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The Effects of Simple Coupled Volume Geometry on the Objective and Subjective Results from Nonexponential Decay

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This project focuses on the individual and interactive acoustic effects of three architectural parameters on the double slope profile from a simple coupled volume system created in the computer modeling program ODEON. The three variables studied are the volume ratio between the main and secondary spaces, the absorption ratio between the two spaces, and aperture size. The resulting energy decay profiles are analyzed using T_{30}/T_{15} coupling coefficient ratios and Bayesian analysis. Coupling coefficient results show general trends in the effects of the three architectural parameters that match previous research results and the predominant interactive effect between the three variables involving a large coupled volume. Similar results are suggested by the Bayesian analysis based on the newly developed quantifiers, decay ratio and ΔdB, although a more complex relationship among the variables may exist at larger volume ratios. A subset of the simulated sound fields have been auralized and used to conduct psychoacoustic testing. The subjective testing results indicate that perceived reverberation increases with greater coupled volume size and aperture size, correlating well with objective results. The outcomes also suggest that higher perceived reverberation coincides with larger decay ratios and smaller ΔdB values. Subjective results based on clarity showed no significant effects.
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I. INTRODUCTION

Coupled volume geometries have been used in several modern concert halls, including Festival Hall in Tampa, FL; the Great Hall in Hamilton, Ontario; Lucerne Concert Hall in Lucerne, Switzerland; the Myerson-McDermott Hall in Dallas, TX; and Verizon Hall in Philadelphia, PA. Architectural acousticians have been particularly interested in halls that utilize coupled volume systems because of their potential for creating a nonexponential energy decay. This paper focuses on the effort to articulate more clearly the effect of certain architectural parameters on the nonexponential decay in coupled volume spaces, both from objective and subjective viewpoints. Objectively the authors explore new ways of quantifying the phenomenon of nonexponential decay, while subjective psychoacoustic tests have been conducted to learn more about its perception.

A system utilizing coupled volumes typically consists of two or more spaces that are connected through an acoustically transparent opening known as a coupling aperture. When the secondary space exhibits a decay time that is longer than that of the main space, sound energy will be fed back into the main space by the auxiliary volume. This late-arriving energy results in the phenomenon known as the double slope decay.

A coupled volume system consisting of two spaces is shown in Fig. 1, where V_i is the volume of the ith space, A_i is the total surface area of the ith space, S is the surface area of the coupling aperture separating the two spaces, and P is the power of the sound source in room 1. Also, α_i is defined as the average absorption coefficient of the ith space, c as the speed of sound, and E_i as the energy density in the ith space. The power balance equations for the two rooms can be written as

$$P-\frac{A_1\alpha_1 cE_1}{4}-\frac{ScE_1}{4}+\frac{ScE_2}{4}=0 \tag{1}$$

$$-\frac{A_2\alpha_2 cE_2}{4}-\frac{ScE_2}{4}+\frac{ScE_1}{4}=0. \tag{2}$$

These equations give us the steady state response of the system. By setting $P=0$, we can obtain the differential equations representing the reverberant sound decay in the two spaces:

$$\frac{c}{4}(A_{1S}E_1-SE_2)=-V_1\frac{dE_1}{dt}, \tag{3}$$

$$\frac{c}{4}(-SE_1+A_{2S}E_2)=-V_2\frac{dE_2}{dt}, \tag{4}$$

where $A_{iS}=A_i\alpha_i+S$, and exponential decay is assumed such that

$$E_i=E_{i0}e^{-2\delta t}, \tag{5}$$

where E_{i0} is the initial energy density in the ith space and δ is the decay constant of the space. By substituting Eq. (5) into Eqs. (3) and (4), and simplifying, we can rewrite the system of equations in matrix form:

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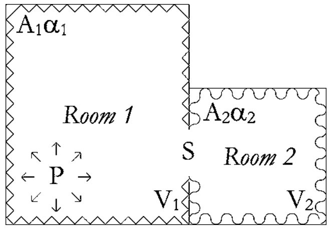

FIG. 1. Plan view of a coupled volume system used in theoretical development.

$$\begin{bmatrix} \left[\left(\frac{c}{4}\right)A_{1S}-2\delta V_1\right] & -\left(\frac{c}{4}\right)S \\ -\left(\frac{c}{4}\right)S & \left[\left(\frac{c}{4}\right)A_{2S}-2\delta V_2\right] \end{bmatrix} \cdot \begin{bmatrix} E_{10} \\ E_{20} \end{bmatrix} = 0. \quad (6)$$

This system can only be valid for a single value of δ if the ratio E_{10}/E_{20} is the same for both equations, which requires the determinant of the coefficients of E_{10} and E_{20} to equal zero, producing the following polynomial equation:

$$4V_1V_2\delta^2 - \left(\frac{c}{2}\right)(A_{1S}V_2 + A_{2S}V_1)\delta + \left(\frac{c^2}{16}\right)(A_{1S}A_{2S} - S) = 0. \quad (7)$$

The eigenvalues of this quadratic equation give the decay constants for each room, δ_1 and δ_2, as a function of the volumes, surface areas, and absorptions of the two rooms, and the speed of sound. The decay time (T_i) for room i alone can then be calculated from δ_i using

$$T_i = \frac{6.9}{\delta_i} \quad (8)$$

in S.I. units.

To visualize the reverberant process as a temporal phenomenon, it is helpful to plot the sound level as a function of time. Cremer *et al.*[1] give the relationship between T_i, E_{i0}, and a reference sound energy density, E_{ref}, as

$$L_i(t) = -\left(\frac{60}{T_i}\right)t + 10\log_{10}\left(\frac{E_{i0}}{E_{\text{ref}}}\right), \quad (9)$$

where $L_i(t)$ is the sound pressure level in the ith space. The early and late temporal components of the decay in the first room can be derived from this equation. For the case where $\delta_1 > \delta_2$, the early portion is dominated by the autonomous effects of room 1 and the late portion is driven by the effects of room 2 on room 1, given by the following:

$$L_{1\text{early}}(t) = -\left(\frac{60}{T_1}\right)t + 10\log\left(\frac{E_{10}}{E_{\text{ref}}}\right), \quad (10)$$

$$L_{1\text{late}}(t) = -\left(\frac{60}{T_2}\right)t + 10\log\left(\frac{E_{21}}{E_{\text{ref}}}\right), \quad (11)$$

where the initial energy density in room 1 caused by the energy transferred from room 2 to room 1 is given as

$$E_{21} = k_1k_2E_{10} \quad (12)$$

and k_1 and k_2 are coupling factors characterizing the transfer of energy between the rooms based on their geometry and absorption, defined as

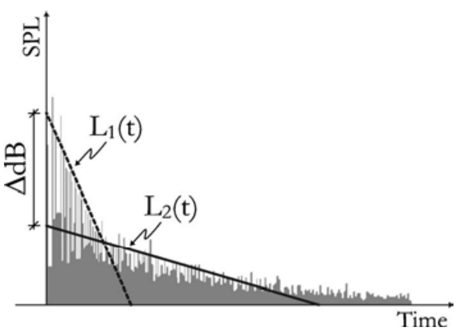

FIG. 2. Representative energy decay curve of a space exhibiting a double slope profile.

$$k_1 = \frac{S}{A_{1S}}, \quad k_2 = \frac{S}{A_{2S}}. \quad (13)$$

The term E_{ref} in Eqs. (10) and (11) may be chosen to be equal to E_{10}, thereby normalizing the sound level in room 1, $L_1(t)=0$ dB at $t=0$. Then substitution of Eqs. (12) and (13) into Eq. (11) and further simplification yields the following early and late decay equations for room 1:

$$L_{1\text{early}}(t) = -\left(\frac{60}{T_1}\right)t, \quad (14)$$

$$L_{1\text{late}}(t) = -\left(\frac{60}{T_2}\right)t - 10\log\left(\frac{A_{1S}A_{2S}}{S^2}\right). \quad (15)$$

These equations are in $y=-mx+b$ form, from which we can see that the early and late portions of the decay are two lines, each with a slope and y intercept that can be calculated from geometrical parameters of the coupled volumes. The above-mentioned theoretical development is a modified version of that given in Cremer *et al.*[1] For a more detailed description of this analysis, the reader is referred to Refs. 1–5.

The above-mentioned results allow for the decay slope of the main room in a coupled volume system to be analyzed based on its two temporal components, early decay and late decay. Many room acousticians believe that a steep early decay slope allows for a high level of perceived clarity, whereas a shallow late decay slope leads to a more lingering reverberance.[4] Clarity describes the definition of a sound in a space, or how easily different parts of a sound signal can be differentiated from one another. Reverberance describes the fullness of tone, and refers to sound energy that persists in a room after a sound is suddenly stopped. Both clarity and reverberance are desired quantities in concert hall acoustics, but seem to have contradictory decay slope criteria. This apparent contradiction may be alleviated in coupled volume systems because $L_1(t)$ and $L_2(t)$, which can exhibit differing slope characteristics, dominate the decay at different times. Figure 2 shows a representative decay outline for this situation, where $L_1(t)$ is shown by the dashed line and $L_2(t)$ by the solid line.

This so-called double slope decay, or double slope effect (DSE), shown by the bold lines in Fig. 2, may result in high clarity and reverberation. The term DSE will be used in this paper to refer to the phenomenon in which the secondary decay is longer than the first, and in which it dominates the decay profile during the latter portion of time. Although coupled volume systems can be designed to exhibit a variety of acoustic effects, both with and without DSE, this work will focus on those which incorporate auxiliary volumes external to a main space connected through apertures specifically designed to obtain DSE. The authors refer to such systems as dedicated coupled volume systems.

II. PREVIOUS RESEARCH

As can be seen from the above-noted theoretical development, the DSE in a coupled volume system is affected by several variables, namely main volume size, coupled volume size, aperture size, and level of absorption in the two spaces. The effects of these variables, as well as source-receiver positions and the relationship between theoretical and computational analyses of coupled systems, have been explored to a limited degree in previous studies.

Eyring conducted a study of several coupled room configurations in an attempt to develop empirical modifications to theoretical formulas.[6] Eyring's configurations examined three distinct variables: absorption in both rooms, aperture size, and the source-receiver positions. Specifically, Eyring found that the double slope effect is most noticeable when a large absorptive room is coupled with a smaller live room. Additionally, for his particular configurations, Eyring posited that the coupled volume effect occurs only for absorption coefficients of less than 0.07 in the coupled room. Finally, he suggested that DSE only occurs when the source and receiver are placed in the same room, particularly, the less reverberant room. This work began to shed light on the relationship between the architectural parameters of the system and the DSE; however, more systematic analysis of these variables was needed.

Harrison and Madaras utilized computer modeling of coupled volumes to study the effects of aperture size and coupled volume size.[7] They quantified the coupled volume effect with a ratio of T_{30}/T_{15}, referred to as the "coupling coefficient." This ratio gives the relationship between two different portions of the reverberant decay. T_{30} is defined as the decay time from −5 to −35 dB in the energy decay function, multiplied by a factor of 2, such that T_{30} is on the same order of magnitude as T_{60}. Similarly, T_{15} is given as the decay time from −5 to −20 dB, multiplied by a factor of 4. Although T_{30}/T_{15} is an adequate indicator of DSE, it does not rigorously differentiate between different double slope profiles. Particularly, a specific coupling coefficient can be used to describe several different double slope profiles. This ambiguity compromises the effectiveness of this descriptor as a metric of the DSE.

Harrison and Madaras showed that T_{30}/T_{15} is maximized for smaller aperture opening sizes and generally increases exponentially with increasing coupled volume sizes. The researchers also pointed out that the shape, location, and configuration of the aperture openings had noticeable, but less significant effects on the acoustic performance than did the volume of the coupled space and aperture size.

Ermann and Johnson studied how the architectural variables, aperture size, and coupled volume absorption affect the decay slope of a coupled system.[8] To quantify the coupled volume effect, the investigators used the ratio of T_{60}/T_{15}, referred to as the "coupling constant," which is a slight variation on Harrison and Madaras' coupling coefficient. This ratio has problematic issues, similar to the coupling coefficient, namely that it is not a rigorous measure of DSE. Additionally, this method obtains a T_{60} value from a straight line fit, which is a poor representation of the actual double sloped decay function and could lead to great inaccuracies. Thus, coupling constant is only a rough indicator of DSE.

The Ermann and Johnson study showed that having an average absorption coefficient in the coupled space less than 0.02 ($\bar{\alpha} < 0.02$) produced significantly higher coupling constants than $0.02 < \bar{\alpha} < 0.05$. The coupling constant approached infinity as $\bar{\alpha}$ decreased to zero, and asymptotically approached unity for $\bar{\alpha} \geqslant 0.10$. The results regarding the effect of aperture size indicate that the coupling constant peaks when the aperture size is 1% of the total main volume surface area, while the coupled volume effect declines dramatically for aperture sizes larger than 1% of the total main volume surface area.

Summers extended the earlier theoretical work into the computational domain using CATT Acoustic computer models of coupled spaces.[9] His work analyzed the limits of statistical models by comparing them to computer modeled results, subsequently making refinements to the basic formulas of statistical coupling. Results showed that high levels of coupling caused large deviations between statistical predictions and computational values. Additionally, the results showed that all assumptions for single rooms, such as a diffuse sound field with even distribution of absorption, must hold true in the coupled system so that theoretical and computational methods may agree.

Research on subjective perception of nonexponential decays has also been undertaken. Atal *et al.* conducted an investigation to determine the correlation between the subjective feeling of reverberation and nonexponential decays.[10] In this study, nonexponential decays were produced by manipulating reverberated signals obtained from computer-simulated reverberators using combfilters connected in parallel. The decays exhibited a flat response across frequency, indicating that each normal mode had an equal decay rate. The researchers carried out pairwise comparison subjective tests between classic exponential decays and the artificially generated nonexponential decays. The reverberation time corresponding to the exponential decay chosen as most similar to a particular nonexponential decay was designated as the subjective reverberation (T_s). The investigators found that this T_s was highly correlated with a new reverberation time metric T(160 ms). T(160 ms) was found by fitting a straight line through the first 160 ms of the nonexponential decay and extrapolating to 60 dB down. This result indicates that the most important factor in determining the subjective evaluation of reverberation in nonexponential decays is the early temporal content of the decay. Further investigation, though, found that introducing the complexity of frequency dependence found in typical concert halls confounded the correlation between the subjective impression of reverberation and the early portion of non-exponential decay.

Picard also constructed a series of artificial nonexponential decays by splicing together pairs of impulse responses exhibiting exponential decay profiles.[11] Subjective testing was conducted in an effort to determine just noticeable differences between exponential and nonexponential decays. The results showed that subjects could more easily perceive a difference between the decays when the y intercepts of the two slopes were minimized. Additionally, difference perception increased as the difference between the two slopes in the nonexponential decays was increased. In the study, Picard suggested a quantitative descriptor for nonexponential decay based on the amount of energy from the latter secondary decay that is added above the original exponential decay.

Ermann conducted a similar subjective perception study, using computational geometric acoustics modeling to generate exponential and nonexponential decays.[12] The results from subjective tests utilizing these decays corroborated those found by Picard, showing that subjects are more likely to recognize a difference between two decay curves when the difference between the first and second slopes in the nonexponential decay is increased.

The research in this paper uses room acoustic computer modeling techniques to extend the work on quantifying the effect of coupled room geometry by more comprehensively analyzing the individual and interactive effects of three architectural parameters found to be significant in previous work: the volume ratio between the main and secondary spaces, the absorption ratio between the two spaces, and the aperture size. In this paper, the coupling coefficient is used to compare against previous research, and new DSE quantifiers are additionally introduced to differentiate the nonexponential decays with greater precision. The study then continues to extend the previous work by correlating subjective responses with the coupled geometry itself. Together, the objective and subjective analyses help to provide a better understanding of how architectural parameters affect DSE in coupled volume systems.

III. DEVELOPMENT OF NEW DSE QUANTIFIERS

A. Decay ratio and ΔdB

As discussed previously, the coupling coefficient obtained from the ratio of T_{30}/T_{15} does not uniquely represent the double slope effect apparent in an energy decay curve associated with a coupled volume system, and thus may not accurately indicate its perception. A more accurate method of characterizing the DSE may be achieved by analyzing the decay as a composite of two distinct slopes. The energy decay curve shown in Fig. 2 will be used as an illustrative example. The steep slope characterizing the early decay, shown as a dashed line in Fig. 2, will be referred to as slope 1. The shallower slope representative of the late decay, depicted as a solid line in Fig. 2, will be referred to as slope 2 (noise floor is disregarded during slope calculation). A time decay, which is inversely proportional to the slope, can be calculated for each line, giving Decay1 and Decay2. The quotient of the two time decay quantities will be defined as a parameter called decay ratio:

$$\text{decay ratio} = \frac{\text{Decay 2}}{\text{Decay 1}} = \frac{T_{60_2}}{T_{60_1}}. \tag{16}$$

A second parameter, ΔdB, can be defined by the difference between the y intercepts of each of the two slopes. The dB level of slope 1 is given by the starting level of its energy decay curve. The dB level of slope 2 is found by extending the slope to the ordinate and determining the level at time zero of its energy decay curve. ΔdB is found by subtracting the second level from the first. In the earlier theory section, the early and late decay functions were normalized such that

$$\Delta dB = 10\log\left(\frac{A_{1S}A_{2S}}{S^2}\right). \tag{17}$$

The decay ratio and ΔdB provide more information about the double slope characteristic of a particular energy decay curve than the coupling coefficient; thus, they can be used to more distinctly describe the DSE produced by a coupled volume system. Additionally, if one of the decay rates is specified, then the DSE may be uniquely defined.

Previous subjective research indicated that the difference between exponential and nonexponential decays was more easily perceived when the two slopes of the nonexponential decays were more different,[11,12] and when the y intercepts of the two slopes were minimized.[11] Note that these two trends correspond to the two proposed quantifiers: more perceptible DSE with larger decay ratio and smaller ΔdB.

The calculation of decay ratio and ΔdB requires four quantities from an energy decay function: the two slopes of the decay, and the y intercepts of those slopes. Obtaining these data from virtual and real coupled volume rooms can prove to be problematic. Particularly, determining the presence of nonexponential decay from visual inspection of the energy decay curve resulting from the Schroeder backwards integration method can be difficult. This is because background noise can alter the shape of the Schroeder curve, depending on the upper time limit of integration.[13] An analytical method is necessary to obtain accurate slope and relative dB level data for the new DSE quantifiers. One such method can be found through the use of Bayesian statistics.

B. Bayesian analysis

Xiang and Goggans developed Bayesian analysis for use in room acoustics and specifically for studying multirate decay impulse responses.[13,14] Bayesian model-based parameter estimation is used to produce an algorithm for the evaluation of multirate decay functions. The method allows for the estimation of the number of decay rates present in a multirate decay curve without requiring an initial guess on the number of slopes inherent in the decay. Additionally, this analysis can be used to determine the parameters of the decay profile, namely the slopes of the decays and ordinate intercepts of those slopes. Bayesian analysis also provides a major improvement over the classical least-squares approach. In the least-squares method, the initial estimations of parameters

TABLE I. Dimensions of main and coupled volumes with volume ratio percentage values used in computer model configurations.

	l (m)	w (m)	h (m)	V(m^3)	Ratio level	Percentage of main volume
Main	34	28	26	24752		
Coupled 1	21	18	13	4914	1	20%
Coupled 2	23	22	17	8602	2	35%
Coupled 3	26	24	20	12480	3	50%

that describe an impulse response must be relatively close to actual values for iteration convergence to occur. The Bayesian method avoids this problem, as careful estimates of the initial values are not required.

In Xiang and Goggans' work, the Bayesian analysis method is shown to be a suitable approach to predicting the decay rates from sound energy decay functions. There is found to be little dependence on the upper time limit of integration or signal to noise ratio. Bayesian analysis can therefore be used to determine the slopes and intercepts in a multirate decay function, and when applied to double slope decay data, it yields the values necessary to calculate the parameters of decay ratio and ΔdB.

IV. COMPUTATIONAL ANALYSIS

This research project uses computer modeling of room acoustics to determine the individual and interactive effects of three architectural parameters on DSE: the volume ratio between the main and secondary spaces, the absorption ratio between the two spaces, and the aperture size. These three variables were modified in a simplified coupled volume system composed of two rectangular boxes connected through one acoustically transparent aperture. The coupled system was realized as a three-dimensional space and studied using the ODEON v5 computational modeling program.[15,16] Although it is beyond the scope of this paper to verify computer modeling as a tool to study DSE, ongoing work by the authors has validated this methodology.[17]

The ranges of values for the architectural parameters in question were developed based on data from existing halls that utilize dedicated coupled volume systems in their design. Additionally, consideration was taken from the values given by the spaces studied in the previous research mentioned earlier.

The first architectural parameter, volume ratio, quantifies the size of the coupled volume with respect to that of the main volume, and is represented as a percentage in this research. Existing dedicated coupled volume halls reviewed for this study have volume ratios ranging from 28% to 50%. Therefore, three volume ratios closely matching this range have been used in this study: 20%, 35%, and 50%. In the computer models, the main volume was held constant at 24 752 m^3, and the coupled volume was varied to obtain the respective volume ratio percentage. Details are provided in Table I.

The second architectural parameter is the absorption ratio. This ratio depicts the equivalent absorption area in the coupled volume as a percentage of the equivalent absorption area in the main volume. The equivalent absorption area in each space, with units of m^2-sabins, is calculated by multiplying its total surface area by its average absorption coefficient, which in this study has been applied uniformly to all surfaces in the space. Absorption ratio percentage values range from 3% to 14% in the existing halls reviewed for this study; four levels across this range have been modeled. The main volume's absorption coefficient in the computer models was held constant at 0.25. Since the surface area in the coupled space changes as its volume changes, the resulting absorption coefficients in the coupled space varied across volume ratio in order to achieve the desired absorption ratio. Values for the four levels of absorption ratio used in this study, labeled as a, b, c, and d, and the respective coupled volume absorption coefficients are shown in Table II.

The final varying parameter is aperture opening size. In this study, aperture opening size is defined as the surface area of the opening between the main and coupled volumes, given as a percentage of the available aperture area. The available aperture area is defined by the surface area of the intersection of the two volumes, which equals the surface area of one face of the coupled volume, and thus varies with each volume ratio. Five percentages of this available area have been used for the aperture opening size, as shown in Table III. The aperture opening size can also be given as a percentage of the main volume surface area, a quantity commonly used in earlier research; those corresponding percentages are also listed in Table III.

Varying the three parameters resulted in a total of 60 different combinations. Each combination was created as one of 60 model configurations in the ODEON program. Figure 3 shows a representative model configuration.

In addition to the previously listed absorption coefficients, the surfaces in each computer model were assigned a uniform scattering coefficient of 0.3. This value is usually a

TABLE II. Absorption ratio values used in computer model configurations.

				Absorption ratio percentages			
				(a) 3%	(b) 7%	(c) 11%	(d) 15%
Volume ratio	Main surface area (m^2)	Main absorption coefficient	Coupled surface area (m^2)	Coupled absorption coefficients			
(1) 20%	5128	0.25	1770	0.02	0.05	0.08	0.11
(2) 35%	5128	0.25	2542	0.01	0.03	0.05	0.07
(3) 50%	5128	0.25	3248	0.01	0.03	0.04	0.02

TABLE III. Aperture opening size as a percentage of available surface area used in computer model configurations.

		Percentage of available aperture				
		2%	5%	10%	20%	40%
Volume ratio	Available aperture area m^2	Percentage of main surface area				
(1) 20%	234	0.09%	0.23%	0.46%	0.91%	1.83%
(2) 35%	374	0.15%	0.36%	0.73%	1.46%	2.92%
(3) 50%	480	0.19%	0.47%	0.94%	1.87%	3.74%

satisfactory, average scattering coefficient for use in room acoustic computer modeling,[15,16] although with a low level of geometric model detail, as used in this study, it may result in a less diffuse sound field.[18] Each of the 60 models was processed in ODEON to produce an impulse response for each configuration. An omni-directional source was used with an impulse response length of 4000 ms and 1705 rays. Generally, a higher number of rays will lead to denser reflection information in the impulse response, since more rays are able to travel in and out of the secondary space. Current research incorporating computer modeling of coupled volume spaces shows a tendency to use a higher number of rays,[19] and future research should further follow this issue in a more systematic way. For the study discussed here, though, inspection of ray-tracing graphics in ODEON produced satisfactory results showing that a sufficient amount of rays had traveled between the main and secondary spaces.

Objective measures, such as T_{15} and T_{30} values, were subsequently extracted from each impulse response. Finally each impulse response was evaluated with Bayesian analysis to obtain its decay slopes and y intercepts.

V. COMPUTATIONAL RESULTS

A. Coupling coefficient results

The coupling coefficients for the 60 models were found by calculating the respective T_{30}/T_{15} ratios. These coupling coefficients are plotted as a function of absorption ratio and aperture size for the three volume ratios in Fig. 4. A few general trends can be noted regarding the individual effects of the architectural parameters on the quantifier, coupling coefficient.

Looking across volume ratio levels, we see that coupling coefficient generally increases as the size of the coupled volume increases. This effect is in accordance with physical expectations. A larger coupled volume would allow sound energy to dissipate more slowly, feeding back into the main volume over a longer period of time, increasing the late energy in the main volume.

Across absorption ratio, coupling coefficient is shown to decrease as the level of absorption in the coupled volume increases. Again, this result is corroborated by the behavior of sound in the physical world. For larger values of α in the coupled volume, sound energy would be absorbed more rapidly. Therefore, the disparity between the early and late energy would decrease due to the smaller amount of late energy being fed back into the main hall.

Finally, in observing the general effect of aperture size, note that coupling coefficient peaks at a particular opening size for each volume/absorption combination, which is typically a small percentage of the total surface area of the main volume. In the physical world, smaller aperture sizes would force sound energy from a relatively reverberant coupled volume to leak back into the main volume more slowly, thus increasing the late decay time. Aperture sizes that are too small, however, would reduce the probability of the sound reentering the space before dissipation, and would therefore decrease the chances of obtaining a double slope decay profile. These observations are in line with the results found in previous research.

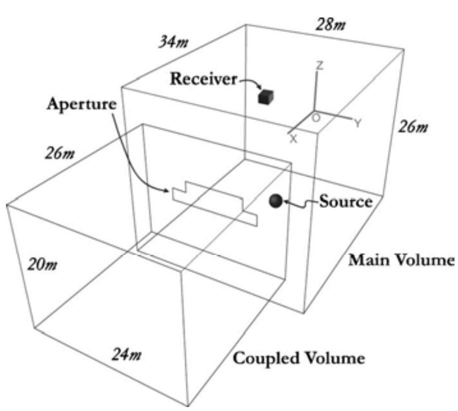

FIG. 3. Representative model configuration: volume ratio 3 and aperture size 10% of available area, with source at coordinates (25, 5, −5) and receiver at (5, −5, 5).

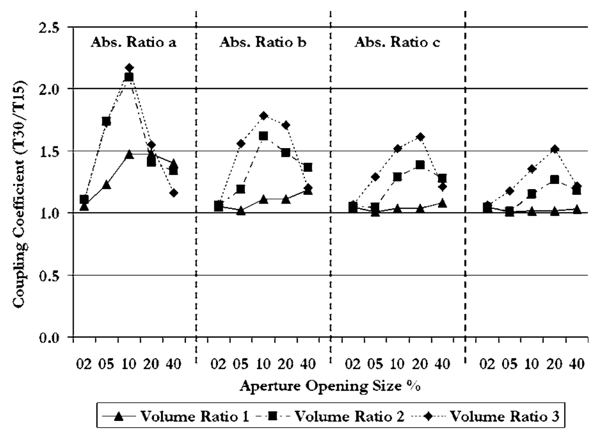

FIG. 4. Coupling coefficient data for all 60 configurations.

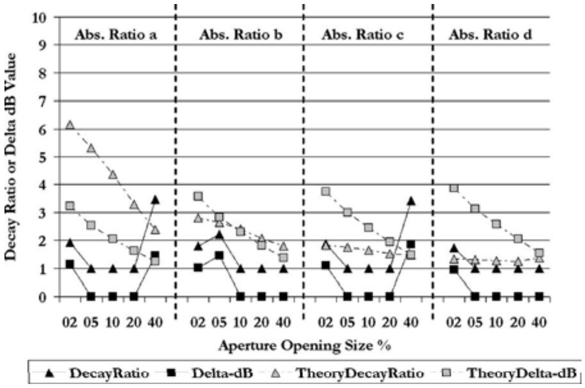

FIG. 5. Computer modeled and theoretical decay ratio and ΔdB data for volume ratio 1.

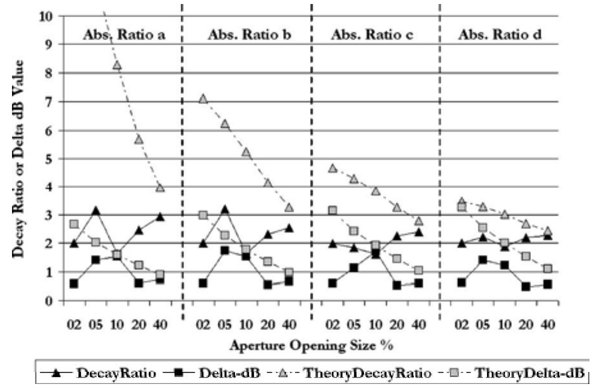

FIG. 7. Computer modeled and theoretical decay ratio and ΔdB data for volume ratio 3.

The results in Fig. 4 also provide insight into the interactions between the three parameters studied. Note that larger coupled volumes with aperture sizes approximately 10%–20% of the available aperture area (or roughly 1% of main volume's surface area) produce high coupling coefficient values. This effect is maximized with lower absorption ratios. A small volume ratio, though, only results in higher coupling coefficients if the absorption ratio is also quite low. In summary, the coupled volume size seems to have the largest effect on coupling coefficient; if it is not large enough, then high coupling coefficient can only be achieved with very low absorption ratios.

B. Decay ratio and ΔdB results

The results based on the newly suggested parameters of decay ratio and ΔdB, determined from Bayesian analysis, are more difficult to interpret than the coupling coefficient results because they provide more information about the double slope decay curves that has not been well-linked to subjective perception of DSE yet. The black data points and lines in Figs. 5–7 show the computer modeled results, with each graph representing a different volume ratio. The ΔdB values shown are normalized by dividing each value by the average ΔdB value across all configurations: 10.5 dB. This normalization was conducted in order to display the ΔdB data in the same range as the decay ratio data; this procedure has no influence on the overall trend of the data.

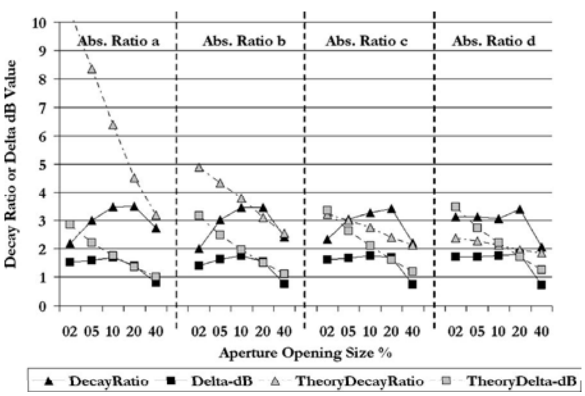

FIG. 6. Computer modeled and theoretical decay ratio and ΔdB data for volume ratio 2.

For the smallest volume ratio 1 (Fig. 5), decay ratio and ΔdB follow the same pattern across configurations with the majority of cases demonstrating decay ratio values of one and ΔdB values of zero. Such decay ratio and ΔdB values indicate an energy decay function from which the Bayesian analysis detected no double slope. In general, the results from this volume ratio suggest that smaller coupled volume sizes do not readily produce DSE. A nonexponential decay curve seems to be found at lower and higher aperture sizes, but not in a consistent or expected fashion across the cases. It is possible that the simplified geometry or selection of calculation parameters have compromised the results in these cases.

For volume ratio 2 (Fig. 6), decay ratio and ΔdB follow the same general pattern with all combinations of aperture size and absorption ratio producing some degree of nonexponential decay. Both decay ratio and ΔdB values peak at 10%–20% of the available aperture area (or roughly 1% of the main volume's surface area) for each absorption ratio. These results match the coupling coefficient trends in Fig. 4 for this volume ratio and those obtained from previous research.

As with volume ratio 2, the results for largest volume ratio 3 (Fig. 7) show all combinations of aperture size and absorption ratio producing some degree of nonexponential decay; however, the decay ratio and ΔdB do not behave similarly across configurations for this large coupled volume size. For example, there is a distinct difference at 10% aperture opening for the lower absorption ratios. Decay ratio and ΔdB do not necessarily have to show the same trends, though; as a matter of fact, that is why decay ratio and ΔdB are being suggested as more accurate quantifiers in this paper. Having both of these descriptors provides a more distinct description of nonexponential decays within coupled volume systems. Figure 8 demonstrates a number of cases of different combinations of decay ratios and ΔdB, where decay ratio may be large while ΔdB small and vice versa. What remains unclear at this point is how these different nonexponential decays are perceived. Subjective testing is required to study further how decay ratio and ΔdB may be linked to perception of DSE.

The theoretical derivation given earlier in this paper showed how decay ratio and ΔdB can be calculated from

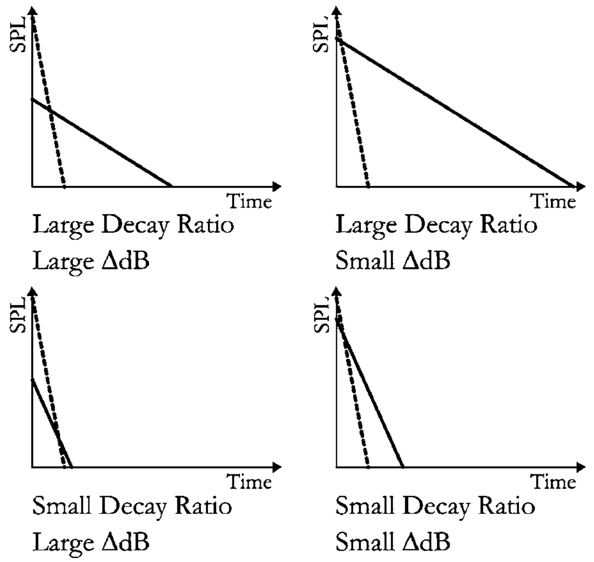

FIG. 8. Four examples of double slope decay outlines showing different combinations of decay ratio and ΔdB.

knowledge of the geometrical parameters. These theoretical values have been calculated for each of the 60 configurations used in the computer simulations, and are shown by the gray data points and dashed lines in Figs. 5–7. The theory expects decay ratio to decrease as aperture size increases, decrease as absorption ratio increases, and increase as the coupled volume increases. Recall that previous research found that nonexponential decays with larger decay ratios were more easily differentiated from exponential decays; the theory finds that larger decay ratios are obtained with smaller aperture openings, smaller absorption ratios, and larger volumes (with some values off the y-axis range in Figs. 6 and 7). The computer modeled decay ratio data generally matches these trends, but not completely across all configurations. The main differences are that the computer modeled behavior at the smallest aperture openings always demonstrates smaller decay ratios. Also the computer modeled results do not show a clear increase in decay ratio when increasing from volume ratio 2 to 3.

The theoretical ΔdB values decrease as aperture opening increases, increase as absorption ratio increases, and decrease as the coupled volume increases. Recall that previous research found that nonexponential decays with smaller ΔdB were more easily differentiated from exponential decays; the theory finds that smaller ΔdB are obtained with larger aperture openings, smaller absorption ratios, and larger volumes. The computer modeled ΔdB results show similar tendencies for aperture size and volume, but the trend with absorption ratio is not found.

The differences between the theoretically expected decay ratio and ΔdB values and the computer modeled results obtained from Bayesian analysis could be due to the following factors. Summers' work indicated that results between computer models and statistical theory deviate when diffuse fields are not achieved, and also when there is a high level of coupling.[9] The computer modeling parameters, particularly the scattering coefficient and number of rays used, may have detracted from the diffuseness of the generated sound fields. Another limitation in both the theory and the computer modeling analysis involves the behavior at small aperture openings. In the physical world, impedance effects at the boundary and considerations of wavelength compared to opening size could reduce the decay ratio values, so that the high values of decay ratio predicted theoretically would be invalid. Furthermore, high decay ratios are usually accompanied by higher uncertainty values in the Bayesian analysis, as explained by Xiang *et al.*[20]

Although the decay ratio and ΔdB results from theory and from Bayesian analysis of the computer modeled impulse responses do not perfectly match, it is still of interest to use the simulated results to learn more about the subjective perception of DSE.

VI. SUBJECTIVE ANALYSIS

To help reach the goal of having an objective measure of DSE that correlates to subjective response, psychoacoustic testing has been conducted. Another purpose of this testing was to determine the effect of the architectural parameter variation on subjective response to DSE. A subset of the impulse responses generated in the computational phase of this study was used in the subjective testing phase. The subset consisted of the sound fields simulated in the combinations of the volume ratio and aperture opening size variables, producing 15 impulse responses in total. The absorption ratio for these combinations was held constant at the lowest level of 3%, since lower levels are most likely to produce DSE according to theory and previous research. Subjects were asked to rate the perceived reverberation and clarity from a series of sound tracks. These two acoustic qualities were chosen as the subjective variables because DSE is allegedly related to having both high reverberance and clarity. Combining the results from subjective perception of these two qualities would indicate the broader psychoacoustic response to DSE.

Auralizations were produced in ODEON by convolving the impulse responses for each configuration with an anechoic music sample. The anechoic piece used in this study was a selection from Beethoven's 9th symphony performed by the Osaka Philharmonic Orchestra.[21] A KEMAR head related transfer function was applied to the convolved impulse responses in ODEON. A series of 15 sound tracks were created from the 15 configurations. The tracks were presented over headphones to 30 human test subjects, and the presentation order was randomized to reduce bias error. Subjects were members of the University of Nebraska community, with the majority being between 20 and 30 years of age. Each subject was determined to have normal hearing thresholds in both ears. Most of the subject pool indicated that they had limited exposure to classical music, such as that played during the testing.

Subjects were given a brief training session that covered the definitions of the acoustical qualities of reverberation and clarity. After the training period, subjects were asked to rate each track on a 9 point scale for reverberation (dead to live) and clarity (unclear to clear).

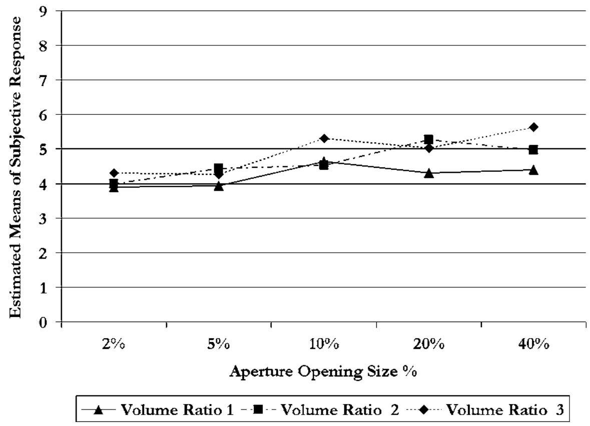

FIG. 9. Statistically estimated means of subjective ratings for perceived reverberance.

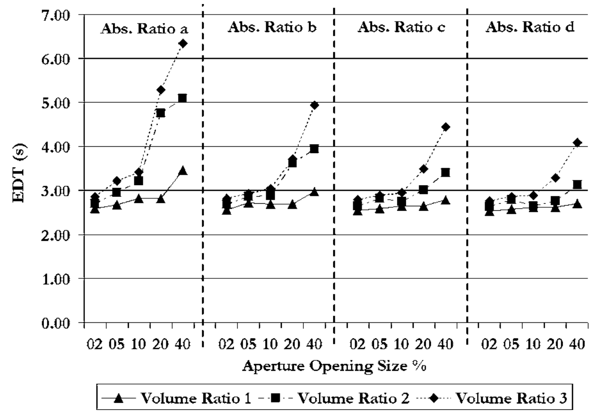

FIG. 11. Objective early decay time (EDT) values for the subset of configurations used in subjective testing.

VII. SUBJECTIVE RESULTS

This study used a repeated measures design with two independent variables: volume ratio with three levels, and aperture size with five levels. There were two dependent variables: perceived reverberation and perceived clarity.

A two-way repeated measures ANOVA was performed on these data. Results indicate that there was a significant effect of volume ($p < 0.0001$) and aperture size ($p < 0.0001$) on perceived reverberation. Additionally, perceived reverberation showed a significant effect due to the interaction between volume and aperture size ($p = 0.028$). There were no observed significant effects on perceived clarity. Figures 9 and 10 show the statistically estimated mean values of perceived reverberation and perceived clarity for each combination of volume and aperture size.

The nonparallel lines in Fig. 9 do indicate a reaction between the independent variables, volume ratio, and aperture size. In other words, as both independent variables change, the perceived reverberation results change in a manner different from when the independent variables are changed separately. It appears that the main cause of the nonparallel lines is the volume ratio 2 data in the midrange of aperture opening sizes tested, implying that this is a region where the subjects may find it difficult to differentiate between their perceptions of reverberation.

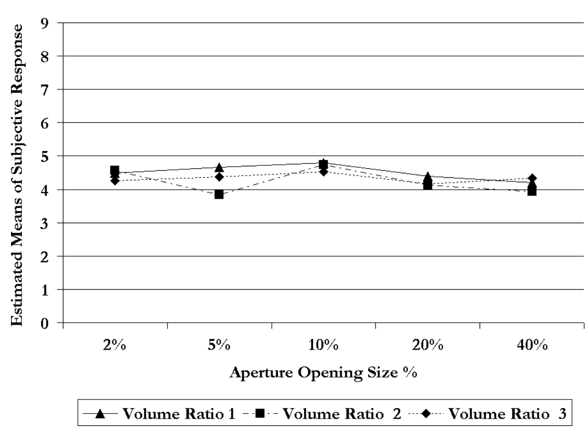

FIG. 10. Statistically estimated means of subjective ratings for perceived clarity.

Post-hoc pairwise Bonferroni comparison tests revealed that, for perceived reverberation, volume ratios 2 and 3 were significantly different from volume ratio 1. Additionally, for perceived reverberation, the 10%, 20%, and 40% openings significantly differed from the 2% opening. Also, the 20% and 40% openings significantly differed from the 5% opening ($p < 0.05$ for all significant tests).

In summary, the volume ratio and aperture size have been found to affect perceived reverberation in this study. Listeners generally perceived a higher reverberation as volume and aperture size were increased independently. Figure 11 shows the objective early decay time (EDT) results from the computational analysis of the impulse responses of each model. These data show a clear tendency for EDT to increase as volume and aperture size are increased. The early portion of the reverberant decay has long been regarded as a good indicator of subjective impression of reverberance.[22,23] Therefore, comparative analysis between the EDT values and the subjective response to reverberance shows good agreement between the objective and subjective results.

The lack of significant effects with clarity is most likely due to the nature of the simplified forms of the modeled coupled volume systems. Clarity may be objectively quantified by the clarity index, a ratio of early to late sound energy received at a given position in a space. The clarity index is given in dB as

$$C_{80} = 10 \log\left(\frac{\int_{t=0}^{80\ \mathrm{ms}} g^2(t)dt}{\int_{t=0}^{\infty} g^2(t)dt} \right), \tag{18}$$

where $g(t)$ is the impulse response of the space in time. This measure is affected greatly by early reflections. In this study, the models were comprised of two simple rectangular boxes, a geometry that does not produce a high number of early reflections that are distinguishable from other reflections. As the coupled space's volume and the aperture size are changed, the early reflections are unfortunately not directly influenced. Objective clarity (C_{80}) results from the 60 configurations studied in this paper are shown in Fig. 12. The greatest differences in the C_{80} values were found when comparing across volume ratios, giving an average difference of 0.68 dB, which is on the order of the just noticeable difference: $0.67 \pm 0.13\ dB$.[24] These objective results

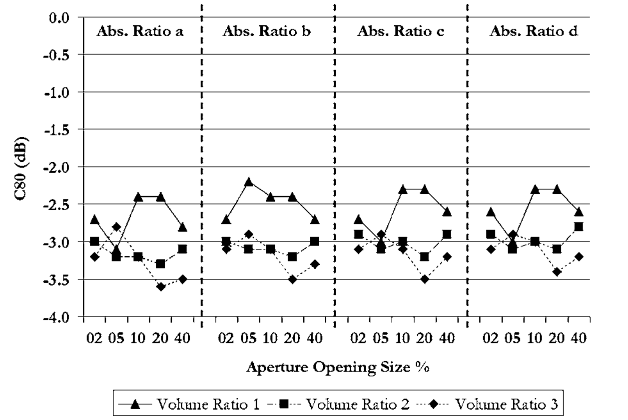

FIG. 12. Objective clarity (C_{80}) values for the subset of configurations used in subjective testing.

suggest no perceptual change in clarity across configurations, agreeing with the subjective results obtained.

The original hypothesis of the authors was that higher perceived reverberation and perceived clarity resulted from nonexponential decay curves that demonstrated greater degrees of nonexponential decay, and that this degree of nonexponential decay could be objectively quantified by coupling coefficient or more precisely with the new combination of decay ratio and ΔdB. Unfortunately, since perceived clarity did not result in any significant results in this study, it is not possible at this time to delve further into the prospect of nonexponential decays producing both high reverberation and high clarity.

Even though the perceived clarity results are not obvious, comparison of the perceived reverberation results to the coupling coefficient, decay ratio, and ΔdB results show very good correlations. As indicated previously, volume ratios 2 and 3 produced statistically higher perceived reverberation values than volume ratio 1. These larger volume ratios also produced higher values of coupling coefficient, decay ratio, and ΔdB than volume ratio 1. The same is true for the subjective results of aperture size; aperture sizes around 20% produced significantly higher perceived reverberation values than those from the lower apertures of 2% and 5%. For volume ratio 2, these larger aperture sizes generally produced higher values of coupling coefficient, while the decay ratio and ΔdB results were more varied depending on the volume ratio.

Recall that the decay ratio and ΔdB data from volume ratio 3 of the computer simulations exhibited different trends across aperture opening size (Fig. 7). To analyze how each of the new quantifiers may relate to subjective perception of reverberation, a comparison of the volume ratio 3 data is made against the statistically estimated means of reverberation from the subjective tests (Table IV). Comparing cases that were found to be significantly different from the Bonferroni tests, we see that in comparing the 2% and 20% cases, decay ratio increases while ΔdB remains about the same, correlating to higher perceived reverberation. Similarly in comparing the 2% and 40% cases, decay ratio increases while ΔdB increases slightly, again correlating to higher perceived reverberation. Comparing the 5% and 20% cases, decay ratio and ΔdB both decrease, resulting in higher perceived reverberation. A comparison of the 5% and 40% cases shows decay ratio decreasing slightly and ΔdB decreasing a larger amount, producing higher perceived reverberation. Comparison between 2% and 10%, however, shows a different trend not expected from previous research; decay ratio decreases and ΔdB increases, producing higher perceived reverberation. The authors believe this anomaly may be due to the issues discussed earlier with the computer modeling, concerning the diffuseness and density of the reflection diagram.

TABLE IV. Computer modeled decay ratio and ΔdB data, and statistically estimated perceived reverberation data for volume ratio 3 and five aperture opening size percentages.

Aperture size (%)	Decay ratio	ΔdB	Statistically estimated reverb
2	2.00	0.58	4.30
5	3.16	1.40	4.27
10	1.64	1.53	5.30
20	2.46	0.60	5.03
40	2.94	0.72	5.63

In summary, the results mostly confirm findings from previous research: larger decay ratios and smaller ΔdB values generally lead to greater perceived reverberation. It still remains unclear, though, which of the two parameters has the more severe influence on DSE overall, since no significant relationships were found with perceived clarity ratings.

VIII. CONCLUSIONS AND FUTURE WORK

Computer modeling results for 60 coupled volume configurations with varying volume ratio, absorption ratio, and aperture size have been studied objectively using coupling coefficient and the new parameters of decay ratio and ΔdB. There is a general increase in coupling coefficient as coupled volume size increases, a general increase in coupling coefficient as coupled volume absorption decreases, and coupling coefficient peaks at a particular opening size for each volume/absorption combination. Coupling coefficient results also suggest that among the three parameters, having larger coupled volume sizes is the most significant in producing higher coupling coefficient values. Smaller coupled volume sizes can only achieve high coupling coefficients if the absorption ratio is also very low. The proposed parameters of decay ratio and ΔdB provide more specific information about each nonexponential decay curve, and have been found to show similar trends as coupling coefficient. However, the new parameter data for the highest volume ratio tested were harder to interpret without further information about subjective perception.

Subjective testing using the results from 15 of the configurations showed a general increase in perceived reverberance with increasing volume and aperture size. Higher perceived reverberance also matched well with objective coupling coefficient results, and generally with higher decay ratios and lower ΔdB values. No significant effects on perceived clarity were found, which may be due to the lack of distinguishable early reflections in the simplified geometry studied. Due to the lack of this significant effect, further extrapolation by combining the subjective results of reverberation and clarity could not be carried out to determine overall subjective response to DSE.

Further study in this area could focus on advancing a metric for describing DSE that will allow for the effects of the architectural parameters on coupled volume systems to be more clearly understood. Several issues can be improved in ongoing work. The simplified geometry of the models may have adversely affected the results due to the lack of strong early reflections. Consequently, models with more realistic geometries can be built to produce results that better articulate the clarity of the systems. Additionally, certain computer modeling calculation parameters may have had an adverse effect on the results. The number of rays used, for example, could have been significantly higher, allowing for a more accurate exploration of the coupled volume geometry. Future work could incorporate these modifications, and more concrete conclusions regarding DSE prediction and perception may be made with further research.

ACKNOWLEDGMENTS

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[1]L. Cremer, H. A. Müller, and T. J. Schultz, *Principles and Applications of Room Acoustics* (Applied Science, New York 1982), Vol. 1, pp. 189–193, 261–292.
[2]H. Kuttruff, *Room Acoustics* (Spon, New York 2000), pp. 142–145.
[3]A. D. Pierce, *Acoustics: An Introduction to Its Physical Principles and Applications* (Acoustical Society of America, New York 1989), pp. 281–283.
[4]A. Munro and R. Johnson, "Calculating reverberation time in coupled rooms," *Workshop on Design of Concert Chambers, Tanglewood '99 Concert Hall Research Group Summer Institute, 1999*.
[5]A. H. Davis, "Reverberation equations for two adjacent rooms connected by an incompletely soundproof partition," Philos. Mag. **50**, 75–80 (1925).
[6]C. F. Eyring, "Reverberation time measurements in coupled rooms," J. Acoust. Soc. Am. **3**, 181–206 (1931).
[7]B. Harrison and G. Madaras, "Computer modeling and prediction in the design of coupled volumes for a 1000-seat concert hall at Goshen College, Indiana," J. Acoust. Soc. Am. **109**, 2388(A) (2001).
[8]M. Ermann and M. Johnson, "Pilot study: Exposure and materiality of the secondary room and its impact on the impulse response of coupled-volume concert halls," J. Acoust. Soc. Am. **111**, 2331(A) (2002).
[9]J. E. Summers, R. R. Torres, and Y. Shimizu, "Statistical-acoustics models of energy decay in systems of coupled rooms and their relation to geometrical acoustics," J. Acoust. Soc. Am. **116**, 958–969 (2004).
[10]B. S. Atal, M. R. Schroeder, and G. M. Sessler, "Subjective reverberation time and its relation to sound decay," *Proceedings of the Fifth ICA, Liege, Paper G 32, 1965*.
[11]D. Picard, "Audibility of non-exponential reverberation decays," *Rapport de Stage D'Option Scientifique* (Rensselear Polytechnic Institute, Troy, NY, 2003).
[12]M. Ermann, "Coupled volume/double slope subjective listening test," J. Acoust. Soc. Am. **114**, 2331(A) (2003).
[13]N. Xiang and P. M. Goggans, "Evaluation of decay times in coupled spaces: Bayesian parameter estimation," J. Acoust. Soc. Am. **110**, 1415–1424 (2001).
[14]N. Xiang and P. M. Goggans, "Evaluation of decay times in coupled spaces: Bayesian decay model selection," J. Acoust. Soc. Am. **113**, 2685–2697 (2003).
[15]C. L. Christensen, *ODEON Room Acoustics Program User Manual V6.5 Industrial*, Auditorium and Combined Editions, Odeon A/S, Lyngby, Denmark, 2003.
[16]C. L. Christensen, "ODEON-a design tool for auditorium acoustics, noise control and loudspeaker systems," *Proceedings of the Institute of Acoustics: Reproduced Sound 17, Measuring, Modeling or Muddling, 2004*, Vol. 23, pp. 137–144.
[17]D. T. Bradley and L. M. Wang, "Comparison of real world measurements and computer model results for a dedicated coupled volume system," J. Acoust. Soc. Am. **116**, 2552(A) (2004).
[18]H. Kuttruff and Th. Strassen, "Zur abhängigkeit des raumnachhalls von der Wanddiffusität und von der raumform," Acustica **45**, 246–255 (1980).
[19]L. Nijs, G. Jansens, G. Venmeir, and M. van der Voorden, "Absorbing surfaces in ray-tracing programs for coupled spaces," Appl. Acoust. **63**, 611–626 (2002).
[20]N. Xiang, P. M. Goggans, T. Jasa, and M. Kleiner, "Evaluation of decay times in coupled spaces: Reliability analysis of Bayesian decay time estimation," J. Acoust. Soc. Am. (to be published).
[21]Osaka Philharmonic Orchestra, *Anechoic Orchestral Music Recording* (Denon Records, 1995)
[22]R. H. Bolt and P. E. Doak, "Tentative criterion for the short-term transient response of auditoria," J. Acoust. Soc. Am. **22**, 507–509 (1950).
[23]M. R. Schroeder, B. S. Atal, G. M. Sessler, and J. E. West, "Acoustical measurements in Philharmonic Hall (New York)," J. Acoust. Soc. Am. **40**, 434–440 (1966).
[24]T. J. Cox, W. J. Davies, and Y. M. Lam, "The sensitivity of listeners to early sound field changes in auditoria," Acustica **79**, 27–40 (1993).