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Writing In Math Class? Written Communication in the Mathematics Classroom

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Abstract

In this action research study of my seventh grade mathematics classroom, I investigated what written communication within the mathematics classroom would look like. I increased vocabulary instruction of specific mathematical terms for my students to use in their writing. I also looked at what I would have to do differently in my teaching in order for my students to be successful in their writing. Although my students said that using writing to explain mathematics helped them to better understand the math, my research revealed that student writing did not necessarily translate to improved scores. After direct instruction and practice on math vocabulary, my students did use the vocabulary words more often in their writing; however, my students used the words more like they would in spelling sentences rather than to show what it meant and how it can be applied within their written explanation in math. In my teaching, I discovered I tried many different strategies to help my students be successful. I was very deliberate in my language and usage of vocabulary words and also in my explanations of various math concepts. As a result of this research, I plan to continue having my students use writing to communicate within the mathematics classroom. I will keep using some of the strategies I found successful. I also will be very deliberate in using vocabulary words and stress the use of vocabulary words with my students in the future.

Introduction

The issue of teaching I researched was improving written communication in the area of problem solving. I believe being able to communicate one's ideas whether it be in math, in science, or in any area of life, is important. Writing out explanations forces one to really understand the problem one is trying to explain. Writing gives students an opportunity to use appropriate vocabulary, break down the steps needed to solve a problem, and think about "Why am I doing this?" I believe all of the above aspects are important in any problem-solving situation. Most of the students I teach struggle with explaining how they solved a problem. They view math as mostly computational. I want my students to discover the wide world of math and how the concepts they learn and the processes they use will carry over into a wide variety of areas in their life. I believe this is an essential life skill.

I wanted to change the definition of what it meant to be successful in my math classes. Instead of focusing on the one right answer to a problem achieved through computational means I wanted to push my students to be able to explain why they did what they did to get an answer. Instead of my students seeing math as purely computational, I wanted them, through their writing, to see the "why" behind what they were doing. I wanted them to be able to explain the steps they used and defend why they used those particular steps. My students usually gave me the response of "because" or "that's just what you need to do" when I asked them why they did what they did to solve a problem. They did not want to think; they just wanted to give answers. And when they did write an explanation, it usually just included numbers and some sort of computation. I could not follow their thought process as to why they solved the problem the way they did. I wanted my students to demonstrate an understanding of the math process they used and have a working knowledge of math vocabulary they could use while writing explanations. I wanted my students to reason and defend the steps they took to solve a problem. Thus, I decided to study written communication in my seventh grade mathematics classroom for my action research project.

My research took place in a middle school in a small rural community in central Nebraska. The middle school consists of fifth through eighth grades with approximately 80 students per grade. Within the entire district of 1,114 students, 93% of the students were Caucasian and 5% were Hispanic. About 20% qualified as students with special needs and another 2% were English Language Learners (ELLs). Approximately 29% of the student population qualified for the free and reduced lunch program. The research took place during the spring semester of the 2008-2009 school year.

Problem Statement

As stated in the National Council of Teachers of Mathematics (NCTM) Principles and Standards for School Mathematics book (2000), in the world today it is important for students to understand and apply the content standards – number and operations, algebra, geometry, measurement, and data analysis and probability. I think it has become equally important to know how to clearly communicate one's mathematical thinking to others. The language of mathematics is important to use in expressing one's mathematical ideas precisely. By practicing communicating to others, one should be able to analyze and evaluate others' mathematical thinking and strategies. Communication is one of the five NCTM process standards (NCTM, 2000). I think this standard, in particular, has far reaching benefits to other areas of my students' lives. Through my experience in the Math in the Middle Institute Partnership, I have come to realize how important communicating my thought process is to my understanding of mathematics. Through my own written explanation of problems I was better able to understand problems and, consequently, I have made more connections to other problems and concepts. Most importantly, I now better understand the why behind what I do as a learner of mathematics. This has helped me in my role as a teacher as I now am better able to communicate mathematical concepts to my students. I have modeled the use of appropriate math vocabulary for my students. I have helped my students become more capable of connecting math concepts. Writing explanations has helped my students to see the relevance of learning math vocabulary.

The teaching principles, offered by the NCTM, also have played a part in my new area of focus. I increased my expectations as far as expecting clear written explanations. However, I was aware many of my students struggled with written communication even with my support. With the time factor and seeing students struggle, I was much more likely to say, “You’ve got the answer; that is good enough.” I realized that was not what was best for my students in the long run. I viewed my job as a teacher should be to encourage my students to develop their potential, not just slide by. I really worked on developing the written explanation of a mathematical problem with my students. The process was taught, modeled, discussed and practiced as a whole class and then individually. Yet I was not where I wanted to be and my students were not where I wanted them to be in terms of written communication. Thus, I saw action research as the perfect fit for me to seriously study my own teaching.

I think written communication within the mathematics classroom is important in my own teaching because it will help me assess whether my students understand a certain concept. If my students cannot explain their mathematical thinking clearly using appropriate vocabulary, then I have not done my job of explaining the concept or the vocabulary term. I want to be able to see what my students are thinking in addition to if they just got the right answer. Practicing clear and precise written communication in math will help my students in other classes as well. My students will have another opportunity to practice writing in complete sentences with correct punctuation. Practicing written communication carries over into reading comprehension, science lab reports, social studies research papers, essays on job application forms, and even leaving a detailed note for someone. The skill of written communication is applicable in a variety of careers.

Literature Review

> "Communication is an essential part of mathematics and mathematics education. It is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process also helps build meaning and permanence for ideas and makes them public. . . they communicate to learn mathematics and they learn to communicate mathematically." (NCTM, 2000, p. 60)

In our society today students are experts at communicating using e-mail, the Internet, and cell phones. Their language consists of short cuts, acronyms, and incomplete sentences. In the math curriculum our district uses, students are asked to explain, justify answers, and provide the rationale for several problems within their daily assignments and in problem-solving situations. I want my students to use written communication not only to communicate their understanding of mathematics, but also to deepen their understanding of the mathematical concepts being taught. Research done in the area of written communication in mathematics argues that writing improves understanding (Pugalee, 2004; Baxter, Woodward, Olson, 2005; Albert, 2000; Draper, Siebert, 2004; Shield, Galbraith, 1998; Burton, Morgan, 2000). The kind of writing (transactional or expressive), the reason for the writing (for the teacher, for the student, or for both), and the task (problem solving, explaining, or note taking) are all themes that emerged as I read the literature. My literature review discussed these themes as part of three subcategories: written communication in mathematics, precise vocabulary within written communication, and written communication improving mathematical understanding.

Written Communication in Mathematics

The kinds of writing that take place in a mathematical classroom are worth thinking about. Kinds of writing might range from general note taking and writing to explain to journaling about attitudes, thinking, and rationale about a certain problem or concept. Using written communication in mathematics needs to be more than just a place for note taking and reiterating what the teacher puts on the board. Fried and Amit (2003) looked at classroom notebooks. Two different high school math classes were targeted to examine student notebooks where student writing took place. Researchers were present for every lesson to video tape and take field notes. In both classes the students copied down what the instructors had written on the board, including all explanations and examples. The students, when interviewed, stated that this helped them study for tests. However, using writing in this setting limited the creativity and expressiveness of the student. It limited any high-level thinking skills. Fried and Amit concluded that this kind of writing (transactional) should be done in balance with expressive writing where the students are encouraged to think independently, creatively, and reflectively about the problems and concepts at hand.

A study done by Shield and Galbraith (1998) aimed to analyze students' expository writing in math and provide an indication of the understanding of particular mathematical ideas held by the students. They looked at the writing of three eighth grade math classes from two different schools. The definition of expository writing in math was that of explaining rather than telling how to do a problem. The researchers worked with the students over a three-month period and collected 290 pieces of writing. The generally accepted idea of understanding seems reasonable when a student is able to write about, show, link the procedure to prior knowledge, and justify the use of that procedure. The student should have an effective understanding of that mathematical procedure. However, as noted by Shield and Galbraith (1998), the restrictions of this model of writing limited the students from any higher levels of thinking about the ideas.

In contrast Albert (2000) noted that "the act of writing serves as a mode for students to reflect on their thinking" (p. 109). This type of communication gives the students an avenue to convey ideas, feelings, and experiences that can lead to developing their critical thinking, sound reasoning, and problem-solving. Her study consisted of exploring the relationship between oral and written thought processes of seven middle school students. The seven students were presented as a single case study. They were all part of a larger study that took place over 14 weeks in three seventh grade classrooms where there was less traditional instruction and more collaborative group activities. The students worked collaboratively to solve problems. Individually the students wrote statements, explaining strategies and procedures they used to solve the problems. Writing in this study included all written work, diagrams, pictures, and figures. Albert wanted to examine the students' thinking processes. Adding writing to mathematics class asks students to examine solutions and results to evaluate the reasonableness of an answer with the goal of helping them focus more on the process and less on getting the correct answer. Albert noted that the students' thinking may be evolving toward the realization that conceptual understanding, ideas and strategies for problem solving are essential to mathematical learning. All of those elements mentioned are high-level thinking skills. Further, all were used in the setting of communicating, specifically using writing to explain problem-solving processes and solutions. Albert (2000) states,

> "The mathematical environment should be constructed in ways that encourage students to find their paths to solutions and connect those paths to their ideas. It is important that students experience mathematics as a process of exploration in which their experiences help empower them as learners and more specifically, as problem solvers." (p. 137)

Writing to explain their thought processes is one way to help connect the students' ideas to their solutions.

One cannot assume that students know how to use written communication to convey understanding in mathematics. In the study done by Draper and Siebert their argument was between their two ideas. Theirs was a form of cooperative inquiry. Students needed to learn to use writing to communicate mathematical understanding and learn mathematical understanding through written communication. Draper came to this study with the expertise in literacy, being a literacy researcher and educator with a background in mathematics. Siebert is a mathematics education researcher and educator. Through their collaboration a conclusion was reached that "every mathematics learning event is also a literacy event, and every literacy event in a mathematics classroom is a mathematics learning event" (Draper & Siebert, 2004, p. 953). As a teacher of math the implications of this statement are not only to use writing as a means to deepen students' understanding of mathematics, but also to teach them how to use writing to communicate their mathematical knowledge. It is not simply exchanging mathematical symbols for English words or vice versus. It is moving away from a formula for writing and using opened problems to allow students to create their own solutions and methods for the problems.

A good argument for using written communication in the form of journals can be found in a study done by Baxter, Woodward, and Olson. Journals used in this study became a place where students were encouraged to write about their mathematical ideas and reasoning. This study took place in the Pacific Northwest in a seventh grade mathematics class. There were 28 students, with one-third of them qualifying for special education services. The classroom teacher developed writing activities that would relate to the math topics being studied, improve students' awareness of their own thought processes, and help students take personal ownership of their knowledge. The study followed the students in this classroom for one school year. Students wrote in their journals at least once a week. “Perhaps the greatest promise of writing in mathematics is that it will forge connections with students who typically drift or run rapidly away from mathematics. Writing offers a means for students to relate mathematical ideas to their own lives” (Baxter, Woodward, and Olson, 2005, p. 132). The teacher of the classroom found reading her students’ journals and responding was a way for her to privately encourage her students. Journal writing was also a way for the student to communicate his or her thoughts to the teacher privately. This teacher saw journal writing as another way to assess and modify her teaching. She could adjust her pacing, explanations, and activities to accommodate the understanding or lack of understanding of her students. One interesting comment made in this study was building a good relationship with the students and being friendly rather than formal resulted in improved performance. The students, through their writing and the teacher responding, were able to develop a deeper relationship with the teacher and be more successful in the classroom.

Precise Mathematical Vocabulary within Written Communication

Just because students write in the mathematics classroom does not mean their writing is formal or contains appropriate math language and correctly used terms. It is up to the teachers to construct different settings for the writing to take place. The teachers should model appropriate writing, offer feedback, and a chance to revise writing to guide the students in expectations in written communication. “It is important to avoid a premature rush to impose formal mathematical language” (NCTM, 2000,p. 63). As students begin to write about their mathematical understanding, they begin to use familiar words. This can be built upon as they make a connection to the more formal mathematical language. The ability to use generalized language and to see a process in general terms is worth developing, and it is not expected that all middle school students will have developed such an ability (Shield, Galbraith 1998). In Draper and Siebert's study (2004), one of the goals of the specific math lesson was that of having students participate in creating mathematics. In order for the students to do this they needed to have a rich understanding of the meanings of the words. In Siebert's analysis of his own lesson he comments on all of the things he did to help his students write better explanations: he provided them with many opportunities to write; his students had frequent opportunities to observe, discuss, and write conceptually oriented explanations within their small groups and the whole class; he regularly graded and offered feedback on their written explanations; and he also presented several solutions and discussed with the whole class whether it was a good explanation and why. His main focus was on the mathematical ideas. He viewed helping his students write good, conceptually-oriented explanations were equal to mathematical understanding. "Mathematics cannot be separated from the texts in which meanings are created, conveyed, and negotiated, just as literacy cannot be separated from the content that defines how the texts are to be read and written" (Draper & Siebert, 2004, p. 952).

The acquisition of mathematical-linguistic language is a necessary and crucial component of the educative process (Burton & Morgan, 2000). This statement concurs with communication goals suggested by the NCTM (2000). The process of learning to write mathematically has to be practiced with guidance and nurtured along. "As students practice communication, they should express themselves increasingly clearly and coherently" (NCTM, 2000, p. 62)

Written Communication improves Mathematical Understanding

Written communication allows students to express their ideas and thinking in ways not obvious to the classroom teacher. In Baxter, Woodward, and Olson's study, the special education students did not share or participate in class discussions (2005). Their mathematical understanding was not evident from class discussions or participation. However while viewing their journals three of the four students were able to try to explain their mathematical reasoning and describe the steps they used to solve a problem. Through journal writing in mathematics the teacher was able to adjust her teaching and methods to help improve her students' understandings.

Writing offers a place for students to explore their own thinking and reflect on it. Being able to convey ideas, feelings, and experiences can lead to the development of higher cognitive functions, including critical thinking, sound reasoning and problem solving as noted by Albert (2000). "Dynamic mathematical communication is critical to learning and understanding mathematical concepts and ideas in which thought processes are developed 'outside-in' through mediated practices assisted by others – to thought processes developed 'inside-out' through mediated practices assisted by self" (p. 138). Students can develop and refine their ideas by talking them through with a partner and then by writing them down.

Another advocate for using written communication to aid in mathematical understanding can be found in a study done by Pugalee (2004). Pugalee investigated the impact of writing during mathematical problem solving. The setting was a class of 20 ninth grade algebra students. The students alternated between written and oral descriptions of their problem-solving processes. Written descriptions were termed as "thinking aloud on paper" (p. 29). Written communication was a more effective tool then the use of the verbal think-aloud processes. Written communication allowed the students to more effectively predict, plan, revise, select, and classify within the problem-solving process.

Summary of the Literature Review

My students are most adept at the computational part of their assignments and lack clarity and precision in their written communication. As a result of my experiences in Math in the Middle I have seen how writing out a problem and all the thinking behind it has helped me to better understand the problem. I feel more confident not only with my solution, but also with my understanding of the concept. I wanted my students to develop their written communication skills with regard to mathematics as well. I was interested in this area as another tool to help my students understand the mathematical concepts they learn and use. Like Draper & Siebert (2004) I wanted to model and discuss with my students appropriate language and format for their written explanations. I wanted to build relationships with my students and maintain a friendly class environment, one open to discussion and questions. I, too, wanted to create a comfortable atmosphere as was seen as being important in the study of Baxter, Woodward, and Olson (2005). I have been frustrated with my use of journals in the past, especially after reading the work of Baxter, Woodward, and Olson (2005) and of Fried and Amit (2003). It takes time to develop a constructive use of journals as seen in Baxter et al (2005). I wanted my students to see purpose in precise written communication. Being able to communicate clearly in our world today is a life skill. I can help my students to better communicate their mathematical ideas, thoughts, and processes within the mathematics classroom. My study combined several ideas found in previous studies to further support my students' written communication in mathematics.

Purpose Statement

The purpose of my study was to explore the relationship between increased written communication in mathematics and student understanding. I examined the research themes of: the quality of written communication in homework and problem solving, the quality and quantity of precise mathematical vocabulary terms used in students' written communication, and students' mathematical achievement increasing when higher expectations are placed on the quality of written communication. I attempted to answer the following research questions:

- What will happen to students' vocabulary usage in written explanations after they receive specific vocabulary instruction?
- What will happen to students' mathematical understanding through increased practice on written explanations?
- What happens to my mathematics teaching when I increase expectations for the quality of student written explanations, including correct usage of precise mathematical vocabulary?

Method

This research was done in a regular seventh grade mathematics class during the second semester of the 2008-2009 school year. The class consisted of 19 students, nine boys and 10 girls. This class contained no verified special needs students. Two students were classified as English Language Learners; however, both were fluent in the English language and received services on an as-needed basis with no specific help given in math.

Starting the week of January 26 and continuing through April 15, I collected two written explanations for math problems from each student in the class. The two problems usually consisted of one problem-solving type problem and one written explanation of a problem selected from the regular 30-problem assignment set from the textbook. (Appendix A). There was one week in which no written explanations were collected due to another project my seventh graders worked on. There were also a couple of weeks where only one written explanation was collected due to vacation days interrupting the flow of the class.

All students were surveyed during the week of February 9 (Appendix B). They were instructed not to put their names on the survey. The survey consisted of three questions dealing with written explanations in math and understanding math vocabulary. The students were to answer strongly agree, agree, undecided, disagree, or strongly disagree. Nineteen students took the survey the first time. Three students strongly agreed that "It helps me understand math problems better when I write out explanations." Thirteen students agreed with the statement, two students were undecided, and one student disagreed. No students strongly disagreed with the statement. Two of the 19 students strongly agreed with the statement: "I like to write out explanations to mathematical problems." Seven students agreed with the statement, six students were undecided, four students disagreed, and two of the 19 students strongly disagreed. "Understanding specific math vocabulary is important in understanding problems" was the last statement for the students to respond to on the survey. Five students of the 19 strongly agreed with the statement, 12 students agreed, two students were undecided, and no students disagreed or strongly disagreed. There were also two open-ended questions for the students to share their opinions. This same survey was given to all students as a post-survey during the middle of April. The comparison of the results can be found in Appendices G and H.

In order to conduct student interviews a sixth grade teacher came into my classroom to explain to my students about the consent process as directed by the Internal Review Board (IRB) of the university for which this action research project took place under. The teacher passed out the permission slips and asked the students to return them to her. She later confirmed permission of the students I chose to interview. I chose six students, two high-ability students, two medium-ability students, and two low-ability students. Two sets of individual student interviews were conducted. One interview session took place at the beginning of the research period in February, and the second session took place in April. I asked students questions ranging from attitudes in math, opinions about writing versus computation problems to asking for ideas or suggestions on ways teachers could help students be more successful in math (Appendix C). Six students were interviewed during the first round of interviews. Those same six students participated in a second interview as well.

I photocopied each piece of student writing that I collected and then placed the copies in a folder. I used a rubric to score each piece of writing (Appendix D). I kept track of individual total scores for each piece of data. I wanted to see if the scores improved over time. I also tallied the number of students who scored at each level for each section of the rubric (Appendix E). I did this to help me identify any particular areas that many of my students were struggling with on their written explanation.

After using my rubric several times, I became frustrated by the lack of precision it allowed me to have. Each category was not worth enough points to really distinguish between a good written explanation and a poor one. For example, a student could miss a half of a point for missing punctuation and capital letters on sentences but then have everything else explained well and labeled. This student might score 4.5 points. Whereas another student may have been missing one key element in the explanation area and lose a point, but have complete and well-punctuated sentences. That student would receive a 4. The half of a point difference between the two scores really did not show the true picture between a good write-up and a poor one.

I also kept a teacher journal to record happenings during each week of the research period. I included frustrations I felt, quotes from my students in the area of written explanations, and the highlights as well. This was a very worthwhile practice. I used my journal to adjust my daily and weekly teaching strategies.

Findings

A typical math lesson in the seventh grade Saxon Course 2 book (Hake, 2007) took me approximately two days to cover. I spent part of the first day of each lesson checking my students' assignment due on that day and answering any questions about it. The next task was for my students to do a five-minute power up over a basic skill we had been practicing. The power up might have consisted of changing 10 improper fractions to mixed numbers and then vice versa. We would check this in class and students would record their score on their record keeping sheets. Eight mental math problems were then read orally twice for the students to work in their heads and write down only the answer. We would check these in class as well and discuss several mental strategies for working the various problems. The students would again record their scores on their individual record keeping sheets. The students then spent time on a problem-solving problem. How this time was used for problem solving varied depending on the different strategies I tried with my students to help them be successful. Often we would run out of time on the first day to finish the problem-solving problem and then would carry this activity over to the beginning of the second day of the lesson. I would spend most of the second day teaching the mathematical concept for the lesson, working examples, discussing problems with my students, answering questions, and allowing them time to work on their assignment.

My Mathematics Teaching

One of the questions I wanted to answer was *what happens to my mathematics teaching when I increase expectations for the quality of student written explanations, including correct usage of precise mathematical vocabulary?* After completing this research project, I found out that I tried a variety of teaching strategies. My students completed different kinds of written explanations. I also restructured my lessons in an attempt to increase the quality of my students' written explanations.

I began by letting the students work with a partner with whom they felt comfortable. In giving oral directions for their written explanations, I told students I wanted to see complete sentences and as much math vocabulary as appropriate. Students immediately got together with their partner as soon as the problem-solving problem was assigned. The students worked several minutes together. I roamed around the room listening, watching, and occasionally explaining the directions. When I noticed most pairs were about finished, I stopped the class. I asked my students to share the methods and strategies they used to solve the problem. As a class, we asked questions of those who shared about the various methods and solutions as well as asked, for additional clarification on things we did not understand. The students were then given a few minutes to wrap up their thoughts and ask any last-minute questions before we proceeded to the lesson of the day.

Writing explanations for problem-solving problems was a new experience for my students. They were not experienced at writing down everything they needed to do and everything they did to arrive at a solution to a particular problem. I noted this in my journal at the beginning of my study.

> As I walked around the room, some students were more concerned about the writing up of the problem than actually solving it. Several wanted me to read over what they had written to see if it was "right." I spent some time questioning them on what they were doing and what the different numbers on their paper meant. I found several of my students could verbalize what they had done, but hadn't

> written any of what they told me on their paper. (Personal Journal, February 5, 2009)

I realized I had a lot of work to do with my students to help them explain their work using writing and to see writing as a tool to communicating their ideas.

After scoring the first written explanation my students turned in, I was frustrated with the outcome, but I was not surprised based on the conversation I had with my students during their actual work time on the problem. The class average was 2.5 points out of a total of five points. Of the six students shown on the scatter plot four scored below the average and one did not even turn in the assignment. (Appendix F)

The student work shown below was a typical example of a written explanation after the first assignment. Bob[1] used words like “just take 21 5/8 and make it 22.” He did not explain the task he was doing or why he decided to make the number 22. I had no idea what Bob was trying to accomplish, or what each of the numbers stood for.

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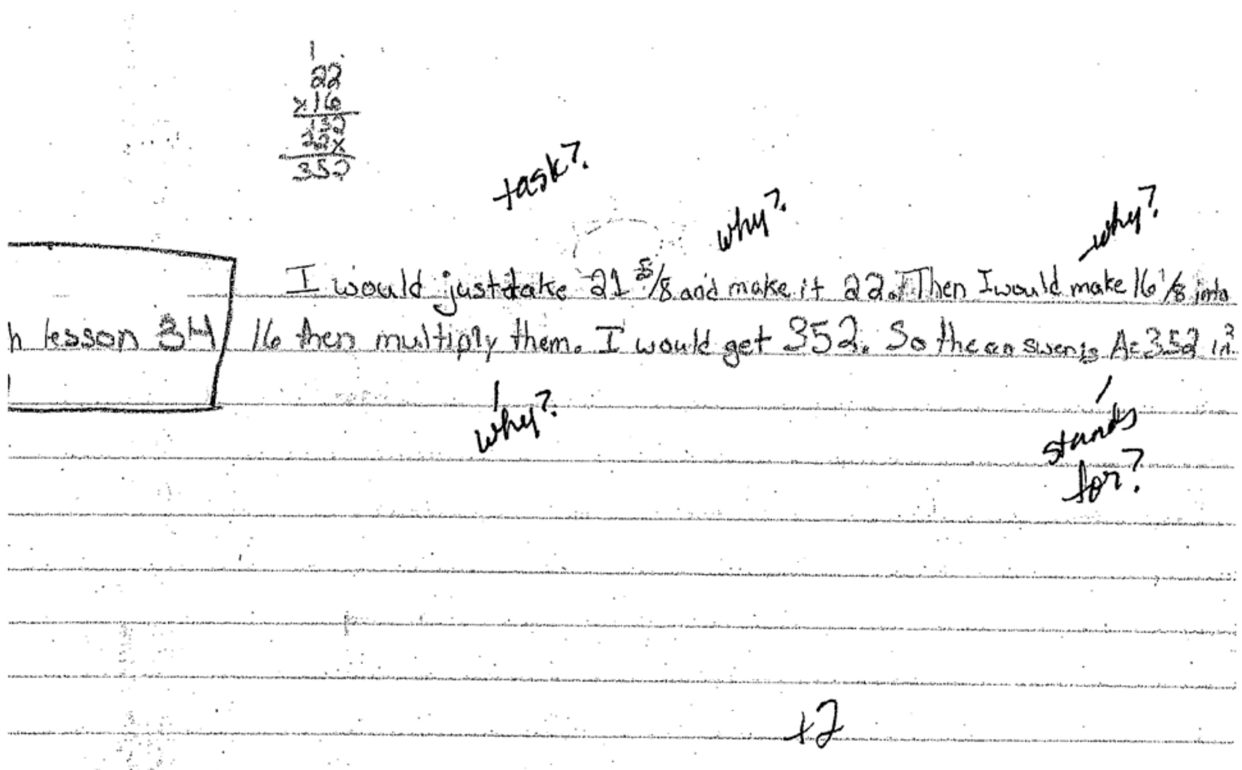
[1] All student names are pseudonyms.



I realized after reading Bob's written explanation, along with the written explanations by other students, my students did not know what I expected. They needed some guidance and modeling. I needed to provide them with an example of what I expected in a written explanation.

I tried to change several different things in my teaching before we had another round of problem solving and written explanations. I emphasized correct vocabulary usage both in my teaching of new concepts and in explanations of the example problems I shared with my students. I gave my students a copy of the scoring rubric. We discussed each part. We looked at examples of a "good" write up (i.e., one that would score five points) and a "poor" write up (i.e., one that would score a one or two points). I used two examples I had written. I led a discussion where we talked about each piece of the two examples and why it received the rubric score it did. We also discussed how to improve the "poor" one. We used the rubric to identify what was left out and how it could be made better.

I had my students brainstorm possible pieces of information they thought was important to include in their written explanations. We did this as a class before they worked with their partner. We discussed labeling our math work and what that would look like. We wrote a sample sentence together to state the task of the problem. Our sample sentence read, "I need to find the number Silvia was thinking of." We followed this procedure for a couple more written explanations on problems I had assigned. I noted in my teacher journal, after trying this strategy, I was beginning to see changes in my students' written explanations.

> I have seen some changes in my students' work. They are more conscientious about putting in the task – what it is they are supposed to do to solve the problem. They are writing more explanations. Some are really thinking about the writing process and how to best say what they want to communicate. I was excited when

> a student asked, "Can we use what we learned the other day on factoring to help us with this problem?" They are starting to make connections. I did notice a more concentrated effort to use math language and appropriate vocabulary. That is a plus! (Personal journal, February 12, 2009)

I was encouraged to continue to look for ways to help my students be successful in their written explanations.

The scores on written explanations for the first six problems averaged 3.5 points. I decided to go back to the basics and model a written explanation for a problem-solving problem for my students. I wanted my students to verbalize what each part would sound like, how to solve the problem, and how to explain it. We worked problem number seven together (Appendix A). The students were still required to write up the explanation as I wrote it up on the ELMO. As part of their score on this problem I wanted them to take it home and get feedback from someone who had not worked the problem. The students were directed to ask an adult to read through the problem and tell if they could understand what was done to solve the problem. The problem was worth five points; the student got one point for the adult's signature. The class average was just below five out of five. The reason two of the target students' scores were fours was due to losing a point for no adult signature (Appendix F).

Another strategy I used in helping my students become more successful in their written explanations was to have them self-evaluate their writing after they thought they were finished. I led them step by step through the rubric and had them underline the different pieces found in their written explanations. I then had my students pair up with a partner to help them edit their work. I encouraged my students to use this procedure on future write ups. I observed mixed results:

> I've seen a little improvement with all of these students, but there continues to be those that really don't care. It doesn't matter whether it is a 30-problem assignment or one written explanation. I do think they tend to slough off more on the written explanation because they are only worth five points. (Personal journal, March 17, 2009)

I decided to show them their overall problem-solving scores so far and what kind of a grade those scores would be. This helped my students become aware of how their scores or lack of scores can add up quickly and translate into a grade they really were not happy with.

I tried allowing my students at least five minutes of quiet thinking time before they were allowed to start working through the problem with a partner. During this time my students read the problem, thought about how to start, and actually started working on it. The students found this to be of real benefit as noted both in my journal and in the second interview done with the students.

> This time I asked them (the students) about the structure of working on this problem yesterday. The same comments were brought out in each class – 'the five minutes by myself gave me time to read the problem and think about it before meeting with my partner. So many times my partner already has ideas and knows what to do and I haven't even had time to read the problem myself.' (Personal journal, March 15, 2009)

This data showed me that this strategy was worthwhile and one I would continue to use. The individual thinking time allowed my students to formulate their own ideas and methods to solve the problem. The students were able to make their own connections based on what they already understood about math and how it might apply to the problem. During a post-interview, Sue commented:

> "I liked when we worked by ourselves and then with a partner and then by ourselves again because you could think of things and then you could check with a partner to see if they were thinking the same thing and then you could finish up by yourself."

Two other students, during their post-interviews stated similar comments about the quiet thinking time. I continued this strategy as hearing comments like these reconfirmed that my students needed this time to think and process their own understanding of the problem before they communicated their thinking to others.

When I increased my expectations of my students no matter what the area, I was willing to try new things and find new ways to help them meet those expectations. I tried a variety of strategies and teaching methods to help as many students as possible become successful on their written explanations.

Vocabulary Usage

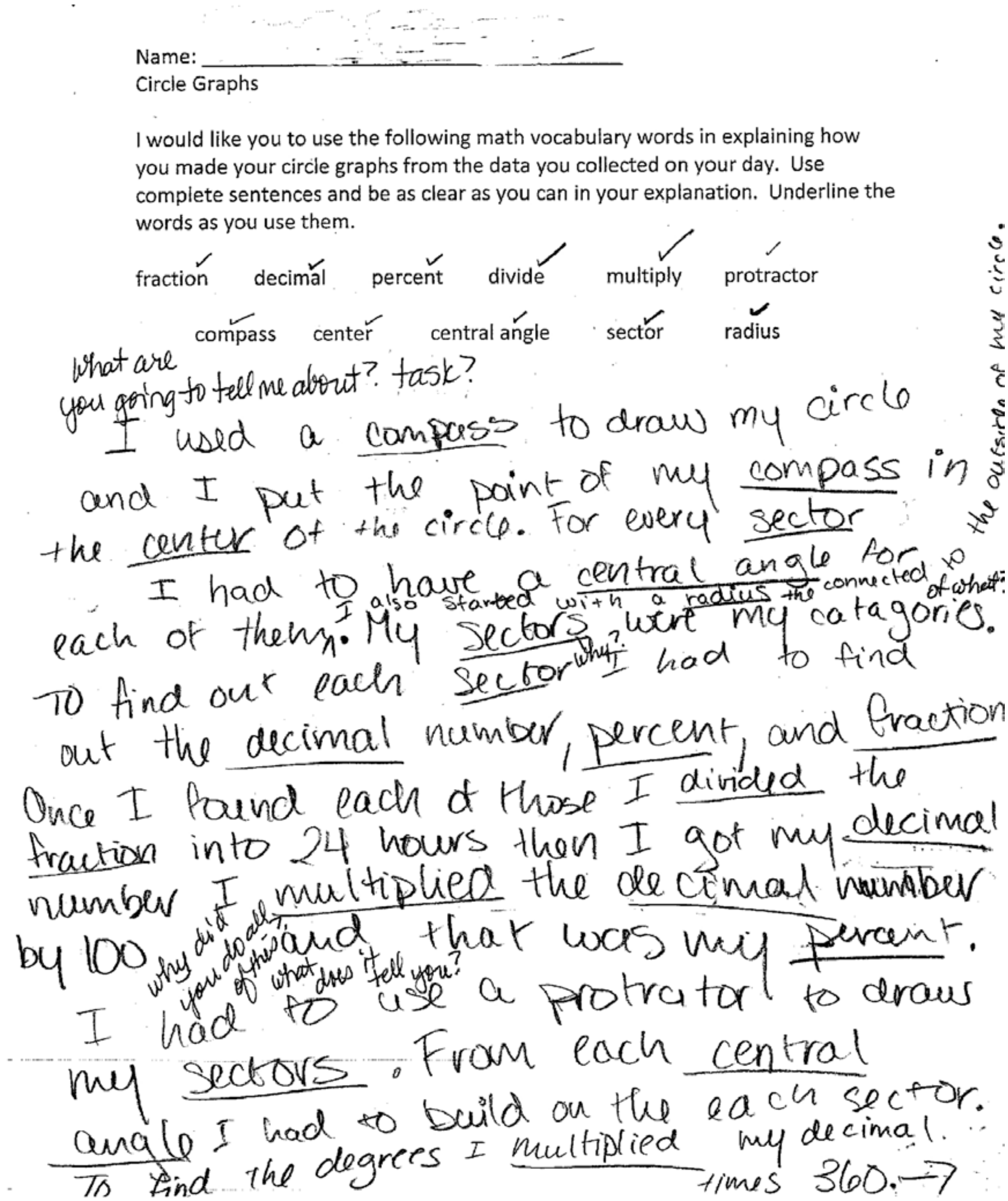
What will happen to students' vocabulary usage in written explanations after they receive specific vocabulary instruction? This is another question I wanted to explore with through this action research project. I continued to stress correct vocabulary in my teaching and explanations. I also wanted my students to not only use correct vocabulary when explaining and talking about math, but also use appropriate vocabulary while writing their explanations. At the beginning of this project, few of my students used vocabulary words in their written explanations. My students may have known the meanings, but were unable to successfully use them in their written explanations. After specific vocabulary instruction, my students used the vocabulary words more often in their writing. I found, however, my students still were not always able to explain what the vocabulary words meant or use them appropriately in their writing.

As already noted early in this paper, according to the pre- and post-survey results, 17 out of 19 students on the pre-survey and 17 out of 18 students on the post-survey said understanding specific math vocabulary was important in understanding problems (Appendix G). From my students responses on the survey I knew they thought vocabulary was important. This point was brought out again in all six individual student interviews. I asked Sally if she thought understanding math vocabulary was important. She commented, “If you don’t know what it (the vocabulary word) means, you’ll get the problem wrong.” The rest of the students interviewed made similar comments. Even though my students think they need to understand the vocabulary words used in problems, they still struggled with finding a way to use the words in their written explanations. I had my students brainstorm a list of possible vocabulary words they thought should be included in their written explanations for several of the problems. I tried assigning a certain number of vocabulary words to be used in their writing and actually gave them the words. My working hypothesis of just knowing the meanings of words does not mean students use them appropriately in their written explanations was reconfirmed by a piece of data from problem number 16 (Appendix A). I assigned my students a written explanation on how they made a circle graph of the data they collected on time spent during a typical school day. I noted the following observation in my teacher journal:

> They (the students) could use them (the vocabulary words) in a sentence, but that didn’t necessarily tell me they knew what they were, what information the words told, or how they applied to the problem. I got sentences in their writing like: “I changed my data into a fraction, decimal, and a percent.” “I made a circle with a

center and a radius." "I made central angles and colored them in for my sectors." These types of comments don't really tell me they knew what they were doing and why. (Personal journal, April 10, 2009)

The following piece of student work is Sally's from problem number 16.


Name: ____________________
Circle Graphs

I would like you to use the following math vocabulary words in explaining how you made your circle graphs from the data you collected on your day. Use complete sentences and be as clear as you can in your explanation. Underline the words as you use them.

fraction decimal percent divide multiply protractor

compass center central angle sector radius

What are you going to tell me about? task?

I used a compass to draw my circle and I put the point of my compass in the center of the circle. For every sector I had to have a central angle for each of them. I also started with a radius the outsides of my circle. My sectors were my catagories. connected to the of what? To find out each sector why? I had to find out the decimal number, percent, and fraction Once I found each of those I divided the fraction into 24 hours then I got my decimal number I multiplied the decimal number by 100 why did you do all of this? what does it tell you? and that was my percent. I had to use a protractor to draw my sectors. From each central angle I had to build on the each sector. To find the degrees I multiplied my decimal times 360. →

Sally was able to use all of the assigned words in her written explanation, but from her sentences I could not tell if she understood, for example, why she needed to change the percent to a decimal or what that percent told her about the data she collected.

Having completed this project, I believe more time was needed to improve the writing process. I will need to continue to model and guide my students in their written explanations. I need to show them more examples on how to appropriately use vocabulary words in their writing. I need to give my students time and opportunities to revise and rewrite their explanations to better use the appropriate words in the appropriate way. Acceptable vocabulary usage may improve with practice.

Mathematical Achievement

The last question I explored with my research project was *what will happen to students' mathematical achievement through increased practice on written explanations?* The data suggests increased practice with written explanations does not directly relate to mathematical achievement. The class average score for each written explanation did not steadily increase. The scores of each of the six students did not show steady improvement either (Appendix F). The class average rubric score for problem number one was 2.5. The class average from the last problem was 3.4. Yet the class average from problem number 13 was 4.4. When the rubric scores were grouped according to the type of problem each score was from, the problem-solving problem scores were a little higher than the rubric scores from the problems from the book. This data, however, does not show any noticeable improvement either (Appendix I). The data did not surprise me. I believe my expectations of my students' written explanations increased with each one. I found in order to see a correlation between written explanations and mathematical achievement I might have needed to collect other kinds of data, such as test scores.

I do not think my students would agree with the data showing no direct relationship to mathematical achievement. From both the individual interviews and the pre- and post-surveys, my students' perceptions indicated that writing problems out helped them understand the problems better. On the pre-survey, when asked to respond to the statement, "it helps me understand math problems better when I write out explanations," 16 out of 19 students strongly agreed or agreed. On the post-survey 14 out of 18 strongly agreed or agreed (Appendix H). Two of the students were undecided on the pre-survey and one disagreed; on the post-survey, two students disagreed, one was undecided and one student strongly disagreed.

I noted, early on in my research, the kind of problem the students were asked to explain made a difference in the scores that were obtained. The two types of problems my students were asked to write explanations for were regular computation problems from the book and more open-ended problem-solving type problems (Appendix A). When I separated the average scores into these two groups, I confirmed this observation. On the graph the average rubric scores from the book problem explanations tended to be lower than the rubric scores from the more open ended problems (Appendix I).

This same happening was also very apparent in the student interviews. I asked one particular question to all six students during the second interview, "Which type of problem, book or problem solving, was harder for you to explain and why?" Four of the six students said it was harder for them to write an explanation for a problem from the book. Their reasons varied, but are worth noting in the table below.

Table 1: Student Responses as to which type of problem was harder to explain

It depends on the problem; it depends on if it is hard or not. Like some of the problems were really hard and I didn't understand them – it was hard to write about it. (Keri)
Writing out the problem was harder (from the book) because you had to like take a lot more time to write out an explanation. (Nate)

I thought that the ones from the book were harder because you had to write out what you were thinking, but you already knew how to do it and so it was harder. You just know how to do it so you don't know how to explain it. (Sue)
If I already knew how to do the problem it was easier for me to explain. It didn't help me understand the problem more by writing it out. (Sally)

I could tell from these comments my students could not write about a problem they did not understand in the first place. My students needed to understand how to solve the problem before they could begin to write an explanation of what they did to solve the problem. I believe understanding of the problem and how to solve it is needed first before any written explanation can be done. I realized after hearing these comments from my students, if they understood the problem really well, they also had a hard time finding the words to explain the problem, because 'they just know it'. The class average rubric scores and the individual students' scores on the graph were inconsistent (Appendix F). The scores did not show a steady improvement over time and amount of practice. Based on how students responded both in the interviews, on the surveys, and their written explanation scores, I think they believed writing problems out helped them to understand problems better, but it depended on if they had a grasp of the problem before they wrote it out.

Conclusion

As I reflect back over all we did this semester I realized it was quite a journey. I learned some important things about myself and my teaching. I think that by raising the expectations of my students I have raised the expectations of myself. I have pushed myself to look for ways to help all my students be successful. I have tried a variety of strategies and made a concerted effort to be a more thorough teacher. I do agree with the NCTM communication standard that states, "[Writing] is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment. The communication process also helps build meaning and permanence for ideas and makes them public" (NCTM, 2000, p. 60). I think my students wrote to share and clarify their thinking, but they needed to be pushed to the next level of reflecting on what they wrote and building upon that to explore the mathematics behind the problem.

I agree with the comment made by Draper and Siebert (2004): "Every mathematics learning event is also a literacy event, and every literacy event in a mathematics classroom is a mathematics learning event" (p. 953). But where in the math classroom does one have time to develop the writing process when there is so much math to teach? This continues to be a dilemma with which I struggle. The data I have collected is not conclusive enough to show the transfer of understanding. We did not spend enough time on each piece of writing to actually see what would happen to student understanding if the students were pushed to revise their explanations.

Writing offers a place for students to explore their own thinking and reflect on it. Being able to convey ideas, feelings, and experiences can lead to the development of higher cognitive functions, including critical thinking, sound reasoning and problem solving (Albert, 2000). I believe my students felt that by writing explanations they did have a chance to get their thoughts and ideas down on paper, but it was not obvious from the data collected that higher-level thinking skills were being developed.

After conducting this research project, NCTM (2000) brings out a point with noting when it states, "it is important to avoid a premature rush to impose formal mathematical language" (p. 63). I think with the time I had allotted to the writing experience for my students and the use of vocabulary, I did push my students before some of them were ready. I also strongly believe what was stated in an article by Shield and Galbraith (1998): "The ability to use generalized language and to see a process in general terms is worth developing, and it is not expected that all middle school students will have developed such an ability" (p. 45). Maybe I expected too much too quickly from some of my students.

Implications

As a result of my study I will continue to emphasis correct and precise vocabulary both in my own teaching and language and that of my students. I will expose my students to and have them practice a wide list of vocabulary words over the course of the year. As part of our school improvement plan vocabulary pre- and post-tests have been developed for each grade level. This will be another avenue to stress mathematical vocabulary.

I will continue to require written explanations on a variety of problems. We will work on the writing process as a means for my students to communicate mathematical ideas and to evaluate their thinking. Taking the explanations a step further and having students revise and improve their explanations is another thing I would like to try.

I know I will continue to have high expectations for my students, not only for their daily work, but in their written explanations. Those expectations will apply to me as well. I plan to not only use the strategies I found helpful, but continue to look for more ways to help my students be successful. After all, that is what teaching is all about.

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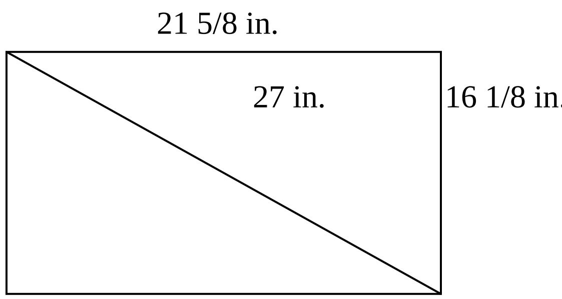
Appendix A

Collected Problems

1. Lesson 34, #7 (1/26)

   Explain: Tom's 27-inch television screen has the dimensions shown. Describe how to estimate its area.



2. Lesson 36 Problem Solving (2/04)
   Silvia was thinking of a number less than 90 that she says when counting by sixes and when counting by fives, but not when counting by fours. Of what number was she thinking?

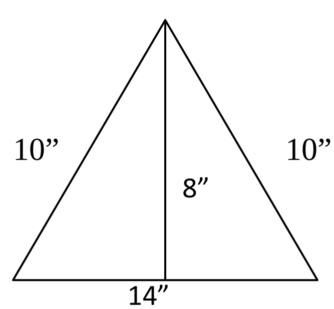
3. Lesson 37, #4 (2/09)
   Consuela went to the store with $10 and returned home with 3 gallons of milk and $1.30 in change. How can she find the cost of each gallon of milk?

4. Lesson 38 Problem solving (2/10)
   In the four class periods before lunch, Michael has math, English, science, and history, though not necessarily in that order. If each class is offered during each period, how many different permutations of the four classes are possible?

5. Lesson 39 Problem Solving (2/18)
   Jamaal glued 27 small blocks together to make this cube. Then he painted the six faces of the cube. Later the cube broke apart into 27 blocks. How many of the small blocks had 3 painted faces? . . . 2 painted faces? . . . 1 painted face? . . . no painted faces?

6. Vocabulary (2/16)

   Explain how to find the area of a triangle.

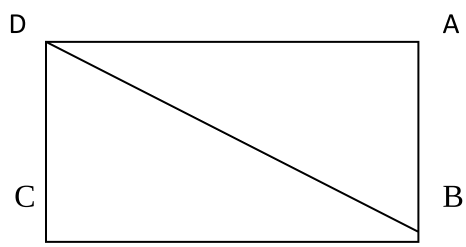


7. Lesson 41, Problem solving (3/4)
Use simpler, but similar problems to find the quotient of 1 divided by 50,000,000,000,000.

8. Lesson 42, Problem solving (3/9)
Darla noticed that each of the numbers in this sequence is a prime number: 31; 331; 3,331; 33,331; 333,331; 3,333,331; 33,333,331; . . . Does the pattern of prime numbers continue?

9. Lesson 43, Problem solving (3/11)
Mavy sees three faces on each of the three number cubes, for a total of nine faces. If the sum of the dots on the three faces of each number cube is different, and Mavy sees forty dots altogether, then which faces must be visible on each number cube? (*note - visible sides are all adjacent as shown in the picture in the book.)

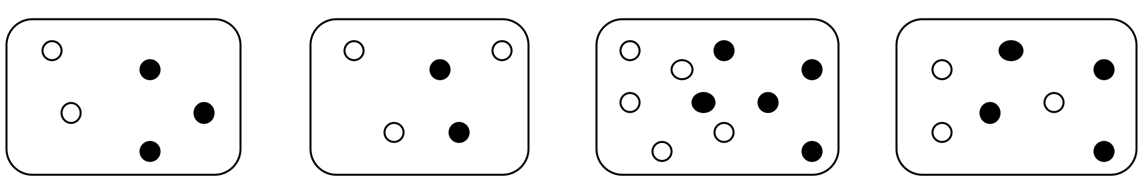
10. Lesson 44, #30 (3/16)
Figure ABCD is a rectangle. The measure of angle ADB is 35 degrees. Find the measure of each angle below. Explain how you found each measure.



Angle ABD Angle CBD Angle BDC

11. Lesson 45 – Student choice on problem from the written assignment. (3/17)

12. Lesson 46, Problem Solving (3/23)
White and black marbles were placed in four boxes as shown. From which box is the probability of choosing a black marble the greatest?



13. Lesson 48, Problem Solving (3/25)

The Teacher asked for two or three volunteers. Adam, Blanca, Chad, and Danielle raised their hands. From these four students, how many possible combinations of two students could the teacher select? How many different combinations of three students could the teacher select?

14. Lesson 49, Problem Solving (3/31)
    Javier used a six-yard length of string to make a rectangle that was twice as long as it was wide. What was the area of the rectangle in square feet?

15. Lesson 50, Problem #7 (4/2)
    Forty percent of the 30 members of the drama club were members of the senior class.

    a) How many members of the drama club were seniors?
    b) What percent of the drama club were not members of the senior class?

16. Circle Graphs (4/14)
    Students will use specific vocabulary to explain how they created their circle graphs from their data. Vocabulary – fraction, percent, decimal, center, central angles, protractor, sectors, radius, multiply, divide, compass

Appendix B

Pre and Post Survey

Please respond to the following items by drawing a circle around the response that most closely matches your opinion: strongly agree (SA), agree (A), undecided (U), disagree (D), strongly disagree (DS)

1. It helps me understand math problems better when I write out explanations.

SA A U D DS

2. I like to write out explanations to mathematical problems.

SA A U D DS

3. Understanding specific math vocabulary is important in understanding problems.

SA A U D DS

Please respond to the following questions using short answers.

4. Given the option, would you rather give one written explanation for a problem or do more computation problems? Why?

5. What could teachers do to help students understand math better?

Appendix C

Student Interview questions *Second interview questions additions in italics*

Student interviews will be focused on a subset of these questions.

What does it look like when you justify answers on a homework assignment?
What are the benefits of justifying your answers on your homework assignments if any?
Given the option, would you rather give one written explanation for a problem or do more computation problems? Why?
When working a word problem do you think you know the meanings of most of the vocabulary words in each problem? Please give some examples.
Why is it important to know the meanings of vocabulary words you see in math?
Was there anything specifically we did this quarter that helped you learn math vocabulary? What would that be?
Has your attitude changed about having to explain a math problem this year? Why or why not?
What makes math easy or difficult for you?
Have you ever had a really bad experience with math? If so, what happened?
What could teachers do to help students in math?
Did you enjoy working word problems before this year?
What do you like most about math? Least about math? This semester I have changed some of my teaching practices. What advice would you give me about continuing these changes next year?
I tried different ways of having you students work on the problem solving problems – with a partner, individuals and then a partner, brainstorming words and strategies, and giving hints of things to note or remember. Which if any of these things did you find the most beneficial?
How do you feel when you are required to give a written explanation for a problem?
We did 16 different written explanations. How did you feel over the course of the quarter the more we had to do?
Some were problem solving problems, others were just explaining problems from the assignment. Which were easier for you to write an explanation for? Why? Which ones were the hardest?
Is there any benefit to writing out an explanation for a problem? If so what are the benefits?
What specific benefits did you get out of writing math explanations?
Comment on the progress you made on your written explanations.
Which part of the explanation was the hardest for you? The easiest?

Appendix D

Name: ____
Problem: ____

Rubric for scoring written explanations in homework and problem solving

1 point for solution
- 0 point – for no solution
- .5 point – incomplete solution
- 1 point – complete and correct solution with label

1 point for stating what they know about the problem and the task
- 0 point – no statement about the problem and the task
- .5 point – partial statement, necessary vocabulary words undefined, task not included
- 1 point – complete statement about the problem in their own words, necessary vocabulary explained, and task defined

1 point for overall sentence structure
- 0 point – no sentences
- .5 point - missing capital letters and punctuation, incomplete sentences, run-on sentences
- 1 point – Clear, complete sentences

2 points for explaining their work / showing their work
- 0 point – no work
- 1 point – some relevant information, computation only, incomplete explanation, missing labels
- 2 points – thorough explanation, work shown with words, drawings, numbers etc., and complete labeling of work

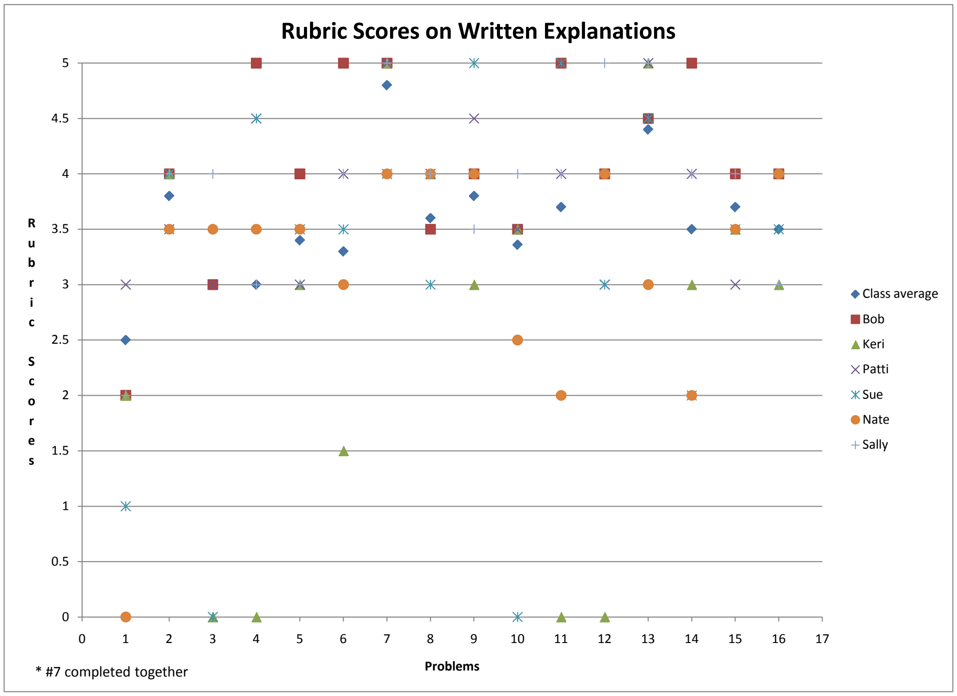
TOTAL – 5 points

Appendix E

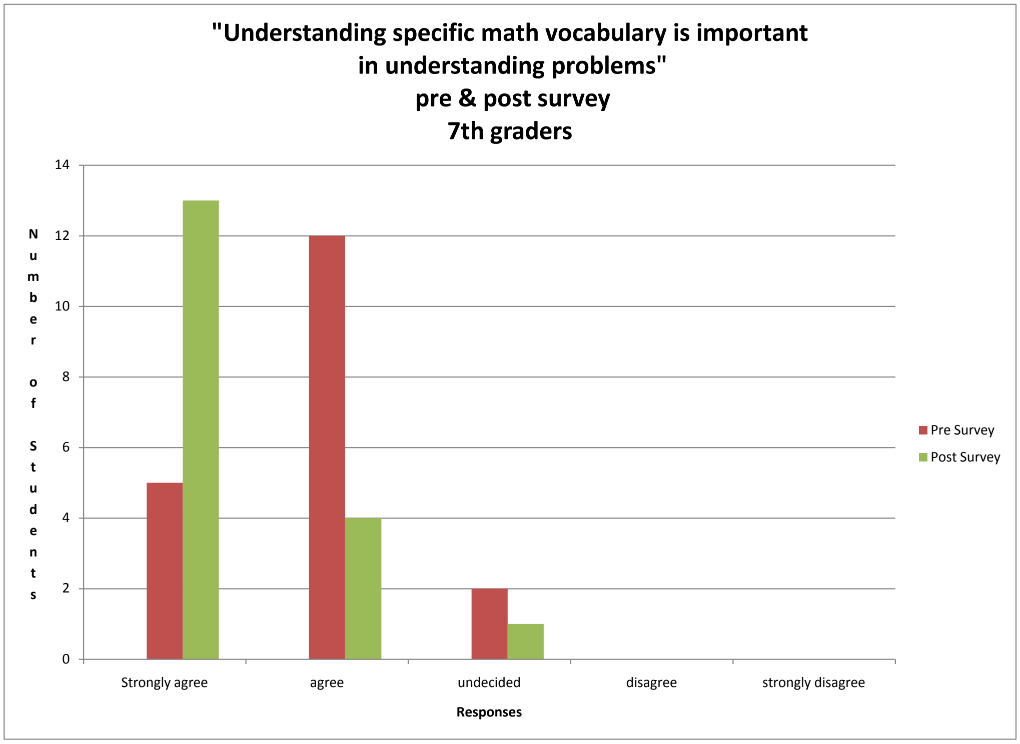
Problem Scoring Rubric Tallies
Problem Solving_________________
Date_____________

	0	.5	1	2
SOLUTION				
TASK & WHAT YOU KNOW				
SENTENCE STRUCTURE				
EXPLAINING & SHOWING WORK				

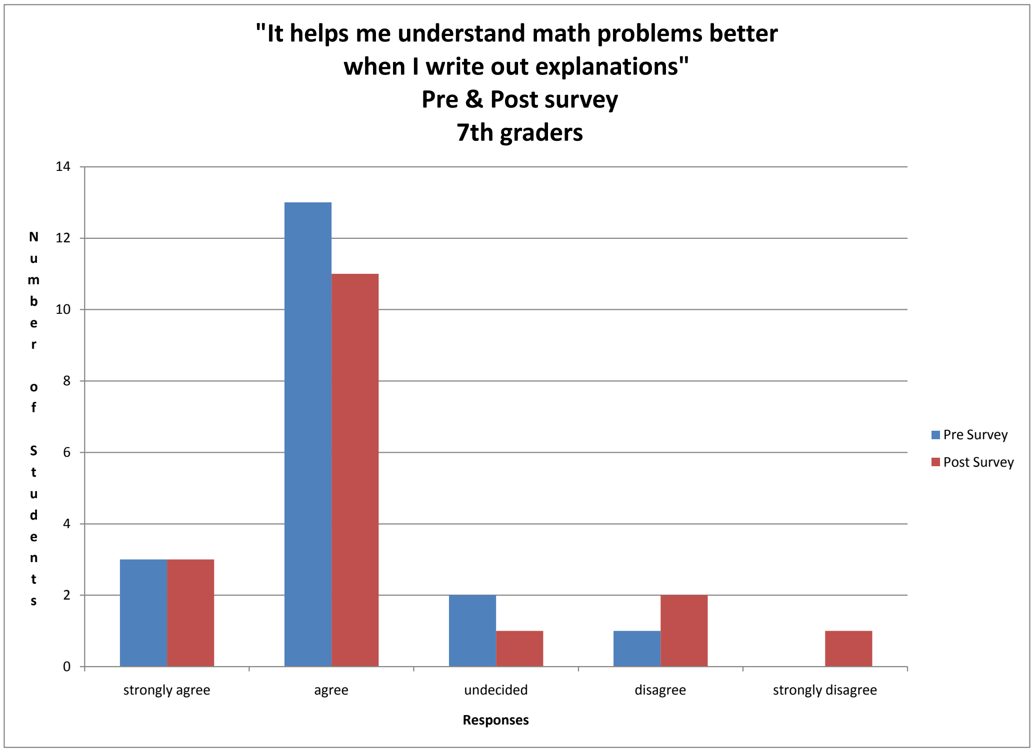
Appendix F


Rubric Scores on Written Explanations
Rubric Scores
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17
Problems
Class average
Bob
Keri
Patti
Sue
Nate
Sally
* #7 completed together

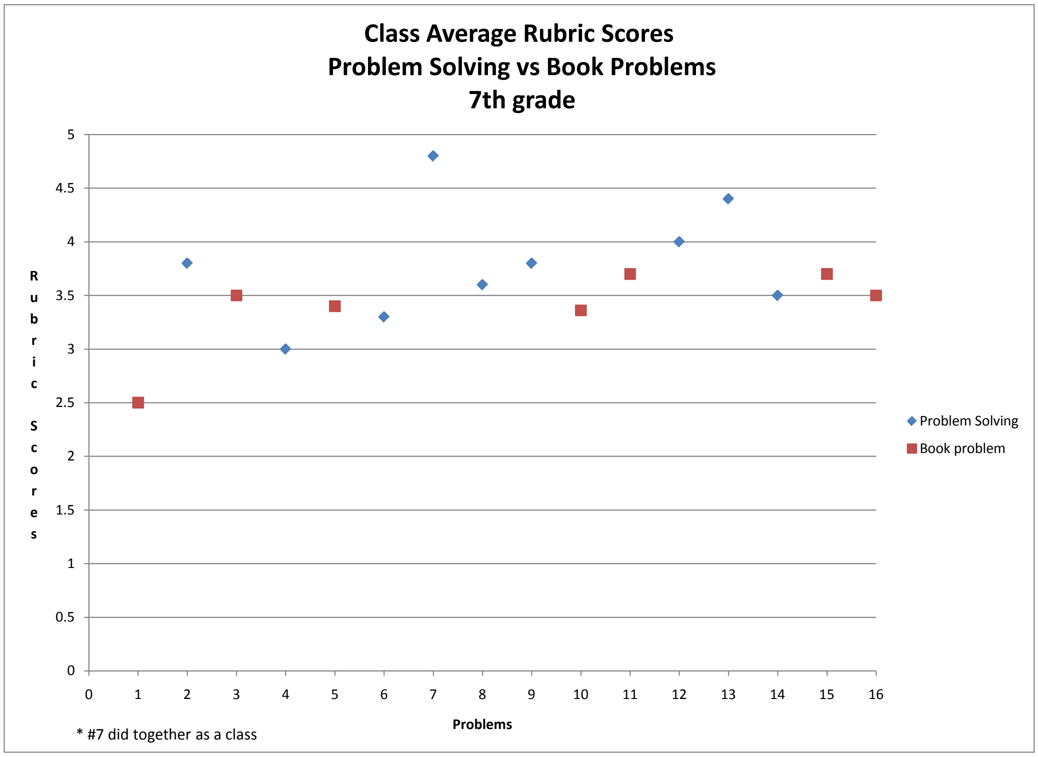
Appendix G


"Understanding specific math vocabulary is important in understanding problems"
pre & post survey
7th graders
Number of Students
14
12
10
8
6
4
2
0
Strongly agree
agree
undecided
disagree
strongly disagree
Responses
Pre Survey
Post Survey

Appendix H


"It helps me understand math problems better
when I write out explanations"
Pre & Post survey
7th graders
Number of Students
14
12
10
8
6
4
2
0
strongly agree
agree
undecided
disagree
strongly disagree
Responses
Pre Survey
Post Survey

Appendix I


Class Average Rubric Scores
Problem Solving vs Book Problems
7th grade
Rubric Scores
Problems
Problem Solving
Book problem
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
* #7 did together as a class