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Assessing the Forecasting Accuracy of Alternative Nominal Exchange Rate Models: The Case of Long Memory

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Abstract
This paper presents an autoregressive fractionally integrated moving-average (ARFIMA) model of nominal exchange rates and compares its forecasting capability with the monetary structural models and the random walk model. Monthly observations are used for Canada, France, Germany, Italy, Japan, and the United Kingdom for the period of April 1973 through December 1998. The estimation method is Sowell's (1992) exact maximum likelihood estimation. The forecasting accuracy of the long-memory model is formally compared to the random walk and the monetary models, using the recently developed Harvey, Leybourne, and Newbold (1997) test statistics. The results show that the long-memory model is more efficient than the random walk model in steps-ahead forecasts beyond 1 month for most currencies and more efficient than the monetary models in multi-step-ahead forecasts. This new finding strongly suggests that the long-memory model of nominal exchange rates be studied as a viable alternative to the conventional models.
Keywords: fractional integration; forecasting; foreign exchange rate

Introduction

Foreign exchange rates have been intensively studied since major industrial countries switched to the floating exchange rate system in the early 1970s. Numerous studies have been conducted on the determination and the statistical behavior of the exchange rate. Time series properties of the exchange rate are crucial in understanding the models of exchange rate determination and the forecasting performance of those models. This proliferation of studies has been in part aided by the availability of rich data sets of different collection intervals (i.e., daily, weekly and monthly) and by recent development of econometric methodologies. Much of the model building is based on the monetary approach to exchange rates, in which the exchange rate is treated as a price of one currency in terms of another.

The monetary models of exchange rate determination have gone through a vast amount of scrutiny in forecasting and efficiency tests. These models provided fairly good forecasting results in the early years of the floating rate system, but their forecasting accuracy has deteriorated markedly from the 1980s. The reduced-form equations of the model yielded coefficient estimates that were both imprecise and statistically insignificant. This is best demonstrated in the results of Meese and Rogoff (1983a), who showed that the performance of the monetary models was inferior to a simple random walk model of exchange rates.[1]

The poor forecasting performance of the structural models has been attributed to many factors. First, exchange rates may diverge from the levels dictated by economic fundamentals due to "rational bubbles" (see, Meese, 1986, and West 1987). Second, Frankel and Froot (1990) and Taylor and Allen (1992) argue, using survey responses, that while the dealers in the foreign exchange market rely more on the economic fundamentals in the long run, they rely more heavily on charts and trends in forming expectations in the short run. Using an error correction model, Kim and Mo (1995), for example, show that, while the random walk model outperforms the monetary structural model in short-run forecasting, the latter outperforms the former in the long run, lending support to the arguments made by Frankel and Froot (1990) and Taylor and Allen (1992).

In this paper, we introduce yet another model of exchange rates for forecasting, i.e., the long-memory model, and compare the forecasting performance of this model with that of monetary models and the random walk model. We first investigate the time series dynamics of nominal exchange rates using the fractionally integrated autoregressive moving average (ARFIMA) model. The long-memory process was first applied to economic time series by Granger (1980), Granger and Joyeux (1980), and Hosking (1981).[2] It generalizes the traditional autoregressive integrated moving average (ARIMA), by allowing the differencing parameter to take non-integer real values.

Since a long memory in the foreign exchange rate was first suggested by Booth *et al.* (1982), there have been several applications of the ARFIMA model to foreign exchange rates. Cheung (1993, pp. 93–101), using weekly spot exchange rates of five currencies, finds evidence of the presence of long memory and estimates the ARFIMA models using time-domain and frequency-domain maximum likelihood methods.[3] In a comparison of relative magnitudes of forecast statistics, he finds no difference between the forecasting accuracy of ARFIMA models and that of random walks.

This study extends Cheung's (1993) analysis of the forecasting accuracy of the ARFIMA model by providing a statistical comparison with not only the random walk model but also with a set of monetary structural models. This is a significant extension along the lines of Meese and Rogoff's (1983a) analysis, as we add a different class of models for comparison, i.e., the ARFIMA model, whose concept was barely available at the time of their study. We also employ Sowell's (1992) recursive exact maximum likelihood estimation method, which is shown to be superior to the Fox and Taqqu (1986) method used by Cheung (1993).

1 This well-publicized result was, however, challenged by Wolff (1987). He argued, using a recursive Kalman filter model for forecasting, that the structural monetary models compare favorably with the random walk model for the dollar/DM exchange rate. Schinasi and Swamy (1989) reported similar results.

2 Recent applications of this model include aggregate output (Diebold and Rudebusch, 1989), monetary aggregates (Porter-Hudak, 1990), consumption smoothness (Diebold and Rudebusch, 1991a), foreign exchange rates (Cheung, 1993), purchasing power parity (Cheung and Lai, 1993), forward premium (Baille and Bollerslev, 1994), inflation rates (Hassler and Wolters, 1995), stock prices (Lo, 1991, Cheung and Lai, 1995, 2001), consumption and income (Gil-Alana and Robinson, 2001), to list a few.

3 We thank Fallaw Sowell for additional insight on the difference between Cheung (1993) and Sowell (1992) techniques.

This paper is organized as follows: the next section presents a brief description of the monetary models and a random walk model. The third section explains a long-memory model of exchange rates and the estimation procedures of the ARFIMA model. The fourth section presents the empirical estimation of these models and comparisons of their forecasting performances. The fifth section concludes the paper with a summary and conclusions.

Traditional Models of Exchange Rates

The Monetary Models

Early studies of exchange rate determination were largely based on monetary models. In this paper, we consider three such models: (i) the "flexible-price"' model of Frenkel (1976) and Bilson (1978); (ii) the "sticky-price" model of Dornbusch (1976) and Frankel (1979); and (iii) the "stickyprice/current-account" model of Hooper and Morton (1982). Meese and Rogoff (1983a) compared the forecasting performance of the above three models with that of the random walk for dollar/DM, dollar/pound, dollar/yen, and the trade-weighted dollar exchange rates, using data for the period March 1973 through June 1981. Their results showed that the random walk model outperformed the three structural monetary models in out-of-sample forecasts.

A general form of the monetary models of exchange rate determination can be written as follows:

$$S_t = \alpha_0 + \alpha_1(m_t - m_t^*) + \alpha_2(i_t^{ST} - i_t^{ST*}) + \alpha_3(i_t^{LT} - i_t^{LT*}) + \alpha_4(w_t - w_t^*) + \alpha_5(\mathrm{CTB}_t - \mathrm{CTB}_t^*) + \alpha_6(y_t - y_t^*) + \varepsilon_t \quad (1)$$

where S_t is the log of the spot exchange rate, m_t is the log of money supply, i_t^{ST} is the level of short-term interest rate, i_t^{LT} is the level of the long-term interest rate, w_t is the log of real wealth, CTB_t is the cumulated trade balance, and y_t is the log of real income. The asterisk denotes the foreign country variable.

The various structural monetary models of exchange rates can be classified by imposing certain restrictions on coefficients:

Frenkel–Bilson model:

$$\alpha_1 = 1, \alpha_0 a2 > 0, \alpha_3 = \alpha_4 = \alpha_5 = 0, \alpha_6 < 0$$

Dornbusch–Frankel model:

$$\alpha_1 = 1, \alpha_2 < 0, \alpha_3 > 0, \alpha_4 = \alpha_5 = 0, \alpha_6 < 0$$

Hooper and Morton model:

$$\alpha_1 = 1, \alpha_2 < 0, \alpha_3 > 0, \alpha_4 = 0, \alpha_5 < 0, \alpha_6 < 0$$

In this paper, the forecasting performance will be compared among the three monetary exchange rate models, the random walk model, and the long-memory model. This will be done by statistically comparing the multi-step-ahead forecast performance of these models.

The random walk model

The general form of the random walk model for exchange rates is written as

$$S_t = S_{t-1} + c + v_t \tag{2}$$

where c is a drift term and v_t is a white-noise error term.

The Long-Memory Model of the Foreign Exchange Rates

The parameter estimates in any regression equation are sensitive to the class of models considered and may be misleading when the model is misspecified. While the issue of model misspecification is hardly settled conclusively, employing a larger class of models will certainly help with the accuracy of the inferences drawn from the parameter estimates. The ARFIMA model is a larger class of stochastic processes, of which random walks are special cases. In this vein, the use of the ARFIMA model is preferred to the conventional ARIMA model in modeling the long-run behavior of a macro time series.

Many previous studies have shown that foreign exchange rates exhibit high persistence and behave like a martingale. Standard unit root tests such as the augmented Dickey–Fuller test and the Phillips–Perron test cannot reject the null hypothesis that there is a unit root in exchange rates (see, for example, Meese and Singleton, 1982). However, the ARFIMA model is preferred to the standard unit root model, as it has been shown that the unit root tests have low power against the ARFIMA alternatives (see Diebold and Rudebusch, 1991b, and Sowell, 1992).

A foreign exchange rate, $S_{t,}$ follows an ARFIMA (p, d, q) process if

$$\Phi(L)(1 - L)^d S_t = \Theta(L)\varepsilon_t \tag{3}$$

where $\Phi(L) = 1 + \varphi_1 L + \varphi_2 L^2 + \ldots \varphi_p L^p$ is an autoregressive polynomial and $\Phi(L) = 1 - \theta_1 L - \theta_2 L^2 - \ldots - \theta_q L^q$ is a moving average polynomial, and

$$(1-L)^d = \sum_{K=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(k+1)\Gamma(-d)} \tag{4}$$

is the fractional differencing operator and $\Gamma(.)$ is the usual gamma function. The roots of $\Phi(L)$ and $\Theta(L)$ are assumed to be outside the unit circle. The parameter "d" is known as the memory parameter and is allowed to take any fractional values (fractional integration).

The stochastic process S_t is both invertible and stationary if all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle and $|d| < 0.5$. If $d > 0.5$, the process is not stationary, because it exhibits an infinite variance (Granger and Joyeux, 1980; Sowell, 1992; Cheung, 1993). Hosking (1981) showed that, for $d \in (0, 0.5)$, the autocorrelation function for the ARFIMA process, $\gamma(k) = E(Z_t Z_{t-k})$, satisfies the condition that, as $k \to \infty$, $\gamma(k) \sim k^{2d-1}$. This slow hyperbolic decline is characteristic of ARFIMA processes and contrasts with the faster geometric decay inherent in stationary ARMA models. Therefore, long-memory or long-range dependency between observations widely separated in time can be examined by fractional integration. If $0 < d < 0.5$, $\sum_{K=0}^{\infty} \gamma(k)$ diverges as $k \to \infty$ and the stochastic processes exhibit

long-memory or long-range dependency between distant observations. For $d \in$ (-0.5, 0), the processes are said to exhibit intermediate or negative long-range dependency. For $d = 0$, the processes exhibit short-memory characteristics of stationary ARMA models.

The increased evidence of fractional integration in macroeconomic variables has generated new interest and a host of new techniques for estimating ARFIMA models (see, for example, Diebold and Rudebusch, 1989, 1991a; Cheung and Lai, 2001; and Gil-Alana and Robinson, 2001). Sowell (1992) provides an excellent survey of past estimation methods along with benefits and problems associated with each method. In a rather complex procedure, Sowell outlines a recursive maximum likelihood method for a joint estimation of ARFIMA model parameters. He provides comparative evidence and shows the superior efficiency of his method. For example, in a simulation exercise, Sowell (1992, p. 180) argues, "Generally, the maximum likelihood estimator was superior to the procedures presented in Fox and Taqqu (1986) and in Geweke–Porter–Hudak." The efficiency gains from Sowell's method arise from the use of the interchange integral and the recursive nature of the procedures as described on p. 175. Thus, to our knowledge, this paper is the first to use Sowell's (1992) method to assess the forecasting accuracy of alternative models of nominal exchange rates. Our results extend findings in Cheung (1993) and provide additional insights on alternative estimation and forecasting methodologies. The recursive maximum likelihood procedure is, therefore, used here and is briefly described below.

The exact joint maximum likelihood method

The long-memory aspect in the nominal exchange rate is captured by estimating the fractionally integrated ARIMA models. The earliest study by Geweke–Porter–Hudak (1983), which is a two-step semi-parametric estimation process, has been demonstrated to have a larger bias and mean squared error than the maximum likelihood estimation method (see Sowell, 1992; Robinson, 1994; and Smith et al., 1997). Therefore, we estimate the ARFIMA model using Sowell's (1992) method of a joint estimation of the time series model parameters.[4]

Consider a normal distribution for the fractionally integrated time series S_t, generated by the process expressed in equation (3). Assume further that a sample of size T observations is available such that $S_t = (S_1, S_2, \ldots, S_T)$ and $S_t \sim N(0, \Sigma)$, where Σ is a $T \times T$ covariance matrix. The normal probability density function is given by the following expression:

$$f(S_t, \Sigma) = (2\pi)^{-T/2} |\Sigma|^{-1/2} \exp\left\{ -\frac{1}{2} S_t' \Sigma^{-1} S_t \right\} \tag{5}$$

A detailed derivation of equation (5) is available in Sowell (1992). The joint estimation of the model parameters requires the specification and estimation of the covariance matrix in (5) above. Sowell noted that $\Sigma = [\gamma(i - j)]$ for $i, j = 1, 2, \ldots, T$, and wrote the autocorrelation function in terms of model parameters and then derived the autocovariance function as

$$\gamma(k) = \frac{1}{2\pi} \int_0^{2\pi} f_s(\lambda) e^{i\lambda k} d\lambda \tag{6}$$

4 Sowell's (1992) estimation method has so far not been employed in the literature.

From a given set of parameter values, the maximization routine involves the evaluation of the likelihood function for a global maximum. The spectral density of S_t is used in the specification of the autocovariance function and can be derived using the spectral density of $U_t = (1 - L)^d S_t$. Details on the derivation of the ML function and the recursive algorithm are available in Sowell (1992).

In the empirical implementation of the above recursive procedure, the maximization of the likelihood function was performed using a Fortran program provided by Fallaw Sowell and made available at his website. The program estimates ARFIMA (p, d, q) with 16 specifications corresponding to three AR and MA parameters. The Fortran subroutine calls the Davidon Fletcher Powell (DPF) algorithm, which is available in the GQOPT package obtained from Richard Quandt of Princeton University. Starting values were generated by using the GPH spectral regression procedure suggested by Sowell (1992). Several other initial parameter values were tried to ensure that a global maximum was reached. The Akaike information criterion (AIC) and the Schwartz information criterion (SIC) were used in the final model selections (Diebold *et al.*, 1991; Sowell, 1992; and Cheung, 1993). Both criteria were generally consistent in the selections. The results are presented below.

Empirical Results

In this section, we investigate the forecasting performance of the monetary structural models, the random walk model, and the ARFIMA model of nominal exchange rates for the G-7 countries.[5] The G-7 countries are Canada, France, Germany, Italy, Japan, and the UK. The time period studied is April 1973 through December 1998 for all countries, except France and Italy owing to data availability. France's time period is from December 1977 through December 1998, while Italy's is from December 1974 through December 1998.

The data

The monthly data were taken from the IFS CD-ROM. The monetary aggregates are all M1, except M2 for Italy and M0 for the UK. The income measures are the seasonally adjusted industrial production and prices are the consumer price indexes for all countries. Short-term interest rates are Treasury-bill rates for Canada, France, Germany, UK, and USA, and the call money rates for Germany, Italy, and Japan. Long-term interest rates are the government bond yields for all countries. Trade balance is the difference between exports and imports for all countries.

Test of long memory in exchange rates

We report in Table I the results of Sowell's test for long memory in the first-differenced exchange rate series for six currencies, which are defined as the units of national currencies per US dollar.[6] The table also reports the parameter estimates of the fractionally integrated ARMA models. The estimate of the fractional integration parameter, d, is given in the first column. All estimates have the expected

5 No previous study has compared the forecasting efficiency among the ARFIMA models, the monetary structural models, and the random walk model. Cheung (1993) compared the forecasting efficiency between the ARFIMA and random walk models without a statistical test. The well-publicized Meese and Rogoff (1983a, b) studies compared forecasting efficiency between the monetary models and the random walk model.

6 We also estimated d using the two-stage Geweke–Porter–Hudak (1983) method. Three of the six estimates based on the value $T^{1/2}$ were very different from those obtained using the exact maximum likelihood method, while fewer estimates were statistically significant. The estimated results may be obtained from the authors upon request.

Table I. Sowell's joint maximum likelihood estimation of ARFIMA model.

Currency	Memory parameter	Autoregressive parameters			Moving average parameters		
	d	φ_1	φ_2	φ_3	θ_1	θ_2	θ_3
CD	0.1562***	–0.1967***	0.1498***	–0.3069	–0.2238	0.1927***	–0.5502**
	(4.63)	(8.16)	(4.01)	(1.47)	(9.49)	(4.73)	(2.51)
FF	0.1459**	0.113	0.1907**		0.9777***		
	(1.95)	(1.21)	(2.10)		(39.0)		
DM	0.0679*	0.1725***	–0.749***		–0.525***	0.167**	
	(1.73)	(4.47)	(7.08)		(7.14)	(2.44)	
IL	0.1143*	0.0337***					
	(1.91)	(4.65)					
JY	0.0797**	0.1415***	–0.450***		–0.456***	0.068	
	(2.07)	(15.5)	(5.61)		(4.16)	(1.05)	
BP	0.009	–0.8976	–0.0096	0.9917			
	(0.09)	(0.90)	(0.09)	(0.99)			

Notes:
1. CD, Canadian dollar; FF, French franc; DM, German mark; IL, Italian lira; JY, Japanese yen; BP, British pound. All exchange rates are foreign currency unit per US dollar.
2. Asymptotic t-values in absolute values are in parentheses.
3. Memory parameter d is significant at 5% or better in the one-tailed test.

*Significance at 10% level; **significance at 5% level; ***significance at 1% level in two-tailed test.

sign and magnitude consistent with the existence of long memory. Except for the British pound, the estimates are statistically significant and show strong memory at the 5% level or higher. We used these models to generate the out-of-sample forecasts below.

Most estimates of the autoregressive and moving average parameters are statistically significant at the 5% level. The French franc exchange rate structure is characterized by a second-order autoregressive and first-order moving average, whereas the German mark and Japanese yen are characterized by second-order autoregressive and moving averages. The Canadian dollar is shown to have a third-order autoregressive and moving average. The Italian lira is best specified by a first-order autoregressive structure. The British pound is characterized by insignificant autoregressive parameters and no moving average term. Furthermore, the British pound is the sole currency with a weak or insignificant memory parameter estimate. The L'jung Box statistics, used to test for autocorrelation, all indicate rejection of the null hypothesis of no serial correlation in the exchange rate series examined.

Comparison of Forecast Accuracy: Test of Equality of Forecast Errors

We now compare the performance of the monetary, random walk, and long-memory models. The three monetary models are estimated using equation (1), with proper parameter restrictions imposed for each model. The estimation results are not reported here to conserve space. These estimated models are used to generate the out-of-sample forecasts and the statistics for forecast errors.

The out-of-sample forecasts are obtained as follows: First, we estimated a regression equation for each of the three monetary models and the random walk model for the period 1974:1–1996:12 (1978:2–1996:12 for France and 1975:2–1996:12 for Italy because of data availability) and then forecasts are generated by a rolling regression, which updates the parameter coefficients as each additional observation is added. Out of-sample forecast errors are generated for the period 1997:1–1998:12. Secondly,

the ARFIMA model parameters, which are estimated for the same sample estimation period, are used to generate the out-of-sample forecast errors for the ARFIMA models during the same evaluation period. Thus, for all models, the out-of-sample forecasts are made for 1, 2, 3, 6, 9, and 12 steps-ahead forecasts for the period 1997:1–1998:12.[7] The relative forecast efficiency of alternative exchange rate models is evaluated by use of a statistical method proposed by Harvey *et al.* (1997).

The Harvey, Leybourne, and Newbold test statistic

In this section, we use a methodology developed by Harvey *et al.* (1997) (HLN hereafter) to compare the forecasting accuracy of the three groups of exchange rate models. The methodology is briefly described as follows.

Consider that the h-step-ahead forecast from the ARFIMA model has produced forecast errors e_{it}, while that from the random walk (or monetary) model has produced forecast errors e_{2t} for t =1, 2..., n. Following HLN, the forecast accuracy may be judged by a forecast error function $g(e_t)$. Define $\delta_t = g(e_{1t}) - g(e_{2t})$ as the difference between the two forecast error functions. The null hypothesis of equal expected forecast errors is $E[g(e_{1t}) - g(e_{2t})] = 0$. We write the mean difference as

$$\bar{\delta} = n^{-1}\Sigma\delta_t \tag{7}$$

where $\delta_t = g(e_{1t}) - g(e_{2t})$. The HLN test assumes that the h-step-ahead forecast is autocorrelated up to lag h and all autocorrelation or order h or higher are zero.[8] The variance of $\bar{\delta}$ is then computed as

$$V(\bar{\delta}) \approx n^{-1}\left[\gamma_0 + 2\sum_{k=1}^{h-1}\gamma_k\right] \tag{8}$$

where γ_k is the kth autocovariance of δ_t. This autocovariance can be estimated by

$$\hat{\gamma}_k = n^{-1}\sum_{t=k+1}^{n}(\delta_t - \bar{\delta})(\delta_{t-k} - \bar{\delta}) \tag{9}$$

The Diebold–Mariano test statistic is then

$$S_l = [\hat{V}(\bar{\delta})]^{-1/2}\bar{\delta} \tag{10}$$

where $\hat{V}(\bar{\delta})$ is obtained by substituting the estimates in (9) into (8). After several approximations and a little algebra, HLN use an unbiased estimator of $\bar{\delta}$ to derive a modified Diebold–Marino estimator:

$$S_l^* = \left[\frac{n+1-2h+n^{-1}h(h-1)}{n}\right]^{-1/2} S_l \tag{11}$$

7 For the HLN statistics of Table 2, we have generated statistics for all 12 steps. However, we report the results only for steps, 1, 2, 3, 6, 9, and 12 to conserve space. Results for all steps are obtainable from the authors upon request.

8 David Harvey provided insights, in his correspondence with us, on the selection and use of the parameter h. The value of h is based on the assumed autocorrelation of lag length $h - 1$. He recommends using h equal to the forecast horizon. He also commented on the evidence of the relative efficiency of the HLN statistic. We are thankful to him for his help.

The statistic in (11), known as the HLN test statistic, is compared to the critical values of the Student t distribution with n degrees of freedom. It has some desirable properties. HLN (1997, p. 293) show that the statistic does not rely on the assumption of unbiased forecasts since an approximately unbiased variance of difference of forecast errors, δ_t, was used in the statistical derivation. Furthermore, in simulation experiments using h = 1 to 10 and sample sizes ranging from n = 8 to 526, the HLN statistic is shown to outperform the Diebold–Mariano statistic and is applicable beyond one-step-ahead forecast (HLN 1997, p. 285).

In addition, according to HLN (1997), the modifications to the Diebold–Mariano statistics do not add extra computational costs, while they prove a valuable tool in testing the equality of forecast performance. The modifications correct the test oversize problem inherent in the original Diebold–Mariano statistic. The HLN statistic can be applied to any error function, not necessarily a quadratic loss function.

Finally, HLN (1997) note that their statistic is a modified Diebold–Mariano statistic that "constitutes the best available approach to assessing the significance of observed differences between the performance of two forecasts" (HLN 1997, p. 291). Thus, we use the HLN statistic and the results are discussed below.

The Harvey, Leybourne, and Newbold test results

Table II presents the HLN test results. The HLN test statistics indicate that the ARFIMA model significantly outperforms the random walk model in all forecast steps beyond 2 months, while the long-memory models are more efficient than the monetary models in all multi-step-ahead forecasts for most countries included in the study. In other words, the HLN statistics indicate, at the 5% level, that the forecast mean square errors from ARFIMA models are significantly smaller than those from random walk models. This finding is consistent for most countries beyond the 2-months-ahead forecasts.

Similarly, the HLN statistics show, at the 5% level, that the forecast mean squared errors from ARFIMA models are significantly smaller than those from monetary models, beyond the 1-month-ahead forecasts in most countries. For all exchange rates analyzed, the superior performance of ARFIMA models becomes more evident as forecast steps lengthen. The HLN test statistics show significant differences between the two forecast error functions at longer horizons. Thus, the long-memory model becomes convincingly superior in the forecasting accuracy at longer forecast steps.

Summary and Conclusions

In this paper, we evaluated the forecasting performance of nominal exchange rates using the fractionally integrated ARMA model as an extension of previous studies. We employed Sowell's exact maximum likelihood estimation method and compared the forecasting performance of the ARFIMA model with that of the three monetary structural models and the random walk model with a drift. While structural and random walk models have been amply compared in the past (see Meese and Rogoff, 1983a,b), no comparison has yet been made between monetary and ARFIMA models. Cheung (1993) did not compare the forecasting accuracy of ARFIMA models with the monetary structural models. He did not offer a formal statistical comparison of the forecasting accuracy either, when comparing the ARFIMA models with the random walk models.

Our findings indicate that the ARFIMA model outperforms structural monetary models in short-run forecasting. Further, the ARFIMA model is also shown to outperform the random walk model

Table II. The HLN test of equality of forecast performance: comparison of equality of forecast mean square errors for pairs of models by currency and forecast steps.

Models	Steps ahead forecasts	CD	FF	GM	IL	JY	BP
LM v RW	1 step	0.9119	1.8900	1.1966	2.1108	1.2438	1.7737
	2 step	1.5350	3.1810	2.0142	3.5530	2.0936	2.9855
	3 step	1.9345	4.0090	2.5384	4.4777	2.6385	3.7625
	6 step	2.6587	5.5097	3.4887	6.1540	3.6263	5.1710
	9 step	3.0245	6.2677	3.9687	7.0008	4.1252	5.8825
	12 step	3.1590	6.5464	4.1451	7.3121	4.3087	6.1441
LM v B-F	1 step	1.2718	1.2509	1.1850	1.6450	1.1686	1.6139
	2 step	2.1408	2.1056	1.9946	2.7689	1.9771	2.7167
	3 step	2.6979	2.6535	2.5137	3.4896	2.4791	3.4237
	6 step	3.7079	3.6469	3.4547	4.7959	3.4071	4.7054
	9 step	4.2181	4.1487	3.9301	5.4558	3.8759	5.3528
	12 step	4.4056	4.3332	4.1048	5.6984	4.0483	5.5908
LM v F-D	1 step	1.4109	3.6838	1.1112	3.8058	1.2750	1.8691
	2 step	2.3749	6.2008	1.8705	6.4060	2.1461	3.1462
	3 step	2.9930	7.8145	2.3573	8.0732	2.7046	3.9650
	6 step	4.1135	10.7400	3.2398	11.0956	3.7172	5.4494
	9 step	4.6795	12.2178	3.6855	12.6223	4.2286	6.1992
	12 step	4.8875	12.7611	3.8494	13.1835	4.4167	6.4748
LM v H-M	1 step	1.7319	0.8712	0.8273	0.9176	0.9002	1.2764
	2 step	2.9152	1.4665	1.3926	1.5445	1.5153	2.1485
	3 step	3.6739	1.8482	1.7550	1.9465	1.9097	2.7077
	6 step	5.0492	2.5400	2.4120	2.6752	2.6246	3.7213
	9 step	5.7440	2.8895	2.7439	3.0433	2.9857	4.2333
	12 step	5.9994	3.0180	2.8659	3.1786	3.1185	4.4216

Notes:
1. An HLN 'S' statistic greater than 1.47 indicates significant difference at 10% level; that greater than 1.65 indicates significant difference at 5% level; and that greater than 2.47 indicates significant difference at 1% level.
2. LM, long-memory model; RW, random walk model.
3. See Table I notes.

nearly unanimously. The HLN statistical test of equality between forecast mean square errors, in general, supports the findings that the long-memory model provides forecast performance superior to that of the random walk, especially at the longer forecast horizons.

Our study demonstrates the existence of significant long memory in the nominal exchange rates in five of the six currencies studied and that the long-memory model outperforms both the monetary structural models and the random walk model in short-run forecasting. This strongly suggests that the long-memory model be seriously studied in future work as an alternative to the monetary and/or random walk models.

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