less than the wild type sample. The results for the area moment of inertia about the second axis are similar. The high bone mass sample has a moment of inertia that is 142% and 279% larger than the wild type and knock out samples respectively. The knock out sample has a moment of inertia that is 56% less than the wild type sample. Since only three bones, one from each genotype were analyzed, statistics cannot be performed on the data.

**Figure 73:** Moment of inertias about principal axis 1, $I_{x'x'}$. 

![Graph showing moment of inertias about principal axis 1, $I_{x'x'}$.](image)
Once the principal area moment of inertias have been calculated and the location from the centroid to the data points and centroid to load axis are known, the elastic modulus can be calculated. As presented in the previous section, the stresses due to axial compression and bending about the x’ and y’ (principal axes) must be calculated. Finally once all three are known, they can be summed to give the total stress at a specific location (corresponding to the data point selected in ARAMIS). Then, the elastic modulus is determined, by dividing the stress by the strain at the point. Figure 75 shows the elastic modulus for the high bone mass sample, the wild type sample, and the knock out sample. The values and error bars were calculated based on the average and standard deviation from multiple points on the surface of each sample. The numerical data is presented in Table 6.

**Figure 74:** Moment of inertias about principal axis 2, \(I_{y'y'}\).
**Figure 76**: Elastic modulus estimations for the three genotypes (elastic modulus average and standard deviation determined from tables 7-9).

**Table 6**: Elastic modulus for mouse bone samples.

<table>
<thead>
<tr>
<th>Bone Code</th>
<th>Elastic Modulus (GPa)</th>
<th>Std Dev (GPa)</th>
<th>Genotype</th>
</tr>
</thead>
<tbody>
<tr>
<td>A2100</td>
<td>24.42</td>
<td>6.31</td>
<td>HBM</td>
</tr>
<tr>
<td>A2132</td>
<td>18.01</td>
<td>8.89</td>
<td>WT</td>
</tr>
<tr>
<td>A2107</td>
<td>8.90</td>
<td>5.95</td>
<td>KX</td>
</tr>
</tbody>
</table>

Figure 76 shows the vertical strain overlay for the HBM, WT, and KX mouse genotypes. All of the images were obtained for the maximum load condition of 6N. The HBM overlay appears to have the smallest strains while the WT and KX genotypes have larger strains.
The total data set is presented showing the strain, stress, and Elastic Modulus for the high bone mass, wild type, and knock out genotypes in the following tables. For A2100 (HBM), 13mm-Point3, the data point falls approximately along the neutral axis which is why the stress is so low, and the strain is at the limit of ARAMIS’s capabilities. For A2132 (WT), 9mm-Point1 should be in compression, however due to noise, the strain value is in tension and 9mm-Point3 is approximately along the neutral axis and should not have been used for analysis. For A2107 (KX) 7mm-Point1 is clearly an outlier with a modulus of 61.8 GPa and was not used in calculations. In the following tables, the data point naming method is (cross section height)-(data point number), where the data point

**Figure 76:** Vertical strain overlay for three mouse genotypes.
number distinguishes which point on a given cross section was used (multiple data points were taken from each cross section).

**Table 7**: High Bone Mass (A2100) elastic modulus calculations (red denotes outliers).

<table>
<thead>
<tr>
<th>Data Point</th>
<th>Strain (%)</th>
<th>Stress (MPa)</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7mm-Point1</td>
<td>0.1259</td>
<td>37.8</td>
<td>30.1</td>
</tr>
<tr>
<td>7mm-Point2</td>
<td>-0.0801</td>
<td>-24.3</td>
<td>30.4</td>
</tr>
<tr>
<td>9mm-Point1</td>
<td>-0.0632</td>
<td>-14.9</td>
<td>23.6</td>
</tr>
<tr>
<td>11mm-Point1</td>
<td>-0.0667</td>
<td>-19.4</td>
<td>29.1</td>
</tr>
<tr>
<td>11mm-Point2</td>
<td>-0.0605</td>
<td>-16.3</td>
<td>26.9</td>
</tr>
<tr>
<td>11mm-Point3</td>
<td>0.0853</td>
<td>10.3</td>
<td>12.1</td>
</tr>
<tr>
<td>13mm-Point1</td>
<td>-0.0825</td>
<td>-15.5</td>
<td>18.8</td>
</tr>
<tr>
<td>13mm-Point2</td>
<td>-0.0745</td>
<td>-2.5</td>
<td>3.4</td>
</tr>
<tr>
<td>13mm-Point3</td>
<td>-0.0518</td>
<td>0.527</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

**Table 8**: Wild Type (A2132) elastic modulus calculations (red denotes outliers).

<table>
<thead>
<tr>
<th>Data Point</th>
<th>Strain (%)</th>
<th>Stress (MPa)</th>
<th>E (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7mm-Point1</td>
<td>-0.0794</td>
<td>-27.4</td>
<td>34.6</td>
</tr>
<tr>
<td>7mm-Point2</td>
<td>0.2757</td>
<td>53.2</td>
<td>19.3</td>
</tr>
<tr>
<td>7mm-Point3</td>
<td>-0.2338</td>
<td>-63.6</td>
<td>27.2</td>
</tr>
<tr>
<td>9mm-Point1</td>
<td>0.1490</td>
<td>-50.8</td>
<td>-34.0</td>
</tr>
<tr>
<td>9mm-Point2</td>
<td>0.1648</td>
<td>49.0</td>
<td>29.7</td>
</tr>
<tr>
<td>9mm-Point3</td>
<td>0.0820</td>
<td>-11.7</td>
<td>-14.2</td>
</tr>
<tr>
<td>9mm-Point4</td>
<td>0.1523</td>
<td>29.9</td>
<td>19.7</td>
</tr>
<tr>
<td>9mm-Point5</td>
<td>0.1408</td>
<td>11.7</td>
<td>8.3</td>
</tr>
<tr>
<td>11mm-Point1</td>
<td>-0.1713</td>
<td>-20.8</td>
<td>12.2</td>
</tr>
<tr>
<td>Data Point</td>
<td>Strain (%)</td>
<td>Stress (MPa)</td>
<td>E (GPa)</td>
</tr>
<tr>
<td>-------------------</td>
<td>------------</td>
<td>--------------</td>
<td>---------</td>
</tr>
<tr>
<td>7mm-Point1</td>
<td>0.0946</td>
<td>58.5</td>
<td>61.8</td>
</tr>
<tr>
<td>7mm-Point2</td>
<td>0.1244</td>
<td>17.3</td>
<td>13.9</td>
</tr>
<tr>
<td>9mm-Point1</td>
<td>-0.4685</td>
<td>-47.4</td>
<td>10.1</td>
</tr>
<tr>
<td>9mm-Point2</td>
<td>0.2844</td>
<td>49.9</td>
<td>17.5</td>
</tr>
<tr>
<td>9mm-Point3</td>
<td>0.3500</td>
<td>31.8</td>
<td>9.1</td>
</tr>
<tr>
<td>11mm-Point1</td>
<td>0.1865</td>
<td>30.5</td>
<td>16.4</td>
</tr>
<tr>
<td>11mm-Point2</td>
<td>0.1782</td>
<td>4.8</td>
<td>2.7</td>
</tr>
<tr>
<td>11mm-Point3</td>
<td>-0.1118</td>
<td>-14.5</td>
<td>13.0</td>
</tr>
<tr>
<td>13mm-Point1</td>
<td>-0.5943</td>
<td>-22.1</td>
<td>3.7</td>
</tr>
<tr>
<td>13mm-Point2</td>
<td>0.4855</td>
<td>7.3</td>
<td>1.5</td>
</tr>
<tr>
<td>13mm-Point3</td>
<td>-0.6384</td>
<td>-7.1</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Table 9: Knock Out (A2107) elastic modulus calculations (red denotes outliers).
4.8 – Conclusions and Discussion

The tests conducted on the mouse bone tibias were very successful in proving that DIC (Digital Image Correlation) strain measurement methods are capable of performing the necessary measurements. This new technique offers a huge advantage over current methods. Full field displacements and strains offer the unique advantage of being able to see the deformation over the entire surface area. Optical measurement techniques have been used in the past few years but none have created a 360 degree deformation view of the mouse bone. Our technique allows for analysis of the data gathered in ARAMIS and can be used to determine material properties as well as characterize the strain and displacement fields.

There is still a great deal of work that needs to be done to replace existing testing methods with the proposed stereo-optical method. Although the experimental procedure was proven to work, it must be modified to obtain data with smaller deviations. There are several areas that can be improved upon:

i. Patterning

**Issue:** Spray paint was used in this study and led to some inconsistencies in the application of the pattern.

**Resolution:** A high precision air brush could be used that is capable of consistently producing uniform dot sizes.

ii. Field of View Size

**Issue:** A large field of view was used so that the entire bone including reference pattern was able to be seen. This decreased the spatial resolution of the system.
Resolution: ARAMIS can be calibrated using a micro-ARAMIS calibration panel (size: 10x8mm). Although the entire bone would not be able to be viewed, there would be higher resolution of the bone in the images. This method will increase test time and was not used due to the number of tests run (test took ~3 hours per sample, not including calculations and analysis).

iii. Reduce the number of tests

Issue: Four tests had to be performed on each bone to obtain test results, possibly leading to damage of the specimen and inconsistencies when creating the 360 degree view

Resolution: If the 360 degree view is not important, than tests should only be performed from one view. This will reduce the chances of damaging the bone during testing. If the 360 degree view is needed, it can be compiled from images acquired at no load and then the test can be performed in only one of the views.

iv. Reduce handling of samples

Issue: The samples were very difficult to handle, especially while wearing latex gloves. When the samples were positioned in the load frame tweezers were used, however they would consistently damage the pattern where they gripped the bone.

Resolution: A pair of tweezers with a small, smooth surface area could help ensure that the pattern is not damaged during loading.
v. **Ensure the sample is loaded properly**

**Issue:** The samples were not subjected to a small preload which could have led to movement of the sample during testing if it was not properly seated.

**Resolution:** It is important to ensure that the mouse bone does not move during testing. This can be done by loading the sample to two Netwons and then reducing the load to 0.1 Newtons before the test begins. This will help ensure that the bone is properly fixed inside of the grips.

vi. **Increase the load on the samples**

**Issue:** The small loads applied lead to strain measurements that were close to the limit of the ARAMIS system.

**Resolution:** The loads applied for the tests were quite small (max: 6N). Most tibia compression tests go to larger loads. Increasing the load will increase the strain in the bone and will therefore be easier to distinguish from the noise.

vii. **Sample Storage and Age**

**Issue:** Samples were stored in saline solution and kept in a freezer when tests were not being performed. The set of specimens was approximately one year old at the time of testing.

**Resolution:** The samples should not be frozen in saline as this could cause damage to the mouse bones and therefore yield results that don’t represent the real bone. If the samples must be stored in this way,
quantification of the difference between bones that have been frozen and bones that are fresh should be determined. An ideal test situation would involve the mouse being sacrificed, the bone being prepared (tissue removed), and then patterned with paint and tested immediately. If the bone could be tested in a saline solution that would be even better, however, difficulties with the optical measurement would arise. Reducing the time between the sacrifice and testing would help ensure that the most realistic test is being performed.

Improving the technique as outlined above should yield much cleaner data, however there will always be somewhat substantial deviations in the results due to the inconsistencies in sample geometry. The tests performed yield insight into the capabilities of the ARAMIS system in testing small biological samples, and the type of analysis that can be performed. The application of DIC strain measurement techniques to mouse bones helps to solve some problems related size and shape when testing mouse tibias with traditional techniques. Strain gages are large compared to the mouse tibia and the gage increases the stiffness of the bone at the location it is applied. The strain gage only provides one strain measurement in an attempt to characterize a very complex deformation. This is where ARAMIS offers the unique ability to provide the full field strain and displacements. With this method the deformation and loading of the bone can be understood and the results visualized much more easily.
Using ARAMIS in analyzing the quasi-static compression of mouse tibias was very successful. A full 360 degree view has been compiled from four test runs (on each bone, ~90 degrees apart) by using a specially designed reference point pattern. Creating the 360 degree model the elastic modulus was calculated for each of the three genotypes based on the geometrical and strain data provided by ARAMIS. As was expected the high bone mass genotype (HBM) had the largest cross sectional areas and moment of inertias. The HBM bone was also the stiffest compared to the wild type (WT) and knock-out (KX) bones. The WT bone had properties in between the HBM and KX bones. This method could be fundamental in better characterizing bone disorders and is an important step forward. Osteoporosis and bone related diseases need to be understood and the development of this technique is a step forward in better understanding the effects of disease on bone properties.
Chapter 5 – Reflected Wave Ultrasonics

5.1 Introduction

The goal in developing an ultrasonic experimental technique is to determine the dynamic response of soft materials. This technique is applicable for unloaded or loaded samples which gives it the unique ability to perform ultrasonic materials characterization under load. The characterization of soft materials especially tissue and tissue simulants is crucial in blast traumatic brain injury research (bTBI). Through the accurate characterization of these materials, better simulants can be used in experimentation and more realistic properties can be used in creating models. A system was designed based on the setup used by Madsen [20] which included a silica block, ultrasonic transducer, pulser-receiver, high-speed digitizer, and computer control program. The system that was developed is able to accurately measure the longitudinal and shear ultrasonic wave speeds in unknown materials. This system lends itself well to the characterization of soft materials with non-uniform and unknown geometries.

Multiple materials were tested including PDMS (polydimethylsiloxane), PC (polycarbonate), Sillyputty®, and bovine liver. The ultrasonic wave speeds were compared to previous experimental results (pulse echo technique) or results obtained from literature values and matched quite well. The reflected wave technique can also measure the attenuation coefficient for the unknown materials, but results for this could not be measured due to the experimental setup and limitations of the current system. This method is important because it allows for the dynamic response of soft materials including tissues to be characterized relatively easily.
5.2 Background & Motivation

The University of Nebraska-Lincoln has been researching blast traumatic brain injury through funding from the Army Research Office for the past several years and this research is crucial for both military and civilian populations. An estimated 1.7 million people in the US sustain traumatic brain injuries annually. These injuries result in 52,000 deaths, 275,000 hospitalizations and over 1.3 million emergency room visits [21]. A total of 43,779 US soldiers were diagnosed with TBI between the years 2003 and 2007 in Operation Iraqi Freedom and Operation Enduring Freedom [22]. From advances in protective armor more soldiers are returning alive from what would have previously been fatal injuries. As more soldiers return from active duty and readjust to civilian life, understanding how to treat TBI is becoming more important.

A proposed $669 million will be used for TBI treatment and psychological health of US military, and $250 million will be used for continued support of TBI research and PTS (post-traumatic stress) in 2011 [1]. With the large number of TBI injuries sustained in both the civilian and military populations, it is vitally important that the mechanism of injury is understood. In order to understand how the blast waves affect the brain, the materials properties must be well characterized so that accurate simulators can be used in experimentation and accurate properties used in modeling.

Traumatic brain injuries occur in varying degrees from mild to severe. Mild TBI is not easily diagnosed, and usually takes months before symptoms are noticed. Symptoms of mild TBI can very common ailments such as headache, dizziness, irritability, fatigue or poor concentration. Assessing soldiers in the field for injuries becomes even more difficult and many service men think they are okay after a
concussion and don’t seek medical attention. Mild TBI however may occur without the soldier losing consciousness. The study of TBI, especially the causes and effect of mild TBI is a vitally important research area to the army because of its prevalence among soldiers. Mild TBIs sustained in the military account for nearly 95% of all TBI related injuries and Figure 77 shows TBI diagnosis (with respect to severity) among military personnel from 2005-2008 [23].

![TBI Severity Diagram](image)

**Figure 77:** Traumatic brain injuries in the military severity chart [23].

With the increase in prevalence of soldiers suffering traumatic brain injuries, especially mild TBI, it is important to understand the mechanism of injury so that better protective armor and physical and pharmacological therapies can be developed. One method in understanding how the injury occurs is through the development of the RED Head as discussed in Chapter 3. The transmission and propagation of the shockwave in the head can be measured by subjecting the RED Head to blast waves. This information
will help increase the understanding of what the brain is exposed to during a blast event. To more precisely measure the acoustical properties of the brain and how these waves are transmitted and how they propagate, it is crucial to use a simulant material that accurately represents the real properties of the skin, skull, and brain. There is little data on dynamic properties of brain and what is available has large deviations. The results are also highly dependent on experimental setup, ex vivo time, temperature etc. An experimental technique that could accurately measure the dynamic response of soft materials could greatly advance the ability to perform more accurate experiments and develop more accurate models.

The following section will provide a description of this ultrasonic technique and its theoretical basis followed by a section on the experimental setup and considerations that must be taken into account for each experiment. The following chapter will then present experimental results for several materials including, PC (polycarbonate), PDMS (polydimethylsiloxane), and Sillyputty®.
5.3 Ultrasonic Theory and Modeling

5.3.1 Theoretical Background to Wave Propagation

Measuring incident and reflected waves from an interface between two materials can provide mechanical properties for the unknown material beyond the interface. In this section we use one-dimensional time harmonic attenuating plane shear waves to show how this method works for a linear viscoelastic material.

[Diagram: Reflection boundary between two media with normal incidence.]

**Figure 78:** Reflection boundary between two media with normal incidence.

We first consider the interface problem in which an incident wave is partially transmitted through the interface and partially reflected from it. Figure 78 shows a typical interface between two Media $a$ and Media $b$. The general solution to this problem is in terms the spacial variable $x$ along the direction of the wave and the time variable $t$ will be characterized by a stress field $\sigma(x,t)$ and a displacement field $u(x,t)$ which are connected through the constitutive equation for the material. We write the balance of momentum for each media and then enforce continuity conditions for stress and displacement at the interface of the two media to connect the solution for the two sides.
Let $x < 0$ denote \textit{Media} \textit{a} and $x > 0$ denote \textit{Media} \textit{b}. The balance of momentum is given by

\[
\begin{align*}
\nabla \cdot \sigma + \rho \mathbf{b} &= \rho \mathbf{a} \quad \text{for } x < 0, \\
\nabla \cdot \sigma + \rho \mathbf{b} &= \rho \mathbf{a} \quad \text{for } x > 0,
\end{align*}
\]  

(21)

where $\rho$ is the density, $\mathbf{b}$ is the body force and $\mathbf{a} = \ddot{\mathbf{u}}$ is the acceleration. The continuity in the displacement and in the normal traction are written by

\[
\begin{align*}
\mathbf{u}(-0,t) &= \mathbf{u}(+0,t), \\
\sigma(-0,t) \mathbf{n} &= \sigma(+0,t) \mathbf{n},
\end{align*}
\]  

(22)

where $\mathbf{n}$ is the normal to the surface.

We will now solve these equations for shear waves ignoring the body force. The only non-trivial balance equations then become

\[
\begin{align*}
\frac{\partial \sigma}{\partial x} &= \rho \frac{\partial^2 u}{\partial t^2} \quad \text{for } x < 0, \\
\frac{\partial \sigma}{\partial x} &= \rho \frac{\partial^2 u}{\partial t^2} \quad \text{for } x > 0,
\end{align*}
\]  

(23)

where $\sigma(x,t)$ is the shear stress and $u(x,t)$ is the transverse displacement. The only non-trivial continuity conditions for the shear wave become

\[
\begin{align*}
\mathbf{u}(-0,t) &= \mathbf{u}(+0,t), \\
\sigma(-0,t) &= \sigma(+0,t).
\end{align*}
\]  

(24)

We can separate the displacement in \textit{Media} \textit{a} into that of the incident wave $u_i(x,t)$ and the reflected wave $u_r(x,t)$, and the displacement in \textit{Media} \textit{b} is $u_t(x,t)$. We can then rewrite the above continuity equation for displacement as

\[
u_t(-0,t) + u_r(-0,t) = u_t(+0,t),
\]  

(25)
5.3.2 Attenuating Waves in a Linear Viscoelastic Media

We will now consider the problem of attenuating harmonic shear wave packages traveling in a single viscoelastic media and solve for the response and then connect the two media through the continuity equations shown above. For a single media we have the balance law

$$\frac{\partial \sigma(x,t)}{\partial x} = \rho \frac{\partial^2 u(x,t)}{\partial t^2}. \quad (26)$$

We assume harmonically attenuating plane shear waves given by the equation

$$u(x, t) = u_0 e^{-\alpha x} \cos \left[ \omega \left( t - \frac{x}{c} \right) \right], \quad (27)$$

where $u_0$ is the amplitude, $\alpha$ is the attenuation coefficient, $\omega$ is the circular frequency, and $c$ is the wave speed. Figure 79 shows a shear sinusoidal wave that has constant amplitude at each point in space, but attenuates as you move forward. Taking the second derivative of the displacement equation the acceleration is described by

$$\frac{\partial^2 u(x,t)}{\partial t^2} = -e^{-\alpha x} \omega^2 \cos \left[ \omega \left( t - \frac{x}{c} \right) \right]. \quad (28)$$

**Figure 79:** Infinitesimal shear sinusoidal attenuating wave in time (note: the wave package consists of many frequencies, not just one).
The material is assumed to be viscoelastic and is defined by a constitutive equation of form

\[ \sigma(t) = G(0)\gamma(t) + \int_0^t G'(t - \tau)\gamma(\tau)d\tau, \]  

(29)

where \( G(t) \) is the relaxation function having a form like that given in Figure 80, where \( G'(t) \) is its derivative, and \( \gamma(t) \) is the shear strain given in this problem as

\[ \gamma = \frac{\partial u}{\partial x}. \]  

(30)

![Diagram of shear relaxation function](image)

**Figure 80:** General shear relaxation function.

The shear strain can be calculated from the shear displacement by taking the derivative with respect to position to get

\[ \gamma(t) = \frac{\partial u(x,t)}{\partial x} = u_o e^{-ax} \left\{ - \alpha \cos \left( \frac{\omega (t - \frac{x}{c})}{c} \right) + \frac{\omega}{c} \sin \left( \frac{\omega (t - \frac{x}{c})}{c} \right) \right\}. \]  

(31)
Then we substitute the shear strain back into the constitutive equation to obtain the shear stress associated with the harmonically attenuating shear wave being considered. We then take its derivative to obtain

\[
\frac{\partial \sigma(x, t)}{\partial x} = u_0 e^{-\alpha x} \left( G(0) \left( \alpha^2 - \frac{\omega^2}{c^2} \right) \cos \left( \omega \left( t - \frac{x}{c} \right) \right) - \frac{2\alpha \omega}{c} G(0) \sin \left( \omega \left( t - \frac{x}{c} \right) \right) \right) + \left( \alpha^2 + \frac{\omega^2}{c^2} \right) \int_0^t G'(t - \tau) \cos \left( \omega \left( \tau - \frac{x}{c} \right) \right) d\tau - \frac{2\alpha \omega}{c} \int_0^t G'(t - \tau) \sin \left( \omega \left( \tau - \frac{x}{c} \right) \right) d\tau.
\]

Now we can substitute this and the acceleration equation into the balance law, and obtain the equation

\[
\left[ G(0) \left( \alpha^2 - \frac{\omega^2}{c^2} + \rho \omega^2 \right) \right] \cos \left( \omega \left( t - \frac{x}{c} \right) \right) + \left[ -2G(0) \frac{\alpha \omega}{c} \right] \sin \left( \omega \left( t - \frac{x}{c} \right) \right)
+ \left( \alpha^2 - \frac{\omega^2}{c^2} \right) \int_0^t G'(s) \cos \left( \omega \left( t - s - \frac{x}{c} \right) \right) ds
+ \left( -2 \frac{\alpha \omega}{c} \right) \int_0^t G'(s) \sin \left( \omega \left( t - s - \frac{x}{c} \right) \right) ds = 0,
\]

which must be satisfied for each point \( x \).

For times that are large compared to the relaxation time of the material, the integrals approach constant values and one can separate this equation into a sine and cosine term. Setting the coefficients equal to zero, results in the following

\[
\int_0^\infty G'(s) \sin(\omega s) ds = \frac{-2 \rho c^3(\omega) a(\omega) \omega^3}{[c^2(\omega) \alpha^2(\omega) + \omega^2]^2},
\]

\[
\int_0^\infty G'(s) \cos(\omega s) ds = -G(0) + \frac{-\rho c^2(\omega) \omega^2[c^2(\omega) \alpha^2(\omega) - \omega^2]}{[c^2(\omega) \alpha^2(\omega) + \omega^2]^2},
\]

where the wave speed \( c(\omega) \) and attenuation \( a(\omega) \) are functions of frequency \( \omega \). The
above equations can be inverted to give the wave speed and attenuation coefficients in terms of the relaxation function.

5.3.3 Reflection and Transmission of Waves at Interfaces

For the application we are studying, there are two materials in contact with each other as seen in Figure 81. The silica plate acts as the reference material and the soft tissue is what is being characterized. The silica is Media $a$ and the soft tissue is Media $b$. A plane sinusoidal wave package travels along the $y$ direction and is partially transmitted and partially reflected off of the interface between the silica and soft tissue.

Figure 81: Idealized reflected wave ultrasonic technique test setup.

For the application of a single frequency, we can write the incident, reflected, and transmitted waves for one frequency, and for the case of the wave package, using superposition, it simply becomes the summation over all frequencies.

\[ u_i(y,t) = e^{-\alpha_i y} \cos(\omega(t - \frac{y}{c_i})) \]  \hspace{1cm} (36)

\[ u_r(y,t) = A_r e^{-\alpha_r (d-y)} \cos(\omega(t - \frac{d-y}{c_r}) + \phi_r) \]  \hspace{1cm} (37)
where the location \( y = d \) is the interface between the two materials and \( i \) denotes incident, \( t \) denotes transmitted, and \( r \) denotes reflected.

We can use superposition to obtain the response to the wave packet from the results for a single frequency. For the single frequency the interface displacement continuity at \( y = d \) is written as

\[
u_i(d, t) + u_r(d, t) = u_t(d, t).
\] (39)

After substitution of the wave equations into the displacement continuity requirement, we get one equation from the coefficient of the sine and one from the coefficient of the cosine terms. They are

\[
e^{-\alpha d} \cos \left( \frac{\omega d}{c_i} \right) + A_r \cos(\phi_r) = A_t \cos(\phi_t),
\] (40)

\[
e^{-\alpha d} \sin \left( \frac{\omega d}{c_i} \right) + A_r \sin(\phi_r) = A_t \sin(\phi_t).
\] (41)

In order to write the stress continuity, we first must know the total shear strains in Media a and Media b. For Media a we must consider both the incident and reflected wave, and in Media b we only need to consider the transmitted wave. From the displacements we obtain the shear strains to be

\[
\gamma_i(y, t) + \gamma_r(y, t) = \alpha_i e^{-\alpha Vy} \cos \left[ \omega t - \frac{\alpha Vy}{c_i} \right] + \frac{\omega}{c_i} e^{-\alpha Vy} \sin \left[ \omega t - \frac{\alpha Vy}{c_i} \right] + A_r e^{-\alpha_i (d-y)} \alpha_i \cos \left[ \omega t - \frac{\alpha_i (d-y)}{c_i} + \phi_r \right] - A_r \left( \frac{\omega}{c_i} \right) e^{-\alpha_i (d-y)} \sin \left[ \omega t - \frac{\alpha_i (d-y)}{c_i} + \phi_r \right],
\] (42)
\[
\gamma_t(y, t) = A_t e^{-\alpha_t(y-d)} \left\{ -\alpha_t \cos \left( \frac{\omega \left( t - \frac{y-d}{c_t} \right) + \phi_t}{c_t} \right) + \frac{\omega}{c_t} \sin \left( \frac{\omega \left( t - \frac{y-d}{c_t} \right) + \phi_t}{c_t} \right) \right\}. \tag{43}
\]

If we now substitute these expressions for the strain into the expression for stress, we obtain the stress fields in Medias \textit{a} and \textit{b} as

\[
\sigma_a = G_i(0) \left\{ -\alpha_t e^{-\alpha_t y} \cos \left( \frac{\omega \left( t - \frac{y}{c_t} \right)}{c_i} \right) + e^{-\alpha_t y} \frac{\omega}{c_is} \sin \left( \frac{\omega \left( t - \frac{y}{c_t} \right)}{c_i} \right) + A_r e^{-\alpha_r(d-y)} \alpha_t \cos \left( \frac{\omega \left( t - \frac{d-y}{c_r} \right) + \phi_r}{c_r} \right) \right\} \tag{44}
\]

\[
= -A_r e^{-\alpha_r(d-y)} \frac{\omega}{c_i} \sin \left( \frac{\omega \left( t - \frac{d-y}{c_r} \right) + \phi_r}{c_r} \right) \]

\[
+ \int_0^t G_i(t - \tau) \left\{ -\alpha_t e^{-\alpha_t y} \cos \left( \frac{\omega \left( t - \frac{y}{c_t} \right)}{c_i} \right) + e^{-\alpha_t y} \frac{\omega}{c_is} \sin \left( \frac{\omega \left( t - \frac{y}{c_t} \right)}{c_i} \right) \right\} dt,
\]

\[
\sigma_b = G_i(0) A_t e^{-\alpha_t(y-d)} \left\{ -\alpha_t \cos \left( \frac{\omega \left( t - \frac{y-d}{c_t} \right)}{c_i} \right) + \frac{\omega}{c_t} \sin \left( \frac{\omega \left( t - \frac{y-d}{c_t} \right) + \phi_t}{c_t} \right) \right\} \tag{45}
\]

\[
+ \int_0^t G_i(t - \tau) A_t e^{-\alpha_t(y-d)} \left\{ -\alpha_t \cos \left( \frac{\omega \left( t - \frac{y-d}{c_t} \right) + \phi_t}{c_t} \right) + \frac{\omega}{c_t} \sin \left( \frac{\omega \left( t - \frac{y-d}{c_t} \right) + \phi_t}{c_t} \right) \right\} dt.
\]

At the interface, the stresses are equal, and by equating the above two equations at the interface we obtain one equation from the coefficient of the sine term and one equation from the coefficient of the cosine term. They are

\[
A_t \sqrt{(-\alpha_t)^2 + \left( \frac{\omega}{c_t} \right)^2} \left[ h_t(\omega) \cos(\phi_t + \eta) + f_t(\omega) \sin(\phi_t + \eta) \right] \tag{46}
\]

\[
= \sqrt{(-\alpha_t)^2 + \left( \frac{\omega}{c_t} \right)^2} \left\{ h_t(\omega) \left[ e^{-\alpha_t d} \cos \left( \frac{\omega d}{c_t} - A_r \cos(\phi_r + \theta) \right) \right.
\]

\[
\left. + f_t(\omega) \left[ e^{-\alpha_t d} \sin \left( \frac{\omega d}{c_t} - A_r \sin(\phi_r + \theta) \right) \right] \right\}.
\]

\[
A_t \sqrt{(-\alpha_t)^2 + \left( \frac{\omega}{c_t} \right)^2} \left[ h_t(\omega) \sin(\phi_t + \eta) + f_t(\omega) \cos(\phi_t + \eta) \right] \tag{47}
\]
\[ = \sqrt{(-\alpha t)^2 + \left(\frac{\omega}{c_f}\right)^2} \left\{ h_t(\omega) \left[ e^{-a_i d} \sin \left( \theta - \frac{\omega d}{c_i} \right) - A_r \sin (\phi_r + \theta) \right] + f_i(\omega) \left[ e^{-a_i d} \cos \left( \theta - \frac{\omega d}{c_i} \right) - A_r \cos (\phi_r + \theta) \right] \right\}, \]

where,

\[ f_i(\omega) = \int_0^\infty G_i'(s) \sin (\omega s) \, ds, \quad (48) \]

\[ h_t(\omega) = G_{ts}(0) + \int_0^\infty G_t'(s) \cos (\omega s) \, ds, \quad (49) \]

\[ f_i(\omega) = \int_0^\infty G_i'(s) \sin (\omega s) \, ds = -\frac{2\rho I c_f^2 \alpha_i \omega^3}{(c_f^2 \alpha_i^2 + \omega^2)^5}, \quad (50) \]

\[ h_t(\omega) = G_t(0) + \int_0^\infty G_t'(s) \cos (\omega s) \, ds = \frac{-\rho I c_f^2 \omega^2 (c_f^2 \alpha_i^2 - \omega^2)}{(c_f^2 \alpha_i^2 + \omega^2)^2}, \quad (51) \]

and,

\[ \cos(\theta) = \frac{\left(\frac{\omega}{c_i}\right)}{\sqrt{(-\alpha i)^2 + \left(\frac{\omega}{c_i}\right)^2}}, \quad (52) \]

\[ \sin(\theta) = \frac{-\alpha_i}{\sqrt{(-\alpha_i)^2 + \left(\frac{\omega}{c_i}\right)^2}}, \quad (53) \]

\[ \cos(\eta) = \frac{\left(\frac{\omega}{c_f}\right)}{\sqrt{(-\alpha_f)^2 + \left(\frac{\omega}{c_f}\right)^2}}, \quad (54) \]

\[ \sin(\eta) = \frac{-\alpha_f}{\sqrt{(-\alpha_f)^2 + \left(\frac{\omega}{c_f}\right)^2}}. \quad (55) \]

Let \( x_1 \) and \( x_2 \) be intermediary variables defined as

\[ x_1 = -\alpha_t h_t(\omega) - \frac{\omega}{c_f} f_t(\omega), \quad (56) \]

\[ x_2 = -\alpha_t f_t(\omega) - \frac{\omega}{c_f} h_t(\omega). \quad (57) \]

Upon reorganizing the four equations which include two for displacement and two for stress, we obtain from continuity, the equations
Solving the above four equations results in formulas for $x_1$ and $x_2$ given by

$$x_1 = \frac{-2\rho_t \omega^3 c_i A_r e^{-\alpha t d} \sin(\Phi_r \cdot \frac{\omega d}{c_i} - \rho_t \omega^2 c_i^2 \alpha_i (A_r^2 - e^{-2\alpha t d})}{e^{-2\alpha t d} + A_r^2 + 2A_r e^{-\alpha t d} \cos(\Phi_r \cdot \frac{\omega d}{c_i})}(c_i^2 \alpha_i^2 + \omega^2),$$

$$x_2 = \frac{-\rho_t \omega^3 c_i (A_r^2 - e^{-2\alpha t d}) + 2A_r e^{-\alpha t d} \rho_t \omega^2 c_i^2 \alpha_i \sin(\frac{\omega d}{c_i}) \Phi_r}{e^{-2\alpha t d} + A_r^2 + 2A_r e^{-\alpha t d} \cos(\Phi_r \cdot \frac{\omega d}{c_i})}(c_i^2 \alpha_i^2 + \omega^2).$$

Upon back substitution, the attenuation coefficient and wave speed in the soft tissue can be determined as

$$\alpha_t = \frac{\omega^2 \rho_t x_1}{x_1^2 + x_2^2},$$

$$c_t = \frac{x_1^2 + x_2^2}{\omega \rho_t x_2}.$$
5.3.4 Application to Experiments

Through the development of the theory in the above sections, we now have the equations we need to evaluate material parameters experimentally. The experiments are performed in two steps. We consider the experimental setup with no sample in order to evaluate the silica and then consider the setup with the silica and specimen to characterize the test material.

If we consider the case where we only have Media a (silica), which is the reference configuration, shear waves can’t be transmitted into air at the media-air interface. This configuration allows us to characterize Media a. Since the shear stress in air is zero, we obtain the following equations from the stress continuity condition (equation 22) at the interface (note, a (’) is used to denote the reference configuration)

\[(c_i^2 a_i^2 - \omega^2) A_r' \cos(\phi_r' + \theta) + 2c_i a_i \omega A_r' \sin(\phi_r' + \theta)\]

\[= (c_i^2 a_i^2 - \omega^2) e^{-\alpha_i d} \cos\left(\theta - \frac{\omega d}{c_i}\right) + 2c_i a_i \omega e^{-\alpha_i d} \sin\left(\theta - \frac{\omega d}{c_i}\right),\]  

(66)

\[(c_i^2 a_i^2 - \omega^2) A_r' \sin(\phi_r' + \theta) - 2c_i a_i \omega A_r' \cos(\phi_r' + \theta)\]

\[= (c_i^2 a_i^2 - \omega^2) e^{-\alpha_i d} \sin\left(\theta - \frac{\omega d}{c_i}\right) - 2c_i a_i \omega e^{-\alpha_i d} \cos\left(\theta - \frac{\omega d}{c_i}\right),\]  

(67)

If we solve these we obtain the amplitude and phase shift of the reflected wave in terms of the thickness of the silica, the wave speed, the attenuation coefficient, and the frequency of the signal. The relationships are

\[A_r' = e^{-\alpha_i d},\]  

(68)

\[\phi_r' = -\frac{\omega d}{c_i}.\]  

(69)
The transducer is located at the position $y = 0$ and not at the interface ($y = d$). We can use the above parameters in the harmonic wave equation to obtain the displacement of the reflected wave at the origin. By doing this for two different silica thicknesses, we obtain

$$u'_r(t) = e^{-2\alpha_i d_2} \cos\left(\frac{2\omega_o d_1}{c_l}\right),$$  \hspace{1cm} (70)

$$u''_r(t) = e^{-2\alpha_i d_2} \cos\left(\frac{2\omega_o d_2}{c_l}\right),$$  \hspace{1cm} (71)

where $\omega_o$ is the center frequency of the transducer, $u'_r$ and $u''_r$ are the reflected waves resulting from the distances $d_1$ and $d_2$ respectively. Now, integrating them over one quarter of the period of the cosine wave, we obtain

$$I[u'_r] = \int_T^{T+\frac{\pi}{2\omega_o}} e^{-2\alpha_i d_1} \cos(\omega_o t - \frac{2\omega_o d_1}{c_l}) \, dt = e^{-2\alpha_i d_1},$$  \hspace{1cm} (72)

$$I[u''_r] = \int_T^{T+\frac{\pi}{2\omega_o}} e^{-2\alpha_i d_2} \cos(\omega_o t - \frac{2\omega_o d_2}{c_l}) \, dt = e^{-2\alpha_i d_2},$$  \hspace{1cm} (73)

where $T$ is any time that a wave starts from zero. The attenuation for the silica using equations (72) and (73) can be written as

$$\alpha_i(\omega_o) = \frac{\ln\left(\frac{|I[u''_r]|}{|I[u'_r]|}\right)}{2(d_1 - d_2)}. \hspace{1cm} (74)$$
We note that wave packages are a composition of many frequencies so we can use a Fourier series to represent the signal in the time domain. Since this is a linear system we can use superposition to obtain the total response for all frequencies. The signal in the time domain, \( u(t) \), can be represented by the Fourier series

\[
 u(t) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T} \right),
\]  

(75)

where the coefficients are

\[
a_0 = \frac{1}{T} \int_{T/2}^{T} u(t) dt,
\]

(77)

\[
a_n = \frac{2}{T} \int_{T/2}^{T} u(t) \cos \frac{2n\pi t}{T} dt, \quad n = 1, 2, \ldots,
\]

(76)

\[
b_n = \frac{2}{T} \int_{T/2}^{T} u(t) \sin \frac{2n\pi t}{T} dt, \quad n = 1, 2, \ldots
\]

(78)

After some manipulation we obtain

\[
 u(t) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos \frac{2n\pi t}{T} + b_n \sin \frac{2n\pi t}{T} \right),
\]

\[
= a_0 + \sum_{n=1}^{\infty} \left[ \sqrt{a_n^2 + b_n^2} \cos \left( \frac{2n\pi t}{T} + \phi_n \right) \right],
\]

\[
= a_0 + \sum_{n=1}^{\infty} \left[ \sqrt{a_n^2 + b_n^2} \cos (n\omega_o t + \phi_n) \right],
\]

(80)

where,

\[
\omega_o = \frac{2\pi}{T}, \quad \cos(\phi_n) = \frac{a_n}{\sqrt{a_n^2 + b_n^2}}, \quad \sin(\phi_n) = \frac{-b_n}{\sqrt{a_n^2 + b_n^2}}.
\]

(79)
The fast Fourier transform (FFT) gives the coefficients for the Fourier series (equation (75)). We can use the magnitude of the frequency response based on the results of the FFT in place of the integrals calculated in equations (72) and (73). Using Fourier series and superposition due to the fact that it is linear, we can determine the material response for a given frequency range. Figure 82 shows a wave package and a corresponding magnitude plot in the frequency domain after the FFT was performed.

![Figure 82: Wave package in the time domain transformed using the FFT into the frequency domain.](image)

Now we can write the attenuation as a function of the FFT magnitude and equation (74) takes on the form

\[
\alpha_i(\omega) = \frac{\ln\left(\left\|F\left[u'\right]\right\|\right)}{2(d_1-d_2)}
\]

(81)

where \(F\) denotes the FFT.
For the case of the silica and soft tissue, we can perform a similar procedure. Using a silica plate of known thickness, we obtain the reference signal. Then the test material is placed on the top surface of the silica and the signal is recorded. As discussed in a later section, it is important that the ultrasonic wave be in the far field for measurements due to the variability of the amplitude in the near field. This means that for the silica used, we must use the second and third reflections for analysis. The second and third reflected wave packages received by the transducer at \( y = 0 \) are

\[
\begin{align*}
 u_{r2}(t) &= \int_{\omega_1}^{\omega_2} \{ A_r^2 B_1 u_{i,0} e^{-2a_i d} \cos \left[ \omega t - \frac{2\omega d}{c_i} + 2\phi_r + \theta_i \right] \} d\omega, \\
 u_{r2}(t) &= \int_{\omega_1}^{\omega_2} \{ A_r^3 B_1^2 u_{i,0} e^{-3a_i d} \cos \left[ \omega t - \frac{3\omega d}{c_i} + 3\phi_r + 2\theta_i \right] \} d\omega, \\
 u'_{r2}(t) &= \int_{\omega_1}^{\omega_2} \{ A_r^2 B_1 u_{i,0} e^{-2a_i d} \cos \left[ \omega t - \frac{2\omega d}{c_i} + 2\phi'_r + \theta_i \right] \} d\omega, \\
 u'_{r2}(t) &= \int_{\omega_1}^{\omega_2} \{ A_r^3 B_1^2 u_{i,0} e^{-3a_i d} \cos \left[ \omega t - \frac{3\omega d}{c_i} + 3\phi'_r + 2\theta_i \right] \} d\omega.
\end{align*}
\]

We can again use the FFT in order to calculate the attenuation and phase shift in the material. The equations are

\[
\begin{align*}
 A_r &= \left[ \left| \frac{F(u_{r2})}{F(u_{r2})} \right| \right] e^{-\alpha_i d}, \\
 \phi_r &= \arctan \left[ \frac{\text{Im} F(u_{r2})}{\text{Re} F(u_{r2})} \right] + \frac{\omega d}{c_i}.
\end{align*}
\]
Once we have the values for $A_r$ and $\phi_r$, we can substitute them back into the equations for $x_1$ and $x_2$ and then substitute $x_1$ and $x_2$ into the equations for the wave speed and attenuation of the sample.

Solving for the wave speed and attenuation in a sample material allows for the unknown material’s properties to be determined. If only the wave speed can be determined the wave modulus can be determined through

$$E = \rho v^2.$$  \hfill (88)

The wave modulus tends to be significantly higher than the equilibrium modulus derived from static tests. With both the wave speed and attenuation values the relaxation function for a linear viscoelastic material can be determined. The following section will describe the experimental technique and what the advantages are.
5.4 Experimental Methods

5.4.1 Introduction to Method

The technique as described in the theory section was developed by Madsen et. all [20] in the early 1980’s. They proved that from the comparison of a reflected ultrasonic wave from a reference (no sample) and one with a sample that the unknown test sample wave speed and attenuation could be determined. This technique provides an important advantage over traditional ultrasonic techniques since the material tested does not have to have a known or even uniform geometry and the ultrasonic wave does not have to pass through the material (just reflect from the contact surface). Any material can be tested as long as sufficient contact is made between the silica reference block and the sample.

Traditionally contact ultrasonic transducers are used to characterize stiff materials using time of flight measurements that require the wave to travel through the material a known distance. The problem in using this technique in characterizing soft materials is (a) the signal attenuates quickly (does not pass through the material) and (b) thickness measurements are not accurate. Since soft materials and tissues are highly attenuating, ultrasonic waves, especially shear waves will not pass through the material, which means that no measurements can be taken. In order to determine the time of flight (which is needed for the calculation of the modulus) a very accurate thickness measurement is needed. It is however, difficult to measure accurate sample thicknesses because it is hard to determine when contact between the measurement device and sample has been made. There are also challenges in cutting consistent samples with parallel surfaces, and performing the test without causing large deformations in the sample.
The technique developed by Madsen [20] compensates for these problems because the ultrasonic wave does not pass far into the test material before it is reflected. This means that samples with extremely high attenuation coefficients can be tested with no problems since the wave does not have to travel in the sample. The goal of the current work is to improve this technique and extend it to materials testing under load. The dynamic response of materials such as brain matter is important in the study of wave propagation inside of the skull. Results are not presented for materials under different loading conditions due to the extensive trouble shooting needed to get the testing method operational.
5.4.2 General Test Setup and Equipment

The general test setup is shown in Figure 83. The system consists of the sample, a fused silica block, ultrasonic transducer, pulser-receiver, high speed digitizer, and computer program to acquire the data. Several fused silica blocks were used in testing, including a 1.5 inch thick block and 1 inch thick block. The silica serves two purposes: first to create a planar surface for the sample to rest on, and secondly, to act as a delay and separate the reflected signals. The silica block with no sample also acts as the reference configuration. The ultrasonic transducers used were a 7.5 MHz longitudinal wave transducer and a 5 MHz shear wave transducer. The transducer was fixed to the fused silica block using tape; the optimum configuration would be to permanently fix the transducer to the silica, but it was not possible for these tests.

![Figure 83: Ultrasonic test setup schematic.](image)

The data acquisition system consists of a National Instruments PXI-1042Q chassis with a 2 channel, 100 MHz, 14 bit digitizer (oscilloscope) (NI PXI-5122) and a high speed eight channel DAQ unit (NI PXI-6133). The two channel high speed digitizer acts
as the oscilloscope and the high speed DAQ is used to measure the voltage output from a load frame so that the position and the load can be measured (this is only needed for tests under load). Figure 84 shows some images of the test setup. There could be many improvements to the current setup including a fixture for the silica and transducer to ensure constant and consistent contact by having the transducer permanently fixed to the silica. Having thermal isolation and vibration isolation would help improve the ability to obtain phase shift data, which in the current set of tests has not been possible to measure accurately as it is on the order of 0.004 radians.

Figure 84: Pictures of ultrasonic test setup. Reference configuration (top left), Sillyputty® after being pressed onto silica (bottom left), and PC being tested with weight on top to ensure sufficient contact (right).
5.3.3 Experimental setup and characterization

All of the tests for this thesis were performed without application of load in order to ensure that the experiments were working and accurate results were obtained, and to characterize the response without load before application of load. Future tests, especially on soft materials will be performed under different strains and strain rates. Before the tests can be performed under different loading conditions however, the sensitivity of the measurement must be known. If there is not enough accuracy, then small differences in the signal may not be captured. According to Yang [24] (who also used a 100 MHz oscilloscope), the phase shift caused by the sample material should be on the order of 0.004 radians (for Sillyputty® with longitudinal ultrasonic waves) or about 0.16 nanoseconds. This phase shift is extremely small and is very difficult to measure. As will be discussed in later sections, the phase shift for our setup could not be accurately measured and therefore, the attenuation coefficient for the sample material was not measured.

Some important notes about the setup are as follows:

1. The electronics including the pulser-receiver need a warm up time of approximately 30 minutes.
2. A reference signal should be acquired for every test performed
3. The pulser-receiver should be adjusted to minimize noise in the signal, but ensure reflections 1-3 are large enough to have analysis performed on them
4. The transducer should be fixed to the silica block so it does not move between the reference test and the sample test
5. Couplant should not be used between the sample and the fused silica unless sufficient contact cannot be made
There are several test parameters related to the ultrasonic wave packet that must be addressed. These are the location of the near and far field regions and the beam spreading angle. The ultrasonic signal goes through a series of maxima and minima and once it reaches the far field (distance greater than $N$), it smoothly decays to zero. Figure 85 shows a schematic of the wave amplitude as a function of distance [25].

**Figure 85:** Near vs. Far field ultrasonic waves [25].
The equation for the field distance \( N \) is given by

\[
N = \frac{D^2 f}{2c},
\]  

(89)

where,

\( N \ldots \) Near field distance,

\( D \ldots \) Element diameter,

\( f \ldots \) Element frequency,

\( c \ldots \) Material sound velocity.

The near field region was calculated for the 7.5 MHz longitudinal transducer and the 5 MHz shear transducer. The Element Frequency (center frequency) was determined by plotting the FFT magnitude and finding the maximum. The following near field distances were found:

**Table 10:** Near field distances for the 7.5 MHz longitudinal and 5 MHz shear transducers.

<table>
<thead>
<tr>
<th>Center Frequency</th>
<th>5 MHz, 1/2 in dia. shear</th>
<th>7.5 MHz, 3/8 in dia. longitudinal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Near Field Distance</td>
<td>4.600575 MHz</td>
<td>6.858122 MHz</td>
</tr>
<tr>
<td></td>
<td>98.46 mm</td>
<td>52.43 mm</td>
</tr>
</tbody>
</table>

Therefore, the near field is reached for both the shear and longitudinal transducers used as long as the second or later reflections are used and the silica block is at least one inch thick.
Another concern for the experimental results is the effect of beam spreading (seen in Figure 86). All ultrasonic beams spread as the distance the wave travels increases (in the far field only). When the beam comes in contact with any surface, part of the wave is reflected and part is transmitted (except for certain special cases where it can be totally transmitted or totally reflected). Other wave modes due to this surface interaction such as shear waves can also be produced. These reflections create unclear and complicated signals in which no data can be obtained. It is important that the wave only interacts with the silica-sample surface and not with the side walls of the silica. Therefore, the width of the beam must be known. The beam spreading is a function of the transducer (element) frequency, transducer diameter and material sound velocity (silica in this case) given by

\[ \alpha = 2 \sin^{-1}\left(\frac{0.514c}{fD}\right) \]

where

\[ f \ldots \text{Element frequency}, \]

\[ D \ldots \text{Element diameter}, \]

\[ c \ldots \text{Material sound velocity}. \]

As it turns out, the beam spreading is relatively insignificant for the setup that is being used. The beam spreading angle, alpha was 3.5 degrees and 7.5 degrees for the shear and longitudinal transducers respectively. Table 11 also shows the wave diameter for the third and fourth reflection at the front wall (location of the transducer). This table shows that the sample diameter must be at least 13.08 mm. If this diameter criteria is not
met part of the wave will be reflected from a region where there is no sample, and only air. Therefore the results will not accurately characterize the sample material. It is important to understand the limitations of the system so that proper tests can be performed. If the material to be tested is very small, and does not meet the diameter criteria, the transducer selected should be of higher frequency. By increasing the frequency of the transducer, the near field length will increase so a thicker silica block should be used.

**Table 11:** Beam width for the fourth reflection at the transducer interface for both the 7.5 MHz longitudinal and 5 MHz shear transducers.

<table>
<thead>
<tr>
<th></th>
<th>5MHz, 1/2 in shear</th>
<th>7.5MHz, 3/8 in longitudinal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Center Frequency</td>
<td>4.60 MHz</td>
<td>6.86 MHz</td>
</tr>
<tr>
<td>Alpha</td>
<td>3.50 Degrees</td>
<td>4.89 Degrees</td>
</tr>
<tr>
<td>Wave Diameter (3\textsuperscript{rd} reflection)</td>
<td>9.35 mm</td>
<td>13.08 mm</td>
</tr>
<tr>
<td>Wave Diameter (4\textsuperscript{th} reflection)</td>
<td>12.46 mm</td>
<td>17.44 mm</td>
</tr>
</tbody>
</table>

**Figure 86:** Beam spreading as a function of near field distance [25].
5.5 Ultrasonic Test Results

5.5.1 Experimental Results for PDMS

PDMS (polydimethylsiloxane) was selected for testing due to accessibility of samples and the ability to compare test results to literature values. PDMS is a silicone rubber that is elastic in nature. In testing this material, no couplant was needed between the silica block and the PDMS. A total of 50 tests were performed to obtain a statistically significant sample set. In order to satisfy the needed inputs for the model, the density of PDMS had to be determined. The density was measured by weighing the sample in air and then in water at a known temperature. If \( A \) is the weight of the sample in air and \( B \) is the weight of the sample in water and \( \rho_o \) is the density of water, for the density of the sample is given by

\[
\rho = \frac{A}{A-B} \rho_o. \tag{91}
\]

Table 12 provides the results of the density measurements. The longitudinal wave speed measured with a pulse echo technique on the same PDMS used for the reflected wave tests was 1001 m/s. The literature results for PDMS properties are a longitudinal wave speed of 1020 +/- 20 m/s at 1 MHz and a density of 1045 kg/m\(^3\). Therefore the wave modulus is approximately 1.1 GPa, which is approximately \(10^4\) higher than the equilibrium modulus [26].
Table 12: PDMS density measurements and results

<table>
<thead>
<tr>
<th>Test</th>
<th>A (grams)</th>
<th>B (grams)</th>
<th>Density (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.0024</td>
<td>0.207</td>
<td>1.033191</td>
</tr>
<tr>
<td>2</td>
<td>6.0023</td>
<td>0.2059</td>
<td>1.032995</td>
</tr>
<tr>
<td>3</td>
<td>5.9971</td>
<td>0.2078</td>
<td>1.033366</td>
</tr>
<tr>
<td>4</td>
<td>5.9977</td>
<td>0.2068</td>
<td>1.033184</td>
</tr>
<tr>
<td>5</td>
<td>5.9967</td>
<td>0.2068</td>
<td>1.03319</td>
</tr>
<tr>
<td>6</td>
<td>5.9989</td>
<td>0.2068</td>
<td>1.033177</td>
</tr>
<tr>
<td>7</td>
<td>5.9978</td>
<td>0.2075</td>
<td>1.033308</td>
</tr>
<tr>
<td>8</td>
<td>5.9982</td>
<td>0.2081</td>
<td>1.033413</td>
</tr>
<tr>
<td>9</td>
<td>5.9981</td>
<td>0.2083</td>
<td>1.033449</td>
</tr>
<tr>
<td>10</td>
<td>5.9979</td>
<td>0.2081</td>
<td>1.033415</td>
</tr>
<tr>
<td>11</td>
<td>5.9982</td>
<td>0.2081</td>
<td>1.033413</td>
</tr>
<tr>
<td>12</td>
<td>5.9987</td>
<td>0.2082</td>
<td>1.033428</td>
</tr>
<tr>
<td>13</td>
<td>5.9985</td>
<td>0.2085</td>
<td>1.033482</td>
</tr>
</tbody>
</table>

H2O (density - 23C) : 0.99756 (g/cm$^3$)

Average (g/cm$^3$) 1.033309
Std Dev 0.000147
Error 0.014%

Figure 87 shows the longitudinal wave speed in PDMS as calculated by the reflected wave method. The wave speed for each data point is averaged from 6.35 to 7.35 MHz. This range was selected because the center frequency is 6.85 MHz so a 1 MHz band was selected and averaged to determine the wave speed. The average wave speed, shown on the figure as the red line, is 977.4 m/s with a standard deviation of 29.5 m/s. This equates to an error of slightly over 3% based on the standard deviation. This value is approximately 2.4% less than the measured value via the pulse echo technique and 4% less than the literature value. The wave modulus calculated from the 50 data points is 0.988 GPa with an error of slightly more than 6%. This is approximately 10%
less than the reported wave modulus and is due again to the lower wave speed and slightly smaller density.

The tests were performed over four days which shows the repeatability of the measurement. The system is quite robust and it is simple to perform several experiments in a short amount of time in order to obtain statistically significant results. This method quite accurately determines the wave speed and elastic modulus values for PDMS when compared to the given literature values.

![Figure 87: PDMS longitudinal wave speed as determined from ultrasonic reflection method](image)
5.5.2 Experimental Results for Polycarbonate

Tests were performed on polycarbonate because a great deal of measurements have already been conducted on it over the past several years by our group. The wave speed in uncompressed PC is approximately 2225 m/s (obtained by using a pulse-echo technique) [27]. The densities were once again determined using the same technique as described for PDMS and the results are shown in Table 13. The average density for the PC plate was 1.1918 g/cm$^3$.

Table 13: Polycarbonate density measurements and results.

<table>
<thead>
<tr>
<th>Test</th>
<th>A (grams)</th>
<th>B (grams)</th>
<th>Density (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.69190</td>
<td>0.27939</td>
<td>1.19488</td>
</tr>
<tr>
<td>2</td>
<td>1.69115</td>
<td>0.27745</td>
<td>1.19335</td>
</tr>
<tr>
<td>3</td>
<td>1.69182</td>
<td>0.27739</td>
<td>1.19321</td>
</tr>
<tr>
<td>4</td>
<td>1.69176</td>
<td>0.27391</td>
<td>1.19029</td>
</tr>
<tr>
<td>5</td>
<td>1.69182</td>
<td>0.27487</td>
<td>1.19108</td>
</tr>
<tr>
<td>6</td>
<td>1.69189</td>
<td>0.27369</td>
<td>1.19008</td>
</tr>
<tr>
<td>7</td>
<td>1.69194</td>
<td>0.27391</td>
<td>1.19026</td>
</tr>
<tr>
<td>8</td>
<td>1.69171</td>
<td>0.27443</td>
<td>1.19073</td>
</tr>
<tr>
<td>9</td>
<td>1.69172</td>
<td>0.27750</td>
<td>1.19331</td>
</tr>
<tr>
<td>10</td>
<td>1.69149</td>
<td>0.27493</td>
<td>1.19118</td>
</tr>
</tbody>
</table>

H2O (density – 23°C) : 0.99756 (g/cm$^3$)

<table>
<thead>
<tr>
<th>Average (g/cm$^3$)</th>
<th>1.191838</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std Dev</td>
<td>0.001694</td>
</tr>
<tr>
<td>Error</td>
<td>0.142%</td>
</tr>
</tbody>
</table>
A total of 25 measurements were performed on the polycarbonate over three days. The reflected wave ultrasonic method provided a wave speed of 2219 m/s with a standard deviation of 47 m/s which gives an error of 2.1%. The difference between the reflected wave and pulse-echo technique is 0.27%. The standard deviation for the pulse-echo technique is less however at 0.47% [27].

Figure 88: Polycarbonate longitudinal wave speed as determined from ultrasonic reflection method.
5.5.3 Experimental Results for Sillyputty®

Sillyputty® was used in testing the reflected wave ultrasonic method because it is a viscous material that creates near perfect contact with the silica block. This excellent contact allows shear wave tests to be performed. Again, the density of the Sillyputty® was determined using the aforementioned technique and is presented in Table 14.

Table 14: Sillyputty® density measurements and results.

<table>
<thead>
<tr>
<th>Test</th>
<th>A (grams)</th>
<th>B (grams)</th>
<th>Density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8967</td>
<td>0.841</td>
<td>1.4058</td>
</tr>
<tr>
<td>2</td>
<td>2.8967</td>
<td>0.8404</td>
<td>1.40539</td>
</tr>
<tr>
<td>3</td>
<td>2.8968</td>
<td>0.8412</td>
<td>1.40592</td>
</tr>
<tr>
<td>4</td>
<td>2.8965</td>
<td>0.8414</td>
<td>1.40612</td>
</tr>
<tr>
<td>5</td>
<td>2.8969</td>
<td>0.8411</td>
<td>1.40583</td>
</tr>
<tr>
<td>6</td>
<td>2.897</td>
<td>0.8402</td>
<td>1.4052</td>
</tr>
<tr>
<td>7</td>
<td>2.8974</td>
<td>0.8406</td>
<td>1.40539</td>
</tr>
<tr>
<td>8</td>
<td>2.8932</td>
<td>0.8391</td>
<td>1.4052</td>
</tr>
</tbody>
</table>

H20 (density – 22.5C) : 0.99766 (g/cm³)

<table>
<thead>
<tr>
<th></th>
<th>Average (g/cm³)</th>
<th>Std Dev</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.40561</td>
<td>0.00033</td>
<td>0.0235%</td>
</tr>
</tbody>
</table>

Yang [24] conducted tests on Sillyputty® with both longitudinal and shear wave transducers using the same reflected wave technique as presented here. The values obtained at 23 C were 1061 ± 8.6 m/s for longitudinal waves (6.4 MHz) and 85.1 ± 13.1 m/s for shear waves (4.1 MHz). At 23.5 C, the longitudinal wave speed was 1035 ± 28.6 m/s and the shear wave speed was 78.3 ± 6.1 m/s [24].
The results obtained for the shear wave speed was 68.1 ± 7.82 m/s at 23 C which is an error of ±11.5%. Comparing these values to that of Yang’s [24] at 23 C, ours is 20% lower, but the error in the measurement is approximately 4% less. The shear transducer has a center frequency of 4.6 MHz so data was averaged between 4.1 and 5.1 MHz to obtain the results shown in Figure 89. The difference between these measurements and those of Yang [24] could be the Sillyputty® used. For the current tests a Sillyputty® purchased from a retail store was used. The exact makeup of it is not known.

![Figure 89: Sillyputty® shear wave speed as determined from ultrasonic reflection method.](image-url)
The results obtained for the longitudinal wave speed in silly putty averaged between 6.35 and 7.35 MHz is 1026 ± 18.2 m/s. This is an error of 1.78% and differs from the results obtained by Yang at 23 C by about 3.3%. The results for the 20 tests performed are shown in Figure 90 with the red line being the average longitudinal wave speed.

**Figure 90:** Sillyputty® longitudinal wave speed as determined from ultrasonic reflection method
5.4.4 Experimental Results for Bovine Liver

The reflected wave ultrasonic technique presented was originally developed by Madsen et al. in order to characterize tissues and tissue simulants. In order to show that the current system has the capability to perform accurate measurements on tissue, bovine liver was tested using both longitudinal and shear waves. The results aligned nicely with measurements made by Madsen [20]. The density of liver as reported Madsen [20] was 1070 kg/m$^3$. Independent density measurements were not performed on the samples used in testing. Madsen only performed shear wave measurements on his samples. Pereira [28] performed longitudinal measurements on bovine liver and determined the wave speed to be 1630 m/s and Hayashi [27] measured the wave speed to be $1538 \pm 29$ m/s.

The results obtained correspond quite well for both the shear and longitudinal measurements. For shear wave measurements averaged from 4.1-5.1 MHz the average wave speed was $16.6 \pm 3.7$ m/s. When the results are averaged from 10 points around 5 MHz, the average is $23.4 \pm 4.3$ m/s. Results obtained by Madsen for 5 MHz shear waves were approximately $22 \pm 3$ m/s. Figure 91 shows the results for three reflected shear wave tests performed on bovine liver. Two tests were performed previous to the three shown, however their wave speeds were around 3000 m/s and 100 m/s. The reason for this is that the liver still had fluid on it during testing and therefore the liver tissue was not being tested. Figure 92 shows the results for the current tests and the good correlation to Madsen’s results.
Figure 91: Shear wave speed measurements for bovine liver.

Figure 92: Shear wave speed results for bovine liver compared to Madsen's measurements.
The reflected longitudinal wave test results obtained for the current work also align well with values obtained by both Pereira [28] and Hayashi [29]. The average value obtained was 1609.9 ± 42.9 m/s. This deviation gives an error of 2.67%. This value is about 1 percent less than Pereira’s who used a ultrasonic back scatter method. Note, that the first test value was not included in the average and deviation due to its extremely large deviation.

**Figure 93**: Longitudinal wave speed for bovine liver (6.35-7.35 MHz average).

The results for shear and longitudinal ultrasonic waves match well to reported literature values. It was also shown that the reflected wave ultrasonic technique is capable of obtained accurate measurements for biological specimens. For these experiments unlike the cited ones, frozen (not fresh) bovine liver was used due to availability and for demonstration of the technique.
5.5 Conclusions and Discussion

Table 15 provides the test results for the three materials tested and Figure 94 plots the results and error bars for the three materials. The results for the materials tested align well with literature values. The errors are rather small and for soft materials, measurements are obtained that would not otherwise be possible. The reflected wave ultrasonic method offers a robust measurement for the wave speed. Unfortunately, results for the attenuation were not able to be obtained due to noisy phase signals. To increase the ability of the system to measure these small phase shifts (on the order 0.003 radians [24]) the system should be isolated thermally and from vibrations which could both cause errors larger than the phase shift trying to be measured. Yang’s method of permanently fixing the dual element transducer (can produce both shear and longitudinal ultrasonic waves) to the silica eliminates errors from transducer placement.

**Table 15**: Compilation of test results for PC, PDMS, and Sillyputty®.

<table>
<thead>
<tr>
<th>Material</th>
<th>Number of Tests</th>
<th>Wave Type</th>
<th>Transducer Frequency (MHz)</th>
<th>Center Frequency (MHz)</th>
<th>Wave Speed (m/s)</th>
<th>Std Dev (m/s)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sillyputty®</td>
<td>20</td>
<td>Longitudinal</td>
<td>7.5</td>
<td>6.85</td>
<td>1025.9</td>
<td>18.2</td>
<td>1.8%</td>
</tr>
<tr>
<td>Sillyputty®</td>
<td>34</td>
<td>Shear</td>
<td>5</td>
<td>4.6</td>
<td>68.1</td>
<td>7.8</td>
<td>11.5%</td>
</tr>
<tr>
<td>Polycarbonate</td>
<td>25</td>
<td>Longitudinal</td>
<td>7.5</td>
<td>6.85</td>
<td>2219</td>
<td>47.1</td>
<td>2.1%</td>
</tr>
<tr>
<td>PDMS</td>
<td>50</td>
<td>Longitudinal</td>
<td>7.5</td>
<td>6.85</td>
<td>977.4</td>
<td>29.5</td>
<td>3.0%</td>
</tr>
<tr>
<td>Bovine Liver</td>
<td>10</td>
<td>Longitudinal</td>
<td>7.5</td>
<td>6.85</td>
<td>1609.9</td>
<td>42.9</td>
<td>2.7%</td>
</tr>
<tr>
<td>Bovine Liver</td>
<td>3</td>
<td>Shear</td>
<td>5</td>
<td>4.6</td>
<td>23.4</td>
<td>4.3</td>
<td>18.4%</td>
</tr>
</tbody>
</table>
Reflected wave ultrasonics is a promising new technique that will allow for the characterization of biological tissues and soft samples in the MHz range. Traditional measurement techniques are not able to obtain results for these types of materials. Results from this method will help to create more accurate models simulating shock wave impact to the human head and will lead to the ability to select more realistic tissue simulants for use in experiments.

**Figure 94:** Wave speeds for tested materials.
Chapter 6: Concluding Remarks

The work in this thesis has explored three experimental techniques that give unique capabilities to characterize materials that with traditional techniques would not be possible. ARAMIS, a stereo optical displacement and strain measuring system uses digital image correlation to resolve displacements down to $1/30000$ the field of view and can measure surface strains from 0.05% up to several hundred percent. Both the low speed and high speed ARAMIS systems offer robust measurements and allow the user to have the unique ability to visualize the full field deformation. The high speed ARAMIS system allows for dynamic characterization of materials. The reflected wave ultrasonic method presented in Chapter 5 allows dynamic characterization of non-uniform, soft materials.

The low speed ARAMIS system was used to characterize the quasi-static compression tests of mouse tibia and the results were used to obtain material properties. Four views of the same test were stitched together to create a full 360 degree view of the specimen. From the 360 degree view, cross sectional information including area and area moment of inertia values were calculated. Strain measurements from user defined points on the cross sectional surface and elastic beam theory were used to calculate the elastic modulus for all three of the genotypes tested, high bone mass (HBM), wild type (WT), and knock out (KX). The HBM mouse with the mutant gene that causes bones to be larger and stronger exhibited the largest elastic modulus and the WT (non-transgenic) bone exhibited properties between the HBM and KX bones. The method proposed allows for a unique analysis using only information gathered with the ARAMIS system.
With refinement of the testing procedure more accurate and precise results can be obtained. This method provides much more information compared to current methods used to characterize mouse bone properties. Hopefully this method will help researchers gain knowledge about bone properties and how different bone diseases such as osteoporosis affect them.

Tests were performed with the high speed ARAMIS system to characterize the 9-inch shock tube in UNL’s Trauma Mechanics Shockwave Facility. The deflection and surface strains of glass and polycarbonate windows mounted in the shock tube were measured to determine the lens effect created by the window’s curvature and the large errors resulting in the ARAMIS measurements. Next, through the tube window tests that studied the errors caused by the shock front light distortion were analyzed. For shock waves with moderately high overpressures (~30 psi) the shock front distortion caused errors on the order of 2% strain, which are very large. There is not a good way to remove these errors from the measurement, and therefore tests using ARAMIS should be performed at overpressures where the shock front is not visible. The high speed system was also important in performing experiments on samples subjected to shock loading in order to verify results from modeling. A thick and thin polycarbonate cylinder was subjected to overpressures of approximately 20 psi and the strains near the leading edge of the cylinders where measured and presented. Finally, ARAMIS results from experiments subjecting the RED Head to shock waves outside of the tube were studied. It was determined that for testing the PDMS skin of the model head had to be fixed to the skull and if it wasn’t delamination occurred, and therefore the experiment did not accurately represent the head. This high speed ARAMIS system is extremely useful in
performing high speed testing of materials but its limitations must be understood. It can provide a unique ability to perform experiments that would not otherwise be possible.

The third and final experimental method presented was a reflected wave ultrasonic technique that allows for the characterization of soft materials with non-uniform geometries. Three materials were analyzed and the results for wave speed in the test material were compared to literature values. In all of the tests, the wave speeds compared well to reported values. Attenuation values were not measured due to the extremely small phase shifts between the reference configuration and the test configuration. By isolating the system thermally and from vibrations, and permanently fixing the transducer to the silica block via epoxy, it would greatly increase the likelihood of being able to accurately measure the phase shift and therefore attenuation values for the test material.
Appendix A: Ultrasonic evaluation program

Main.m

%% Program Description
% This program is used to analyze ultrasonic waveform data. It is
% performed in three steps: signal modification, FFT analysis and
% mathematical model computation. The outputs of the program are wave
% speed and attenuation coefficients which can be used to find the
% viscoelastic relaxation function of the test material. The test setup
% is comprised of a fused silica block with an ultrasonic transducer
% attached to the bottom surface. This serves as the reference
% configuration. The sample is placed on top of the silica block.
% Ultrasonic signals are obtained for both reference and test
% configurations and based on the analysis of both signals, the
% desired parameters are obtained. A Olympus pulser-receiver is used
% with the ultrasonic transducer and a National Instruments High Speed
% Digitizer and LabView program to record the data

% This code was programmed by Jonathan Hein, MS student in the department
% of Engineering Mechanics, University of Nebraska-Lincoln, revised
% February 2011

% Throughout the code, notations with 'i' stand for incident wave (silica)
% and 't' stands for the transmitted wave (sample). The notation
% test (which is the transmitted/sample material) and ref (which is the
% incident/reference block) is also used.

%% Set the values for constants in the analysis
sampling_freq=100000000;  % Sampling freq = 100MHz
d=25.55;               % 1 inch silica thickness
d=38.1;                 % 1.5 inch silica thickness
d=d/1000;               % Thickness of silica (mm) converted to (m)
alpha_il=0;             % Attenuation of silica (nepers/m)
c_il=3768.7;            % Shear wave speed in silica (m/s)
c_il=5935;              % Longitudinal wave speed in silica (m/s)
rho_i=2200.6;           % Silica density (kg/m^3)

% Test material densities (kg/m^3)
rho_t=1033;             % PDMS
rho_t=1403;             % Silly Putty
rho_t=1192;             % PC

lower_freq=1;           % Lower frequency point to return data
upper_freq=15;          % Upper frequency point to return data
n1=10;                  % Number of files to analyze

%% Set the parameters to be used when searching for the reflections in
%% the test signals
min_amplitude=125;      % This is the amplitude criteria
A1=800;                 % Location of Reflection 1 (in # of points)
A2=1700;                % Location of Reflection 2
A3=2560;                % Location of Reflection 3
A4=3400;                % Location of Reflection 4
%% Initialize the program by displaying what it does and how to name the files to analyze
disp('This program will analyze and solve for the wave speed and attenuation');
disp('coefficients for an unknown material based on a reference signal and a test signal.');
disp('Place the test files with the reference having file names:');
disp('(0.txt, 1.txt, etc) and the test files having file names:');
disp('(0-1.txt, 1-1.txt, etc) in the current directory folder.');
disp('');

%% File name and sheet for data export
% If exporting data to an excel file, remove '%' from the next three lines. This is where the name and sheet are specified for the xlsx document.
file_name=input('Enter the file name to save the output data in (in single quotes): ');
disp('');
Sheet=input('Enter the material tested, test number and reflection number (i.e. AL_Test2_Ref1): ');
%Display to the user that the files have been imported and variables created
disp('Files successfully created:');
disp('==============================================================================');

%% Import reference data
% Set the file name and extension for the reference data.
% A string is created by concatenating the file number, 0, 1, 2,...,n1 and the file extension, '.txt'. This is then passed to the function 'Import Reference Data' which does just that. This loop iterates n1 times, where n1 is the user input of how many files to analyze
for i=1:n1
    name='';
    iter=i-1;
    name_iter=int2str(iter);
    extension='.txt';
    fileToRead=cat(2,name,name_iter,extension);

    % Call Function to Import Reference Data
    % 'fileToRead' and 'name_iter' are the inputs and 'ref_opened' and 'ref_signal' are the outputs
    [ref_opened, ref_signal]=ImportRefData(fileToRead,name_iter);

    % Display the reference file name to the user, showing that the file was successfully imported
    disp(ref_opened);

    % Create an array, 'ref_wave' by evaluating the imported signal and then adding it to 'ref_array' which is an array that contains all of the ref data
    [ref_wave]=eval(ref_signal);
    ref_array(:,i)=ref_wave(:,2);
end

disp('');
disp('******************************************************************');
disp('');

%% Import test data
A string is created by concatinating the file number, 0, 1, 2, etc 'I'
(indicating that it is the test and not the reference) and the file
extension, '.txt'. This is then passed to the function 'Import
Test Data' which does just that. This loop iterates n1 times,
where n1 is the user input of how many files to analyze
for i=1:n1
    name='';
    iter=i-1;
    name_iter=int2str(iter);
    extension='-1.txt';
    fileToRead=cat(2,name_iter,name,extension);

    Call Function to Import Test Data
    'fileToRead' and 'name_iter' are the inputs and 'test_opened' and
    'test_signal' are the outputs
    [test_opened, test_signal]=ImportTestData(fileToRead,name_iter);

    Display the test file name to the user, showing that the file was
    successfully imported
disp(test_opened);

    Create an array, 'test_wave' by evaluating the imported signal then
    adding it to 'test_array' which is an array that contains all of the
    test data
    [test_wave]=eval(test_signal);
    test_array(:,i)=test_wave(:,2);
end

Compute the Average Signals
The reference and test signals are averaged together to create one
variable for each. These variables are not used in the analysis as
each signal for both the reference and test are analyzed so that
information about the error can be obtained
ref_ave=mean(ref_array'); %Compute Ref Average
ref_ave=ref_ave'; %Transpose the array (so its vertical)
test_ave=mean(test_array'); %Compute Test Average
test_ave=test_ave'; %Transpose the array (so its vertical)

Determine the size of the length of the array (this value is used
when determining the reference locations
n=size(ref_array);

Determine the reflection position
The function will return four reflections. Normally only the second
and third reflection are needed, but to allow for flexibility the
first and fourth are provided as well. The location of each
reflection must be provided (to ensure the same reflection is found).
AA is a dummy variable which passes the reflection location to the
function 'Peak'. x is also a dummy variable to import into the
function 'Peak' and n is the size of the averaged waveform.

The function 'Peak' is called; y is a dummy variable containing the the
reference array, n is the length of the reference array, AA is the
reflection location and min_amp is the search criteria variable.
'peak_amp' is returned which includes the desired reflection and is 100
% pts long and 'peak_shifted' which is the point where 'peak_amp' starts in
% 'ref_array'. This is performed four times (to find four reflections
% with AA being set to the location of the desired peak, i.e.

% Set the dummy variable equal to the reference array
y=ref_array;
AA=A1;
    [peak_amp, peak_shifted]=Peak(y, n, AA, min_amp);
    ref_reflection1=peak_amp;
    ref_peak_start1=peak_shifted;
AA=A2;
    [peak_amp, peak_shifted]=Peak(y, n, AA, min_amp);
    ref_reflection2=peak_amp;
    ref_peak_start2=peak_shifted;
AA=A3;
    [peak_amp, peak_shifted]=Peak(y, n, AA, min_amp);
    ref_reflection3=peak_amp;
    ref_peak_start3=peak_shifted;
AA=A4;
    [peak_amp, peak_shifted]=Peak(y, n, AA, min_amp);
    ref_reflection4=peak_amp;
    ref_peak_start4=peak_shifted;

% The function 'Peak' is called; y is a dummy variable containing the the
% test array, n is the length of the test array, AA is the reflection
% location and min_amp is the search criteria variable. 'peak_amp' is
% returned which includes the desired reflection and is 100 pts long
% and 'peak_shifted' which is the point where 'peak_amp' starts in
% 'test_array'. This is performed four times (to find four reflections
% with AA being set to the location of the desired peak, i.e.

% Set the dummy variable equal to the test array
y=test_array;
AA=A1;
    [peak_amp, peak_shifted]=Peak(y, n, AA, min_amp);
    test_reflection1=peak_amp;
    test_peak_start1=peak_shifted;
AA=A2;
    [peak_amp, peak_shifted]=Peak(y, n, AA, min_amp);
    test_reflection2=peak_amp;
    test_peak_start2=peak_shifted;
AA=A3;
    [peak_amp, peak_shifted]=Peak(y, n, AA, min_amp);
    test_reflection3=peak_amp;
    test_peak_start3=peak_shifted;
AA=A4;
    [peak_amp, peak_shifted]=Peak(y, n, AA, min_amp);
    test_reflection4=peak_amp;
    test_peak_start4=peak_shifted;

%% Assemble the reference arrays
% Four arrays are created, one for each reflection. Since the point
% location where the original signal was extracted from, an array of
% zeros is created that is one less than the reflection location. Then
% another array of zeros is created that is the total length of the
% signal minus the length of the first array of zeros and the 100
% points from the reflection plus one. These three arrays are then
% concatenated into one array. Now the reflection signal is the same
% length as the original waveform and it is at the exact same location
% in time.

first=zeros(ref_peak_start1-1,n1);
last=zeros((n-100-ref_peak_start1+1),n1);
final=zeros(10000,n1);
REFERENCE1=vertcat(first, ref_reflection1, last, final);

first=zeros(ref_peak_start2-1,n1);
last=zeros((n-100-ref_peak_start2+1),n1);
REFERENCE2=vertcat(first, ref_reflection2, last, final);

first=zeros(ref_peak_start3-1,n1);
last=zeros((n-100-ref_peak_start3+1),n1);
REFERENCE3=vertcat(first, ref_reflection3, last, final);

first=zeros(ref_peak_start4-1,n1);
last=zeros((n-100-ref_peak_start4+1),n1);
REFERENCE4=vertcat(first, ref_reflection4, last, final);

%% Assemble the test arrays
% Four arrays are created, one for each reflection. Since the point
% location where the original signal was extracted from, an array of
% zeros is created that is one less than the reflection location. Then
% another array of zeros is created that is the total length of the
% signal minus the length of the first array of zeros and the 100
% points from the reflection plus one. These three arrays are then
% concatenated into one array. Now the reflection signal is the same
% length as the original waveform and it is at the exact same location
% in time.

first=zeros(test_peak_start1-1,n1);
last=zeros((n-100-test_peak_start1+1),n1);
TEST1=vertcat(first, test_reflection1, last, final);

first=zeros(test_peak_start2-1,n1);
last=zeros((n-100-test_peak_start2+1),n1);
TEST2=vertcat(first, test_reflection2, last, final);

first=zeros(test_peak_start3-1,n1);
last=zeros((n-100-test_peak_start3+1),n1);
TEST3=vertcat(first, test_reflection3, last, final);

first=zeros(test_peak_start4-1,n1);
last=zeros((n-100-test_peak_start4+1),n1);
TEST4=vertcat(first, test_reflection4, last, final);

n=n+10000;

%% Perform FFT on reference signal
% The FFT is performed by passing the number of points in the
% in the waveform signal, and the newly created reflection arrays. The
% function is called four times, once for each reflection array.
% This user define function returns the real and complex numbers for each
% reflection array, the frequency array. The outputs for each reflection
% are then formed into one array that should be half the length of the
% original wave form (since the FFT repeats only the first half of it is
% kept) by four times the number of files analyzed

y=REFERENCE1;
[FT FREQUENCY]=FFT_Analysis(n,y);
REF_FFT_FULL1=FT;
REF_FFT_FULL1=FT(1:(n/2),:);

y=REFERENCE2;
[FT FREQUENCY]=FFT_Analysis(n,y);
REF_FFT_FULL2=FT;
REF_FFT_FULL2=FT(1:(n/2),:);

y=REFERENCE3;
[FT FREQUENCY]=FFT_Analysis(n,y);
REF_FFT_FULL3=FT;
REF_FFT_FULL3=FT(1:(n/2),:);

y=REFERENCE4;
[FT FREQUENCY]=FFT_Analysis(n,y);
REF_FFT_FULL4=FT;
REF_FFT_FULL4=FT(1:(n/2),:);

REF_FFT_FULL = [REF_FFT_FULL1 REF_FFT_FULL2 REF_FFT_FULL3 REF_FFT_FULL4];

%% Perform FFT on test signal
% The FFT is performed by passing the number of points in the
% in the waveform signal, and the newly created reflection arrays.  The
% function is called four times, once for each reflection array.
% This user define function returns the real and complex numbers for each
% reflection array, the frequency array.  The outputs for each reflection
% are then formed into one array that should be half the length of the
% original wave form (since the FFT repeats only the first half of it is
% kept) by four times the number of files analyzed

y=TEST1;
[FT FREQUENCY]=FFT_Analysis(n,y);
TEST_FFT_FULL1=FT;
TEST_FFT_FULL1=FT(1:(n/2),:);

y=TEST2;
[FT FREQUENCY]=FFT_Analysis(n,y);
TEST_FFT_FULL2=FT;
TEST_FFT_FULL2=FT(1:(n/2),:);

y=TEST3;
[FT FREQUENCY]=FFT_Analysis(n,y);
TEST_FFT_FULL3=FT;
TEST_FFT_FULL3=FT(1:(n/2),:);

y=TEST4;
[FT FREQUENCY]=FFT_Analysis(n,y);
TEST_FFT_FULL4=FT;
TEST_FFT_FULL4=FT(1:(n/2),:);

TEST_FFT_FULL = [TEST_FFT_FULL1 TEST_FFT_FULL2 TEST_FFT_FULL3 TEST_FFT_FULL4];
FREQ=FREQUENCY;

%% Rearrange arrays and create needed variables
% Set the variable ‘q’ to the size of the FFT and initialize variables
% 'countA' and 'HIGH' to 0. The count variables are used to mark the points were the 'lower_freq' and 'upper_freq' start. This is done so that only the user selected frequency range is used.

q=size(REF_FFT_FULL);
q=q(1);
n2=size(REF_FFT_FULL);
n2=n2(2);

LOW=1;
HIGH=1;

% Search through the frequency array, 'FREQ' and determine the location in the array where the user selected frequency range is. LOW will give the position for the lower frequency and HIGH will give the position for the upper frequency
for i=1:q
    if FREQ(i)<lower_freq
        LOW=LOW+1;
    end
    if FREQ(i)<upper_freq
        HIGH=HIGH+1;
    end
end

%% Trim the data
% Call the function 'Trim' to cut of the data that is not wanted. The parameters needed to input into the model are returned as well as the trimmed frequencies so they can be used in graphing data
m=HIGH-LOW+1; % Number of points contained between LOW & HIGH

[T_MAG REF_MAG, TEST_MAG, SINE_PHI, COSINE_PHI, RATIO, FREQUENCY, OMEGA]=Trim(LOW, HIGH, q, n2,n1, REF_FFT_FULL, TEST_FFT_FULL, FREQ);

% Calculate the average and standard deviations for the values returned from the Trim function
REF_MAG_AVE=mean(REF_MAG,2);
TEST_MAG_AVE=mean(TEST_MAG,2);
RATIO_AVE=mean(RATIO,2);
RATIO_STD_DEV=std(RATIO,0,2);
ANGLE=asin(SINE_PHI);
PHASE_DIFF_AVE=mean(ANGLE,2);
PHASE_DIFF_STD_DEV=std(ANGLE,0,2);
SINE_PHI_AVE=mean(SINE_PHI,2);
SINE_PHI_STD_DEV=std(SINE_PHI,0,2);
COSINE_PHI_AVE=mean(COSINE_PHI,2);
COSINE_PHI_STD_DEV=std(COSINE_PHI,0,2);

% Calculate the Attenuation Coefficient based on the ratio
A=RATIO_AVE*exp(-alpha_il*d);

% Determine the upper and lower bound for phase (+/- 1 StdDev)
X=PHASE_DIFF_AVE+PHASE_DIFF_STD_DEV;
Y=PHASE_DIFF_AVE-PHASE_DIFF_STD_DEV;
Perform Model Calculations for wave speed and attenuation

Calculate x1 and x2 (intermediate model variables so that their values can be used to determine the wavespeed and attenuation in the sample material. Note: x1 and x2 have no physical meaning and are only used to make the equations cleaner.

for i=1:m

Full equation for the numerator ('x1_n', 'x2_n') and the denominator ('x1_d', 'x2_d'). (Use this one if the reference material has a non-zero attenuation coefficient)

\[
x1_n(i,1) = (-2*\rho_i*\Omega(i)^3*c_{il}*A(i)\exp(-\alpha_{il}*d)*)*\sin(\phi_{AVE}(i)) - \\
(\rho_i*\Omega(i)^2*\alpha_{il}*A(i)^2\exp(-2*\alpha_{il}*d));
\]

\[
x1_d(i,1) = (\exp(-2*\alpha_{il}*d)+A(i)^2+2*A(i)\exp(-\alpha_{il}*d))*(c_{il}^2*\alpha_{il}^2+\Omega(i)^2);
\]

\[
x2_n(i,1) = (\rho_i*\Omega(i)^3*c_{il}*(A(i)^2\exp(-2*\alpha_{il}*d)));+ \\
(2*A(i)\exp(-\alpha_{il}*d)*\rho_i*\Omega(i)^2*c_{il}^2*\alpha_{il} \sin(\phi_{AVE}(i)));
\]

\[
x2_d(i,1) = (\exp(-2*\alpha_{il}*d)+A(i)^2+2*A(i)\exp(-\alpha_{il}*d))*\cos(\phi_{AVE}(i)))*c_{il}^2+a_{il}^2+2*OMEGA(i)^2);
\]

\[
x(i,1)=x1_n(i,1)/x1_d(i,1);
\]

\[
x2(i,1)=x2_n(i,1)/x2_d(i,1);
\]

Equations for 'x1' and 'x2' based on the assumption that the attenuation in the silica is zero => \alpha_{il}=0.

\[
x1(i,1)=(-2*OMEGA(i)^3*c_{il}A(i)^2\exp(-\alpha_{il}*d)+2*A(i)\exp(-\alpha_{il}*d)*)/(\Omega(i)^2+1+2*A(i)\cos(\phi_{AVE}(i)));
\]

\[
x2(i,1)=(\Omega(i)^2*\rho_i*c_{il}*(1-A(i)^2))/(\Omega(i)^2+1+2*A(i)\cos(\phi_{AVE}(i)));
\]

Equations to determine the attenuation (alpha Ts) and wavespeed (c Ts) in the sample material. [c Ts is in (m/s^2) and alpha Ts is in (dB/mm)]

\[
\alpha_{ts}(i,1)=(((\Omega(i)^2*\rho_t*x1(i)))/((x1(i)^2+x2(i)^2)))*(8.6858896381/1000);
\]

\[
c_{ts}(i,1)=(x1(i)^2+x2(i)^2)/(\Omega(i)*\rho_t*x2(i));
\]

Calculate the measurement sensitivity

Set the Phase and Ratio standard deviations equal to delta_theta and delta_F which are used in the evaluation of the error based on a Taylor Series expansion
delta_theta(i)=PHASE_DIFF_STD_DEV(i); delta_F(i)=RATIO_STD_DEV(i);

CALCULATE THE PARAMETERS THAT RELATE TO THE WAVE SPEED

\[
\text{speed}_{\text{thteta numerator}}(i)=\rho_i*\Omega(i)*c_{il}*(8*RATIO_AVE(i)^2)*\sin(\phi_{AVE}(i))^2*\cos(\phi_{AVE}(i));
\]

\[
\text{speed}_{\text{denominator}}(i)=\rho_t*(1+\text{RATIO}_A VE(i)^2)^2*\sin(\phi_{AVE}(i))^2*\cos(\phi_{AVE}(i));
\]

\[
\text{speed}_{\text{F numerator}}(i)=\rho_i*\Omega(i)*c_{il}*(4*RATIO_AVE(i)^2); \]

\[
\text{speed}_{\text{F denominator}}(i)=\rho_i*\Omega(i)^2*\sin(\phi_{AVE}(i))^2*\cos(\phi_{AVE}(i));
\]
RATIO_AVE(i)^2)+2*RATIO_AVE(i)*alpha_il*SINE_PHI_AVE(i))+2*(1+RATIO_AVE(i)^2+2*RATIO_AVE(i)*
COSINE_PHI_AVE(i))*(COME_GAS(i)*RATIO_AVE(i)+alpha_il*SINE_PHI_AVE(i)));

% CALCULATE THE PARAMETERS THAT RELATE TO ATTENUATION
alpha_F_numerator(i)=
-2*rho_t*((OMEGA(i)*SINE_PHI_AVE(i)+c_il*alpha_il*RATIO_AVE(i))*(1+RATIO_AVE(i)^2+2*RATIO_AVE(i)
*COSINE_PHI_AVE(i))*(2+OMEGA(i)*RATIO_AVE(i)*SINE_PHI_AVE(i)+c_il*alpha_il*(RATIO_AVE(i)^2-1))
*(RATIO_AVE(i)*COSINE_PHI_AVE(i)))*((4*RATIO_AVE(i)*SINE_PHI_AVE(i))+(RATIO_AVE(i)^2+2*RATIO_AVE(i)-2)
+4*RATIO_AVE(i)*rho_t*(2+OMEGA(i)*RATIO_AVE(i)*SINE_PHI_AVE(i)+c_il*alpha_il*(RATIO_AVE(i)^2-1))
*4*RATIO_AVE(i)^2*SINE_PHI_AVE(i)^2+(RATIO_AVE(i)^2-1)^2)+4*RATIO_AVE(i)*rho_t*(2+OMEGA(i)*RATIO_AVE(i)
*SINE_PHI_AVE(i)+c_il*alpha_il*(RATIO_AVE(i)^2-1))*(4*RATIO_AVE(i)^2*SINE_PHI_AVE(i)^2+(RATIO_AVE(i)^2-
1)^2);  

alpha_denomonomator(i)=rho_i*c_il*(4*RATIO_AVE(i)^2*SINE_PHI_AVE(i)^2+(RATIO_AVE(i)^2-1)^2)^2;

alpha_theta_numerator(i)=
-2*OMEGA(i)*RATIO_AVE(i)*COSINE_PHI_AVE(i)*(1+RATIO_AVE(i)^2+2*RATIO_AVE(i)*COSINE_PHI_AVE(i))
*(-2*RATIO_AVE(i)*SINE_PHI_AVE(i)*(2+OMEGA(i)*RATIO_AVE(i)+c_il*alpha_il*(RATIO_AVE(i)^2-1))
*8*RATIO_AVE(i)^2*SINE_PHI_AVE(i)*COSINE_PHI_AVE(i));

% CALCULATE THE DERIVATIVE TERMS RELATED TO THE WAVE SPEED
dSpeed_dTheta(i)=abs(speed_theta_numerator(i)/speed_denomonomator(i))));
dSpeed_dF(i)=abs(speed_F_numerator(i)/speed_denomonomator(i));

% CALCULATE THE DERIVATIVE TERMS RELATED TO ATTENUATION
dAlpha_dTheta(i)=abs(alpha_theta_numerator(i)/alpha_denomonomator(i))/1000;
dAlpha_dF(i)=abs(alpha_F_numerator(i)/alpha_denomonomator(i))/1000;

% CALCULATE THE ERRORS USING A FIRST ORDER TAYLOR SERIES EXPANSION
delta_speed(i)= dSpeed_dTheta(i)*abs(delta_theta(i))+dSpeed_dF(i)*abs(delta_F(i));
delta_alpha(i)=dAlpha_dTheta(i)*abs(delta_theta(i))+dAlpha_dF(i)*abs(delta_F(i));

c_upper=c_ts+delta_speed';
c_lower=c_ts-delta_speed';

alpha_upper=alpha_ts+delta_alpha';
alpha_lower=alpha_ts-delta_alpha';
c_ts_ave=mean(c_ts);
c_ts_std_dev=std(c_ts);
alpha_ts_ave=mean(alpha_ts);
for i=1:m
    c_ts_average(i)=c_ts_ave;
    alpha_ts_average(i)=alpha_ts_ave;
end

% Plot the data
% Create a 2x2 plot for the sample wave speed, the sample attenuation, the
% phase angle difference between the incident and transmitted material and
% the values of x1 and x2.
% subplot(2,2,1); plot(FREQUENCY, c_ts, 'k', FREQUENCY, c_ts_average, 'r', FREQUENCY, c_lower, 'k',
% FREQUENCY, c_upper, 'k');
% %xlim([lower_freq upper_freq]);
% %ylim([1500 2500]);
% title('sample wave speed');
% xlabel('frequency (MHz)');
% ylabel('sample wave speed, c_t_s (m/s)');
% legend('wave speed', 'lower limit', 'upper limit');
% subplot(2,2,2); plot(FREQUENCY, alpha_ts, 'k', FREQUENCY, alpha_ts_average, 'r', FREQUENCY, alpha_lower, 'k:', FREQUENCY, alpha_upper, 'k:');
% xlim=[lower_freq upper_freq]);
% title('sample attenuation');
% xlabel('frequency (MHz)');
% ylabel('sample attenuation, alpha_t_s (dB/mm)');
% legend('sample attenuation', 'lower limit', 'upper limit');
% subplot(2,2,3); z=plot(FREQUENCY, PHASE_DIFF_AVE, 'k');
% hold on
% subplot(2,2,3); plot(FREQUENCY, X, 'k:',FREQUENCY, Y, 'k:', FREQUENCY, ANGLE, 'r');
% hold off
% xlim=[lower_freq upper_freq]);
% ylim([-0.2 0.2]);
% title('phase angle difference');
% xlabel('frequency (MHz)');
% ylabel('radians');
% legend('Average Phi', 'Lower Limit', 'Upper Limit', 'All Phi');
% subplot(2,2,4); plot(FREQUENCY,RATIO_AVE); grid on;
% set(gca,'XTick',0:1:15)
% set(gca,'XTickLabel',{'0', '1','2','3','4','5','6','7','8','9','10','11','12','13','14','15'})
% xlim=[lower_freq upper_freq]);
% title('Magnitude Ratio (Reflection 3/2)');
% xlabel('frequency (MHz)');
% ylabel('reference', 'sample');
% subplot(2,2,4); plot(FREQUENCY,REF_MAG,'k', FREQUENCY, TEST_MAG,'r'); grid on;
% set(gca,'XTick',[0:1:15])
% set(gca,'XTickLabel',{'0', '1','2','3','4','5','6','7','8','9','10','11','12','13','14','15'})
% xlim=[lower_freq upper_freq]);
% title('FFT');
% xlabel('frequency (MHz)');
% legend('reference', 'sample');
% ZZ = [c_ts_ave c_ts_std_dev]

% Outputs
% These outputs can be written to an excel file
OUT0_A={'Silica Wave Speed (m/s)';'Silica Attenuation';'Silica Thickness (mm)';'Sample Density';'A1';'A2';'A3';'A4'};
OUT0_B=[c il; alpha il; d; rho_t; A1; A2; A3; A4];
OUT1={'frequency', 'Reference Magnitude', 'Test Magnitude', 'Magnitude Ratio', 'Sample Wave Speed', 'Wave Speed Error'};
OUT2=[FREQUENCY, REF_MAG_AVE, TEST_MAG_AVE, RATIO_AVE, c ts, delta_speed'];

xlswrite(file_name, OUT0_A, Sheet);
xlswrite(file_name, OUT0_B, Sheet, 'B1');
xlswrite(file_name, OUT1, Sheet, 'A10')
xlswrite(file_name, OUT2, Sheet, 'A11');
ImportRefData.m

function [ref_opened, ref_signal]=ImportRefData(fileToRead, name_iter)
% This function imports data from the specified file
DELIMITER='t';
HEADERLINES=3;

wave=importdata(fileToRead, DELIMITER, HEADERLINES);

name='ref_waveform_';
ref_signal=cat(2,name,name_iter);

% Assign the data to the variable waveform
assignin('caller',ref_signal, wave.data);
ref_opened=cat(2,ref_signal,' created successfully from ',fileToRead);
end

ImportTestData.m

function [test_opened, test_signal]=ImportData(fileToRead, name_iter)
% This function imports data from the specified file
DELIMITER='t';
HEADERLINES=3;

wave=importdata(fileToRead, DELIMITER, HEADERLINES);

name='test_waveform_';
test_signal=cat(2,name,name_iter);

% Assign the data to the variable waveform
assignin('caller',test_signal, wave.data);
test_opened=cat(2,test_signal,' created successfully from ',fileToRead);
end
function [peak_amp, peak_shifted]=Peak(y, n, AA, min_amp)

%% Determine the number of reflections
%% This function will determine the number of reflections that were found in
%% the signal, ut_array which is an array for all of the imported signals

count=1;  % Initialize count variable to 1
j=1;     % start variable for reflection detection

% Determine the location of the peaks
for i=AA:n
    % Determines if there is a peak based on the amplitude
    % criterion set above. Absolute value of signal is used
    % in case largest value is negative.
    if abs(y(i))>min_amp
        reflection=i;
        break
    else
        end
end

% Shift 30 points earlier in time from the location of the 'peak'. Allows
% for the signal created in peak_amp to be centered
peak_shifted=reflection-30;

%% Create Reflection Arrays
% For each signal, 1:n1 a 100xreflections array will be created. 100
% points is selected for the signal length and reflections = the number of
% reflections found in the ut_array array.
peak_amp=y(peak_shifted:(peak_shifted+99),:);

end
**FFT_Analysis.m**

```matlab
function [FT,FREQUENCY]=FFT_Analysis(n,y)

% Determine the signal length of peak_amp (normally 100)
N=size(y);
N=N(1);

% Set the number of total points to analyze in the FFT analysis. The
% number of points kept from the FFT will be half of the value in the
% variable 'NumPoints' because
% zero_pad=(NumPoints-N(1))/2;

% Create an array of zeros the size of zero_pad
% P=zeros(zero_pad,1);

signal2analyze=y;

specturm=fft(signal2analyze);

% Determine the frequency step
freq_step=(100000000/n);
FREQUENCY(1)=0;
for i=1:(n-1)
    FREQUENCY(i+1,1)=FREQUENCY(i)+freq_step;
end
FREQUENCY=FREQUENCY/(10^6);  % Put the frequency in terms of MHz

FT=spectrum;

end
```
Trim.m

function [T_MAG, REF_MAG, TEST_MAG, SINE_PHi, COSINE_PHi, RATIO, FREQUENCY, OMEGA] = Trim(LOW, HIGH, q, n2, n1, REF_FFT_FULL, TEST_FFT_FULL, FREQ);

% Magnitude and Magnitude Error of the FFT for the reference test
z=1;
for i=1:n2
    for j=1:q
        R_MAG(j,i)=abs(REF_FFT_FULL(j,i));
        T_MAG(j,i)=abs(TEST_FFT_FULL(j,i));
        R_SINE(j,i)=imag(REF_FFT_FULL(j,i))/R_MAG(j,i);
        R_COSINE(j,i)=real(REF_FFT_FULL(j,i))/R_MAG(j,i);
        T_SINE(j,i)=imag(TEST_FFT_FULL(j,i))/T_MAG(j,i);
        T_COSINE(j,i)=real(TEST_FFT_FULL(j,i))/T_MAG(j,i);
        if i<n2
            z=2*i+1;
        end
    end
    REF_MAG(:,i)=R_MAG(LOW:HIGH,i);
    %REF_PHASE(:,i)=R_PHASE(LOW:HIGH,i);
    REF_SINE(:,i)=R_SINE(LOW:HIGH,i);
    REF_COSINE(:,i)=R_COSINE(LOW:HIGH,i);
    TEST_SINE(:,i)=T_SINE(LOW:HIGH,i);
    TEST_COSINE(:,i)=T_COSINE(LOW:HIGH,i);
end

sin2p=R_SINE(:,n1+1:2*n1);
sin3p=R_SINE(:,2*n1+1:3*n1);
cos2p=R_COSINE(:,n1+1:2*n1);
cos3p=R_COSINE(:,2*n1+1:3*n1);

sin2=T_SINE(:,n1+1:2*n1);
sin3=T_SINE(:,2*n1+1:3*n1);
cos2=T_COSINE(:,n1+1:2*n1);
cos3=T_COSINE(:,2*n1+1:3*n1);

for i=1:n1
    for j=1:q
        SIN_PHi32(j,i)=(sin3(j,i)*cos2(j,i)-sin2(j,i)*cos3(j,i))*(cos3p(j,i)*cos2p(j,i)+sin3p(j,i)*sin2p(j,i))- (sin3p(j,i)*cos2p(j,i)-sin2p(j,i)*cos3p(j,i))*(cos3(j,i)*cos2(j,i)+sin3(j,i)*sin2(j,i));
        COS_PHi32(j,i)=(cos3(j,i)*cos2(j,i)+sin3(j,i)*sin2(j,i))*(cos3p(j,i)*cos2p(j,i)+sin3p(j,i)*sin2p(j,i))+ (sin3(j,i)*cos2(j,i)-sin2(j,i)*cos3(j,i))*(sin3p(j,i)*cos2p(j,i)-sin2p(j,i)*cos3p(j,i));
    end
end

T_MAG_RATIO_32(j,i)=T_MAG(j,n1*2+i)/T_MAG(j,n1*1+i);
R_MAG_RATIO_32(j,i)=R_MAG(j,n1*2+i)/R_MAG(j,n1*1+i);
MAG_RATIO_32(j,i) = T_MAG_RATIO_32(j,i) / R_MAG_RATIO_32(j,i);
end

RATIO(:,i) = MAG_RATIO_32(LOW:HIGH,i);
SINE_PHI(:,i) = SIN_PHI32(LOW:HIGH,i);
COSINE_PHI(:,i) = COS_PHI32(LOW:HIGH,i);
end

FREQUENCY = FREQ(LOW:HIGH);
OMEGA = FREQUENCY * 10^6 * 2 * pi;
Works Cited


