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A SET OF COMPLETELY INDEPENDENT POSTULATES FOR THE LINEAR ORDER η^* .

BY PROFESSOR M. G. GABA.

(Read before the American Mathematical Society September 4, 1919.)

PROFESSOR E. V. HUNTINGTON has published[†] three sets of completely independent postulates for serial order. His set A involves four postulates, which is as high a number of postulates as had been proved completely independent. In the present paper are given seven postulates which form a categorical and completely independent set for the linear order.

Our basis is a class of elements $[p]$ and an undefined dyadic relation (called 'less than') among the elements. If we are given two elements $p_1 p_2$ and if the relation p_1 less than p_2 holds, we will symbolize it by $p_1 < p_2$. If the relation p_1 less than p_2 does not hold, we will symbolize it by $p_1 \nless p_2$.

Our postulates are:

- I. If $p_1 < p_2$, then $p_2 \nless p_1$.
- II. If $p_1 \nless p_2$, then $p_2 < p_1$; p_1, p_2 distinct.
- III. If $p_1 < p_2$ and $p_2 < p_3$, then $p_1 < p_3$.
- IV. If $p_1 < p_2$, then there exists a p_3 such that $p_1 < p_3$ and $p_3 < p_2$.
- V. For every p_1 there exists a p_2 such that $p_2 < p_1$.
- VI. For every p_1 there exists a p_2 such that $p_1 < p_2$.
- VII. The class of elements $[p]$ form a denumerable set.

That the set is categorical follows from the fact that the seven postulates stated are the necessary and sufficient conditions for the linear order η . To show complete independence it will be necessary to cite 128 (2^7) examples showing all possible combinations ($\pm \pm \pm \pm \pm \pm \pm$) of our postulates holding and not holding. This is done by giving eight definitions of $<$, and sixteen sets of points such that each definition is applicable to every one of the sets, and every combination

* The linear order η is an ordered set equivalent to that of all the rational numbers.

[†] "Sets of completely independent postulates for serial order." This BULLETIN, March, 1917. This paper contains a bibliography of complete independence.

of definition of $<$ and set yields a different example. The eight definitions give the eight $(\pm\pm\pm)$ groups of cases for the implicational postulates I, II and III, whereas each of the sixteen sets gives all the eight cases where any particular set $(\pm\pm\pm\pm)$ of the existential postulates IV, V, VI and VII hold or do not hold.

For the independence examples, the set $[p]$ consists of points on a line such that

| | IV | V | VI | VII | |
|-----|----|---|----|-----|---|
| 1) | - | - | - | - | $p = -3, -2 \leq p \leq 2, p = 3$ and p real. |
| 2) | - | - | - | + | $p = -3, -2 \leq p \leq 2, p = 3$ and p rational. |
| 3) | - | - | + | - | $p = -3, -2 \leq p < 3,$ and p real. |
| 4) | - | - | + | + | $p = -3, -2 \leq p < 3,$ and p rational. |
| 5) | - | + | - | - | $-3 < p \leq 2, p = 3$ and p real. |
| 6) | - | + | - | + | $-3 < p \leq 2, p = 3$ and p rational. |
| 7) | - | + | + | - | $-3 < p \leq 3/2, 2 \leq p < 3,$ and p real. |
| 8) | - | + | + | + | $-3 < p \leq 3/2, 2 \leq p < 3,$ and p rational. |
| 9) | + | - | - | - | $-3 \leq p \leq 3,$ and p real. |
| 10) | + | - | - | + | $-3 \leq p \leq 3,$ and p rational. |
| 11) | + | - | + | - | $-3 \leq p < 3,$ and p real. |
| 12) | + | - | + | + | $-3 \leq p < 3,$ and p rational. |
| 13) | + | + | - | - | $-3 < p \leq 3,$ and p real. |
| 14) | + | + | - | + | $-3 < p \leq 3,$ and p rational. |
| 15) | + | + | + | - | $-3 < p < 3,$ and p real. |
| 16) | + | + | + | + | $-3 < p < 3,$ and p rational. |

A definition of $<$ requires that whenever we are given two numbers of our set $p_1 p_2$ we have a criterion whereby we can tell whether the relation $p_1 < p_2$ holds or does not hold. In all the eight definitions of $<$ the relation holds for any pair of numbers $p_1 p_2$ if it holds in the case of ordinary linear order,

| | I | II | III | |
|-----|---|----|-----|---|
| 1') | - | - | - | except $0 \nless 1, -1 < -2, 0 < -1$ and $0 < -2$. |
| 2') | - | - | + | except $1 < -1, 1 < 0, 0 < -1, p_1 < -1, p_1 < 0, p_1 < 1,$ $-1 \nless p_2, 0 \nless p_2$ and $1 \nless p_2; p_1 \neq -1, 0, 1; p_2 \neq -1,$ $0, 1.$ |
| 3') | - | + | - | except $0 < -m/2^n, n$ positive integer and m odd positive integer. |
| 4') | - | + | + | except $p_1 < -1, p_1 < 0, p_1 < 1, -1 \nless p_2, 0 \nless p_2,$ and $1 \nless p_2; p_1 \neq 3; p_2 \neq -1, 0, 1.$ |
| 5') | + | - | - | and $p_2 - p_1 < 1/3.$ |
| 6') | + | - | + | and $p_2 - p_1 = m/2^n, n$ positive integer and m odd integer. |
| 7') | + | + | - | except $0 < -m/2^n$ and $-m/2^n \nless 0, n$ positive integer and m odd positive integer. |
| 8') | + | + | + | with no exceptions. |

To illustrate: The independence example where postulates II, III, V, and VII hold and postulates I, IV and VI do not hold $(-+ +- + - +)$ is definition 4' used on set 6.

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