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INTEGRATION OF A REAL OPTION OF AN ENVIRONMENTAL PROJECT

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INTEGRATION OF A REAL OPTION OF AN ENVIRONMENTAL PROJECT
AND FINANCIAL OPTION OF CARBON CREDITS
UNDER A CARBON CAP-AND-TRADE SCHEME

By

Qi Yang

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INTEGRATION OF A REAL OPTION OF AN ENVIRONMENTAL PROJECT
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This thesis explores a problem of how to evaluate environmental projects under the
Carbon Cap-and-Trade Scheme (CCTS) from the perspective of an individual firm. The
focus is the integration of a real option approach for project investment timing and
financial carbon option approach for carbon credit management. The real option is for a
mid or long term decision on when to invest on the environmental project, and the
financial option is for a supplementary decision to hedge short-term residue risks. Two
uncertainties (the carbon price and carbon credit demand) are considered, and their
changes are represented by a lattice of multiple orders. Dynamic programming and linear
stochastic programming are used to solve this integrated option problem under the carbon
uncertainty. Numerical sensitivity analyses are also conducted by changing the
parameters for the carbon price and carbon credit demand. The integrated approach can
help a firm to make environmental investment decision and hedge carbon credit risks
effectively under the CCTS. This research can also helps examine the effects of different
regulatory tools related to implementing environmental projects and financial options by manufacturing firms under the CCTS so that policy-makers could design more effective environmental regulations reducing GHG emissions and manufacturers’ financial risks.
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CHAPTER 1
INTRODUCTION

Chapter 1 provides background information about climate change and the regulatory mechanism of the carbon cap-and-trade scheme. Then, the thesis topic of environmental investments is described and discussed in terms of its complex dynamic decisions. The necessity of considering real and financial carbon options for regulatory compliance is also described. This chapter also presents the objectives and contributions of the thesis.

1.1 Climate Change and Carbon Cap-and-Trade Scheme (CCTS)

The recent decades have witnessed increasingly stringent environmental regulations about climate change (Sarkis & Tamarkin, 2005). Climate change-related issue of sustainability has been a popular topic among academia and policy makers (Carey, 2004). Relevant research literature has revealed that a substantial portion of climate change can be attributed to a huge amount of greenhouse gases (GHG) emissions by human activities (Houghton et al., 2001). The cap-and-trade scheme is emerging as a better alternative than the conventional command-and-control approach. It will be expected to be adopted for the reduction of GHG emissions in the United States (Coggins & Swinton, 1996 and Stavins, 1995).
A carbon cap-and-trade scheme (hereinafter referred to as “CCTS”) is a market-based regulatory mechanism and is defined as an administrative approach used to control pollution by providing economic incentives for achieving reductions in the emissions of pollutants (Stavins, 2000). A carbon credit is a limited, transferable right issued by the government and it can give its holder the right to emit some amount of carbon dioxide equivalent (Gupta & Maranas, 2003). Because the marginal GHG emission abatement costs vary from different sources, the carbon cap-and-trade scheme can help the GHG emitter achieve the emission reduction targets at the minimum abatement cost (Montgomery, 1972). From the perspective of a regulator, the CCTS can deal with the externality of GHG emissions effectively (Harry & Ted, 2008).

Historically, the cap-and-trade scheme was applied to reduce the SO$_2$ emissions under the Acid Rain Program of the 1990 Clean Air Act. It was proved to be effective and cost-efficient to reduce SO$_2$ emissions (Coniff, 2009). In December 2002, the EU created a compulsory carbon cap-and-trade system in order to meet tough targets specified by the Kyoto Protocol (Wagner, 2004). In the United States, there has been no national regulation implemented for GHG emissions so far, but there are several regional or state-level regulations to combat GHG emissions. The Chicago Climate Exchange (CCX) is North America’s only voluntary, legally binding GHG emissions’ reduction and trading
system. CCX has been trading carbon credits since 2003. CCX had an aggregate baseline of 680 million carbon credits and the highest price of the carbon credit is $7.50 per metric ton in May 2008 (http://chicagoclimateexchange.com/).

1.2 Problem Description of Environmental Investments under the CCTS

There is a business need for a firm to make environmental investments to achieve regulatory compliance. First, every firm has the obligation to comply with GHG regulations as one of its social responsibilities. Another powerful factor is the pressure from environmentally conscious customers.

In our problem setting, an individual GHG emitter is considered and it has to comply with the regulations in the context of the carbon cap-and-trade scheme (CCTS). The basic rules of regulatory compliance under the CCTS are as follows. Each GHG emitter receives a certain amount of maximum carbon credits (the carbon cap) for free at the beginning of each compliance period. If the firm’s actual GHG emissions by the end of the compliance period are less than its carbon cap, the unused carbon credit can be traded in the carbon market for profit. If the actual GHG emissions exceed the carbon cap, there are three choices to achieve regulatory compliance: (1) actually reducing GHG emissions by setting up an appropriate environmental project before the end of the compliance period, (2) buying additional carbon credits from a spot carbon market at the end of the
compliance period, or (3) buying an appropriate set of financial options of the carbon credit (hereafter referred to as “a financial carbon option”) at the beginning of the compliance period, which can be exercised at the end of the compliance period.

The decisions on regulatory compliance under the CCTS have to be made in the presence of multiple sources of uncertainty. There are at least two major sources of uncertainty associated with the environmental project. The first one is the price of the carbon credit on a spot carbon market (hereafter referred to as “the carbon price”). The carbon credit market will be a part of financial markets and behave like a stock market with ups and downs. The second is the random demand of the firm for carbon credits (hereafter referred to as “the carbon credit demand”). There are various factors which can account for the random carbon credit demand. The uncertain amount of GHG emissions can be attributed to those variations such as the production activity by the customers’ demand for its products, the energy sources and even the weather conditions (Gupta & Maranas, 2003). The customers’ random demand is treated as the main cause of the uncertain carbon credit demand in our analysis. The fluctuation of the carbon credit demand driven by the random products’ demand leads to the unbalance between carbon cap and carbon credit demand.
All three choices for achieving the compliance under the CCTS are associated with some risks. For the environmental project, it has a high project risk because its investment returns are highly volatile. The investment returns from the project are cost savings from reduced regulatory compliance costs generated after the periods when the project is set up. The investment return in each period is uncertain, depending on the carbon price and the carbon credit demand in the corresponding period. For instance, the environmental project may turn out to be not cost-efficient if the carbon price or the carbon credit demand continues to decrease. Another reason for the high project risk is due to the fact that the environmental project is capital-extensive and usually irreversible (Spangardt, Lucht, & Handschin, 2006).

The firm can rely solely on trading carbon credits on the spot carbon market to offset any surplus or shortfall of GHG emissions. However, such practice might work for the firm in the short run and the compliance costs could be even lower than expected when the carbon market is bullish. However, the firm is subject to the fluctuation of carbon prices in this case and it might be not cost-effective in the long run. Furthermore, such practice could give rise to the customers’ impression of having a poor record of regulatory compliance on the firm. For the choice of the financial carbon option, it is also a short-term tool and there is an additional cost paid for the option premium. Moreover,
transactions on financial carbon options require knowledge or experience about the financial market trend and derivative trading rules. Otherwise, the financial carbon options hedging will lead to be market price’s speculation and more risks can be generated.

As a decision-maker, the biggest challenge in determining the best investment decisions is how to utilize the various instruments, such as the real option of an environmental project and the financial carbon option, to achieve regulatory compliance with the least costs or risks. When to invest on the project is a critical decision for the firm. After the project is set up, how to maximize the investment returns from the project and hedge the project risks are also concerns for the decision-makers. In order to capturing those essential parts of the investment problem, appropriate modeling and a set of sophisticated mathematical tools are needed.

1.3 The Real Option of an Environmental Project, the Financial Option of Carbon Credits, and the Necessity for their Integration to Achieve Regulatory Compliance under the CCTS

The term “real option” was coined by Myers (1977) and a real option analysis is the extension of the pricing theory of the financial option to the evaluation of real, non-traded assets (Cox, Ross, & Rubinstein, 1979). The basic idea of evaluating a project with a real
option is to set up a replicating portfolio consisting of the twin security and the risk-free security and such a portfolio can perfectly correlates with the revenue’s uncertainty (Luenberger, 1998). Such an idea is illustrated in Figure 1.1.

The project with a real option explicitly accounts for the value of management flexibility and enables the decision-makers to make better decisions in the face of uncertainty. Management flexibility is the ability to affect uncertain future cash flows of a project in a way that enhances its expected returns or reduces its expected losses (Brandão, Dyer, &
Hahn, 2005). Management flexibility includes various options for the project such as to defer investment within certain periods of time, to expand the project later, to suspend the operations and to reduce or expand the scale of the operation (Miller & Park, 2002). Specifically for our investment problem, the value of the environmental project can be enhanced from the exercise of its real option. With the utilization of a real option, the project’s downsize risks are minimized and uphill profits are maintained (Copeland & Antikarov, 2005).

Since there is a financial market for carbon credits under a carbon cap-and-cap scheme, some derivative instruments such as the financial carbon option are available on the carbon market. A financial carbon option is a legally binding and negotiable contract, which gives the option’s holder the right, but not the obligation, to purchase or sell a certain quantity of carbon credits at a specific price (a strike price) and specific time (Luenberger, 1998).

The financial carbon option has two basic types: one is call option and the other is put option. The call option gives the buyer of this type of the option the right to buy carbon credits at the strike price. The put option gives the buyer the right to sell carbon credits at the strike price. In terms of the exercise time, there are two styles of financial carbon option: a European style and an American style. The European style is an option that may
only be exercised on expiration. The American option is an option that may be exercised on any trading day on or before the expiration date (Hull, 2008).

The appeal of a financial carbon option lies in the rule that there is no obligation to buy the underlying assets, therefore the holder can profit from a favorable price change while being protected from an adverse one. In exchange for protection from downside risks, the option’s holder has to pay a premium to the option’s writer (Hull, 2008). The financial carbon option is an appealing tool and its availability is a distinctive feature under the CCTS (Antes & Letmathe, 2006).

The real option of the project and financial carbon options can both serve the purpose of enhancing the project value and hedging the project risks (Triantis, 2005). Does it imply that once one alternative is chosen, the other is no longer needed? The answer is no.

There are two major reasons which can justify that both real and financial carbon options are necessary in hedging project risks and achieving regulatory compliance under the CCTS. First, the exercise of a real option inherent in the environmental project might be insufficient to achieve the regulatory compliance. The implementation of the environmental investment can reduce the possibility of exceeding the carbon cap, but not completely. Even if a favorable situation occurs, the liability of complying with carbon cap might still exist, because the reduction of GHG emissions can be less than the
difference between the carbon cap and actual the carbon credit demand. However, the compliance can be guaranteed if the financial carbon options are appropriately included in the compliance portfolios.

Second, financial carbon options can help hedge any residual project risks (Triantis, 2000). Those residual risks affect the value of the investment otherwise. Decisions on financial carbon options, in coordination with those on the real option, can boost the project return and reduce the variability of the investment returns. For instance, when the carbon price is high, there can be some possibility of a substantially low demand of carbon credits. In this case, financial carbon options of a put type can be used to protect the firm from the fluctuations of the carbon price and carbon credit demand to hedge project risks unshielded by the real option. If the goal of the environmental investment is to guarantee regulatory compliances with the least compliance cost and generate the maximum expected returns, the integration of real and financial carbon options is highly necessary in the decision-process of the project evaluation.

1.4 Thesis Objectives and Contributions

The objectives of this thesis are to study the problem of environmental investments under the CCTS, provide a set of methodologies to make the optimal decisions and offer robust solutions for the project evaluation in the face of multiple sources of uncertainty. Our key
focus is on how to make joint use of real and financial carbon options to enhance the project value and hedge project risks.

To achieve these objectives, an integrated mathematical model of evaluating environmental investments under the CCTS is introduced in the thesis. This investment decision model integrates the decisions on financial carbon options with those on a real option in the evaluation process of the project. The stochastic programming formulation is made to derive the optimal decisions on financial carbon options. The objective function for the stochastic programming formulation is to hedge project risks with minimal compliance costs. By using optimal decisions on financial carbon options as inputs to our real option analysis and by applying the technique of dynamic programming, optimal investment time for the environmental project is determined.

Numerical examples are presented to examine the validity and effectiveness of the integrated model. The dynamics of the carbon price and carbon credit demand are approximated as a lattice of multiple orders and the parameters of up and down factors for the carbon price are estimated from historical data. Furthermore, the effectiveness of the joint use of real and financial carbon options on the project value is discussed. The sensitivity analyses of the solutions from our integrated model are conducted as well.

The expected contributions of this research are as follows. First, this research
demonstrates a use of combined real and financial carbon options in evaluating environmental investments under the CCTS. A recent study pointed out that many existing studies focus on either financial hedging or operational hedging options including capacity and sourcing. (Ding, Dong & Kouvelis, 2007). Only a limited number of studies considered such integrations. One example is by (Ding, Dong & Kouvelis, 2007) in which joint optimal decisions are made on capacity options to improve expected profit and financial options to reduce profit variance. This research adopts a similar approach to an environmental investment case. The investment timing for carbon emission reduction project and financial carbon risk management are integrated in our approach. With the dynamic programing for the option analysis of the project evaluation, the best time to invest is determined. With the stochastic programming optimization for the financial carbon option, the optimal investment returns are determined for each period. In addition, numerical case studies show that how the integrated approach can be applied to maximize invest returns from the environmental project and hedge the risks with multiple uncertainties. Therefore, this research demonstrated the feasibility and usefulness of jointly using both options.

Real and financial carbon options are explicitly integrated in our mathematical model. Previous studies on the integration in other areas often treated real and financial options
as two independent decisions (Aytekin & Birge, 2004). This research attempts a more explicit integration. In terms of the model structure, this thesis embeds optimal decisions on financial carbon options into the framework of a real option analysis. In our research, the real option is treated as the focus of the problem and the financial option is a supplementary decision to the real option decision. In other words, the first priority in this thesis’s model is when to invest in the face of various uncertainties. Financial carbon options can serve the purpose of hedging residue risks and realizing the increase of the project’s value by exploiting the short-term trends of the carbon price and carbon credit demand within a period. This approach is different from those of many previous studies. For example, in some studies considered currency exchange rates, the real options involved are usually short-term project, while the environment investments in this thesis have a long term focus (Spangardt & Lucht, 2004).

The second expected contribution is that our research helps achieve regulatory compliance under the CCTS and offer solutions to hedge project risks. Many conventional methods of evaluating the environmental project usually focused on maximizing expected returns over the planning horizon (Spangardt et al., 2006). The solutions by these existing studies concerning expected values, however, cannot hedge the risks that extreme cases happen. Our research also helps hedge such risks and meet
the regulatory compliance more effectively by the joint use of both options. The risks from either carbon price’s increase or decrease are hedged and regulatory compliance under the CCTS can be guaranteed.

Third, our study helps understand the roles of the both options in the regulatory compliance, and can provide some information for the regulators to design effective environmental regulations. In some cases, the residue risks of the project are hedged effectively by financial carbon options (Triantis, 2005), and the effectiveness of the financial carbon option is built upon a fully competitively carbon market. A robust carbon market is a prerequisite for the utilization of financial carbon options (Guay & Kothari, 2003), and the transaction cost on the carbon market could be a significant portion of the compliance costs in the use of financial options (Ding, Dong & Kouvelis, 2007). Our research can be used to examine the effects of different regulatory tools on the compliance under the CCTS and related costs for manufacturing firms. Our findings help enable policy-makers to achieve the overall goal of reducing GHG emissions and impose acceptable compliance costs on the GHG emitters.

1.5 Thesis Organization

The rest of the thesis is organized as follows. In Chapter 2, literature reviews of existing research are provided. In Chapter 3, an integrated model for evaluating the environmental
project is introduced. The stochastic programming formulation of determining decisions on financial carbon options and the procedures to derive the optimal investment time of the project are described. Then Chapter 4 provides numerical examples to illustrate the optimal decisions on real and financial carbon options by solving the integrated model. The implications of the results from these numerical examples are also discussed. Chapter 5 concludes the thesis and suggests directions for future work.
CHAPTER 2

REVIEW OF THE STATE-OF-THE-ART RESEARCH

In this chapter, the methodology of the project evaluation is introduced and the advantage of a real option analysis over the method of discounted cash flow is explained. Then we make the overview of the existing research specifically about the topic of environmental investment from two perspectives. Lastly, the recent research about the integration of two perspectives is discussed and the novelty of our model is described.

2.1 The Methodology of the Project Evaluation: Discounted Cash Flow vs. Real Option Analysis

The conventional method of the discounted cash flow method (DCF) was introduced in firms in the 1950’s. With this method, one project’s value is determined by discounting all future expected cash flows to the current period and summing them as the net present value (NPV). One discount rate is subjectively chosen to reflect the risk of the project. If the project is associated with a high level of uncertainty, a high discount rate will be used (Blyth & Yang, 2006).

The method of the DCF has been widely used for the project evaluation because of its simplicity and intuitiveness. Many practitioners consider it as the model of choice (Miller
& Park, 2002). The project is deemed as “viable” if the NPV of all expected investment returns from the project exceeds the initial capital outlay, otherwise the project is “unviable” (Luehrman, 1998). The main criticism of DCF method lies in its implicit assumption that once the firm commits to a project, the project’s outcome will be unaffected by future decisions of the firm (Brandão, Dyer, & Hahn, 2005). In other words, the investment is regarded as a now-and-never decision under the DCF approach (Dixit, Pindyck, & Davis, 1994).

The real option analysis gets its name from its focus on options associated with real assets rather than with financial assets. A real option of a project, is the right but not the obligation to make investment decisions; for example, the opportunity to invest in the expansion of a manufacturing facility, or alternatively to sell the facility, is a real call or put option, respectively. The real option theory stems from the pricing theory of the financial option. The method of a real option analysis can be applicable to the problem of valuing real investments under uncertainty (Dixit, Pindyck, & Davis, 1994). The real option analysis adds the value of managerial flexibility to the traditional DCF approach, the managerial flexibility typically includes the option to invest, abandon, expand, or contract the project in question (Campbell, 2002).
A standard real option analysis can be described as follows: the investment returns from the project in question is replicated by a self-financing portfolio of riskless free asset such as a treasury bond and future cash flow generated from different investment decisions taken by the firm (Brennan & Schwartz, 1985). The real option analysis is based on standard no-arbitrage arguments. In particular, the approach assumes that asset price movements can be described by geometric Brownian motion so that standard financial tools, such as Black-Scholes and binomial lattice pricing, can be applied (Cox, Ross, & Rubinstein, 1979).

The real option analysis (ROA) can incorporate the value of managerial flexibility in the project evaluation and thus has some advantage over the DCF method (Cox, Ross, & Rubinstein, 1979). For those projects whose values are determined by several random variables and each variable has a high degree of uncertainty, the value of managerial flexibility can contribute significantly to the project value (Herbelot, 1992). For instance, if a project has an option to defer the investment, the project evaluation is no longer determined by the positive NPV only (McDonald & Siegel, 1986); instead an additional step is taken to evaluate whether the project could yield better investment returns if the project is postponed (Dixit, Pindyck, & Davis, 1994). In this regard, the project, which was deemed as “unviable” with DCF method, might become “viable” in future. Even if
the NPV of the project is positive now, it does not follow that it is the best decision to set up the project now (Copeland & Antikarov, 2005).

Enormous literatures have been devoted to the real option analysis of the project evaluation on various areas such as the pharmaceutical R&D and natural resource investments. The pioneering work of Black and Scholes (1973) and Merton (1973) provided the groundwork for the idea of incorporating option pricing methods into the problem of valuing real-asset investments under uncertainty. McDonald and Siegel (1986) determined the optimal timing for investing in a project with irreversible investments with uncertain returns represented by a continuous stochastic process. Dixit, Pindyck, and Davis (1994) proposed the use of dynamic programming to decide the investment time of the project.

2.2 Evaluation of Environmental Investments under the CCTS with a Real Option Analysis

The environment project under the carbon cap-and-trade scheme (CCTS) has been studied intensively by applying the real option approach. For instance, Sarkis and Tamarkin (2005) made a real option analysis of environmental project for the case study of British Petroleum. The carbon credit price was considered as the source of uncertainty and its dynamics were approximated by a binomial lattice. Later, Sarkis and Tamarkin
(2008) evaluated the investment of solar photovoltaic technology under the CCTS with the same approach. Two sources of uncertainty were considered: the exercise price of the technology and the price of carbon credit. One quadrangular lattice was used to represent the dynamics of the two uncertainties. Blyth and Yang (2006) presented a real option model to study the technology investment choice and the investment time for the power industry in the presence of climate policy risks. The uncertainties of the prices of the carbon credit and oil were considered.

None of the above research considers the financial carbon option as a decision variable in the real option evaluation of the investment. Financial derivatives such as option and forward are often used by a firm to hedge short-term exposure to any undesirable market fluctuations. In comparison with a real option, a financial carbon option is much more liquid, which can be easily traded in the market. The financial carbon options enable the firm to change the position in a quick and inexpensive way. Since the carbon credit market is a financial market, financial carbon options are available to the firms and it can be a cost-efficient tool to hedge compliance risks and achieve compliance. Some researches such as Pantzalis, Simkins and Laux (2001) and Guay and Kothari (2002) have revealed that the relationship between the real option and financial carbon options can be complementary in reducing the risks and enhancing the value of the project.
Therefore, the financial carbon option in the evaluation of the investment is highly necessary to be considered in the problem.

**2.3 Study of Environmental Investments from Risk Management’s Perspective**

Another branch of the research on environmental investment is taken from the perspective of risk management. Risk management is an analytical process to analyze the exposure to risks and employ various tools to minimize the impact by those risks (Stulz, 1996). The studies from this perspective have intended to explore the best course of actions with the realization of various uncertainties after the environment project was set up. Some risk metrics such as Conditional-value-at Risk (CVaR) and Conditional-cash-flows-at-risk (CCFAR) were explicitly considered as one term in the objective function or as a form of constraints (Rockafellar & Uryasev, 2000).

The research problems in this direction were solved by stochastic programming (SP) formulations. SP is demonstrated to be appropriate and fit for solving those problems of making decisions before the uncertainty is realized. Gupta and Maranas (2003) studied the problem of the technology choice for reducing GHG emissions under the CCTS. Financial carbon options were considered in the presence of the uncertainty of the customer’s demand for products and the price of the carbon credit. The authors revealed that the utilization of the financial carbon options could lower the expected compliance
cost and reduce the probabilities of exceeding carbon cap. Spangardt and Lucht (2004) also introduced a stochastic optimization model to analyze the investment decision under the uncertainty of the carbon price and project costs. Its objective function was to achieve the maximal expected profit and the minimum compliance risks out of the environmental project. The term of CVaR was used in the objective function for the minimization of compliance risks. Kettunen and Bunn (2010) presented a stochastic optimization model to analyze the best investment in the presence of the uncertainty of the price of the carbon credit and electricity price. The CCFAR was introduced as a risk constraint. The CCFAR is defined as the cash flow measure of the project risk conditional on the lower percentile of the cash position.

Although the research from this perspective provided some useful insights to manage the environmental project effectively under some given risk constraints, the solutions offered are for post-project only and leave the critical question of when to invest unanswered. In this aspect, the real option analysis can fill the gap.

2.4 Integration of Real and Financial Carbon Options in the Evaluation of Environmental Investments

Some literature has studied the integration of the real option and financial carbon options in other areas such as the capacity planning in a global supply chain with the uncertainty
of exchange rate. Triantis (2000) concluded that the real and financial carbon options may be alternative mechanisms of limiting downside risks while allowing for uphill profits. These two alternatives are not mutually exclusive in the project evaluation. Even if the investment on real assets is decided, financial carbon options are still necessary. Aabo and Simikins (2005) explored the relationship between the real option and financial carbon options via the questionnaires on Danish non-financial firms. It was observed that some firms used both the real and financial carbon options in response to exchange rate’s fluctuation. Aytekin and Birge (2004) dealt with the optimal investment of the production facility and retail remarkets across different countries with the uncertain exchange rate. Ding and Kouvelis (2001) analyzed the integration of the production site’s choice and the financial option in the face of currency exchange rate’s uncertainty in a global supply chain. Stochastic programming was applied to solve the models proposed in these two papers. All above-mentioned research focused on the area of exchange rate and had an emphasis on short-term planning of real and financial carbon options. To the best knowledge of the authors, none of the previous studies explicitly considered the integration of real and financial carbon options on environmental investments under the CCTS.
CHAPTER 3

INTEGRATED INVESTMENT DECISION MODEL FOR THE
EVALUATION OF AN ENVIRONMENTAL PROJECT

This chapter describes an integrated investment decision model for an environmental project using real and financial carbon options. The objective of the integrated model is to maximize the discounted future returns of the project and to reduce the project risks. To make decisions on financial carbon options in each independent period, a stochastic programming (SP) is used. By dynamic programming (DP) utilizing these SP solutions, a real option analysis model of an environmental project is presented.

3.1 Definitions of Mathematical Symbols

The mathematical symbols used in this chapter are defined as follows:

**Indexes**

\[
\begin{align*}
    c &= \text{A call financial carbon option} \\
    f, g, h, i, j, k, l, m, n &= \text{States of the investment problem} \\
    o &= \text{The carbon spot market}
\end{align*}
\]
\[ p = \text{A put financial carbon option} \]

\[ t, t' = \text{Time or period in the planning horizon} \]

**Parameters**

\[ B_t = \text{The budget available for the purchase of financial carbon} \]

\[ \text{options in period } t \]

\[ K = \text{Amount of the investment cost on the environmental project,} \]

\[ \text{equivalent to the exercise price for the real option of the} \]

\[ \text{project} \]

\[ K_t^c = \text{The strike price for a call option defined at the beginning of} \]

\[ \text{period } t \text{ (i.e. time } t-1) \]

\[ K_t^p = \text{The strike price for a put option defined at the beginning of} \]

\[ \text{period } t \text{ (i.e. time } t-1) \]

\[ \hat{Q}_t = \text{The maximum amount of carbon emissions assigned in period} \]

\[ t, \text{which is also the carbon cap} \]

\[ Q_t^{jk}(D_t^j, I_t^k) = \text{The amount of carbon emissions in period } t, \text{depending on the} \]

\[ \text{carbon credit demand and the status of the environmental} \]
project

\[ cc_t = \text{The purchasing cost for one unit of a call option occurring at} \]
\[ \text{the beginning of period } t \text{ (i.e. time } t-1); \]
\[ cc_t = cc_t(p^f_{t-1}, K^c_t, \hat{N}(p^f_{t-1})) \]

\[ cp_t = \text{The purchasing cost for one unit of a put option occurring at} \]
\[ \text{the beginning of period } t \text{ (i.e. time } t-1); \]
\[ cp_t = cp_t(p^f_{t-1}, K^p_t, \hat{N}(p^f_{t-1})) \]

\[ d(t,t') = \text{The risk-free factor of discounting the cash flow occurring at} \]
\[ \text{time } t' \text{ (} t' \geq t \text{) back to time } t \]

\[ d_1 = \text{The down factor for the carbon price for one period} \]

\[ d_2 = \text{The down factor for the carbon credit demand for one period} \]

\[ p^i_t = \text{The price of the carbon credit on spot carbon market (also} \]
\[ \text{called the carbon price) observed at the end of period } t \text{ (i.e.} \]
\[ \text{time } t) \]

\[ q_1 = \text{The risk-neutral probability in one period.} \]
\[ q_1 = \frac{r_f - d_1}{u_1 - d_1} \]
\[ r_f = \text{Risk free discount factor for one period; it is equal to } d(t, t+1) \]

\[ s_{t}^{ijk} = \text{One state observed at the end of period } t, \text{ which is represented by a 3-element vector } \{p_t^i, D_t^j, I_t^k\} \]

\[ u_1 = \text{The up factor for the carbon price for one period} \]

\[ u_2 = \text{The up factor for the carbon credit demand for one period} \]

\[ w(s_{t+1}^{lmn} | s_t^{ijk}) = \text{The conditional probability of state transition from time } t \text{ to time } t+1, \text{ which spans over period } t+1 \text{ and } s_{t+1}^{lmn} \in \hat{N}(s_t^{ijk}). \text{ For period } t, \ w(s_t^{ijk} | s_{t-1}^{fgh}) \text{ is used for notational convenience.} \]

\[ \alpha = \text{The conversion factor of customer’s demand for products to carbon emissions.} \]

\[ \beta = \text{The coefficient of emissions’ reduction by the implementation of the environmental project} \]

**Sets**

\[ \hat{N}(s_t^{ijk}) = \text{The set of all immediate successor states at the time } t+1 \text{ transiting from state } s_t^{ijk} \text{ at time } t \]

\[ N(s_t^{ijk}) = \text{The set of all successor states for time } t+1 \text{ to } T \text{ from a common} \]
predecessor state \( s_{ijk}^{t} \) at time \( t \).

\[
S = \text{The set of } S_t \text{'s over the planning horizon, } i.e. \text{ } S = \{S_1, S_2, \ldots, S_T\}
\]

\[
S_t = \text{The set of all possible states at time } t
\]

\[
a_t^{ijk} = \text{The set of all available actions at time } t
\]

**Variables**

\[
A_t(s_{ijk}^{t}) = \text{One stream of the cash flow as the cost savings from the project when it is set up in or before period } t
\]

\[
C_t^{1} = \text{The costs to purchase carbon options at the beginning of period } t \text{ (i.e. time } t-1)\]

\[
C_t^{2} = \text{Compliance costs by the exercise of carbon options and spot market transaction at the end of period } t \text{ (i.e. time } t)\]

\[
C_t = \text{The total compliance costs in period } t, \text{ which include the costs of purchasing financial carbon options at the start of a period, and exercising the financial options and/or buying or selling on the spot carbon market at the end of the same period.}\]
\( D_t^j \) = The firm’s demand for carbon credits resulting from fulfilling the customers’ demand for products, observed at the end of each period; called simply the carbon credit demand hereinafter.

\( I_t^k \) = The status of the project at the end of period \( t \) as a binary variable; \( I_t^k = 1 \) when the project is set up; otherwise zero.

\( V_t(s_t^{ijk}) \) = The optimal value function representing the expected cash flows obtainable from state \( s_t^{ijk} \) at time \( t \) to the end of the planning horizon (period \( T \)) by making optimal decisions on real and financial carbon options.

\( V_t^{inv}(s_t^{ijk}) \) = The project value when it is set up before or in period \( t \) (i.e. before or at time \( t-1 \)).

\( V_t^{wait}(s_t^{ijk}) \) = The project value when it is to be set up in future periods after time \( t \).

\( X_{fgh}^t \) = The set of decision variables \( x_{fghc}^t \) and \( x_{fghp}^t \) representing the first stage variables (decided at the beginning of period \( t \)) in the SP equivalent formulation.
\[ Y_{ijk}^t = \text{The set of decision variables } y_t^c, y_t^p \text{ and } y_t^m \text{ representing} \]
the second stage variables (decided at the end of period } t \text{ in} \]
the SP equivalent formulation \[ x_{fghc}^{t-1} = \text{Number of call options purchased at the beginning of period } t \]
\text{(i.e. time } t-1) \]
\[ x_{fghp}^{t-1} = \text{Number of put options purchased at the beginning of period } t \]
\text{(i.e. time } t-1) \]
\[ y_t^c = \text{Number of carbon credits obtained by the exercise of the call} \]
\text{option at the end of period } t \text{(i.e. time } t) \]
\[ y_t^p = \text{Number of carbon credits sold by the exercise of the put option} \]
at the end of period } t \text{(i.e. time } t) \]
\[ y_t^o = \text{Number of carbon credits sold/purchased directly from the spot} \]
carbon market at the end of period } t \text{(i.e. time } t) \text{ to make the} \]
carbon balance to zero given state } s_{ijk}^{t-1} \text{ at the beginning of the} \]
period (i.e. time } t-1); positive if the purchase is made, negative \]
otherwise. \[ y_t^o = y_t^o(s_{ijk}^{t-1}) \]
\[ z_t = \text{The objective function value for period } t, \text{ calculate at time } t \text{ in} \]
the stochastic programming (SP) equivalent formulation

\[ I(A) = \begin{cases} 
1 & \text{if } A > 0, \\
0 & \text{otherwise} 
\end{cases} \]

3.2 Assumptions, State Definition and State Transition

In this section, the assumptions made for the integrated model are explained. After the assumptions, the state definition is introduced and how one state at one period transiting to another state in its following periods is described.

3.2.1 Assumptions for the Integrated Model

The integrated model in this chapter is based on the following assumptions:

- A deterministic carbon cap should be met at the end of each period and it is known in advance (Triantis, 2005). This is a usual rule specified in the regulation under the CCTS. In EU’s CCTS, the firm is given the information of the carbon cap for the next five years.

- The carbon price and carbon credit demand can be observed at the end of one period, and remain the same by the beginning of its next period (Dixit, Pindyck, & Davis, 1994). With this assumption, there is no gap of time between two neighboring periods and relevant information about the state at the beginning of period \( t \) can be known.
• There are many kinds of real options available for an environmental project such as the options to wait, suspend and expand. The real option of the project in our analysis is limited to the option to wait for investment, which means that the firm has the flexibility to postpone the investment within the planning horizon (McDonald & Siegel, 1986). Under this assumption, we can focus on the comparison of the real option and financial carbon option in terms of the respective effect on the project value. Another reason for this assumption is that the environmental project for GHG emissions reductions is capital-intensive and the decision-maker is more concerned with the time to set up the project (Antes & Letmathe, 2006).

• The environmental project is set up at the beginning of a period. Once it is set up at the beginning, its effect is immediate in carbon emissions reduction in this period and will be also effective for all the remaining periods (Yang & Blyth, 2007).

• The project cannot be divested once it is invested (Spangardt, Lucht, & Handschin, 2006).

• The investment cost of the environmental project remains the same during the planning horizon (Laurikka & Koljonen, 2006).
• No carbon credit is transferable to the next period, and any remaining carbon credits after compliance will be sold on the spot market. This simplifying assumption could be valid, because carbon credits usually expires after specified periods in many regulations. The implication of this assumption is that the decisions on financial carbon options in each period can only have an effect on the compliance costs of this period. The decisions on carbon options in different periods are independent from each other.

• The carbon price and carbon credit demand both follow the stochastic progress of Geometric Brownian Motions (GBM) (Spangardt, Lucht, & Handschin, 2006).

• The combinations of the carbon price and carbon credit demand are represented by a multinomial lattice of order four i.e. quadranomial lattice (Sarkis & Tamarkin, 2008 and Copeland & Antikarov, 2005).

• The carbon price and carbon credit demand are not correlated (Luenberger, 1998). The correlation may exist in actual cases, but it was not considered in this paper for focusing on other important aspects of the problem.

• Financial carbon options are purchased at the beginning of each period, and can be exercised only at the end of the same period. In other words, these financial carbon options are European-styled options. The first reason to choose this option
style is that the premium of a European’s option costs less than that of an American one. What is more, the main purpose of buying the financial option is to hedge the risk of the carbon price and carbon credit demand at the end of the period, which is not known before the end of the period. Lastly, financial carbon options only serve as a short-term tool for hedging risks under this assumption (Triantis, 2000).

- The purpose of financial carbon options is to hedge the risks of carbon prices and the unbalance between the carbon credit demand and carbon cap. They are not used for speculating on market fluctuations (Kettunen & Bunn, 2010). Therefore, no short selling of carbon options is allowed and the limit on the budget for constructing a portfolio of financial carbon options is necessary.

- The strike price for a financial carbon option is treated as a parameter in our model and in reality, the strike prices can be a set of discreet positive values (Moschini & Lapan, 1992). For simplicity, it is assumed that the strike price is determined before a financial carbon option is purchased. One simple rule to determine the value of a strike price will be used in numerical experiments.

- The purchasing costs of financial carbon options, call or put, are determined by option pricing theory under a complete market and there is no risk premium for
the financial carbon option (Ding & Kouvelis, 2001). A risk premium is defined as the minimum amount of money accepted to compensate for taking a risky asset instead of a risk-free one.

3.2.2 State Definition and State Transition

The planning horizon of the problem consists of total $T$ periods, and the time of the problem is defined as time $t = 0, 1 \ldots T$. For a modeling purpose, we differentiate time $t$ and period $t$. Time $t$ means a time point and it refers to the end of period $t$ or the beginning of period $t+1$. Period $t$ is a time interval between times $t-1$ and $t$.

The state of the problem at time $t$ is defined as $s_t = \{p_t^i, D_t^j, I_t^k\}$. The superscripts $i, j, k$ are related to the nodes at time $t$ in a dynamic programming network based on a lattice of the carbon price, the carbon credit demand and the status of the project.

The first element $p_t^i$ of the state represents the carbon price observed at time $t$ (in fact, realized at the end of period $t$ and has remained the same until the beginning of period $t+1$). The index $i$ denotes the total number of upward moves in the carbon price by time $t$, and the range of $i$ is $i = 0, 1, 2 \ldots t$. The carbon price and the number of upward moves could be redundant in information, but both are kept for modeling convenience.

The second element $D_t^j$ represents the carbon credit demand observed at time $t$. The
index $j$ denotes the number of upward moves in the carbon credit demand by time $t$ and the range of the index $j$ is $j=0, 1, 2... t$.

The last element $I_t^k$ stands for the status of the environmental project at time $t$. $I_t^k$ is equal to zero if the facility has been running without an environmental project up to current time point $t$; otherwise it is equal to one when the project is set up before time $t$ (i.e. in or before period $t$); the value of $k$ is either 0 or 1. When $I_t^k = 0$, the real option of the project is available to the decision-maker.

Given two initiate states (at time $t = 0$) $s_0^{000} = (p_0^0, D_0^0, I_0^0)$ or $s_0^{001} = (p_0^0, D_0^0, I_0^1)$, the state $s_t^{ijk}$ is represented as follows:

$$p_t^i = p_0^0 \ast (u_1)^i (d_1)^{(t-i)} \quad D_t^j = D_0^0 \ast (u_1)^j (d_1)^{(t-j)} \quad I_t^k \in \{0,1\}$$

The total number of states at time $t$ is equal to $2 \cdot (t+1)^2$ for $0 \leq t \leq T$. State transition from time $t$ to time $t+1$, namely one state $s_t^{ijk} = \{p_t^i, D_t^j, I_t^k\}$ transiting to

$$s_{t+1}^{lmm} = \{p_{t+1}^l, D_{t+1}^m, I_{t+1}^n\}$$

is related as $l = i$ or $i+1$ and $m = j$ or $j+1$.

According to the information of the carbon price at time $t-1$ and possible carbon prices at time $t$, the purchasing cost of the call or put financial carbon option can be determined by the option pricing formulation under risk-neutral probability

$$cc_t(p_{t-1}^i, K_t^r, \hat{N}(p_{t-1}^i)) = \frac{1}{1+r_f} \sum_{p' \in \hat{N}(p_{t-1}^i)} q_t \cdot 1(p_t^i - K_t^r)$$
\[ cp_t(p_{t-1}^f, K_t^c, \tilde{N}(p_{t-1}^f)) = \frac{1}{1 + r_f} \sum_{p' \in \tilde{N}(p_{t-1}^f)} q_t \cdot 1\{K_t^c - p_t^f\} \]

where \( q_1 = \frac{r_f - d_1}{u_1 - d_1} \) and \( 1\{\ldots\} \) denotes an indicator function. \( K_t^c \) and \( K_t^p \) can be any positive values. In reality, there is always a set of discrete values of strike prices available to the buyer of financial options (Moschini & Lapan, 1992). This study assumes single values of them are given.

When \( I_t^K \) is one \((k = 1)\), \( n = 1 \). This is because when the project has been set up, the project cannot be reversed. The decisions at time \( t \) and beyond are related to financial carbon options only. Those decision variables include the numbers of call and put option units purchased at a predefined strike price.

When \( I_t^K \) is zero \((k = 0)\), \( n = 0 \) or \( 1 \). The decision-maker has the opportunity to decide whether or not to set up the environment project at time \( t \). At the same point of time, decisions on financial carbon options are made as well.

The state definition and state transitions are illustrated with Figure 3.1. Representative states at the first three time points \((i.e.\ two\ periods)\) are considered. The rectangles represent states and the lines represent state transitions. Note that the values of the carbon price and carbon credit demand are relative ones with respect to an initial state.
Figure 3.1 One example of the states and state transition representations in two periods with respect to its initial state
3.3 The Stochastic Programming Formulation for the Optimal Carbon Portfolio

Decisions in Each Period

This section describes a mathematical programming model for carbon emission compliance with minimized costs of carbon options in an individual period. The cash flow is also defined as the savings from compliance costs by the environmental investments on the project and financial carbon options.

3.3.1 The Optimal Decisions on Financial Carbon Options in Each Period

The stochastic programming (SP) is formulated to address the characteristics of the carbon compliance and financial carbon option problem. In our problem setting, some decisions are made before the realization of the uncertainties. Other corrective decisions, which are called recourse actions, are made after the realization of the uncertain variables (Spangardt & Lucht, 2004). For any period $t$, the decisions on purchasing financial carbon options need to be made at the beginning of the period without the complete information of the carbon credit demand in the whole period. However, the information of the carbon price and carbon credit demand from the end of period $t-1$ (i.e. time $t-1$) is available to the decision-maker. Up and down factors for the carbon price and carbon credit demand from time $t-1$ to time $t$ are also known with probabilities. The set of probabilities is obtained from a risk-neutral quadranomial lattice. Once the time frame
moves to the end of period $t$, recourse actions are taken according to the state realized at
the end of period $t$. Those actions include the exercise of financial carbon options and the
transaction of carbon credits on the spot carbon market. The decisions at the beginning of
a period are termed the first stage variables and those at the end are termed the second
stage variables.

The stochastic linear program for each period is shown as follows:

$$
\min z_t = \min \left[ C_t^1 \cdot (1 + r_f) + \sum_{s_t^j \in \mathcal{N}(s_{t-1}^{fgb})} w(s_t^j) \cdot s_t^j \cdot C_t^2 \right] \\
\forall t, f, g, h, i, j, k
$$

(1)

Subject to

$$
C_t^1 = x_{t-1}^{fghc} \cdot cc_t + x_{t-1}^{fghp} \cdot cp_t \\
\forall t, f, g, h, i, j, k
$$

(2)

$$
C_t^2 = y_t^c \cdot K_t^c - y_t^p \cdot K_t^p + y_t^o \cdot p_i \\
\forall t, i
$$

(3)

$$
x_{t-1}^{fghc} \in \{0, Z^{+}\} \quad \forall t, f, g, h
$$

(4)

$$
x_{t-1}^{fghp} \in \{0, Z^{+}\} \quad \forall t, f, g, h
$$

(5)

$$
y_t^c \in \{0, Z^{+}\} \quad \forall t
$$

(6)

$$
y_t^p \in \{0, Z^{+}\} \quad \forall t
$$

(7)
\[ y^c_t = 1\{p^i_t - K^c_t\} \cdot x_{i-1}^{fhc} \quad \forall t, f, g, h, i \]  
(8)

\[ y^p_t = 1\{K^p_t - p^i_t\} \cdot x_{i-1}^{fhp} \quad \forall t, f, g, h, i \]  
(9)

\[ C^t_k \leq B^k_i \quad \forall t, k \]  
(10)

\[ Q^k_t(D^j_t, I^k_t) = \alpha \cdot \beta^t(i^k_i)D^j_t \quad \forall t, j, k \]  
(11)

\[ y^o_t = Q^k_t(D^j_t, I^k_t) - \tilde{Q}_t - y^c_t + y^p_t \quad \forall t, i, j, k \]  
(12)

\[ I^h_{t-1} \leq I^k_t \quad \forall t, h, k \]  
(13)

\[ I^k_t \in \{0, 1\} \quad \forall t, k \]  
(14)

The ranges of the indexes shown above are

\[ 1 \leq t \leq T \quad 0 \leq f, g \leq (t-1) \quad 0 \leq i, j \leq t \quad h \leq k \quad h, k \in \{0, 1\} \]

The equations are explained as follows:

Equation (1), the objective function, is to minimize the total compliance costs in period \( t \).

The compliance costs consist of two terms. The first term is the costs for the purchase of carbon options at the beginning of period \( t \), as its cost term is expressed in Equation (2).

This term is equal to the SP first stage variables multiplied by one risk-free discount factor \( 1 + r_f \). The second term is the expected recourse costs related to the SP second stage variables over possible states realized at the end of period \( t \). The cost terms are
expressed by Equations (2) and (3).

Equations (4)-(7) denote sign restriction of variables, representing the constraints on no short selling of options at each period. Only financial carbon options purchased at the beginning of period $t$ (the first stage) can be executed at the end of period $t$ (the second stage).

Equations (8)-(9) specify how the decision variables $y^c_t$ and $y^p_t$ are determined. The number of carbon options exercised in the second stage is either zero or the same as the number of carbon options purchased in the first stage depending on the value of the carbon price realized at the end of period $t$.

Equation (10) represents the budget constraint in constructing the compliance portfolio of carbon options in the first stage.

Equation (11) determines the amount of carbon emissions for period $t$ depending on the status of the project.

Equation (12) indicates the determination of the variable $y^o_t$. At the end of each compliance period, any surplus or shortfall of the carbon credits after the actions from $y^c_t$ and $y^p_t$ will be leveled off by selling or purchasing in the spot carbon market $y^o_t$.

Equations (13) and (14) determine the irreversibility of the investment. Once the project is set up, there is no more opportunity to reverse the investment in a later period.
By solving the above stochastic programming problem, the set of optimal decisions on the first and second stage variables can be determined for each period.

### 3.3.2 Determination of the Cash Flow of Cost Savings from Environmental Investments in Each Period

The cash flow of compliance costs occurring in period $t$ by utilizing the financial carbon options can be determined as follows:

$$C_t(I^k_t = 1, s^{ijk}_t)$$ is the total compliance costs incurred in period $t$ under the optimal decisions derived by SP formulation with the utilization of carbon options given that the environmental project was set up prior to the beginning of period $t$ and its state at the end of period $t$ is in one state $s^{ijk}_t$.

Further we define the compliance costs incurred if environmental project is not yet set up but some actions on the carbon options are taken in the first stage.

$$C_t(I^k_t = 0, s^{ijk}_t)$$ is the total compliance costs incurred in period $t$ under the optimal decisions derived by SP formulation with the utilization of carbon options given that the environmental project has not been set up yet at the beginning of period $t$ and its state at the end of period $t$ is in one state $s^{ijk}_t$. 
Then the cash flow of cost savings from environmental investments for of the whole
period \( t \). This stream of cash flow is equal to the difference of compliance costs between
when the project is set up and when not.

\[
A_t(s_{t}^{ijk}) = C_t(I_t^{k} = 1,s_{t}^{ijk}) - C_t(I_t^{k} = 0,s_{t}^{ijk})
\]

\[\forall t,i,j,k\]
\[1 \leq t \leq T \quad 0 \leq i,j \leq t \quad k \in \{0,1\}\] (15)

With the optimal decisions on financial carbon options, the series of cash flows \([A_t, A_{t+1},...\]
\(A_t\)] have truncated distributions of investment returns from the environmental project.

Those cash flows will be inputs for the real option analysis of the environmental project
in the following section. The next section presents how these decisions on financial
carbon options can alter the optimal investment time for the environmental project in
addition to the real option.

### 3.4 The Optimal Decision on the Investment Time with a Real Option Analysis

In this section, a real option analysis of the environmental project is conducted when the
financial carbon options are used concurrently. By using the cash flows derived from the
stochastic program in Section 3.3, the project is evaluated with a real option analysis. The
stochastic programming formulation deals with the best decisions on financial carbon
options. The real option analysis can determine the best investment time along each path
of the realization. The real option’s formulations and framework in this section are based on dynamic programming (DP) model of the investment evaluation by Dixit, Pindyck, and Davis (1994), Blyth and YangLucht (2006) and Fortin et al. (2008).

The environmental project is reviewed by comparing the net present value (NPV) of the investment if the project is set up at the beginning of period \( t \) with the discounted expected investment returns if the project has the option to wait for investment in future periods.

The terms in DP formulations are defined as follows. The optimal value function \( V_t(s_{t}^{ijk}) \) of the DP formulation is defined as the maximum expected cash flows obtainable from state \( s_{t}^{ijk} \) at time \( t \) to the end of the planning horizon \( T \) by making optimal decisions on the project investment and carbon options. Here \( s_{t}^{ijk} \) contains the information about the carbon price, the carbon credit demand and the status of the environmental project at time \( t \). Depending on the status of the environmental project in period \( t \), \( V_t(s_{t}^{ijk}) \) has two cases. \( V_t^{inv}(s_{t}^{ijk}) \) denotes the project value when the project is set up in or before period \( t \) given that the state at time \( t \) is \( s_{t}^{ijk} \). Similarly, \( V_t^{wait}(s_{t}^{ijk}) \) denotes the project value when the project is to be set up in future periods after \( t \). \( w(s_{t+1}^{lmn}, s_{t}^{ijk}) \) is the conditional probability of state transition from time \( t \) to time \( t+1 \), which spans over period \( t+1 \), and \( s_{t+1}^{lmn} \in \hat{N}(s_{t}^{ijk}) \). The set of feasible actions at the beginning of period \( t \), denoted by \( a_{t}^{ijk} \),
includes the project implementation and financial carbon options, or financial carbon options only. The feasible actions depend on the status of the project. \( A_i(s_{ij}^k) \) stands for the cash flow as the cost savings from the project occurring at the end of period \( t \) when it is set up at the beginning of period \( t \). \( r_f \) stands for the risk free discount factor for one period. Again, if \( I_t^k = 1, I_{t+1}^n = 1 \), and if \( I_t^k = 0, I_{t+1}^n = 0 \) or 1

The DP functional equation for \( t < T \) is:

\[
V_t(s_{ij}^k) = \max \begin{cases} 
V_t^{inv}(s_{ij}^k) = A_i - K + \frac{1}{1+r_f} \sum_{l,m,n} V_{t+1}(s_{l,m,n}^{l,m,n}) \cdot w(s_{l,m,n}^{l,m,n} | s_t) & I_t^k = 1 \\
V_t^{wait}(s_{ij}^k) = \frac{1}{1+r_f} \sum_{l,m,n} V_{t+1}(s_{l,m,n}^{l,m,n}) \cdot w(s_{l,m,n}^{l,m,n} | s_t) & I_t^k = 0 
\end{cases}
\]

\( \forall t, i, j, k, l, m, n \quad 0 \leq i, j \leq t \quad 0 \leq j, m \leq t \quad k \leq n \in \{0,1\} \) \hspace{1cm} (16)

The boundary condition of optimal value function \( (t = T) \) is determined by:

\[
V_T(s_{ij}^k) = \max \begin{cases} 
A_T(s_{ij}^k) - K & \forall i, j, k 
\end{cases}
\]

\hspace{1cm} (17)

For the final period \( T \), it is the last period for the investment. There are no more cash flows of cost saving obtainable in period \( T+1 \), which is reflected in the following equation:
It is also obvious that there is no more opportunity to invest after period $T$. Therefore, $V_T^{wait}(s_{T}^{ijk})$ is equal to zero. In other words, the investment decision is a ‘now or never’ one (Dixit, Pindyck, & Davis, 1994). The project will be set up in period $T$ as long as $V_T^{inv}(s_{T}^{ijk}) > 0$.

For any period $1 \leq t < T - 1$, $V_t^{inv}(s_{t}^{ijk})$ is the net present value (NPV) of the cash flows generated from the project initiated from the state $s_{t}^{ijk}$. When the project is set up at the beginning of period $t$, a series of cash flows $[A_t, A_{t+1}, ... A_T]$ in terms of cost savings from regulatory compliance are generated along each path from period $t$ to $T$ in the dynamic programming network. The cash flows of cost savings for each period ranging from $t$ to $T$ are uncertain, because they depend on the carbon price and carbon credit demand at the end of each period. Thus, the investment returns from the project are determined as an expectation of the future returns.

In contrast, the project value with a real option to wait $V_t^{wait}(s_{t}^{ijk})$ is equal to the NPV of future cash flows if one chooses not to invest in the project in period $t$ and is determined by the discounted value of the expected future cash flows with the risk-free discount factor based on the expected outcome of the decisions in period $t+1$ and the

\[
\frac{1}{1 + r_f} \sum V(s_{T+1}^{lmn} \ast w(s_{T+1}^{lmn} | s_{T}^{ijk})) = 0 \quad \forall i, j, k, l, m, n
\]
current state in period $t$ with risk-neutral probability.

The optimal value of investment at a given state in each period can be derived by the recursive optimal value function. The optimal value function $V_t(s_{t}^{ijk})$ is the maximum between $V_t^{inv}(s_{t}^{ijk})$ and $V_t^{wait}(s_{t}^{ijk})$. The best investment time over the planning horizon is determined accordingly by working backward to the first period.

The rationale behind the real option can be attributed to the sources of uncertainty, which are the driving force for the environmental project’s value. The option to wait provides the decision-maker with additional momentum for maximizing the investment when a more favorable situation occurs in future periods. According to the real option’s theory, the higher the uncertainty, the higher the project’s option value is.
In Chapter 4, numerical examples are presented in order to verify the validity and effectiveness of our integrated model. First, additional assumptions and parameter values for the specific examples are described. The results of the numerical calculations are discussed in terms of the project value, the boundary of the project value and the effectiveness of real and financial carbon options under the different option hedging conditions. Sensitivity Analyses of the carbon price and carbon credit demand are also conducted.

4.1 Conditions for the Numerical Examples

4.1.1 Additional Assumptions

The following additional assumptions are necessary to set the parameters to be used in the numerical examples:

- The mean value of carbon prices and the volatility of the carbon price are estimated using a historical data. Daily carbon credit price data from European Climate Exchange (ECX) from March 13, 2009 to April 22, 2010 were used for estimating the variability of the credit price
The trend of the carbon price from the data is shown in Figure 4.1.

![Historical Daily Carbon Price in ECX](http://www.ecx.eu/EUA-CER-Daily-Futures)

**Figure 4.1 Daily prices of the carbon credit in the European Climate Exchange**

*Source: European Climate Exchange*

- For simplicity but without loss of generality, it is assumed that the volatility of the carbon credit demand is comparable to that of the estimated carbon price.

- The carbon cap assigned to the manufacturing firm is constant for every period in the planning horizon.

- The financial carbon options purchased at the beginning of period $t$ is available at only one strike price. The options purchased in period $t$ in numerical examples are further assumed to be at-the-money, which means their strike prices are equal to
the spot carbon price at time $t-1$. Although there are many strike prices available for a firm, this assumption is still reasonable when the firm’s intention using the financial carbon options is only to “lock” the carbon price at the time of purchase. For example, the spot market for the carbon credit is highly volatile and a firm has no expertise or is not in a proper position to speculate on the price change. On such a circumstance, the firm will take the spot price on the carbon price as a strike price. According to its moneyness of a financial carbon option, it is classified as in-the-money (ITM), at-the-money (ATM) or out-of-the-money (OTM). ATM and OTM options are the main traded ones in practice. Since OTM has no intrinsic value, ATM is chosen for numerical examples. (Hull, 2008)

- The put and call financial carbon options purchased at the same time have the same strike price. This is one of the common options trading strategies, termed “a long straddle” and can hedge both directions of price movement because the price increase or decrease is hedged by the call or put option respectively. (Moschini & Lapan, 1992)

### 4.1.2 Estimation of the Mean and the Volatility of Carbon Prices

Because the carbon price is the most influential parameter in numerical examples, an
appropriate procedure is highly necessary for estimating its variability. The following estimation procedure is based on the MESSAGE model (Messner & Strubegger, 1995), and it is assumed that the fluctuation of carbon credit prices follows a Geometric Brownian Motion (GBM).

Let \( p_t \) be the price of the carbon credit (the carbon price) at time \( t \) and under the assumption of GBM, \( p_t \) is governed by the following equation:

\[
d\ln p_t = \nu dt + \sigma d\tilde{z} \quad \forall \sigma \geq 0 \quad d\tilde{z} \sim N(0,1)
\]

(19)

Where \( \nu \) is the expected growth rate of the carbon price and \( \sigma \) is the expected volatility of the price expressed as a percentage of change. \( \tilde{z} \) is a standard Wiener process. The parameters \( \nu \) and \( \sigma \) are estimated from the actual data from ECX with the estimation procedure as follows:

First, the lognormal value of carbon prices for two neighboring time points is calculated:

\[
\nu_t = \ln \left( \frac{p_t}{p_{t-1}} \right)
\]

(20)

Then, the average value of \( \nu_t \) among the data set is computed.

\[
\bar{\nu} = \frac{1}{n} \sum_{t=1}^{n} \nu_t
\]

(21)
Then the volatility is calculated by:

\[ \sigma = \frac{1}{\sqrt{dt}} \sqrt{\frac{1}{n-1} \sum_{t=1}^{n} (v_t - \bar{v})^2} \] (22)

The estimated value for \( \nu \) and \( \sigma \) is 0.24 and 0.45, respectively (annual values) and these estimates are very close to the results estimated by Spangardt et al. (2008). According to the binomial lattice model (Cox, Ross, & Rubinstein, 1979), the upward and downward factors can be calculated as:

\[ u_1 = e^{\sigma \sqrt{\Delta t}} = 1.57 \] and \[ d_1 = e^{-\sigma \sqrt{\Delta t}} = \frac{1}{u_1} = 0.64 \]

4.1.3 Other Parameters

Unless it is specified otherwise, the following values are used for the parameters throughout the numerical examples:

\[ p_0^0 = $10 \] the carbon price at the initial state

\[ D_0^0 = 1000 \] the carbon credit demand at the initial state

\[ u_1 = 1.57 \] the up factor for the carbon price change within one period; estimated in Section 4.1.2.

\[ d_1 = 0.64 \] the down factor for the carbon price change within one
Period; estimated in Section 4.1.2.

\[ u_2 = 1.40 \]
the up factor for the carbon credit demand; a value set to be comparable with \( u_1 \)

\[ d_2 = 0.71 \]
the down factor for the carbon credit demand

\[ r_f = 0.05 \]
risk free interest rate within one period

\[ q_1 = 0.44 \]
risk neutral probability for a real option analysis

\[ \alpha = 1 \]
the conversion rate of customer’s demand for products to the amount of carbon emissions

\[ \beta = 20\% \]
the reduction rate of GHG emissions by the environmental project

\[ Q = 1000 \]
the constant carbon cap for every period

\[ K = \$15000 \]
Amount of one time investment cost for the project

\[ T = 6 \]
number of periods in the planning horizon

### 4.2 Results and Discussion of the Numerical Examples

In this section, the solutions from the integrated model are presented. The effectiveness of the model is demonstrated by the project values under different evaluation conditions, the
boundaries of the project values under different conditions. In the following results, the project value is defined as the expected net present values (ENPVs), which are the expected values of all future investment returns discounted to the initial period minus the initial capital outlay. Lastly, sensitivity analyses of the project values are also conducted with respect to the carbon price and the carbon credit demand.

4.2.1 The Project Value under Different Option Hedging Conditions

The ENPVs are calculated for four cases: (1) the project without any options, (2) the real option alone, (3) the project with a real option, and (4) the project with real and financial carbon options. These four cases with different option hedging conditions are compared.

The expected net present values (ENPV) without any real or financial carbon option is calculated based on the conventional method of the discounted cash flow (DCF). The real option alone case is the discounted non-negative cash flows on the condition that the project has the option to wait for investment in future periods. The project with a real option is the optimal value by comparing the ENPVs on two conditions: i.e. the project without any options and the real option alone. The project with real and financial carbon options is the optimal value obtainable on the condition that both options are utilized.

As shown in Figure 4.2, the ENPV for the project without any options has the lowest value in each period. The value from the case of a real option alone is higher than the
ENPV without any options case. The ENPV for the project with a real option is above the ENPV without any options and a real option alone, except that there is one same point in period 1. The reason is that period 1 is the optimal time to invest.

![Figure 4.2 Project values under different option hedging conditions](image)

Figure 4.2 Project values under different option hedging conditions

The trends in the above figure can be explained as follows. First, all these lines follow a decreasing trend from period 1 to period 6. The decrease in the number of the periods remaining for investment can account for such downward trends. Second, if we look at the trend backward from period 6 to period 1, the ENPV for the project without any options starts at a negative value while the value of a real option alone starts at zero. The reason for this is that the real option alone enables the decision-maker to avoid the investment if
the project yields a negative NPV.

It is a general rule that the project value is expected to decrease with fewer periods for investment. However, the rate of decrease in project value varies from different option hedging conditions. The slope of the ENPV for the project without any options is the steepest among all lines. The line, representing the real option alone, follows the same trend, but decreases at a lower rate. The line, representing the project with a real option, captures the relative changes of the project values under above two conditions.

The project value can be further enhanced by the real and financial carbon options. It is reflected in the graph that the line, denoting the ENPV with real and financial carbon options, is always the highest line. As the number of periods for investing increases, the effect of both options on boosting the project value becomes more significant.

As far as the best period for investment is concerned, this period should be the first period from looking backwards, in which the increasing trend is reversed. Given that all the parameters being equal, the investment decisions might be different under two conditions of a real option v.s. real and financial carbon options. In Figure 4.2, period 1 is the best period to invest under the condition that both options are considered, while period 2 is the best period to invest if only the real option is considered. In this regard, the joint use of real and financial carbon options might alter the investment timing. One
implication is that the project value with both options tends to encourage the decision maker to invest earlier than the one with a real option only.

However, the optimal decisions on investment period are very sensitive to the parameters assigned. In Figure 4.3, the upward factor for the carbon credit card is assigned as $u_2=1.05$ (a quite stable condition) and other parameters remain unchanged. It can be seen that the best investment decisions under the conditions of a real option v.s. real and financial carbon options are the same. Period 2 is the optimal period to invest. In comparison with Figure 4.2, the ENPVs of a real option vs. real and financial carbon options become narrower in Figure 4.3. This means that the gains from both options are not significant when the volatility of the carbon credit demand is low.
4.2.2 The Boundaries of the Project Value under Different Option Hedging Conditions

The upper bound of the project value defines the best case of the investment returns and the lower bound of the project value defines the worst case. By examining the upper and lower bounds of the project value under different conditions, a deeper understanding is gained on how the real and financial carbon options can affect the investment returns in different cases. In this numerical example, the lower bounds of the project value are
calculated from the case of the lowest carbon price and the carbon credit demand for each period and the upper bounds of the project value are calculated from the case of the highest carbon price and carbon credit demand.

![Image of Figure 4.4](image-url)

**Figure 4.4** Different boundaries of the project value under different option hedging conditions

As shown in Figure 4.4, the lower bound of the project value without a real option is located in the negative region in most periods. The real option can improve the lower
bound of the project value significantly. Further the lower bound with real and financial carbon options stay moderately higher than the one with a real option only. It can be interpreted as the financial carbon option can enhance the project value in addition to the effect of the real option.

For the upper bound of the project value, that the upper bounds of the project values without and with a real option coincide. This is supported by the real option’s advantage of maintaining upside gains. The upper bound with real and financial carbon options is higher than the upper bound with a real option only. This means that the real and financial carbon options are more effective in hedging the risks for the case of a high carbon price and a high carbon credit demand than the real option only. This tends to support the argument that the financial carbon option serves the purpose of hedging the residue risks left by the real option. However, the benefits of financial carbon option hedging have a less significant effect on the project value than the real option does. Another concern is the purchasing costs of financial carbon options. In comparison, the real option can be obtained without cost if the delay of the investment does not make the firm lose competitiveness (Triantis, 2000). When the additional purchase and transaction costs in the financial carbon options are considered, it is not recommended to use the financial carbon options only to hedge the project risks.
4.2.3 Effectiveness of Real and Financial Carbon Options in the Enhancement of the Project Value

The real and financial carbon options are both effective instruments for the enhancement of the project value. One comparison chart is presented to demonstrate their effectiveness. From the chart, it can be seen that the real option can increase the project value by improving its lower bound. Similarly, the lower bound and the project value are increased with the help of the financial carbon option as well. What is more, the financial carbon option can also improve the upper bound, which cannot be achieved by the real option. These findings are consistent with the trends revealed by Figure 4.2.

It is worth mentioning that the real and financial carbon options vary from the sources of value enhancement. The increase of the project value by the real option comes from the elimination of all negative cash flows and project risk reduction. In contrast, the increase by the financial carbon option arises from the project risk reduction only. It is reflected in the chart that the real option increases the lower bound by 100% while the financial carbon option has no or negligible improvement on the lower bound.

Furthermore, it can be observed that the joint use of real and financial carbon options can enhance the project value significantly. The magnitude of the increase in the project value is much higher than the sum of the separate use of the two instruments. The ratios of the
increase by the joint use versus the summation of the individual uses are 61.70% and 25.00% respectively.

In terms of upper bound of the project value, the use of financial carbon option has not a significant improvement (7.17%). This observation matches the implication from Figure 4.5 that the financial carbon option can be complement to the real option but not its substitution. The reason is transaction costs occurring in the financial carbon options. These costs might be sunken costs for the firm. The firm should also be cautious of falling into option speculations if the number of financial carbon options is not limited.
Table 4.1 A comparison chart of the effectiveness of real and financial carbon options

<table>
<thead>
<tr>
<th>(Lower Bound, Upper Bound) for Period T</th>
<th>Without the Financial Carbon Option</th>
<th>With a Real Option</th>
<th>( (RO,<em>) - (NO \ RO,</em>) ) / ( (NO \ RO,*) )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expected Project Value for Period One</strong></td>
<td>Without a Real Option</td>
<td>With a Real Option</td>
<td>(100%, 0.00%)</td>
</tr>
<tr>
<td><strong>Expected Project Value for Period One</strong></td>
<td>(-$14,947, $154,185)</td>
<td>(0, $154,185)</td>
<td>12.12%</td>
</tr>
<tr>
<td><strong>Expected Project Value for Period One</strong></td>
<td>With a Real Option</td>
<td>(-$14,730, $217,700)</td>
<td>(0, $217,700)</td>
</tr>
<tr>
<td>(1.45%, 7.17%)</td>
<td>(0.00%, 7.17%)</td>
<td>(100%, 41.19%)</td>
<td>61.70%</td>
</tr>
</tbody>
</table>

4.2.4 Sensitivity Analyses of the Effects of the Carbon Price and Carbon Credit

Demand’s Volatility on the Project Value

As revealed by the previous research, the sources of uncertainty are the driving force of the project value (Yang & Blyth, 2007). Both real and financial carbon options can deal with those uncertainties. In the rest of this section, the effects of the carbon price or
carbon credit demand’s volatility on the project value under different option hedging conditions are examined.

The results shown in Figure 4.5 are calculated when the up factor for the carbon credit demand is fixed and assumed to be 1.05. The left side vertical Y-axis is for the column chart representing the ENPV. The right side vertical Y-axis is for the line chart for the rate of the increase in the project value. The meaning of two Y-axes is the same in Figure 4.6. We choose a relative low volatility of the carbon credit demand so that the effect of the carbon price’s volatility becomes dominant. At the same time, the carbon credit demand has a higher weight in determining the cost saving of the regulatory compliance and we can focus more on the effect of carbon price’s volatility.
Figure 4.5 Effect of the carbon price’s volatility on the project value with a real option vs. real and financial carbon options

As shown in Figure 4.5, a higher volatility of the carbon price can make the environmental project more valuable. The project values with a real option only is on the rise with the exception in the range of increase factor 1.05 to 1.20. The effect of a low volatility of the carbon price has an insignificant effect on the increase in the project value. However, the project with real and financial carbon options is always on the rise with the increase of the carbon price’s volatility. The gap between the project values with a real option v.s. real and financial carbon options is slightly widening as the up factor for the
carbon price increases. This can be interpreted as the joint use of real and financial carbon options can deal with the uncertainties more efficiently than a single use of the real option.

However, it can be observed that the relative rate of increase in project value is changing. Starting from 1.05 to 1.15, the relative rate of increase is high, then decreases abruptly to some level from 1.20 to 1.25 and remains to be constant from 1.30 to 1.90. The trend in the change of the relative rate can be explained by the relative effect of the real option and the financial carbon option. When the upward factor is low, the real options can’t have a significant improvement on the project value because of the huge initial investment cost involved. The financial carbon option is a short term tool and can capture the carbon price movement in a short time period effectively. Therefore, the use of financial carbon options are significant for the project value when the upward factor is in a low range. As the value of the upward factor is increasing, the improvement from the real option becomes significant because the initial investment cost can be well compensated by the reduction of the compliance costs from the project. At the same time, more financial carbon options are needed to keep pace with the carbon price movement. Because of the transaction costs incurred in the financial carbon options, its improvement on the project value dwindles. This also validates our previous finding that the financial
carbon option can only be a complementary instrument to the real option.

Similarly, the effect of carbon credit demand’s volatility is studied in Figure 4.6. The parameter for the carbon credit demand’s volatility is fixed at 1.40 and this is a mid-range value for the carbon credit demand’s volatility. A similar trend to Figure 4.5 can be observed in Figure 4.6. The project with real and financial carbon options always yield higher investment returns than the one with a real option only. Regarding the rate of the increase in project value, it follows a upward curve when in a range of the low volatility. Then downward trend follows in a range of the medium volatility. Starting from $u_2 =1.18$, the marginal improvement diminishes almost to zero.
Figure 4.6 Effect of the carbon credit demand’s volatility on the project value with a real option vs. real and financial carbon options

For Figures 4.7 and 4.8, wider ranges for the volatilities of the carbon price and carbon credit demand are chosen and both parameters are changed at the same time. Sensitivity analyses are conducted to examine the effects of the real option vs. real and financial carbon options on the project value. If there is only the real option considered, the project value will be significantly affected by the real option only when the volatility of the carbon price is larger than 1.42 (an upward factor). There is a clear-cut line in the figure
around 1.42 which can separate two regions for each band of the project value. When the real and financial carbon options are both considered, there is no such a dividing line. It is implied that the joint use of the real option and financial carbon option can deal with the uncertainties more effectively and less insensitive to the range of the parameters.

Furthermore, the project value in Figure 4.8 is larger than the one in Figure 4.7 given that the upward factors remain the same. This is again to support our findings in previous sections that joint use of both options can be a significant enhancement of the project value.

![Graph](image)

Figure 4.7 Sensitivity analyses of the carbon price and carbon credit demand on the project value with a real option.
4.3 Further Discussions about the Number of Periods Considered, Carbon Demand-Price Correlation, and Probability of Carbon Credit Demand

This study and its models are based on some simplifying assumptions. Those assumptions are that (1) the number of periods to be considered are given and fixed, (2) the carbon price and carbon credit demand are independent from each other, and (3) the true probability of carbon credit demand is known in advance. These assumptions were necessary to develop the essential relations for the determination of the project value and the real and financial carbon options’ effects. However, these assumptions may not be
realistic in actual situation of environmental investments. This section describes how the results would be different if such assumptions are relaxed.

There is usually a positive relationship between the number of periods for investment and the project value given that the project has the real option at least in the planning horizon. More periods for investing on the project within the planning horizon, a higher project value it should have. In our numerical experiments, there are 6 periods in the planning horizon. If the NPVs, calculated by the method of the discounted cash flow, are reviewed, the project value is located in a positive region in the first 3 periods; however, the project value becomes negative in the latter 3 periods. If we change the planning horizon to more than 6 periods and other parameters remain the same, the project value stays in the positive region for most of the periods. In this case, the variance of the project value becomes smaller and the method of the discounted cash flow is sufficient to evaluate the project. If the planning horizon of less than 6 periods is considered, there will be more periods in which the project value is negative. Due to the limited number of periods, even real and financial carbon options might not make the project effective. The project is “unviable” under any option hedging condition. The 6 periods are chosen for our numerical examples because it can demonstrate whether the effect of a real option or combined real and financial carbon options are significant or not. The decision of the
planning horizon length is another important problem that is beyond the scope of this study.

The correlation between the carbon price and carbon credit demand is an important factor affecting the project value. There was no correlation considered in our calculation. This is not an accurate reflection of the real situation. It is likely that their correlation is positive because of several reasons. For example, when the economy is strong, more carbon credits are needed so as to boost the carbon price. In a strong market, a firm more likely to produce and emit more, and probably requires more carbon credits. In addition, a higher carbon demand driven by individual firms can result in an even higher carbon price in addition to the price increase by the force of a competitive market (Koenig, 2011). In this positive correlation case, the total variance of the project value becomes bigger. Thus, the expected NPV as the project value will be higher. Therefore, the project value will be higher and the firm is encouraged to invest earlier when the positive correlation is considered.

The uncertainties of the carbon price and the carbon credit demand are associated with different types of risks. The carbon price is related to carbon market risks while the carbon credit demand is related to private risks because the demand is firm-specific (Luenberger, 1998). Therefore, the risk-neutral probability is used for the carbon price
while the true probability of the carbon credit demand is for the carbon credit demand in
building the quadranomial lattice (Smith, 2005). In our numerical examples, a
hypothetical firm is assumed, and the true probability was not used. If there is a data,
statistical estimation techniques such as Bayesian inference can be applied. Our solutions
will be more close to the project value on real situations.
CHAPTER 5

CONCLUSIONS AND FUTURE RESEARCH

5.1 Validity and Effectiveness of the Integrated Model

The integrated model proposed in this thesis is capable of incorporating both managerial flexibility and the strategy of financial carbon option hedging into the evaluation of the environmental project. The project evaluated by our model can yield better investment returns of the project and minimize project risks than the ones derived by the DCF or the existing method of considering the real option only. With the increase in the project value, the joint use of real and financial options provides the firm with more incentive to invest earlier in additional to managerial flexibility provided by the real option.

With the numerical examples, the validity and effectiveness of the model are verified. The results from our numerical examples can demonstrate that the joint use of real and financial carbon options can be a significant enhancement of the project value and the proper use of financial carbon options is a necessary and effective tool for the regulatory compliance under the CCTS. More importantly, the interaction and the relationship between the real option and financial carbon options are revealed from our numerical examples. The methodology embodied in our model can facilitate the
decision-maker to make the optimal decisions on the investment out of complex, dynamic situations about the carbon price and carbon credit demand.

5.2 Directions for Future Work

There are some limitations in our research and more research work would be desirable. First, the results are obtained based on the assumption that the carbon price and carbon credit demand both follow Geometric Brownian Motion. This assumption might not hold for an accurate representation of the carbon price and carbon credit demand. The choice of stochastic progress can have a significant effect on the investment evaluation. Even if the mean price and its volatility are comparable, the decisions under different stochastic processes can vary greatly. To obtain sufficient actual data and further study on the carbon price’s trend over a long period can be a potential area for future research.

Second, fluctuations of the carbon price and carbon credit demand are assumed to be independent from each other. This is a valid assumption for a firm who is only a price-taker in a carbon market, but this might not be the case if the firm is an influential market player in a competitive market. It will be more desirable to consider the correlation factor between the carbon price and carbon credit demand.
Third, the results from our numerical examples might be very sensitive to some parameters such as initial investment cost. In this sense, the estimation of the parameters becomes critically important. A better way to estimate the input parameters and to come up with more robust solutions would be directions for future work.

Fourth, we focus on the case that the randomness of the carbon credit demand is attributed only to the customers’ demand for the firm’s products. In reality, the carbon credit demand can be firm-specific or industry-specific and there are a great variety of other crucial factors such as different sources of energy used in the facility. It would be necessary to consider these factors into the function of the carbon credit demand in future work.

Fifth, the strike price for a financial carbon option is predefined and there is only one strike price available to choose in each period in our model. The exercise date of a financial carbon option for the current period coincides with the time point the project is evaluated in the next period, this is not a good reflection of actual situation on the use of a financial carbon option. It would be meaningful to consider the choice of strike prices as one decision variable and multiple expiration dates available to select for financial carbon options.
REFERENCES


APPENDIX A

RAW DATA FOR ESTIMATING THE MEAN AND VOLATILITY OF CARBON PRICES

Table A.1 Daily prices of the carbon credit in European Climate Exchange: data from 3-13-2009 to 4-22-2010

<table>
<thead>
<tr>
<th>Date</th>
<th>Carbon Price (€)</th>
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</tr>
</tbody>
</table>
APPENDIX B

COMPUTER PROGRAMS FOR SOLVING THE INTEGRATED MODEL

MATLAB® AND IBM ILOG® codes listed are created for solving the integrated model proposed in Chapter 3. Code 1 and Code 2 are MATLAB® codes for the implementation of the model introduced in Section 3.4, which can determine the optimal project value and best investment time over the planning horizon by DP. Code 3 is an IBM ILOG® code to solve SP formulation in Section 3.3.

**Code 1: MATLAB® code for the implementation of the integrated model in Section 3.4 (model and data)**

clc;
clear all;
u1=1.57;
u2=1.40;

factor=[u1,u2; % uu factor (price,demand)
u1,1/u2; % ud factor (price,demand)
1/u1,u2; % du factor (price,demand)
1/u1,1/u2]; % dd factor (price,demand)
u=factor(1,1); % up factor of carbon price
d=factor(3,1); % down factor of carbon price
rf=0.05; % risk free interest rate
p=(1+rf-d)/(u-d); % risk-neutral probability
P=[0.5*p,0.5*p,0.5*(1-p),0.5*(1-p)]; % probability vector of state transition
Q=1000; % carbon cap
t=6; % number of periods
\(S\{1\}=[10,1000]; \quad \% \text{initial state (price, demand) in period 1} \\
\beta=0.2; \quad \% \text{reduction factor of emissions by the environ project} \\
d1=1/u1; \\
d2=1/u2; \\
Y\{1\}=\beta*S\{1\}(1,1)*S\{1\}(1,2); \\

\text{for} \ k=2:t \\
\text{for} \ i=1:k \\
\text{for} \ j=1:k; \\
\quad S\{i,j,k\}(1,1)=S\{1\}(1,1)^{*}(u1^{(i-1)})*(d1^{(k-i)})); \\
\quad S\{i,j,k\}(1,2)=S\{1\}(1,2)^{*}(u2^{(j-1)})*(d2^{(k-j)})); \\
\quad Y\{i,j,k\}(1,1)=f*S\{i,j,k\}(1,1)^{*}S\{i,j,k\}(1,2); \\
\quad \% \text{one stream of cash flow of cost saving at state w in period j} \\
\quad \text{end} \\
\text{end} \\
\text{end} \\

\text{for} \ i=1:t \\
\text{for} \ j=1:t; \\
\quad Z\{i,j,t\}=Y\{i,j,t\}; \\
\text{end} \\
\text{end} \\

r=1/1.05; \quad \% \text{discount factor} \\
\text{for} \ k=t:-1:2; \\
\quad m=k-1; \\
\text{for} \ i=1:m \\
\quad \text{for} \ j=1:m; \\
\quad \quad Z\{i,j,m\}=Y\{i,j,m\}+r*P*[Z\{i+1,j+1,k\};Z\{i+1,j,k\};Z\{i,j+1,k\};Z\{i,j,k\}]; \\
\quad \quad \% \text{accumulated cash flows of cost saving starting from state w} \\
\quad \quad \% \text{in period i to period t.} \\
\quad \text{end} \\
\text{end} \\
\text{end}
for k=t:-1:2;
    m=k-1;
    for i=1:m
        for j=1:m;
            OP{i,j,m}(1,1)=r*p*(S{i+1,j+1,k}(1,1)-S{i,j,m}(1,1))/rf;
                % call option premium
            OP{i,j,m}(1,2)=r*(1-p)*(S{i,j,m}(1,1)-S{i+1,j+1,k}(1,1))/rf;
                % put option premium
        end
    end
end

dr=0.50; % sales allocated for regulatory compliance
for k=t:-1:2;
    m=k-1;
    for i=1:m
        for j=1:m;
            q{i,j,m}(1,1)=max(S{i+1,j+1,k}(1,2)*(1-f)-Q,0);
                % number of call options required in node n in period m in preparation
                % for period k
            q{i,j,m}(1,2)=1*max(Q-(S{i+1,j,k}(1,2)*(1-f)),0);
                % number of put options required in node n in period m in preparation
                % for period k
            W{1,1,1}=0;
                % PLS NOTE THIS INITIAL CONDITION
            W{i+1,j+1,k}(1,1)=q{i,j,m}(1,2)*(-OP{i,j,m}(1,2)*rf)+q{i,j,m}(1,1)*(S{i+1,j+1,k}(1,1)-S{i,j,m}(1,1)-OP{i,j,m}(1,1)*rf)+dr*max(S{i+1,j+1,k}(1,2)-Q,0);
            W{i+1,j,k}(1,1)=q{i,j,m}(1,2)*(-OP{i,j,m}(1,2)*rf)+q{i,j,m}(1,1)*(S{i+1,j,k}(1,1)-S{i,j,m}(1,1)-OP{i,j,m}(1,1)*rf)+dr*max(S{i+1,j,k}(1,2)-Q,0);
            W{i,j+1,k}(1,1)=q{i,j,m}(1,1)*(-OP{i,j,m}(1,1)*rf)+q{i,j,m}(1,2)*(-S{i,j+1,k}(1,1)+S{i,j,m}(1,1)-OP{i,j,m}(1,2)*rf)+dr*max(S{i,j+1,k}(1,2)-Q,0);
            W{i,j,k}(1,1)=q{i,j,m}(1,1)*(-OP{i,j,m}(1,1)*rf)+q{i,j,m}(1,2)*(-S{i,j,k}(1,1)+S{i,j,m}(1,1)-OP{i,j,m}(1,1)*rf)+dr*max(S{i,j,k}(1,2)-Q,0);
                % gains from a financial carbon option in one node in period k due to the
                % position taken in period m
        end
    end
end

for i=1:t
    for j=1:t;
        O{i,j,t}=Y{i,j,t}+W{i,j,t};
    end
end

for k=t:-1:2;
    m=k-1;
    for i=1:m
        for j=1:m;
            O{i,j,m}=Y{i,j,m}+W{i,j,m}+r*P*[O{i+1,j+1,k};O{i+1,j,k};O{i,j+1,k};O{i,j,k}];
            % accumulated cash flows of cost saving starting from state w
            % in period i to period t.
        end
    end
end

% REAL OPTION ANALYSIS WITHOUT THE CONSIDERATION OF A
% FINANCIAL CARBON OPTION
k=15000; % strike price for the real option; capital outlay
for i=1:t
    for j=1:t;
        v{i,j,t}=Z{i,j,t}-k;
        v{i,j,t}(v{i,j,t}<0)=0;
        (v{i,j,t}>0);  % investment decision at the last period t
    end
end

for k=t:-1:2;
    m=k-1;
    for i=1:m
        for j=1:m;
            vinv{i,j,m}=Z{i,j,m}-k;
            % Value of the project if investing in one node in period m
            vcon{i,j,m}=r*P*[v{i+1,j+1,k};v{i+1,j,k};v{i,j+1,k};v{i,j,k}];
        end
    end
end
% Value of the project if waiting for investment beyond period j
V{i,j,m} = max(VINV{i,j,m}, VCON{i,j,m});
% The optimal value between investing-now and investing-later
(VINV{i,j,m} > VCON{i,j,m}); % investment decision in period i before t
end
end
end

% REAL OPTION ANALYSIS WITH THE CONSIDERATION OF A FINANCIAL CARBON OPTION
for i=1:t
    for j=1:t;
        VO{i,j,t} = O{i,j,t} - K;
        VO{i,j,t}(VO{i,j,t} < 0) = 0;
    end
end

for k=t:-1:2;
    m=k-1;
    for i=1:m
        for j=1:m;
            VINVO{i,j,m} = O{i,j,m} - K;
            % Value of the project if investing in one node in period m
            VCONO{i,j,m} = r*P*[VO{i+1,j+1,k}; VO{i+1,j,k}; VO{i,j+1,k}; VO{i,j,k}];
            % Value of the project if waiting for investment beyond period j
            VO{i,j,m} = max(VINVO{i,j,m}, VCONO{i,j,m});
            % The optimal value between investing-now and investing-later
            (VINVO{i,j,m} > VCONO{i,j,m}); % investment decision in period i before t
        end
    end
end
Code 2: MATLAB® code for the sensitivity analysis of the integrated model in
Section 3.4

clear all;
clc;
a=1.38:0.02:1.60;
b=1.26:0.01:1.50;
rf=1.05; % risk free interest rate
Q=1000; % carbon cap
t=6; % number of periods
β=0.2; % reduction factor of emissions by the environ project
for i=1:max(size(a))
    for j=1:max(size(b));
        u{i}(j,1)=a(i);
        u{i}(j,2)=b(j);
        factor{i,j}=[u{i}(j,1),u{i}(j,2); % uu factor (price,demand)
                     u{i}(j,1),1/u{i}(j,2); % ud factor (price,demand)
                     1/u{i}(j,1),u{i}(j,2); % du factor (price,demand)
                     1/u{i}(j,1),1/u{i}(j,2)]; % dd factor (price,demand)
        up(i,j)=factor{i,j}(1,1); % up factor of carbon price
        dp(i,j)=factor{i,j}(3,1); % down factor of carbon price
        p{i,j}=(RF-dp{i,j})/(up{i,j}-dp{i,j}); % risk-neutral probability
        P{i,j}=[0.5*p{i,j},0.5*p{i,j},0.5*(1-p{i,j}),0.5*(1-p{i,j})]; % probability vector of state transition
    end
end
for i=1:max(size(a))
    for j=1:max(size(b));
        for m=1:t-1;
            n=m+1; % m is n's previous period
            for w=1:4^(n-1);
                prevstat=ceil(w/4);
                factorlevel=mod(w,4);
                if factorlevel==0
                    factorlevel=4;
                end
            end
        end
    end
end
\[ S_{i,j,1} = [10,1000]; \]
\[ Y_{i,j,1} = \beta S_{i,j,1}(1,1) * S_{i,j,1}(1,2); \]
\[ \text{% the stream of cash flow of cost saving at the initial state} \]
\[ S_{i,j,n}(w,:) = S_{i,j,m}(\text{prevstat,:}) .* \text{factor}_{i,j}(\text{factorlevel,:}); \]
\[ \text{% possible state in period } j \text{ derived from its previous state} \]
\[ Y_{i,j,n}(w,:) = f * S_{i,j,n}(w,1) * S_{i,j,n}(w,2); \]
\[ \text{% one stream of cash flow of cost saving at state } w \text{ in period } j \]
\[ r = 1/1.05; \quad \text{% discount factor} \]
\[ \text{for } i=1:\text{max(size(a))} \]
\[ \quad \text{for } j=1:\text{max(size(b))}; \]
\[ \quad \text{for } m=t-1:2; \]
\[ \quad \text{for } w=1:4^{n-1}; \]
\[ Z_{i,j,t} = Y_{i,j,t}; \]
\[ \text{% boundary value for the last period} \]
\[ Z_{i,j,n}(w,:) = Y_{i,j,n}(w,:) + r * P_{i,j} * Z_{i,j,m}((4*w-3):(4*w,:),:); \]
\[ \text{% accumulated cash flows of cost saving starting from state } w \text{ in period } t. \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{end} \]
\[ \text{for } i=1:\text{max(size(a))} \]
\[ \quad \text{for } j=1:\text{max(size(b))}; \]
\[ \quad \text{for } m=1:t-1 \]
\[ \quad \text{for } w=1:4^{n-1}; \]
\[ \text{OP}_{i,j,m}(w,1) = p_{i,j} * (S_{i,j,n}(4*w-3,1)-S_{i,j,m}(w,1))/r; \]
\[
\text{% call option premium}
\]
\[
\text{OP}(i,j,m)(w,2)=rp*(1-p(i,j))*(S(i,j,m)(w,1)-S(i,j,n)(4*w-1,1))/rf;
\]
\[
\text{% put option premium}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
dr=0.50; \quad \text{% sales allocated for regulatory compliance}
\]
\[
\text{for i=1:max(size(a))}
\]
\[
\text{for j=1:max(size(b));}
\]
\[
\text{for m=1:t-1}
\]
\[
\text{n=m+1;} \quad \text{% i is j's previous period}
\]
\[
\text{for w=1:4^(n-1)}
\]
\[
\text{l=ceil(w/4);}
\]
\[
\text{q}(i,j,m)(l,1)=max(S(i,j,n)(4*l-3,2)*(1-f)-Q,0);
\]
\[
\text{% number of call options required in node n in period i in preparation}
\]
\[
\text{% for period j}
\]
\[
\text{q}(i,j,m)(l,2)=1*max(Q-S(i,j,n)(4*l-2,2)*(1-f),0);
\]
\[
\text{% number of put options required in node n in period i in preparation}
\]
\[
\text{% for period j}
\]
\[
\text{W}(i,j,1)(1)=0; \quad \text{% PLS NOTE THIS INITIAL CONDITION}
\]
\[
\text{W}(i,j,n)(w,:)=q(i,j,m)(l,2)*(-OP(i,j,m)(l,2)*RF+max(-S(i,j,n)(w,1)+S(i,j,m)(l,1),0))+q(i,j,m)(l,1)*(-OP(i,j,m)(l,1)*rf+max(S(i,j,n)(w,1)-S(i,j,m)(l,1),0))+dr*max(S(i,j,n)(w,2)-Q,0);
\]
\[
\text{% gains from financial carbon options in node w in period j due to the}
\]
\[
\text{% position taken in period i}
\]
\[
\text{W}(i,j,n)(W(i,j,n)<0)=0;
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{end}
\]
\[
\text{for i=1:max(size(a))}
\]
\[
\text{for j=1:max(size(b));}
\]
\[
\text{for m=t:-1:2}
\]
\[
\text{n=m-1;} \quad \text{% i is j's next period}
for \( w=1:4^{n-1} \)
\[
O_{i,j,t} = Y_{i,j,t} + W_{i,j,t};
\]
% boundary value for the last period
\[
O_{i,j,n}(w,:) = Y_{i,j,n}(w,:) + W_{i,j,n}(w,:) + r*P_{i,j} * O_{i,j,m}((4*w-3):(4*w,:),:);\]
% accumulated cash flows of cost saving starting from state \( w \)
% in period \( i \) to period \( t \).
end
end
end
end

% REAL OPTION ANALYSIS WITHOUT THE CONSIDERATION OF FINANCIAL CARBON OPTIONS

\( k=15000; \)  % strike price for the real option; capital outlay
for \( i=1:\text{max(size(a))} \)
    for \( j=1:\text{max(size(b))} \)
        for \( m=t:-1:2 \)
            \( n=m-1; \)  % \( i \) is \( j \)'s next period
            for \( w=1:4^{n-1} \)
                \[
V_{i,j,t} = Z_{i,j,t} - k;
\]
                % boundary condition for the project
                \[
V_{i,j,t}((V_{i,j,t} < 0)) = 0;
\]
                % non negative constraint as investment criteria
                \[
V_{INV_{i,j,n}}(w,:) = Z_{i,j,n}(w,:) - k;
\]
                % Value of the project if investing in node \( w \) in period \( j \)
                \[
V_{CON_{i,j,n}}(w,:) = r*P_{i,j} * V_{i,j,m}((4*w-3):(4*w,:),:);
\]
                % Value of the project if waiting for investment beyond period \( j \)
                \[
V_{i,j,n}(w,:) = \text{max}(Z_{i,j,n}(w,:) - k, r*P_{i,j} * V_{i,j,m}((4*w-3):(4*w,:),:));
\]
                % The optimal value between investing-now and investing-later
            end
        end
    end
end
end

\((V_{i,j,t} > 0)\);  % investment decision at the last period \( t \)
for \( m=3:-1:1 \)
(VINV\{i,j,m\}>\text{VCON}\{i,j,m\})' \quad \% \text{investment decision in period i before t}
end

\% \text{REAL OPTION ANALYSIS WITH THE CONSIDERATION OF FINANCIAL CARBON OPTIONS}
for i=1:max(size(a))
\quad for j=1:max(size(b));
\quad \quad for m=t:-1:2
\quad \quad \quad n=m-1; \quad \% i is j's next period
\quad \quad \quad for w=1:4^{(n-1)}
\quad \quad \quad \quad VO\{i,j,t\}=O\{i,j,t\}-k;
\quad \quad \quad \% \text{boundary condition for the project}
\quad \quad \quad VO\{i,j,t\}(VO\{i,j,t\}<0)=0;
\quad \quad \quad VINVO\{i,j,n\}(w,:)=O\{i,j,n\}(w,:)-k;
\quad \quad \quad \% \text{Value of investment if investing in node w in period j}
\quad \quad \quad VCONO\{i,j,n\}(w,:)=r*P\{i,j\}*VO\{i,j,m\}((4*w-3):(4*w,:),:);
\quad \quad \quad \% \text{Value of investment if waiting for investment beyond period j}
\quad \quad \quad VO\{i,j,n\}(w,:)=\max(O\{i,j,n\}(w,:)-k,r*P\{i,j\}*VO\{i,j,m\}((4*w-3):(4*w,:),:)); \quad \% \text{The optimal value between investing-now and investing-later}
\quad \quad \quad end
\quad \quad end
\quad end

(VO\{i,j,t\}>0)'; \quad \% \text{investment decision at the last period t}

for m=3:-1:1
\quad VINVO\{i,j,m\}>\text{VCONO}\{i,j,m\} \quad \% \text{investment decision at the last period t}
end
Code 3: IBM ILOG code for the SP formulation in determining optimal decisions on financial carbon options (model and data)

\{
{int} option={1,2};
float strikeprice[option]=[10,10];
float optioncost[option]=[2.403, 1.927]...;
{int} scenario={1,2,3,4};
float demand[scenario]=[120, -429, 120, -429];
float probscenario[scenario]=[0.218, 0.218, 0.282, 0.282];
float price[scenario]=[15.7, 15.7, 6.40, 6.40];
float totalbudget=[1300, 1300, 1300, 1300];
\}

//variables
dvar float optionuse[option];
dvar float exercise[option][scenario];
dvar float purchase[scenario];

minimize
sum (i in option)( optioncost[i]*optionuse[i])+
sum (s in scenario, i in
option)(probscenario[s]*(strikeprice[i]*exercise[i][s]*((-1)^{(i+1)})+price[s]*purchase[s]))
;

subject to
{sum (i in option) optionuse[i] <= totalbudget;

forall (i in option, s in scenario)
ct01:
((-1)^{(i+1)})*exercise[i][s] + purchase[s] == demand[s];

forall (i in option: i==1, s in scenario:s>2)
ct02:
exercise[i][s]==optionuse[i];

forall (i in option: i==1, s in scenario:s<=2)
\textbf{ct03:} \\
\texttt{exercise[i][s]==0;}

\texttt{forall (i in option: i==2, s in scenario:s>2)} \\
\textbf{ct04:} \\
\texttt{exercise[i][s]==0;}

\texttt{forall (i in option: i==2, s in scenario:s<=2)} \\
\textbf{ct05:} \\
\texttt{exercise[i][s]==optionuse[i];}

\texttt{forall (i in option)} \\
\textbf{ct06:} \\
\texttt{optionuse[i]>=0;}

\texttt{forall (i in option, s in scenario)} \\
\textbf{ct07:} \\
\texttt{exercise[i][s]>=0;}

\texttt{forall (i in option: i==1, s in scenario:s==1)} \\
\textbf{ct08:} \\
\texttt{optionuse[i]<=demand[s];}

\texttt{forall (i in option: i==2, s in scenario:s==2)} \\
\textbf{ct09:} \\
\texttt{optionuse[i]<=-demand[s];}

\}

\textbf{Data:} \\
\texttt{totalbudget=1300;} \\
\texttt{option={1, 2};} \\
\texttt{optioncost=[1.78, 1.64];} \\
\texttt{strikeprice=[10,10];} \\
\texttt{scenario={1,2,3,4};} \\
\texttt{price=[16,16,6.25,6.25];} \\
\texttt{demand=[200,-500,200,-500];} \\
\texttt{probscenario=[0.25,0.25,0.25,0.25];}
APPENDIX C

SOLUTIONS FOR THE NUMERICAL EXAMPLES

By utilizing the computer codes in Appendix B, we solved the problems and the solutions are shown in the following tables. In each table, the numbers 1-6 in the first row denote the period and the numbers 1-36 in the first column stand for the state in each period.

Table C.1 Solutions for the project value with a real option

Table C.2 Solutions of the investment decisions with the consideration of a real option

Table C.3 Solutions for the project value with real and financial carbon options

Table C.4 Solutions of the investment decisions with the consideration of real and financial carbon options

Table C.5 Solutions for the project value with a real option solved by Code 1 and Code 3

The values below (excluding those in the first row and the first column) are the net present value of the project with a real option for a given state. The values in the second row are the expected net present values for its respective period.

<table>
<thead>
<tr>
<th>Period</th>
<th>Expected net present value</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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Table C.6 Solutions of the investment decisions with the consideration of a real option solved by Code 1 and Code 3

In this table, decisions on whether to set up the project in each period are listed out when the real option is considered. Depending on the state realized in each period, the investment decision is different.

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Table C.7 Solutions for the project value with real and financial carbon options solved by Code 1 and Code 3

The values below (excluding those in the first row and the first column) are the net present value of the project with both options for a given state. The values in the second row are the expected net present values for its respective period.

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Table C.8 Solutions of the investment decisions with the consideration of real and financial carbon options solved by Code 1 and Code 3

This table maps out the decisions on the project in each period when both options are considered. Note that the italicized cell means that the decision on the investment time for the project with real and financial carbon options is different from that for the project with real option only.

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