7-2006

Improving Mathematics Problem Solving Through Written Explanations

Janet Schlattmann
Alliance, Nebraska

Improving Mathematics Problem Solving Through Written Explanations

Janet Schlattmann
Alliance, Nebraska

A report on an action research project submitted in partial fulfillment of the requirements for Master of Arts in the Department of Teaching, Learning and Teacher Education, University of Nebraska-Lincoln.

Ruth Heaton, Advisor

July 2006

Improving Mathematics Problem Solving Through Written Explanations

Abstract

In this action research study of two classrooms of 7th grade mathematics, I investigated how requiring written explanations of problem solving would affect students ability to problem solve, their ability to write good explanations, and how it would affect their attitudes toward mathematics and problem solving. I studied a regular 7th grade mathematics class and a lower ability 7th grade class to see if there would be any difference in what was gained by each group or any group. I discovered that there were no large gains made in the short time period of my action research. Some gains were made in ability to problem solve by my lower ability students over the 7 weeks that they did a weekly problem solving assignment. Some individual students felt that the writing had helped them in their problem solving because they needed to think and write each step. As a result of this research I plan to continue implementing writing in my classroom over the entire school year requiring a little more from students each time we problem solve and write.

This study is intended to describe the impact of a problem solving curriculum that focuses on including more problem solving opportunities for students and requiring writing or representing solutions beyond just computation. I observed low achieving students and what effect in particular this type of curriculum had on them. I wanted to see whether writing solutions made students better problem solvers and if attitudes as problem solvers changed as students worked on more problem solving and writing. I also wanted to study if the students became better at communicating their thinking as a result of the increased focus on writing.

This action research study was conducted in two of my 7th grade mathematics classes at Brightstar Middle School where I have taught for 17 years. One class, called Class A, of 12 students consisted of students who have not had much success in regular math classes, have had a hard time getting homework completed, or just struggled and needed more help and time to learn the concepts. Class A followed the same curriculum as the other classes, but the teaching and approach was a little different than the others. The other class studied, Class B, was a regular 7th grade class with 14 students. The reason for studying both was to see if the changes in problem solving had an impact for one group or the other or both.

Problem of Practice

My students have had low scores on problem solving when taking achievement tests over the past several years. Our district has noticed an overall problem with low problem solving scores. My students have not demonstrated to me that they are very confident in their ability to problem solve. When we have a day we use for problem solving I find that some of the students give up before they even start or want to be led through the

whole thing. As I have asked students to explain how they arrived at a solution I found that few can explain clearly what they did or were thinking. The assignments I asked them to turn in earlier in the year had very brief explanations and these were primarily numbers, not explanations of their thinking. This had been a problem at the beginning of this year and also with previous students. This was evident as we piloted a problem solving assessment the previous year and asked students to show all of their work. Students' ideas of problem solving have been very limited to just doing a calculation as a result of being taught that way.

I want my students to be able to develop a "habits of mind" for problem solving. I want my students to be confident in approaching a problem, be able to convince someone else that they know how to solve a problem, and be able to explain their thinking well enough that another person will be able to follow just what they did to solve the problem. Through the classes that I have taken where I was required to show my thinking and all of the steps leading up to a solution, I found I had to understand the problem better before I could explain it. I could get a number answer sometimes without really understanding what all of it meant. I believe through explaining a problem, a deeper understanding for the problem happens and more confidence is built. Justification brings a higher level of understanding and thinking.

Our school's North Central Accreditation goal for our school is problem solving. We have worked on building up students' knowledge of strategies and steps to problem solving and have worked to include more problem solving in our curriculum. No Child Left Behind is requiring that our students be tested each year and show adequate yearly

progress in problem solving. Our students' scores on Terra Nova tests indicate we need to improve our problem solving skills.

Problem solving and communication are two of the process standards adopted by the National Council of Teachers of Mathematics (NCTM). Communication is an area that we have not concentrated on very much in our school or district for mathematics. Students will need to be able to communicate their thinking and justify the computations that they make if they are to use their mathematical skills in any practical situations in the work place. It is important that we change what we do to improve our students' problem solving skills.

Literature Review

Problem solving and communication are mathematical process standards set up by the National Council of Teachers of Mathematics as important for students to develop. Research shows many different ideas on how problem solving and communication should be developed; many times a connection is made between the two. The Principles and Standards (NCTM, 2000, p.53) states "Students should have frequent opportunities to formulate, grapple, and solve complex problems that require a significant amount of effort and should then be encouraged to reflect on their thinking." It is important that students have the opportunity to solve complex problems and also reflect on thinking which is where the communication is addressed.

Students often do not know where to start with problem solving and find it very difficult. Teaching problem solving strategies, teaching Polya four problem solving steps, and modeling graphic organizers are some of the areas that help students learn how to solve problems (Kjos & Long, 1994; Mills& Stevens, 1998; Millard, Oaks, & Sanders,

2002; Ishii, 2002). The study done by Kjos and Long focused on a change in the instructional approach of problem solving with three areas: students were taught problem solving steps, manipulatives were used, and abilities were developed through writing about math skills. They found student attitudes about their own mathematical ability improved and observed an increased ability to write about their thinking. Teaching problem solving steps and specific strategies improves student achievement in problem solving. Students gain insight and knowledge of the problem solving process. They are then able to use that past success and process to know where to start on a new problem (Millard, Oaks, & Sanders, 2002). The steps of problem solving can also be more effective if graphic organizers are used to help students organize their thoughts and work through the steps of what they know and where they are headed. (Stevens, 1998).

Journal writing in mathematics has been used with increased frequency. The NCTM standard of communication in mathematics encourages writing in mathematics as writing can help students consolidate their thinking because they have to reflect on their work and clarify their thoughts. “Teachers should ask students to reflect on, explain, and justify their answers so that problem solving both leads to and confirms students’ understanding of mathematical concepts” (NCTM, 2000, p.121).

Journal writing can take the form of using writing prompts, expository writing, writing explanations, writing stories about problems, writing about attitudes toward math, and writing what students have learned. Student attitudes, thinking, communication and procedural knowledge can be increased and show improvement when journal writing is used in a mathematical classroom. Using student reflections in the form of journals helps communication between student and teacher. Using and creating graphic organizers along

with self and peer editing brings positive results in improvement in achievement and indicating students are more highly motivated (Mills & Stevens, 1998). Journals can be successfully used for the sole purpose of monitoring student attitudes about problem solving. (Millard, Oaks, & Sanders,2002).

Four studies done by teachers using writing in their classes were condensed into a paper by Ishii (2002) that indicated the most positive outcome of journal writing was that it created student generated mathematical discourse. Students were discussing mathematics and thinking in a way they had not before. Since they had written about their thinking, they also wanted to share what they had written and discuss mathematics (Ishii, 2002). Writing can help students increase the amount of communicating that they do about mathematics as they want to share their thoughts with other students and teachers. Jurdak (1998) found that journaling does not lead to growth in problem solving. He writes, "Journal writing has a positive impact on conceptual understanding, procedural knowledge, and mathematical communication but not on problem solving, school mathematics achievement and attitudes toward mathematics". (Jurdak,1998, p. 413). He went on to explain that "the reason journal writing may not have a positive impact on problem solving may be the complex nature of problem solving and the nature of journal prompts" (p. 419). He cited research that showed there are more positive effects on problem solving for expository writing than journal writing. Expository writing "engages students explicitly in the whole process of problem solving (p.149).

Expository writing, where students explain how they solve a problem and the thinking they are doing in problem solving, has not been studied to a great extent. Research does show that students' thinking and ability to create their own thinking and

reasoning is increased by writing. One study by Pugalee (2001) indicated that students' writing about their mathematical problem solving processes showed evidence of a metacognitive framework. His study came after looking at research indicating that verbalization requiring metacognitive statements, especially explanations of actions, enhances problem solving performance. Students' writing showed metacognitive behaviors during orientation (assessing and understanding the problem), organization (plans of action), execution (carrying out the plan and redirection if it does not work) and verification (evaluating decisions and results) phases of problem solving. Students' thinking, reasoning, and mental processes were evident.

Writing can help teachers learn how students learn and think. Writing helps students develop tools they need to think algebraically. "The opportunity to explain thinking in writing helps develop conceptual knowledge…conceptual knowledge and sense making cannot be generated by procedures learned by rote" (Steele, 2005, p.153). Steele found that the use of writing helped students develop schemata knowledge for algebraic thinking.

The research on written communication and problem solving has focused on problem solving strategies, teaching the four problem solving steps and journal writing. Little research has been done exploring what the effects of writing explanations to problem solving might be to problem solving skills or to attitudes students have about problem solving. There is a need to find out if writing about reasoning and thinking will help improve problem solving achievement. My research is intended to see if the writing approach makes a difference for low achieving students and/or average students. Research was not found about how writing in mathematics may affect the low achiever's

problem solving skill, communication skills or attitudes about mathematics. This study will try to provide some insight into using written explanations of thinking and reasoning in problem solving to see what affect it has on problem solving skills, communication and attitudes of students about problem solving among low achieving students and average ability students.

Purpose Statement/Research Questions

The purpose of this study is to determine the effects on lower achieving students as well as other students of using written explanations and/or representations of thinking and reasoning in mathematical problem solving. Data collection took place in my classroom during 9 weeks of the 2006 spring semester. This study will attempt to answer these research questions:

- What is the effect of students expressing their thinking and reasoning through writing/representation in math problem solving on their ability to solve problems?
- How will student attitudes about themselves as problem solvers change when written communication is used?
- What is the effect on students' ability to communicate their thinking when written communication is included with problem solving?
- How will low achieving student's problem solving growth compare to the scores of students in a regular class?

Methods

The data collected to study what the effect of students expressing their thinking and reasoning through writing/representation in math problem solving on their ability to solve problems started with a pre-test. This was a two question pre-test scored with a 4 point

rubric on concepts and computations. (See Appendix A and B) This was given to all 12 students in Class A and 14 students in Class B at the beginning of the nine weeks of my research. A post-test was given at the end of the 9 weeks which also had two problems that were scored using the rubric. (See Appendix C)

A second data collection was done by taking scores on seven daily problem solving assignments given once a week. These were also scored with the concepts rubric. On these scores I found that the difficulty of the problems I gave the students each week made a difference in the kind of scores they received. It may have been the problem difficulty, not the students' ability to problem solve that made the scores vary. There was also a little problem with getting some of the problems completed as it was flu season, and many students were ill during this time. Some made up their work and others did not. Those who were gone found it difficult to do problem solving without having been in the classroom for the beginning explanations.

The third area of data collection came from the problem solving assessments that our district gives. These scores were from the same students' 6th grade assessments and then their scores from this year as they took the 7th grade district assessment. I found a problem with comparing these two years' assessments as we are in a transition from the type of test we gave previously to the one we just wrote last year. The 6th grade test was not as long or difficult. The assessment that they took in 7th grade is longer and was written to better reflect the areas of advanced, proficient, progressing and beginning.

The next data collected aimed to answer the question how will student attitudes about themselves as problem solvers change when written communication is used. The first week of my research I gave a Likert-scale attitude survey to all of the students; the last

week of the research I gave the same attitude survey (See Appendix D). I kept observations in my journal as to what I saw happening each week and even as they took tests. The third type of data I collected was through interviewing three students in each class, asking them about their feelings about mathematics, writing and problem solving. Each interview was recorded.

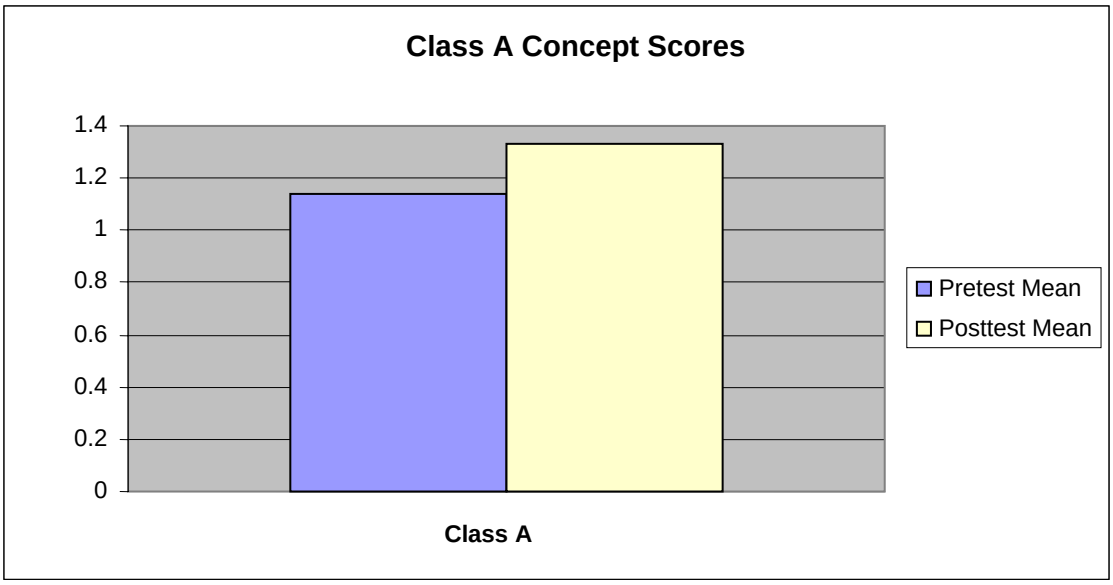
My third question is to study the effect on students' ability to communicate their thinking when written communication is used. I used the pre-test and post-test on problem solving described in my first question and this time scored it using a communication/representation rubric (See Appendix E). It was also a 4 point scale rubric. I scored each of the seven daily problem solving assignments using the communication/representation rubric. The third area of data collection was through observing and journaling what I saw in students' communication.

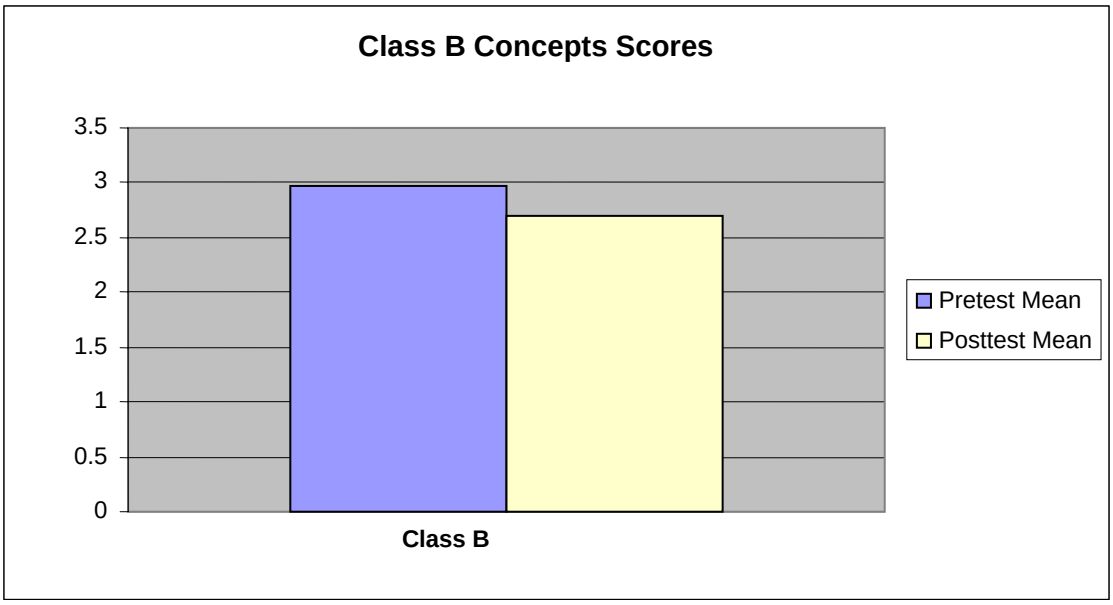
The fourth question was to compare the students in Class A with the students in Class B. I used the pre-test and post-test scores as a comparison of conceptual skills. I compared the scores of the 7 daily problem solving assignments. I also compared the scores on the district assessments from this year and the previous year.

Analysis

Concepts

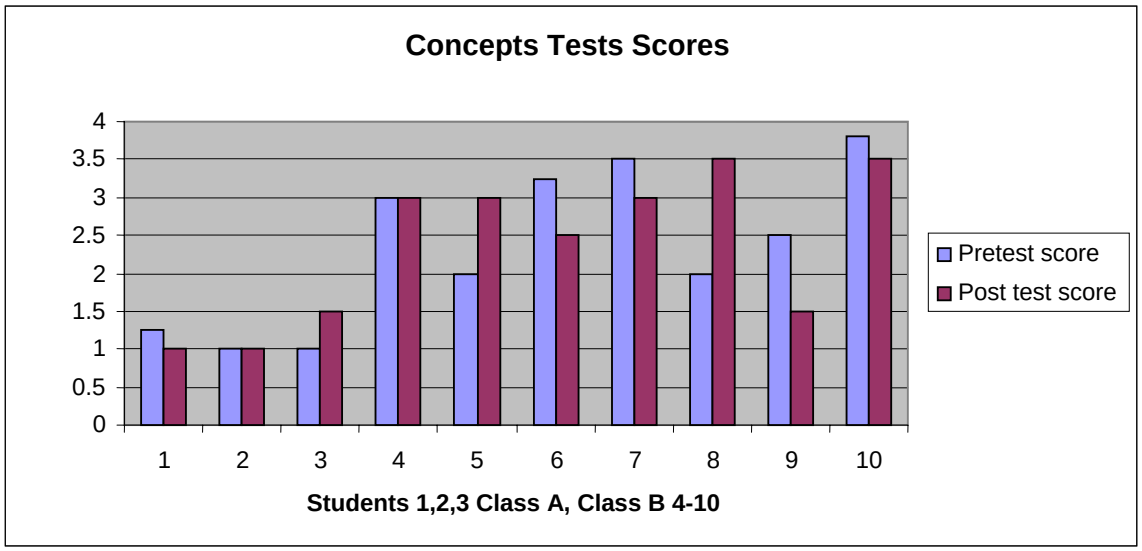
Mathematical Concepts is the first set of data I want to consider. When using the problem solving pre-tests and post-tests as comparison the Class A only raised their mean score by 0.2. The Class B went down by 0.3 from the pre-test mean score. The pre-test and post-test means are depicted in the graphs below (Charts of the actual data for all of the graphs on concepts can be found in Appendix F).





It was a little more interesting to look at a few individuals who gave permission for their scores to be used. I will be discussing the three students in Class A in more depth as I go along since my emphasis was seeing if writing made a difference for the students in this class. The students in Class A consisted of one student I will refer to as Callan who often did not do any work and was a behavior problem. Another student, Jim, had difficulty with written expression. The third student, Kathy, tries hard but does not make

much progress. They are typical of this class as a whole. In Class A, Kathy's scores went down; Callan stayed the same; and Jim made an increase. On the graph below, the first three students are from Class A.

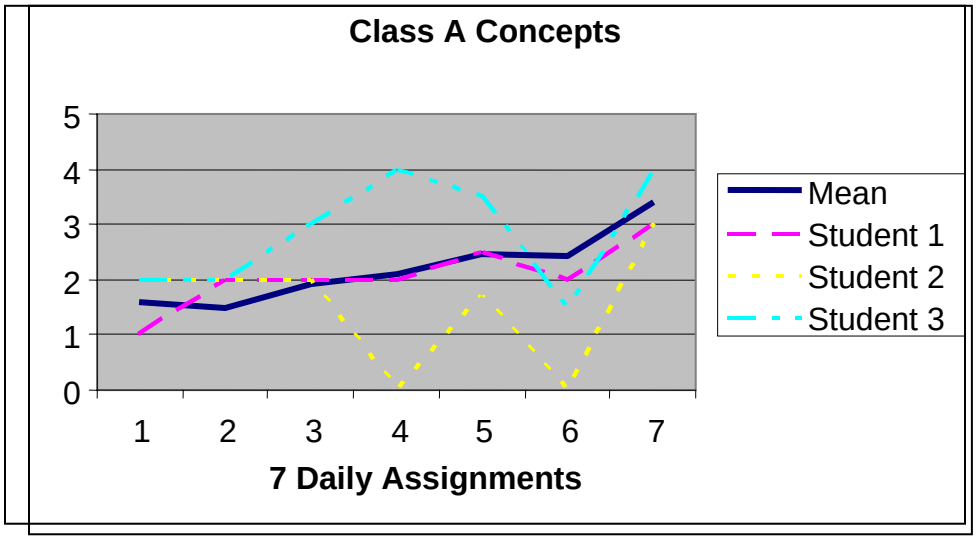


.

The students in Class B were all girls. Three of them are students who do not always turn in homework. Two of these three miss a lot of school. The others are good students. Overall, they are pretty typical of the types of students in my class except that most of the rest of my class turn in homework on a more regular basis. I will identify student 6 as Sue and student 7 as Linda. Sue is an average student who usually turns in assignments; although, during this period she was ill and did not turn in homework. Linda is a strong student who works hard and has good skills.

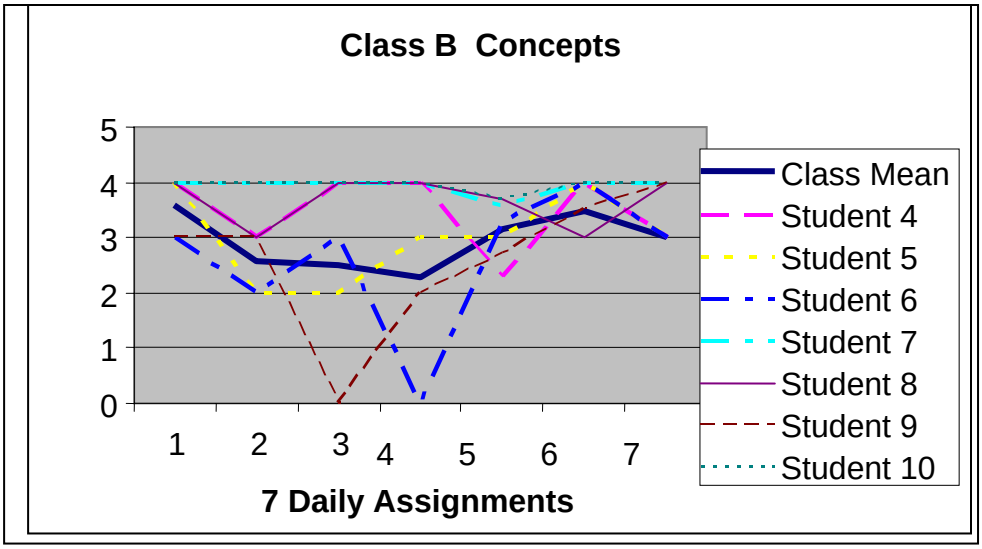
Class B showed one student, student 8, making substantial progress, student 5 showing some progress and then others' scores either decreasing or staying the same. For a few students the problem solving was making a difference. Sue and Linda's scores went down in the post-test.

The biggest growth appeared when looking at the daily assignment scores on concepts taken over the 7 weeks. Class A showed a steady increase in their mean scores. A graph of the three students and the class mean follows.



The graph shows that Callan, student 2, did not turn in any assignment on daily assignment 4 and 6. Jim, student 3, was making progress but went down on assignment 6. Jim was the one who made progress on the concept post-test. Kathy, student 3, did a little better each time we did an assignment, except assignment 6

The scores in Class B seemed to change according to the difficulty of the assignment. For several weeks there were students who did not even do the assignment, which brought the mean score down. This corresponded to the time of flu season; many students were out for several days up to a week with illnesses. Some students never did complete the problem solving assignment after being ill. The graph of their progress follows.



Sue had an assignment that was not turned in and varied a little in her scoring on each assignment. Linda stayed pretty steady except on one assignment. Her scores did not really improve or drop.

The third data that I considered in reference to concepts was my district's assessment on problem solving. Trying to compare this year's district assessment with last year's district assessment was not a very valid comparison. The scores would definitely show a drop in ability to problem solve in both classes. The problem is that we had a shorter, simpler version of the assessment that students were given last year. We then rewrote our school assessments to show adequate yearly progress. When we improved them to show the difference of beginning, progressing, proficient and advanced it made our assessments more difficult. We found as we gave this new assessment this year that we probably made them too difficult. We only gave a trial run of the assessment last year to the 6th graders. Only some of the students took the trial, which means only part of them took the more difficult test so I cannot use those scores to see progress. If I compare last year's scores of the same students taking an easy test compared to the same students

taking a difficult test this year all of my students in both classes show a regression in problem solving ability. A chart follows that shows these scores which I described but will not really consider because of the difference in the assessment difficulty.

Class A Percentage in each category of District Problem Solving Assessments

	Beginning	Progressing	Proficient	Advanced
6th Assessment	8%	42%	25%	25%
7th Assessment	75 %	25%		

Class B Percentage in each category of District Problem Solving Assessment

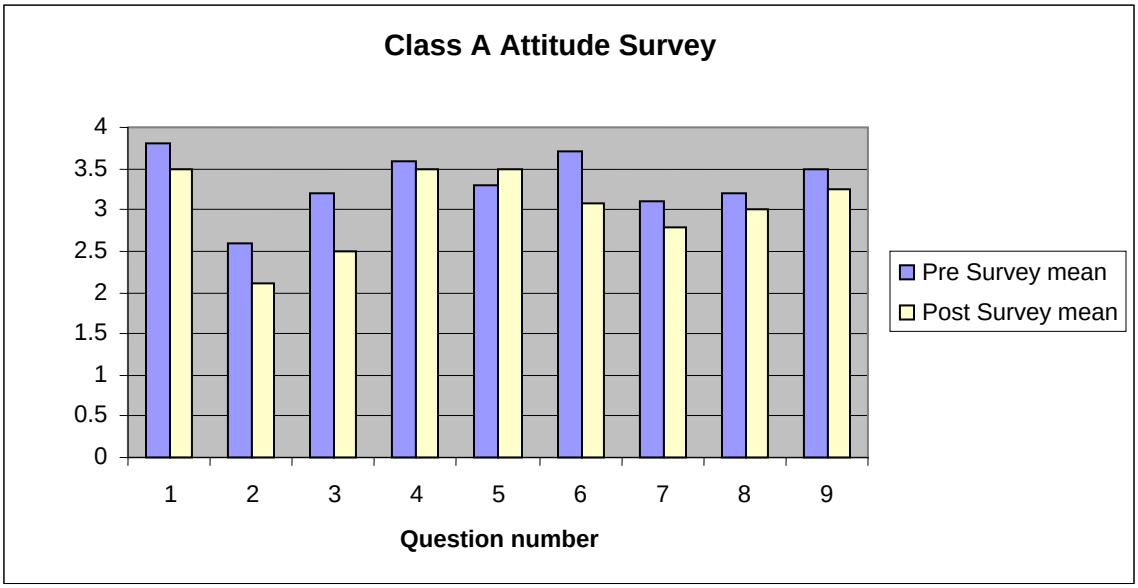
	Beginning	Progressing	Proficient	Advanced
6th Assessment		17%		83%
7th Assessment	14%	50%	36%	

Attitudes

Students' attitudes about themselves as problem solvers are as varied as the students themselves. Some students felt that the writing is making a difference in their ability to problem solve and felt better about the explanations that they wrote. When comparing the attitude scores from the first survey to the survey at the end, most of the scores went down for Class A. The survey is found in Appendix D. The only survey question that was higher was the question that said "I understand more about problem solving than I am able to put down in writing." The scores that went up for Class B were the questions that asked if writing about thinking helps to understand problem solving; if the student was improving in ability to problem solve; I know more about problem solving than I can write; and that others would understand what I write.

The survey was given before the problem solving and writing started and again after we were done. The following graph shows the mean score for each question in the

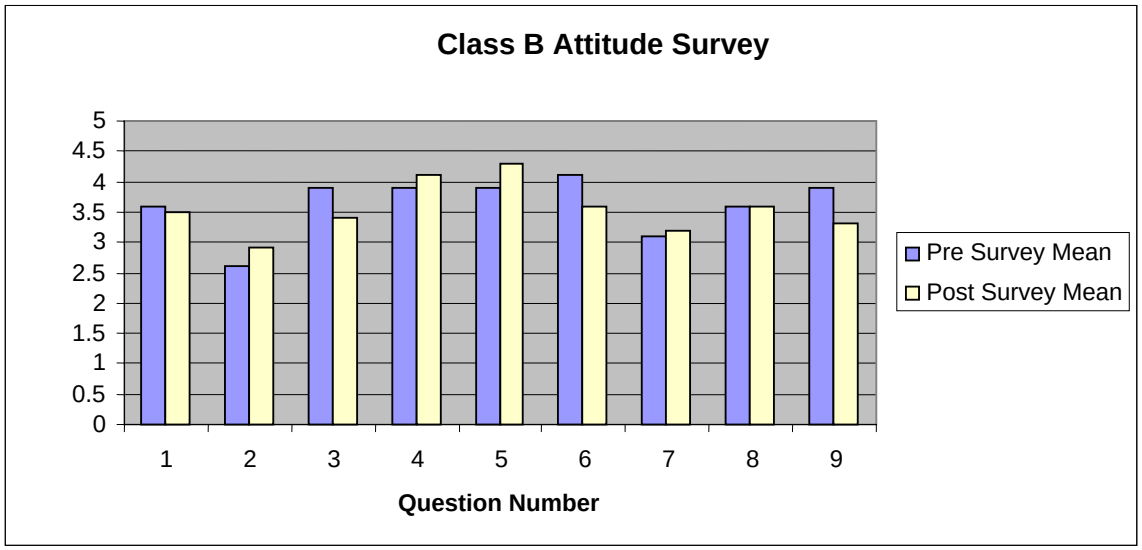
beginning and end for my Class A. I assigned a point value to the questions giving SA a value of 5, A is 4, U is 3, D is 2 and SD is 1. (Actual data can be found in Appendix G



To help understand the chart listed below are the items on the survey:

1. I am pretty good at math problem solving.
2. Writing about my thinking or my reasoning helps me to better understand the problems I am solving.
3. Writing my ideas and reasoning is easy for me.
4. I am improving at problem solving.
5. I understand more about problem solving than I am able to put down in writing.
6. The activities we are doing in class are helping me be more confident in problem solving.
7. Others would clearly be able to understand how I solve problems by reading my explanations and looking at my charts or models.
8. I am good at writing.
9. I like mathematics.

The mean scores for Class B are on the graph below.



To help understand the chart, listed below are items on the survey:

1. I am pretty good at math problem solving.
2. Writing about my thinking or my reasoning helps me to better understand the problems I am solving.
3. Writing my ideas and reasoning is easy for me.
4. I am improving at problem solving.
5. I understand more about problem solving than I am able to put down in writing.
6. The activities we are doing in class are helping me be more confident in problem solving.
7. Others would clearly be able to understand how I solve problems by reading my explanations and looking at my charts or models.
8. I am good at writing.
9. I like mathematics.

I found a difference in attitudes among the students when I interviewed 2 students from each class at the end of our problem solving. I interviewed Jim, Kathy, Sue, and Linda. Three main ideas seemed to surface.

The first idea was whether writing in problem solving had helped the students and if they liked the writing. Kathy and Sue were not very confident at problem solving. Sue did not like the writing she had to do to express thinking, as well as not liking problem solving. She found it difficult or did not want to have to do it. Kathy said she was sometimes confused but was understanding better. She thought it was fun even though sometimes she did not know what to write down. The second of the confident students said that she was fairly confident and that the writing was easy.

A second idea was what had helped them to become a better problem solver. The reasons as to what had helped students become better problem solvers varied. One felt that drawing out pictures or diagrams helped most. Another said figuring out what the question was about or working through the reading process as well as having to put an answer so everyone could understand had helped her. A third said writing it down and going through the steps had helped her the most. She also expressed that it is easy for her to explain to others if she has written it down first. The fourth student felt that the work the teachers had done to ask him questions to help lead him through the questions and the practice we had done was most beneficial.

The third point that came out was if their feeling about problem solving had changed this year. One expressed, “I used to hate problem solving but now because I understand it better it is more fun” (4/21/2006). One felt that it had not changed and said, “story problems are stupid and you won’t have to do them later in life” (4/21/2006). A third student felt doing a lot of problems has helped her feel more sure of herself and was emphatic with a yes when asked if the writing had helped her to understand problems

better. The fourth student felt his feelings had changed a little as he had learned through teachers helping him think through the problems.

Another student who was not interviewed had commented during parent teacher conferences that "writing the explanation has really helped me to understand problem solving better because I can more easily think through the steps that are needed" (3/23/2006).

Observations also showed me a variety of feelings. From week to week a student varied on willingness to try and actual completion of an assignment. The very first week when I gave a pre-test I noticed negative attitudes from Class A. In my journal I noted "After reading the questions a student complained it was too hard, I don't get it, you have to help me. He would not quit complaining and became a behavior problem. I had to set him in the hall to do his pre-test. He did nothing on the pre-test." (2/12/2006). My reaction to this attitude was "Rather than trying and failing, they just do not do anything" (2/12/2006). My 5th period did not show any negative behavior. I noted that "there was no outward panic or complaint- all students appeared to be busy trying to solve the problem" (2/12/2006). I felt that this followed the attitude that I see in Class B on a regular basis where most try to do the work and do not get frustrated enough to act out. It was interesting to me that students' perception of themselves as problem solvers was pretty good on the pre-attitude survey that I gave, but when they actually had to problem solve it was a time of panic and refusal to do the problems.

On various occasions I had to have a student from Class A sit in the hall as he was disturbing others by his negative comments. On several occasions the student was Callan. He would not try on the problems. His outbursts made it impossible for others to

concentrate. As the weeks went along there was less negative behavior and some weeks where all of the students tried the problems without observable outward negative behavior or comments. One student in my class had made some progress by the third week. We had worked on marking the text to understand the problem. As he worked through the problem he was able to see how one step led to another. The success was changing his attitude about writing and problem solving. I wrote about that student "He was very confident that what he had done was correct and reasonable because of the explaining he had to do" (2/26/2006). Many others that same week were still finding work difficult and some refused to do any work. Class B students were still eager to please and get good grades.

By the week of March 3rd I found most of the students were trying the problems. Many students needed some guidance from the teachers but were at least working. A few students needed a little help to know where to get started. That included asking what they knew, what kind of problem it reminded them of, and what the question was asking them. Some could get started but got stuck and needed some guiding questions again to keep them going. At first, students did not know how to write down their thinking, and so we would ask them what they did and help them formulate how to say that on paper. One student started out believing he could not write anything. His writing was poor and he did not see any purpose in writing. As the weeks went by he started right in with his problem solving. I wrote in my journal that "he needs constant reassurance that what he is writing or what he is doing is correct but he did a good job today. He was pleased with what he had done" (3/3/2006). He had a few times of complete shutdown and it took the

second day on the same problem before he would come around to being open to trying again.

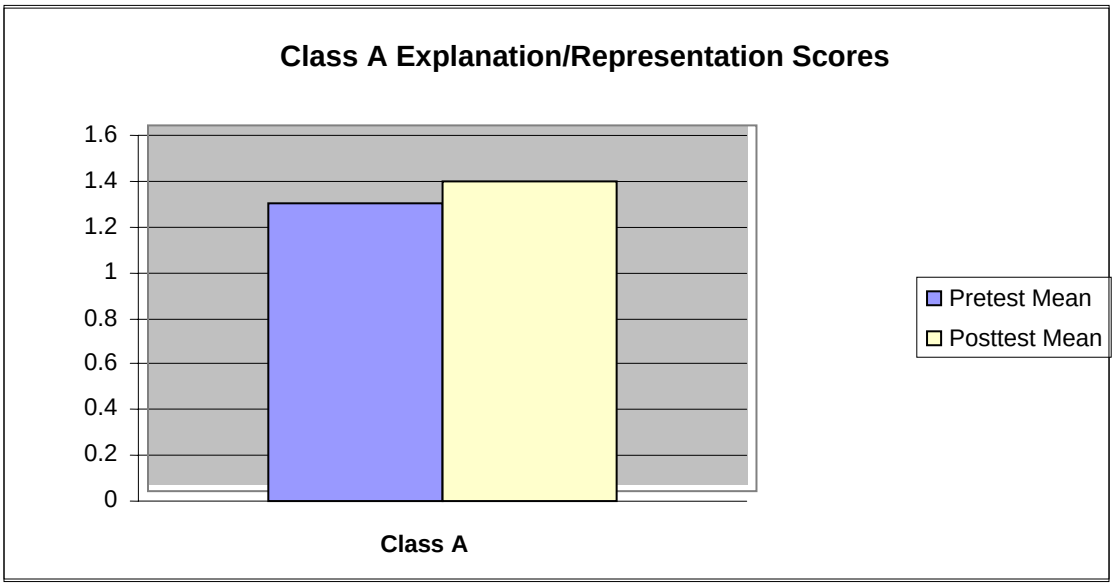
Jim had a hard time problem solving and wanted to be led through everything without trying any thinking on his own. He waited for a teacher to lead him through the thinking by asking him questions. I had written that he "started with an attitude that he could not get started on his own and only relied on another teacher …to get him started. In the last two weeks he has started the problems on his own and only then asked for help or if he was doing it right" (3/24/2006). Toward the end he was starting problems on his own and not even asking many questions. He was confident in his own thinking. In an interview with him he felt that the questions that teachers asked him had made the most difference in his problem solving. He is a student who does not like to write and has difficulty writing. He did not feel the writing had made any difference in how he could problem solve.

The last week of daily problem solving students in Class A were excited about problem solving. We had been working with patterns, and the lesson we were working on dealt with patterns and fractions. I noticed that "As they started dividing several were excited that they found an easy pattern to know how to divide the problem…the behaviors and the amount of effort that students were putting forth was so much better than it has been for over a month…I did not have any student refuse to do this assignment today or have to be set apart from the others" (3/31/2006). Either this assignment was easier or all of the weeks of working through problem solving and writing were starting to make a difference in the confidence level and attitudes these students had about problem solving

Explanation/Representation

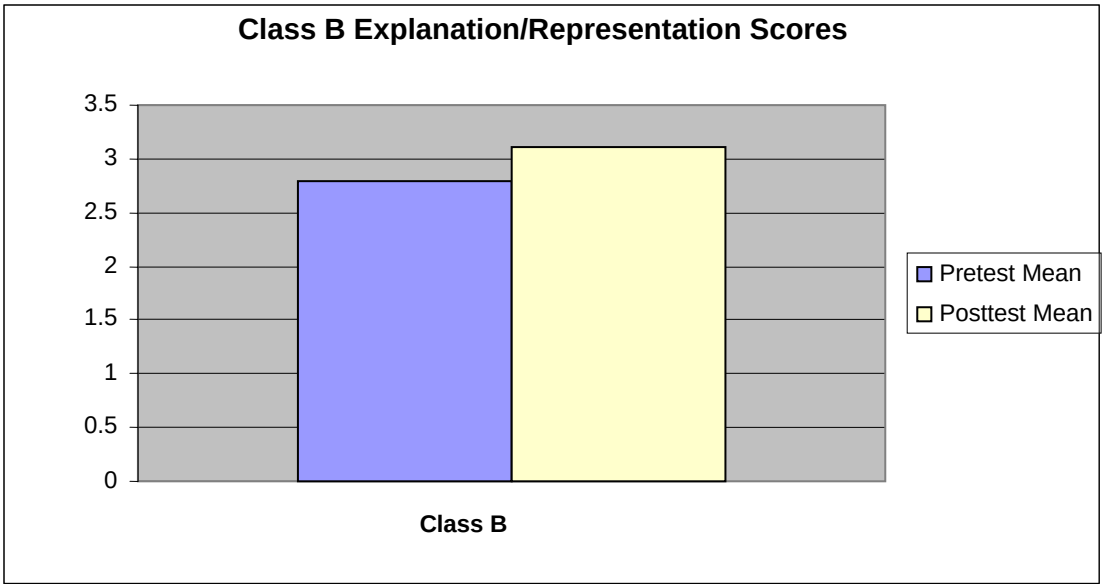
Students are showing little difference in their ability to explain so that others can understand their thinking on daily assignments. The scores on explaining oftentimes parallel the scores on concepts. If the student does not understand the concept the ability to explain the thinking is going to be lower. This again varies widely from student to student.

Pre-test and post-test explanation scores for Class A follow on the next graph. (A chart of mean and standard deviation for all explanation scores can be found in Appendix H.)

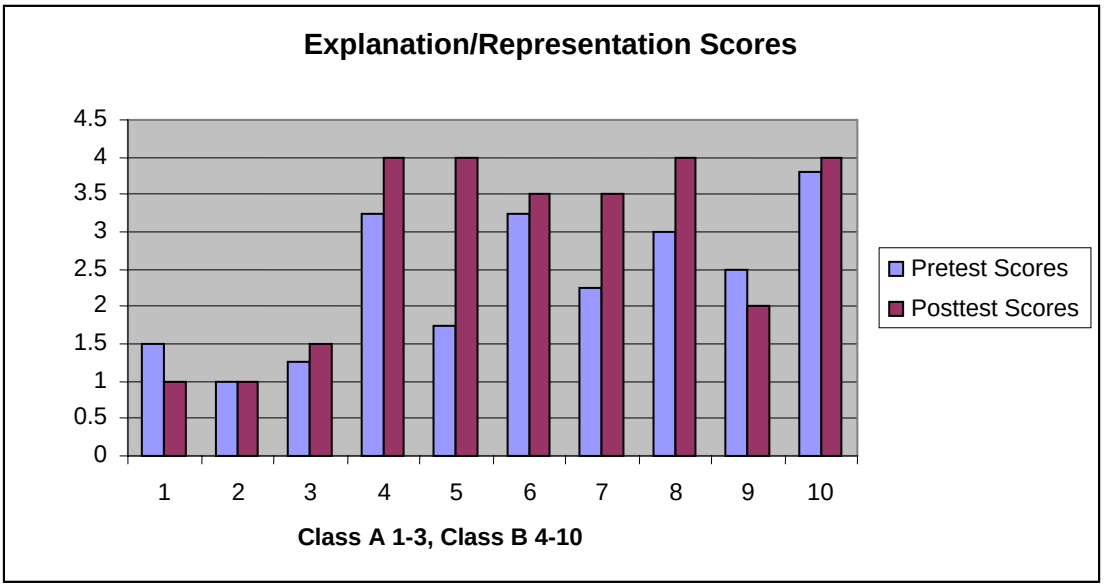


The mean scores for Class A explanation/representation increased only by 0.1. It would indicate that the explanation score stayed pretty steady. The standard deviation for the pre-test was about 0.5 and for the post-test about 1, which shows that my scores were more widespread in the post-test.

The mean scores in Class B showed a growth of 0.3 which is not a large change either. The graph for the scores is shown below.

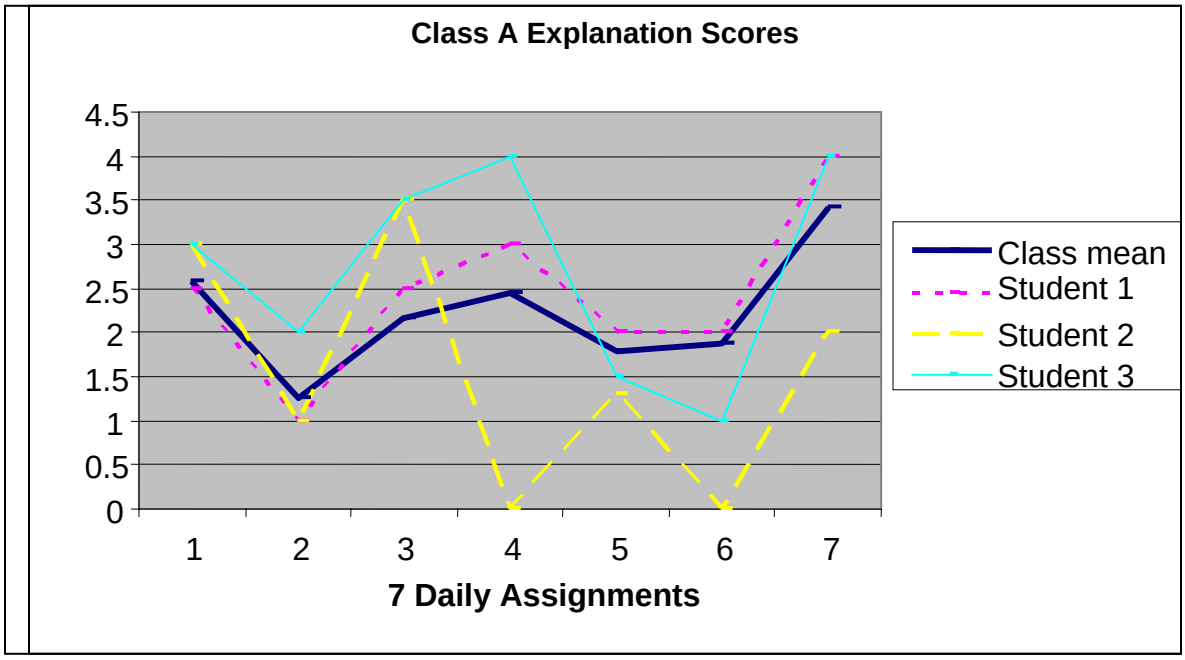


Looking at individual scores for explanation can be seen in the next graph.



There was a big difference in the growth made by students. Student 5 made a huge leap in score. Kathy went down, Callan stayed the same and Jim made a little progress. The students in Class B made more progress as did those in Class A.

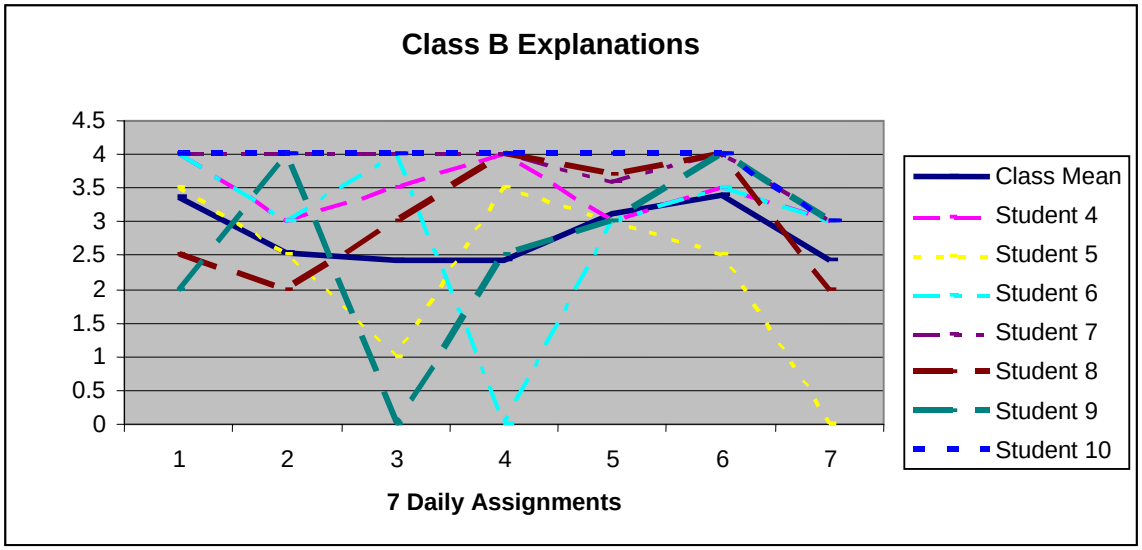
The Class A explanation scores varied on daily assignments from week to week. The averages started at 2.5 and went up and down each week. The average was a 3 on the last daily assignment. Two means scores in week 6 and 7 were pulled down by students who did not complete the assignment. The last daily assignment had a higher score than any previous assignments, which should be encouraging. The chart of daily scores below shows the progress of the whole class and individuals. (Data can be found in Appendix H.)



In my observations of students I found that some of my Class A students were trying to write explanations but found it very difficult to express themselves. In the beginning, when my students had finished the pre-test and we were discussing the problems, I noted that “some of the thinking explained to me was very good. I did not see as much of that

on the paper as they were able to tell me" (2/12/200). I saw the same thing the next week when their oral explanations were good but what they had written on their papers was very vague. I went over the rubric with the classes again to show what kinds of explanations or diagrams were missing. In their surveys many expressed they knew more about problem solving than what they could put down in words. As the weeks went on, students were starting to write more. Jim had difficulty knowing what to write. He disliked putting his thoughts and explanations on paper. He had started the problem but did not know what he had found, only a number. I noted that "I asked him to tell me what he saw. I would tell him to write the first of what he just told me…then asked him how the next part of the problem went…I helped him formulate his thinking by asking him questions… He wrote more today than he has (previously)" (3/3/2006).

In contrast to those struggling to explain I observed one student who "started by writing his explanation how he approached the problem before he ever told me what the fractions were. He was very precise in explaining what I was looking for" (3/312006). It showed me that he was starting to think in steps and to explain as he went to clarify his thinking.



The explanations in the Class B on daily assignments varied according to the difficulty of the assignment. The assignment in week 4 showed a lower score than did the rest of the assignments. This was a problem that had several parts to it. Students may have been able to explain what was happening in the beginning of the problem but did not finish the explanation as to how this pattern would continue or what kind of numbers were in the pattern. Also, during weeks with lots of illness there were students who never completed the assignment and averages went down.

In my observations I see some students in Class B striving to explain things well. They had the same struggles as Class A with knowing more than they could put down in words. As the weeks went on they were striving to meet the requirements on the rubric for explaining and as I noted in my journal they were “asking me if what they wrote was clear enough or if I understood what they were saying” (3/17/2006). By asking me they are reaching out to an audience beyond themselves to make sure it is clear. By the last week, students in Class B were starting to explain their work without relying on my input as to whether it is explained well enough. My notes indicated that “students did not ask

for help in writing the explanation to parents-they were pretty confident that they knew just what to do" (3/31/2006). They were gaining an understanding of what was needed. I also saw several types of explanations emerging for the same problem, showing me that the students are not relying on what one person thinks and then copying that, but are writing about their own perception of the problem and how it made sense to them.

Even though it did not show up as much in scores, I noticed that my students wrote more detail on their post-test than on their pre-test. As an example, Sue's explanation on the pre-test was "I drew it out. Put in fence posts. I counted fence posts. Took it times 3" (2/7/06). Then an explanation on her post-test was "The magic ball bounces half the height it was thrown so if it was thrown 64 take that divided by 2 and you keep on doing that until you get to 8 because it bounces half the height" (4/10/2006).

Linda's explanations were similar. Her pre-test had numbers and the word width written by them. She had a list of numbers and some multiplications of numbers. There was nothing explained in words or shown in a diagram except a rectangle. Her post-test had an explanation that said "on the eighth bounce it would be .5 feet high because it starts at 64 then half of 64 is 32 and….you would do this because that's all of the eight bounces twice because you have to add up and down travel" (4/10/2006). The explanations for Class B were more in depth and longer. They were not always correct, but students had attempted to explain what they thought.

The students from Class A did not show much change. Callan just wrote random numbers on his paper for the pre-test and wrote a few more on the pre-test. Kathy only wrote the answer on both tests. Jim did show a little improvement. He wrote only

numbers and lists on his pre-test but on the post-test wrote sentences telling what he did, not explaining his reasoning.

I also observed that what students who were 7th graders last year wrote on their post-tests as explanations were so limited and brief. This year's 7th graders wrote so much more and more in-depth as to their thinking. I cannot give specific examples of that because all of the assessments had to be turned into the district at the end of the year.

Comparisons

The students with low ability made more progress in problem solving than did those in my regular class. When comparing concepts means for pre-test and post-test Class A mean increased by 0.2 and the 5th period decreased by 0.1.

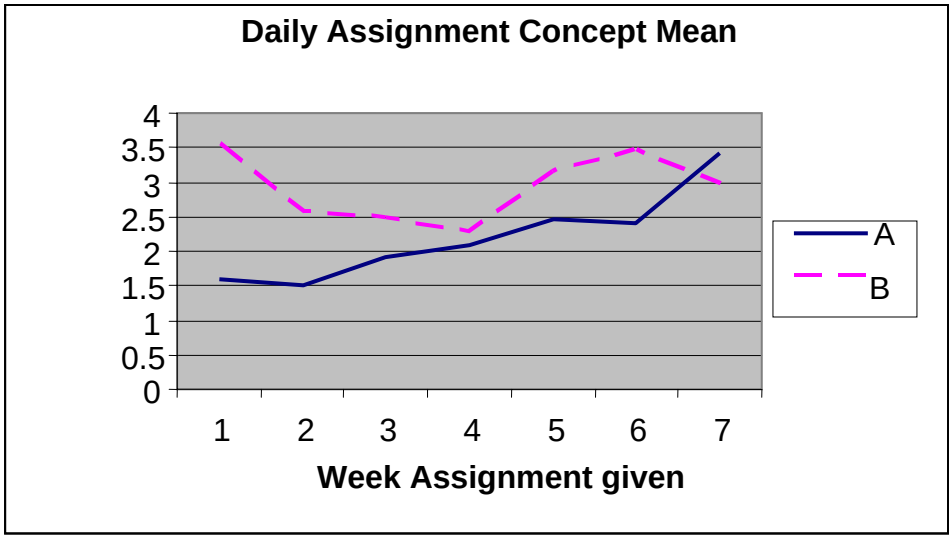
Class A Concepts

	Pre-test	Post-test
Mean score	1.14	1.33
Standard deviation	.36	.61

Class B Concepts

	Pre-test	Post-test
Mean score	2.98	2.7
Standard Deviation	.72	.77

Also in the daily concept scores the graph shows that the mean score for Class A steadily increased where Class B had definite ups and downs. The last daily assignment was different for the two classes as the problem reflected what we had worked on so far this year. Class A had not yet studied proportions so had a problem dealing with fractions instead.



The final assessments were not a very reliable source to use in comparing students as the tests were not the same kind of test from year one to year two. Both classes showed a decline in ability to problem solve given the differences in the tests.

Class A Percentage in each category of District Problem Solving Assessments

	Beginning	Progressing	Proficient	Advanced
6th Assessment	8%	42%	25%	25%
7th Assessment	75 %	25%		

Class B Percentage in each category of District Problem Solving Assessment

	Beginning	Progressing	Proficient	Advanced
6th Assessment		17%		83%
7th Assessment	14%	50%	36%	

While not much growth was shown, it would appear that those in my low ability class made more growth than those in the regular class. It did not seem that way to me at the beginning of my research when the frustrations of these students were so apparent and some did not want to do any of the assignment.

Interpretation

Based on the research findings and my observations, I find that my students who are willing to turn in work and try are improving slightly in their ability to write explanations that others can understand clearly. They are making a slight improvement in their willingness to try things on their own and seeing themselves as problem solvers. Asking students to write when they are not willing to do regular class work can result in negative classroom behaviors or tears of frustration.

My research has shown me that just as my students have different learning styles, students also react to problem solving and writing about problem solving in different ways. For some students, who like to write, the written explanations are not too difficult and may help them to become better problem solvers. Those who find writing difficult in all areas struggle to write explanations for math and can become very frustrated by needing to explain their thinking. Sometimes the behaviors and refusal to do any work comes out when they are pushed to do the writing. I did find though, that over time, the students who struggle can overcome much of the hesitance to write. It takes time to overcome not wanting to do something that is more work. The writing has not done harm to any students and for many has helped them to see step-by-step what they are doing instead of jumping to an answer. When asked what has helped them to become better at problem solving the answers they gave were writing explanations to think through the steps, doing more problems, understanding the question and drawing pictures of what the problem says. Most of these are involved in the explanation and representation that I asked them to do.

I probably got more particular in my scoring of the students' writing in the end as I expected more out of them over time. In the beginning, I gave them some points for drawing any kind of diagram and writing down a few steps. For example, one of the first problems students did was to find a pattern with the perimeter of hexagon trains. The diagrams students drew were just a few hexagons hooked together. If they wrote any kind of sentence, I gave them credit. In the end, I expected a diagram or drawing with labeled parts and the explanations needed to be full sentences that explained why they had done what they had in the problem, not just the process they used. I was looking for and expecting to see the reasoning, not just the process put into words.

I have also seen that even if student scores don't show big gains it cannot be done all at once. The writing process is something that needs to be developed over time. Spreading out more of the activities over the year instead of such a concentrated time frame would be helpful. Starting at the beginning of the year with students understanding what I expect in problem solving would also be helpful.

I found that reading the explanations helped me as a teacher know what misconceptions students had in their thinking or skills. One set of daily problems used area and perimeter and then found the cost of border, paint and carpeting. In writing the explanations students told me that they were adding the dimensions of the walls together to find out how much paint they needed. It was clear that they did not know how to find area or did not understand that being told that paint was sold by how many square feet it covered required finding area. Also, in the same set of problems the room's dimensions were given in feet. The border was sold by the yard and carpeting was sold by the square yard. As explanations were written it was easy to see what students understood about

changing from feet to yard or finding square yards for carpeting. Another example where writing helped me to understand whether a student understood the problem was on the pre-test. There was a question about flower beds and a fence being put around the beds. In the explanations students gave, I found that the way I had seen the problem with one fence, some students had seen as several fences with one around each flower bed. It made a big difference in what I could count as correct. Without explanations, the answers would be counted wrong and I would have wondered where the students would get such an answer.

Implications

I plan to continue writing in my classes with their problem solving. Many of the students improved in problem solving through our work or improved their skills at writing to explain their reasoning. I saw Class A daily scores improve over a period of weeks and behaviors start to subside as they realized that they could do the problems and the writing. I saw growth in the writing and thinking of the students in Class B. The answers were not always correct, but the ideas they were using and expressing were good. Students told me that the writing was making them think through the steps. I believe it has made a difference and is worth trying again over a longer period of time. I will need to help them more step-by-step with more teacher examples, sharing in groups of what they have written, different types of writing organization, and sharing with the class so others can see what good explanations sound like.

Other teachers may find my research helpful because of the importance of communicating, reasoning and justification, which are part of the NCTM reasoning standards. Using written explanations helps some students improve their written

expression, helps others reason through a problem because of the steps they are forced to explain, and for a few it helps improve their problem solving skills. Those who teach classes of students who struggle to succeed can also see that these students can make improvements given the opportunity to problem solve with challenging problems and writing their reasoning. It also may indicate to others that student attitudes are not always positive when something is more difficult or more work is added to what they have to do.

More research needs to be done in the area of written expression and its effects on problem solving skills of low achievers. There was so little research that I could find on this topic. My research was for a limited number of students, over a short time frame. It is important to find out if the communication, reasoning and justification process standards can be accomplished through requiring students to write out solutions and representations of their work. It is important to find out if students who have been low achieving in the past can make positive strides toward being problem solvers using an approach of writing down their thoughts and reasoning.

References

Ishii, D. (2002). Emergent issues in middle school mathematics action research*: A meta-analysis of writing as a pedagogical strategy*. Paper presented at the Annual Meeting of the Mid-Western Educational Research Association.

Jurdak, M. & Abu Zein, R (1998). The effect of journal writing on achievement in and attitudes toward mathematics. *School Science and Mathematics*. 98 (8), 412-423.

Kjos, R. & Long, K. (1994). Improving critical thinking and problem solving in fifth grade mathematics. Master's Research project, Saint Xavier University and IRI/Skylight.

Millard, E., Oaks, T. & Sanders, T. (2002). Improving student achievement through inclusion of problem solving in the Math Curriculum. Master's Research project, Saint Xavier University and IRI/Skylight.

Mills, M.& Stevens, P. (1998). Improving writing and problem solving skills of middle school students. Master's Research project, Saint Xavier University and IRI/Skylight.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: author.

Pugalee, D. (2001). Writing, mathematics, and metacognition: Looking for connections through students' work in mathematical problem solving. *School Science and Mathematics*. 101(5), 236-248.

Steele, D. (2005). Using writing to access students' schemata knowledge for algebraic thinking. *School Science and Mathematics*. 105 (3), p 142-154.

Appendix A

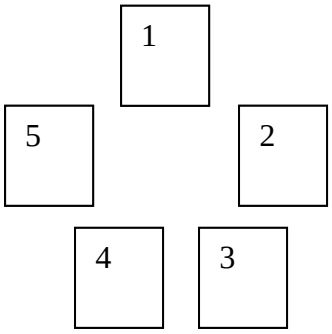
Problem Pre Test 7th grade

Solve the two problems given. Show all of your work and explain your thinking.

1. Berenice wants to put flowers in 4 separate rectangular flower beds. The first flower bed has a width of 2 feet and a length that is twice as much as the width. The 2nd plot's width is double the width of the 1st bed and has the same length as the 1st. The 3rd flower bed has a width that is double the width of the 2nd but keeps the same length as the 2nd. The 4th plot has a width 3 times that of the 1st bed and length 3 times the longest side of the 2nd flower bed. Berenice wants to keep the rabbits out so she is putting a fence around the flower beds. If fence posts cost $3 each and she wants them placed every two feet, how much will her fences cost?

Seating Students in a Group

Students sit in a group of 5. Each day they shift one seat clockwise so that they have a different position. In the drawing Sam starts in seat 1.



Where will Sam be on the 9th day?

Where will Sam be on the 17th day?

Where will Sam be on the 382nd day?

Explain your reasoning or how you figured out the problem.

Appendix B

Math - Problem Solving : Math Concepts and Computation

Teacher Name: **Schlattmann**

Student Name: _____

CATEGORY	4	3	2	1
Mathematical Concepts	Explanation shows complete understanding of the mathematical concepts used to solve the problem(s).	Explanation shows substantial understanding of the mathematical concepts used to solve the problem(s).	Explanation shows some understanding of the mathematical concepts needed to solve the problem(s).	Explanation shows very limited understanding of the underlying concepts needed to solve the problem(s) OR is not written.
Mathematical Errors	90-100% of the steps and solutions have no mathematical errors.	Almost all (85-89%) of the steps and solutions have no mathematical errors.	Most (75-84%) of the steps and solutions have no mathematical errors.	More than 75% of the steps and solutions have mathematical errors.

Date Created: **November 19, 2005**

Appendix C

Post Test 7th grade Problem Solving

Name _____

Solve the two problems. Show all of your work and explain your reasoning.

1. Bobo and the Bouncing Ball from Gary Tsuruda 's POW's

Bobo was walking down the street one day minding his own business when a ball fell right in front of him and bounced straight up. It soured up to a tremendous height and came down again. Bobo caught the ball and climbed to the top of the nearest building to investigate. On the roof, he saw a little girl peering over the edge. "What's wrong little girl?" he asked. The girl answered without turning around, "I dropped my magic ball and now some clown has stolen it."

"I didn't steal it!" claimed Bobo. "I just caught it and brought it back to you."

The little girl turned around and a smile lit up her face. "Oh, thank you! You saved my magic ball! …Oh, and I am sorry about calling you a clown."

"That's OK, "Said Bobo. "But this looks like a regular super ball to me."

"No, this ball is special. It always bounces exactly half the height from which it was dropped, and it'll bounce forever if nobody catches it."

"Wow!" exclaimed Bob. "I'd give anything for a ball like that!"

"I tell you what," said the little girl. "I'll give you the ball if you can figure out two things: how high will it bounce after its 8th bounce if I drop it from here, 64 feet above the sidewalk; and how far will the ball have traveled altogether by the time it reaches its peak on its eighth bounce.

Please help Bobo solve these problems so that he can keep the magic ball.

2. I am putting up a bulletin board and trimming it with a border. My bulletin board is 6 feet by 9 feet. Colored fabric covering is sold for $6.00 a square yard and may be cut to any size. The border is lace that costs $.03 an inch. How much will this bulletin board project cost me?

Show and explain each step.

Appendix D

Student Problem Solving Survey

Do not put your name on this survey.

Please respond to the following items by drawing a circle around the response that most closely reflects your opinion: strongly agree (SA), agree (A), undecided (U), disagree (D), or strongly disagree (SD).

1. I am pretty good at math problem solving **SA A U D SD**

2. Writing about my thinking or my reasoning helps me to better understand the problems I am solving. **SA A U D SD**

3. Writing my ideas and reasoning is easy for me. **SA A U D SD**

4. I am improving at problem solving. **SA A U D SD**

5. I understand more about problem solving than I am able to put down in writing. **SA A U D SD**

6. The activities we are doing in class are helping me be more confident in problem solving. **SA A U D SD**

7. Others would clearly be able to understand how I solve problems by reading my explanations and looking at my charts or models. **SA A U D SD**

8. I am good at writing. **SA A U D SD**

9. I like mathematics. **SA A U D SD**

Please add comments that you would like to make about problem solving, writing or how you feel about the class.

Appendix E

Math - Problem Solving : Explanations and Representation

Teacher Name: **Schlattmann**

Student Name: _____

CATEGORY	4	3	2	1
Explanation	Explanation is detailed and clear. Explanation is clear as to the reasoning used.	Explanation is clear. Explanation explains procedures but not reasoning.	Explanation is a little difficult to understand, but includes critical components.	Explanation is difficult to understand and is missing several components OR was not included.
Diagrams and Sketches	Diagrams and/or sketches are clear and greatly add to the reader's understanding of the procedure(s).	Diagrams and/or sketches are clear and easy to understand.	Diagrams and/or sketches are somewhat difficult to understand.	Diagrams and/or sketches are difficult to understand or are not used.

Date Created: **November 19, 2005**

Appendix F

Class A Concepts

	Pretest	Posttest
Mean score	1.14	1.33
Standard deviation	.36	.61

Class B Concepts

	Pretest	Posttest
Mean score	2.98	2.7
Standard Deviation	.72	.77

Class A individual scores on Concepts

Student	Pretest	Posttest
1	1.25	1
2	1	1
3	1	1.5

Class B individual scores on Concepts

Student	Pretest	Posttest
4	3	3
5	2	3
6	3.25	2.5
7	3.5	3
8	2	3.5
9	2.5	1.5
10	3.75	3.5

Class A Daily Assignments

	Daily 1	*Daily 2*	*Daily 3*	*Daily 4*	*Daily 5*	*Daily 6*	*Daily 7*
Mean	1.5	1.5	1.9	2	2.5	2.4	3.4
Median	2	1.5	2	2	2.5	2.8	3.5
Standard Deviation	.51	1.2	1.1	1.2	1	1.2	.7

Class B Daily Assignments

	Daily 1	*Daily 2*	*Daily 3*	*Daily 4*	*Daily 5*	*Daily 6*	*Daily 7*
Mean	3.6	2.6	1.9	2.1	3.2	3.5	3
Median	4	3	2	2	3.5	4	4
Standard Deviation	.5	1.6	1.1	1.2	1	1.1	1.5

Appendix G

Class A Attitude survey

Question	Pre Survey Mean	Pre Survey Standard Deviation	Post Survey Mean	Post Survey Standard Deviation
1	3.8	1.24	3.5	1.31
2	2.6	1.24	2.1	.9
3	3.2	1.34	2.5	1.45
4	3.6	.67	3.5	1.35
5	3.3	1.16	3.5	1.31
6	3.7	1.14	3.08	1.01
7	3.1	1.02	2.8	.83
8	3.2	1.3	3	1.71
9	3.5	1.34	3.25	1.36

Class B Attitude Survey

Question	Pre Survey Mean	Pre Survey Standard Deviation	Post Survey Mean	Post Survey Standard Deviation
1	3.6	1.01	3.5	1.02
2	2.6	.51	2.9	1.27
3	3.9	.99	3.4	1.19
4	3.9	.99	4.1	1.17
5	3.9	1	4.3	.61
6	4.1	.83	3.6	1.01
7	3.1	1.14	3.2	1.25
8	3.6	1.15	3.6	1.02
9	3.9	.77	3.3	1.33

Appendix H

Class A Explanation/Representation scores

	Pretest	Posttest
Mean score	1.3	1.4
Standard deviation	.5	1

Class B Explanation/Representation scores

	Pretest	Posttest
Mean Score	2.8	3.1
Standard deviation	.7	1.1

Class A Explanation/Representation scores

Student	Pretest	Posttest
1	1.5	1
2	1	1
3	1.25	1.5

Class B Explanation/Representation scores

Student	Pretest	Posttest
4	3.25	4
5	1.75	4
6	3.25	3.5
7	2.25	3.5
8	3	4
9	2.5	2
10	3.8	4

Individual scores on Pre and Posttest Explanation/Representation
Class A Daily Scores

	Daily 1	*Daily 2*	*Daily 3*	*Daily 4*	*Daily 5*	*Daily 6*	*Daily 7*
Mean	2.6	1.25	2.2	2.5	1.8	1.9	3.4
Median	2.5	1	2.5	2.8	1.8	2.3	4
Standard Deviation	.6	1.3	1.2	1.1	.69	1	1

Class A Daily Scores Explanation/Representation

	Daily 1	*Daily 2*	*Daily 3*	*Daily 4*	*Daily 5*	*Daily 6*	*Daily 7*
Mean	3.4	2.5	2.4	2.4	3.1	3.4	2.4
Median	3.8	3	3.3	2.8	3	3.8	3
Standard Deviation	.84	1.5	1.8	1.7	1	1.1	1.1