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ON THE SHORT-_PERIOD TYPE II CEPHEID FIELD STARS

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ABSTRACT

Fourier decomposition parameters for the photelectric light curves of 20 short-period type II Cepheids in the field are compared with the models of Hodson, Cox, and King, also subjected to Fourier analysis. The hydrodynamic light curves display sequences in the Fourier phases very reminiscent of the resonance progressions among classical Population I Cepheids with periods less than 10 days. However a handful of type II models with small radii and relatively high period ratios, $P_2/P_0$, give Fourier phases which do not seem to follow the resonance sequence. Comparing the models with the type II Cepheid observations, we find very good agreement in both the phase-period and phase-phase diagrams. As in the models, there are a number of observed stars which obviously display a resonance progression and others which may not. Thus, based on the Fourier diagrams, we are led to distinguish three classes of short-period type II Cepheids with periods and period ratios as follows: type II S ($1^4 \leq P \leq 1^6$; $P_2/P_0 \geq 0.53$), type II M ($1^4 \leq P \leq 2^4$; $P_2/P_0 < 0.53$), and type II L ($P \geq 2^5$; $P_2/P_0 \leq 0.49$). An apparent resonance sequence among the II M stars leads us to propose that the resonance center ($P_2/P_0 = 0.5$) occurs near 2 days, rather than 1.6 days as suggested previously. We discuss the problems posed for theories of stellar and galactic evolution by the existence of disk, halo, and cluster pulsators with very similar light curves. Comparison of our results with the work of Carson, Stothers, and Vemury, Carson and Stothers, and Petersen and Diethelm, finds a number of points of agreement as well as some contradiction. Further calculations and additional observations, particularly CCD photometry of globular cluster Cepheids, will be necessary to resolve these problems.

Subject headings: stars: Cepheids — stars: pulsation

I. INTRODUCTION

In recent years considerable attention has been given to the short-period type II Cepheids with $1 \leq P \leq 3$ days. Although these stars have been grouped together by many authors under the category "BL Herculis," it remains unclear to what extent such objects actually comprise a homogeneous class (Diethelm 1983). Theoretical models, both linear (Carson, Stothers, and Vemury 1981, hereafter CSV; King, Cox, and Hodson 1981; Petersen 1981) and nonlinear (Carson, Stothers, and Vemury 1981; Carson and Stothers 1982, hereafter CS; Hodson, Cox, and King 1982, hereafter HCK) have been constructed for these stars, but a paucity of precise observational data has hampered the comparison of theory and observation. From a theoretical viewpoint the short-period type II Cepheids are especially interesting because they seem to show a "Hertzsprung progression" akin to that found among the classical, Population I Cepheids with periods between 6 and 18 days (see, e.g., CS and references therein). For the classical Cepheids the Hertzsprung progression has been linked to a near-period resonance ($P_2/P_0 \approx 0.5$) in the models (Simon and Schmidt 1976), and, indeed, likely models for the type II stars show the same approximate resonance. Comparison of the two resonance sequences has led various authors (Petersen 1981; HCK) to conclude that a "break" in light curve structure occurs for the type II objects at $P \approx 1.6$ days, analogous to what is seen for the Population I stars at $P \approx 10$ days. If this analogy holds, then $P_2/P_0 \approx 0.5$ at $P_0 \approx 1.6$ days for the short-period type II pulsators.

CSV and CS have taken a different tack, attempting to compare theoretical models with observations. These comparisons have been qualitative but very detailed for a few stars, and semiquantitative in a broad discussion of the Hertzsprung or "bump" progression. This work has also demonstrated a "break" in the light curves at $P \approx 1.6$ days. While differences exist between the models and conclusions of CS and those of HCK, we shall not discuss them in the present work as there is little we can now contribute in this regard.

However, the question remains as to whether or not the short-period type II Cepheids comprise a single class as seems suggested by the bump sequence. The problem is raised by CS who discuss possible differences in luminosity and metal abundances among these stars. Another approach, strictly observational, has been undertaken by Diethelm (1983). Based upon light curve shape and two-color diagrams, this author has divided the stars in question into three categories which he calls RRd, CW, and BL Herculis. The CW stars in Diethelm's sample have periods between 1.415 and 1.946 days, while the periods of the RRd and BL Her stars fall almost exclusively above or below this range. While this circumstance suggests we are looking at a single class of objects, it could also be coincidence due to the relatively small number of stars observed.

Another technique for treating the light curves of pulsating stars was described by Simon and Lee (1981) who Fourier decomposed the $V$ magnitude variations of a large sample of classical Cepheids. The Fourier fit to the light curves was of the form $A_0 + \sum_{j=1}^{\infty} A_j \cos (j \omega t + \phi_j)$. The presence of the period $P_2/P_0 \approx 0.5$ (and presumably of the Hertzsprung progression) was seen in the Fourier coefficients as a slow rise in the quantities $\phi_{21} \equiv \phi_2 - 2\phi_1$ and $\phi_{31} \equiv \phi_3 - 3\phi_1$ as the period increased from 3 to 8 days, followed by a much sharper rise near 10 days. While this technique has the advantage of providing a quantitative description of the progression of light curve shapes, it requires observations with good phase coverage and high accuracy.
For the short-period type II Cepheids, it is only in the past year or two that sufficient observations have been compiled to make the Fourier technique useful. In a very recent paper, Petersen and Diethelm (1986, hereafter PD) have tabulated Fourier decomposition parameters for a large number of photoelectric and photographic observations of these stars. In the present work we shall use the most accurate of these observations to plot Fourier diagrams and make comparisons with both the classical type I Cepheids and with the models of HCK, also subjected to Fourier decomposition. We shall find that the (presumed) resonance sequence among the type II stars differs in significant ways from its Population I counterpart, but that hydrodynamic models of the type II Cepheids are amazingly successful in depicting the light curve progression for these stars. The comparison of theory and observation will also enable us to draw tentative conclusions regarding these objects, which differ from a number of the viewpoints previously presented in the literature.

II. FOURIER DECOMPOSITIONS

Fourier parameters for photoelectric type II light curves are listed in the first part of Table 1 of PD. These authors rate the accuracy of the Fourier fits on a scale of 1 to 4, with the last being the worst. At least one of the stars with this poorest rating was thrown out in PD's discussion, and a number of the others are suspect. For the present work we have thus selected all of the photoelectric Fourier decompositions for which PD give ratings of 1 to 3. There are 19 such stars, as follows: BX Del, BL Her, HQ CrA, KZ Cen, V2022 Sgr, SW Tau, V745 Oph, V971 Aql, DU Ara, VZ Aql, V839 Sgr, RT TrA, V553 Cen, V465 Oph, V716 Oph, VX Cap, EK Del, UY Eri, and UX Nor. To these we add one further object, XX Vir (Lub 1977) whose light curve we have ourselves Fourier decomposed. Thus our sample consists of 20 stars, the best short-period type II Cepheid light curves presently available in the literature.

The Fourier coefficients for these light curves may be compared with those emerging from Fourier analysis of the models of HCK. In Figure 1 we have plotted the phase differences \( \phi_{21} \), \( \phi_{31} \), and \( \phi_{41} \equiv \phi_4 - 4\phi_1 \) versus linear nonadiabatic period ratio \( P_2/P_0 \), for the first 18 of the HCK models, i.e., those with periods between 1 and 3 days. These plots were made with data kindly supplied to us by S. W. Hodson.

The crosses in Figure 1 represent models A–D and the dots, models E–R (see Table 1 of HCK). One notices at once the striking appearance among the dots of resonance sequences reminiscent of those in the classical Population I Cepheids (Simon and Lee 1981). However, there are also a number of interesting differences among the plots for the two different populations. Whereas \( \phi_{21} \) for the classical Cepheids varies over a range of about \( 3\pi/2 \) radians, the same quantity has a much more modest range in the type II models, about \( \pi/2 \) radians. The range of variation of \( \phi_{31} \) is also smaller for the type II stars. Indeed, for the present objects, the resonance appears most strongly in the quantity \( \phi_{41} \) which varies over nearly \( 2\pi \). Interestingly enough, this \( \phi_{41} \)-period progression turns out to be very similar in form to that found recently among the classical Cepheids with periods less than 10 days (Simon and Moffett 1985).

Models A–D (crosses in Fig. 1) have small radii and high mean densities. Their periods are the shortest and their period ratios, \( P_2/P_0 \), the largest in the HCK sample. We note from Figure 1 that these four models do not seem to follow the resonance sequence but rather appear to lie well above it. This fact will later play a role in possible interpretations of the type II pulsators.

III. THEORY VERSUS OBSERVATIONS

An important difference in period structure between the classical and type II Cepheids was pointed out by HCK. Whereas in the former stars the loci, in the H-R diagram, of constant period \( P_0 \) and constant period ratio \( P_2/P_0 \) nearly

![Fig. 1.—The Fourier phase terms \( \phi_{21} \), \( \phi_{31} \), and \( \phi_{41} \) vs. linear nonadiabatic period ratio for models A–D (crosses) and E–R (dots) of HCK](image-url)
coincide, in the case of the type II Cepheids these two loci diverge. This means that at a given period for the type II stars there will be a range in period ratio and thus, presumably, a scatter in the phase quantities $\phi_{21}$, $\phi_{31}$, and $\phi_{41}$ when they are plotted versus period.

Figure 2 displays the quantity $\phi_{21}$ versus period, plotted side by side for the observed stars of Table 1 and the models of HCK. Figures 3 and 4 show similar plots for $\phi_{31}$ and $\phi_{41}$, respectively. In all three figures the crosses in the observational graphs denote stars in Diethelm's sample (and similar stars not treated by Diethelm 1983) which were classified RRd or BL Her and which have periods shorter than 1.6 days. The dots in Figures 2–4 are Diethelm's (plus some additional stars) CW class, while the open circles represent the four long-period stars in Diethelm's RRd and BL Her categories.

In the opinion of the author, the overall agreement between theory and observation in Figures 2–4 is very good. This is all the more apparent when one considers the poor state of similar comparisons made for the classical Cepheids (Simon and Davis 1983). All three diagrams show what may be interpreted as a scattered resonance sequence among the stars of middle period, perhaps extending to the longer period stars. The shorter period stars do not seem to follow the same sequence.

Despite the general agreement between the HCK and the observational plots, there is enough scatter in the diagrams that it becomes difficult to make a comparison between actual stars and calculated models with great precision. Once more, this is due to the fact, noted above, that the abscissa of all the plots—i.e., the pulsation period—is not a "good variable" for studying the resonance. To remedy this deficiency we shall now introduce the phase-phase diagrams $\phi_{21}$ versus $\phi_{31}$, $\phi_{41}$ versus $\phi_{21}$, and $\phi_{41}$ versus $\phi_{31}$, which are presented, respectively, in Figs. 5–7. Dots, crosses, and circles have the same distinction as in previous plots. In each of Figures 5–7, the period ratio $P_2/P_0 (\times 1000)$ is written in next to each dot representing an HCK model.

The resonance sequence shows up clearly in all of these diagrams, but is particularly striking and beautiful in Figure 7. This is all the more remarkable in view of the fact that a number of the Fourier coefficients for the observational data are only marginally determined due to mediocre phase coverage for some of the stars. We strongly suspect that the observational resonance sequences in Figures 5–7 will be considerably tighter when more data is available.

We note that the same stars occupy similar positions in all of the phase-phase diagrams—objects like HQ Cra, KZ Cen, and V745 Oph at the bottom left, and V465 Oph, EK Del, UY Eri, and UX Nor at the top right. Furthermore, there is a clear tendency for the stars represented by crosses to lie above the resonance sequence in Figures 5 and 6, though not in Figure 7. Comparison of the dot sequence in Figures 5–7 with corresponding phase-phase plots (Simon and Moffett 1985) for the short-period classical Cepheids ($P < 10$ days) again shows a considerable similarity of form.

IV. DISCUSSION

For simplicity in what follows, let us denote by class S the short-period stars in our observational sample (i.e., the crosses), by class M the middle-period stars (dots), and, finally, by class L the longer period stars (circles). Then, the theoretical and observational evidence presented in Figures 2–7 leads us to conclude that the type II S stars have relatively large period ratios ($P_2/P_0 \sim 0.53$), while the type II M objects have smaller period ratios, akin to those in the resonance sequence of classical Cepheids, i.e., $P_2/P_0 < 0.53$. The type II L stars display large values of the Fourier phases $\phi_{21}$, $\phi_{31}$, and $\phi_{41}$. According to Figures 2–7 we may associate such values with either large period ratios ($P_2/P_0 \geq 0.53$) similar to those of the
FIG. 3.—Values of $\phi_{31}$ vs. period for the HCK models and for the observed stars. Notation as in Fig. 2.

FIG. 4.—Values of $\phi_{41}$ vs. period for the HCK models and for the observed stars. Notation as in Fig. 2.
Fig. 5.—Values of $\phi_{31}$ vs. $\phi_{21}$ for the HCK models [numbers in parentheses are period ratios ($P_3/P_{\phi_{21,NA}} \times 1000$)] and for the observed stars. Notation as in Fig. 2.

Fig. 6.—Values of $\phi_{41}$ vs. $\phi_{21}$ for the HCK models and for the observed stars. Notation as in Fig. 5.
II S stars, or small period ratios ($P_2/P_0 \leq 0.50$), similar to those of the longest period II M objects. The latter interpretation is more reasonable, as we shall see below.

In order to make further progress we now refer to Figure 1 of HCK. These authors have constructed an H-R diagram and, on the basis of LNA models of mass $M = 0.55 \, M_\odot$, have plotted blue and (estimated) red edges along with loci of constant period and constant period ratio $P_2/P_0$. The latter loci are inclined very sharply to the former such that at a given period the period ratio is much larger at the blue edge than at the red. In Figure 8 of the present work we have reproduced the lower portion of the HCK diagram and attempted to locate the type II pulsators according to our analysis. The lower hatched region on this figure represents the domain of the II S stars: $1.0 \leq P \, (\text{days}) \leq 1.6, \, P_2/P_0 \geq 0.53$.

The 10 II M stars of our observed sample are plotted on Figure 8 as dots. The period ratios for these stars were estimated by comparing the observed and theoretical parts of the instability strip. While it is difficult to assign an error to our estimated period ratios without constructing a new series of models, we note that even if the values are off by as much as $\pm 0.005$ and in such a way as to spread out the dots, the II M stars will still occupy the center of the strip, albeit in a somewhat wider swath.

Let us turn to the II L stars. As mentioned above, Figures 2–7 do not provide enough evidence to determine whether or not these objects form a longer period extension of the II M sequence. To attack this question we shall use a further quantity generated by the Fourier decompositions—namely, the amplitude ratio, $R_{21} \equiv A_2/A_1$. Based upon experience with the classical Cepheids (Simon and Lee 1981) one expects $R_{21}$ to be influenced both by the resonance and by the amplitude of pulsation. Unfortunately, the latter influence is greatly magnified in the models of HCK which, in general, display far higher limiting amplitudes than those actually observed in the type II Cepheid sample. Figure 9 shows a plot of $R_{21}$ vs. amplitude for the HCK models. The trend of increase among the dots is apparent. It is also interesting to note the six models which fall below this trend. We have written in alongside these points the LNA period ratios ($P_2/P_0)_{\text{LNA}}$ (times 1000). Of the 18 HCK models treated here, these are the ones with the smallest values of $P_2/P_0$ in all cases near 0.5. Thus we discern the tendency for models near the resonance to have lower amplitude ratios $R_{21}$ than they would otherwise. This is certainly not surprising in view of the well-documented fall of $R_{21}$ near 10 days in the classical Cepheids (Simon and Lee 1981). We shall return to this point shortly.

In Figure 10 we display a plot of $R_{21}$ versus $V$ amplitude for the observed stars. Again, there is a trend of increase. However, when $R_{21}$ is plotted versus period (Fig. 11), another picture emerges. Among the type II M stars (dots), $R_{21}$ is seen to drop off with period, attaining quite small values around 2 days. On the other hand, the II L stars (circles) show very high values of $R_{21}$, with EK Del at 2.05 days having $R_{21} = 0.472$, one of the largest values in the sample. This large discontinuity argues against a straightforward extension of the II M stars across the resonance where they become II L objects. Indeed, according to Figure 9, proximity to the resonance, on either side, seems to make $R_{21}$ smaller.

The conclusion that the II L stars do not form a direct extension of the II M class is strengthened by Figure 12 in which we have plotted $R_{21}$ versus $\phi_{41}$ for the observed sample. The abscissa was chosen as the observational variable which

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**Fig. 7.** Values of $\phi_{41}$ vs. $\phi_{31}$ for the HCK models and for the observed stars. Notation as in Fig. 5.
best follows the resonance sequence (see Figs. 4 and 7). For the II M stars, $R_{21}$ is seen to generally decrease as $\phi_{41}$ rises toward $2\pi$. The two dots which lie considerably above this trend are VZ Aql and V839 Sgr, two stars whose light curves are relatively poorly determined. Improved observations could bring these stars closer to the others. However, the four II L stars lie so far from the dots that it is very difficult to imagine they could be part of the same sequence. Indeed these stars fall comfortably among the II S objects (crosses). In fact, on virtually all of the plots which do not involve period, the crosses and the circles fall together.

Since the II L and II S stars show similar values of $\phi_{21}$, $\phi_{31}$, $\phi_{41}$, and $R_{21}$, one might argue that the former, like the latter, ought to have large period ratios, $P_2/P_0 \geq 0.53$. However, one may easily deduce from Figure 1 of HCK, that were this to be the case the masses of the II L stars would have to be very large, of the order of $0.75 M_\odot$ or even greater. It would be very difficult to reconcile such high masses with standard theories of stellar and galactic evolution, particularly for a star like UY Eri which shows halo characteristics (see below). Thus, the best interpretation for the II L stars would seem to be that they have low period ratios, with values that fall safely below those of the longest period II M stars. Assuming a mass of $0.55 M_\odot$ and period ratios $P_2/P_0 \leq 0.49$ yields the upper hatched region on Figure 8 as the domain of the II L objects.

If the above analysis is correct, then we note that UY Eri and, particularly, EK Del ought to lie near the lower vertex of the II L region on Figure 8. In Figure 13, we plot the traditional amplitude-period diagram for the observed stars. The sample has been augmented by a few stars (Diethelm 1983) whose amplitudes are reasonably well determined even though the details of their light curves are not. The low amplitudes of EK Del and UY Eri could be consistent with a location toward the center of the strip, and thus, according to Figure 8, indicating a higher luminosity. A similar conclusion regarding these two stars was reached by CS on somewhat different grounds.
Fig. 10.—Values of $R_{21}$ vs. $V$ amplitude for the observed stars. Notation as in Fig. 2.

Fig. 11.—Values of $R_{21}$ vs. period for the observed stars. Notation as in Fig. 2.

Fig. 12.—Values of $R_{21}$ vs. $\phi_{41}$ for the observed stars. Notation as in Fig. 2.
V. COMPARISON WITH CSV AND CS

The Fourier decomposition technique has two distinct advantages over other methods in comparing observed and theoretical light curves: (1) the parameters are strictly quantitative; and (2) the phase quantities \( \phi_j \) are largely independent of total pulsation amplitude (Simon and Lee 1981; Simon and Moffett 1985; Simon and Aikawa 1986). When the limiting amplitude rises in a theoretical model the higher order Fourier terms are also enhanced, thus distorting the appearance of the light curve. Since the correct limiting amplitude is so uncertain in the models, it is prudent to compare observation and theory in terms of amplitude-independent quantities. The Fourier phases satisfy this requirement very well.

A number of the interpretations offered by CS rest on an observed tendency in their models—namely, an association of a larger limiting amplitude with a higher luminosity (at fixed mass). However, the limiting amplitude is influenced by other effects as well. One of these, pointed out by CS, is location in the instability strip with respect to the red and blue edges. Another property, not mentioned by CS, is proximity to the amplitude of the classical Cepheids at periods near 10 days, and which, according to us, has a similar influence near 2 days among the type II M stars.

A third effect has to do with artificial viscosity in the models themselves. Even if the viscosity coefficient is held fixed from model to model (CS did not indicate what was done in this regard), it is still not safe to assume that the relative limiting amplitudes will be correct. To the extent that the artificial viscosity mimics real physical damping that takes place in pulsating stars, the models will be uncertain because we do not know how much of this damping to include. Clearly, a given value of the viscosity coefficient could not be expected to correctly model such damping in all stars. (It may be added that, consistent with this argument, it is not necessarily true, as is sometimes assumed, that the less artificial damping in a model, the better.) Until hydrodynamic codes are constructed with a more precise treatment of the outer layers, the degree of correspondence between real and theoretical limiting amplitudes will remain unknown.

Given this situation, it is useful to compare the CSV and CS conclusions (hereafter CSVCS) with those based upon our interpretation of the Fourier phases. At short periods, the two approaches yield results which generally coincide. The preferred CSVCS models for BX Del, BL Her, and XX Vir all have period ratios \( P_2/P_0 \sim 0.54 \), and are thus consistent with our basic characterization of the II S stars. The small increase in luminosity \( (\Delta \log L \approx 0.1) \) suggested by CS for XX Vir as opposed to BL Her also produces no conflict with our analysis.

At a somewhat longer period, the star KZ Cen lies, according to our interpretation, in the II M regime and has a period ratio between 0.53 and 0.52. Models 4 and 5 of CS and model 2 of CSV all satisfy this criterion. Indeed, Petersen and Hansen (1984) conclude that a model in between models 4 and 5 would match KZ Cen precisely. However, it should be noted that model 6 of CS, which actually falls between models 4 and 5 on the H-R diagram (although its period is slightly longer), has a far different appearance. This is presumably a function of its large amplitude and points up once more, the caution that must be exercised in making eyeball comparisons.

The rest of the CSVCS models in the range \( 1.4 \leq P \leq 2.0 \) days all have very large amplitudes \( \geq 1.7 \) mag. Indeed, the middle period CS models seem to be less successful in duplicating observed light curves, perhaps due to their high amplitudes. Whereas we assert that stars like RT Tra and V553 Cen ought to have period ratios near 0.50, none of the CSVCS models with appropriate periods lies close to the resonance. However, this circumstance is perhaps fortuitous. To model these stars, we suggest somewhat higher luminosities or smaller masses than those chosen by CS.

At still longer periods CS assert that their models 7 and 8 account for the light curves of UX Nor and V465 Oph, respectively. Since these models have period ratios of 0.51 and 0.49, the former contradicts our suggestions regarding the type II L stars, while the latter does not. However, because models 7 and 8 of CS display enormous amplitudes (1.9 and 2.3 mag, respectively), it is difficult to say for certain how closely they actually reproduce the observed light curves. This comparison is better made by Fourier analysis.

To summarize, we would say that the CSVCS calculations themselves do not significantly disagree with the picture we

![Fig. 13.—\( V \) amplitude vs. period for the observed stars. Notation as in Fig. 2.](image-url)
have presented, since relatively small changes in the parameters could make the two sets of models largely coincide. However, as we shall see below, the present results may be in conflict with the idea advanced by many investigators (Petersen 1981; HCK; CS) that the type II Cepheids between 1 and 3 days comprise in some sense a single “bump sequence” akin to the Hertzsprung progression and certainly disagree with the contention that the center of such a progression lies near 1.6 days.

VI. NATURE OF THE TYPE II CEPHEIDS

In the present work we have divided the type II Cepheids into three classes—S, M, and L—strictly according to their pulsation properties as determined by Fourier decomposition. Although two stars with a similar pulsation period and (inferred) period ratio presumably have a similar mass and luminosity, it does not follow that they necessarily share a common internal structure or evolutionary history. Indeed, the type II Cepheid models show that pulsation in these stars is confined to the outer few percent of the stellar mass. Clearly, then, the pulsational properties of these objects cannot be used as a probe of the detailed physical structure of their interiors.

If the short-period type II pulsators formed a homogeneous group, it would be relatively simple to account for their properties. A single star evolving with increasing (or decreasing) luminosity in Figure 8 could in turn enter the S, M, and L domains and thus take on progressively the characteristics of the different classes. Or, a crossing at lower luminosity, concentrated in the blue and center of the strip, might account for the S and the M objects, while a different penetration of the strip at higher luminosity with a long time scale in the red could explain the L stars. However, there is ample evidence that the type II Cepheids are not monolithic.

Table I lists 10 stars for which abundance and/or velocity data are available. The values of [A/H] and \( V_{\text{pec}} \) are from Harris and Wallerstein (1984). RT TrA is not included among the survey of these authors but is known to have high metals and a low velocity (Lloyd Evans 1983). Among the three II S stars with listed metal abundances we notice a sharp dichotomy between BL Her on the one hand and VX Cap and XX Vir on the other. The former has metals characteristic of the old disk population, while the latter stars have abundances more proper to the halo. There exist a number of distinctions between the light curve of BL Her and those of XX Vir and VX Cap (Diethelm 1983), perhaps the chief one being amplitude (CS). The halo-type stars display larger amplitudes and higher values of \( R_1 \), and are thus rather easily distinguishable, for example on Figures 10 and 13. Following Diethelm (1983) we shall class BX Del, V527 Sgr, and SW Tau with BL Her, and BF Ser and CE Her with XX Vir. While Diethelm (1983) called these two groups BL Her and RRd, respectively, we shall refer to them hereafter as S-BL and S-XX. The five II M stars which appear in Table I all show high metals and moderate to small velocities, i.e., the characteristics of the old disk population. They thus resemble BL Her. Furthermore, the II M objects seem to form a well-defined group with pulsation properties following a resonance sequence. A further interesting fact is that at least two of these pulsators—RT TrA and V553 Cen—are carbon stars (Lloyd Evans 1983). Whether or not others of them will prove to share this property remains to be seen. Up to now, none of the observed II M stars has been found to display halo attributes. If future spectroscopy and photometry on these stars and on the objects we have called S-BL should establish that all of these variables have old disk characteristics, then a 0.55 \( M_0 \) evolutionary track, crossing the strip at \( L/L_0 \approx 2.1 \) could economically give rise to the entire II S-BL and II M classes. The II S-XX pulsators, halo objects of different origin and perhaps with somewhat different masses and luminosities, might then be explained as stars which populate the strip only in the blue.

However, even if future observations confirm a clear population difference between the S-BL and M pulsators, on the one hand, and the S-XX stars, on the other, the existence of short-period type II Cepheids in globular clusters poses conundrums which threaten to throw any simple evolutionary scheme into confusion. A list of globular cluster Cepheids is given in Table 2 of CSV. To the first 14 stars on this list (i.e., those with periods under 3 days), we may add the variable V12 in the cluster M9 (Clement, Ip, and Robert 1984). Fourier coefficients for eight of these 15 objects are included in Table 1 of PD. While these decompositions do not have the accuracy of those for the field stars in our sample, a number of them seem good enough to allow tentative conclusions.

It has already been pointed out by CS that one of these stars, V60 of \( \omega \) Cen, has a light curve which resembles that of BL Her. To these we may add V12 of M9, V1 of M15, and V43 of \( \omega \) Cen. All four stars have periods and Fourier characteristics which place them among the S-BL stars. In addition, the star V1 of M56 seems to have a period and Fourier phases similar to those of the II M objects. Indeed the short-period type II Cepheids in metal-poor globular clusters seem to resemble not the halo-type field stars, but rather the old disk objects with high metals and low space velocities.

The above suggestion is bolstered when one considers the pulsation amplitudes of the cluster variables. To crudely translate blue or photographic amplitudes into visual amplitudes we have calculated the ratio \( A_V / A_B \) for the photometric observations in Table 1 of PD. We find \( \langle A_V / A_B \rangle = 0.73 \). Multiplying by this factor the amplitudes given by CSV for the cluster stars, we obtain estimates of their visual amplitudes. These amplitudes are very low compared to those of the S-XX stars, but are similar to those of the S-BL and M objects.

It is very difficult to understand how, according to standard theories, metal-poor and metal-rich stars could so similarly populate the instability strip that their periods and period ratios would be virtually identical. Horizontal-branch evolutionary tracks (Sweigart and Gross 1976; Gingold 1976) display great sensitivity to input parameters, particularly abundances and core masses. In order to account for the metal-rich RR Lyrae stars, Taam, Kraft, and Suntzeff (1976)
were forced to postulate large amounts of essentially stochastic mass loss in old disk stars. Clearly, the metal-poor halo and cluster stars ought to have histories very different from this. Surely, it is demanding a great deal from coincidence that disk and halo objects arrive in the instability strip with essentially the same masses and luminosities.

The above situation brings to mind an early “solution” offered for the solar neutrino problem (Bahcall 1979). According to this idea (Bahcall and Ulrich 1971), the Sun has a low primordial metal abundance but has accreted metals in its surface layers, perhaps due to passage through dense interstellar clouds (Aumann and McCrea 1976; Newman and Talbot 1976). If the short-period type II disk Cepheids were also essentially stars of Population II composition covered with a thin veneer of metals, then their evolutionary similarity to halo and cluster stars would be easy to understand. Of course, most of the objections raised against metal accretion for the Sun (e.g., Roed 1979) could also be mounted against this suggestion. Indeed, at present it cannot be said that the type II Cepheid data warrants any such desperate hypothesis. Nevertheless, as this data becomes more complete and our models become better, nonstandard solutions to the problems posed by low-mass, metal-rich pulsators may become necessary.

VII. CONCLUDING REMARKS

It is interesting to further compare the resonance sequences in the classical and type II Cepheids. PD conclude that these sequences are basically similar, although they point out a number of small differences. However, the Fourier diagrams of PD include many poorly observed stars and are thus crowded in a way that may obscure otherwise observable trends. In our opinion, such a trend is the resonance progression among the II M stars. Comparing this with the classical Cepheid sequence one also finds a number of similarities. In both cases there is a rise in the Fourier phase quantities $\phi_{21}$ and $\phi_{31}$ as the resonance center is approached from the short-period side. However, as mentioned above, the effect in $\phi_{31}$ seems somewhat muted in the II M stars. In these objects the strongest expression of the resonance appears in the phase term $\phi_{41}$. Turning to the amplitude ratio $R_{31}$, Figures 11 and 12 show a diminution of this quantity near the resonance center in crude agreement with a similar trend among the Cepheids of Population I (Simon and Lee 1981).

When one looks at the actual light curves and compares them by eye, greater discrepancies appear. While we assert that a classical Cepheid like U Sgr (Pel 1976) occupies, with regard to the resonance, a position similar to that of the II M star V745 Oph (Diethelm 1983), the two light curves do not look similar. On the other hand, placing the Type II resonance or “bump-crossing” at $P \approx 1.6$ might seem to imply the similarity in light curve structure of two stars like DR Vel (Pel 1976) and V971 Aql (Diethelm 1983). Although both display, in some sense, a bump on the rising branch, can we really maintain that they look alike? These problems point up the difficulty of making eyeball comparisons. But, in fact, one ought to ask a further question: Why should two stars, one type I and one type II, with similar near-resonant period ratios have similar light curves? We know, for example, that the fourth Fourier term is considerably larger in the II M stars than in the classical Cepheids, a circumstance which is bound to have an effect on the shape of the light curve. Indeed, we have already noted that pulsation in a Type II star is confined to the outer 1% of the mass; on the other hand, in a Population I Cepheid, 50% or even more of the mass participates. Under such circumstances can it really be surprising that the resonance sequence expresses itself somewhat differently in the two classes?

In both PD and the present work, an interpretation of the resonance sequence is crucial to an understanding of the short-period type II Cepheids. As mentioned above, Diethelm (1983) proposed that these stars should be divided into three categories: BL Her, RRd, and CW. While PD could not show strong evidence for a resonance near 1.6 days, they accept its existence in line with the traditional idea of a single class of old, standard Population II stars. Our own analysis differs in our assertion that only the middle-period (II M or CW) stars display a resonance progression which is directly comparable to that of the Population I Cepheids. However, these stars may well be related in an evolutionary sense to the objects with old disk characteristics and periods less than 1.6 days (IIS-BL or BL Her stars).

The masses and luminosities we infer for the short-period Type II Cepheids do not differ greatly from those of CSVCS, but our association of light curve properties with period ratio, if correct, allows a tighter handle on acceptable parameters for the models. This correspondence can be made even more precise with a new, more extensive series of calculations. At present, relying only upon the single-mass models of HCK, our scheme still makes a number of testable predictions, particularly involving the stars SW Tau and EK Del. If our interpretation is correct then the former star must lie in the upper vertex of the lower hatched region on Figure 8 and thus have a higher temperature than stars of similar period in the sample (e.g., V745 Oph). Similarly, we require that EK Del be located near the lower vertex of the upper hatched domain in Figure 8 and thus be cooler than other stars near two days, e.g., RT TrA and V553 Cen.

At the moment, we cannot agree with the statement of CS that the “BL Herculis stars comprise a well-defined subgroup of type II Cepheids.” Due to the presence in the period range 1–3 days of disk-type, halo-type, and cluster stars, this group cannot be logically coherent unless the high-metal component has been accreted (see below) or there exist some other drastic departures from standard ideas regarding stellar and galactic evolution. We propose that the name “BL Herculis” no longer be applied to the group as a whole. For the moment, they are better called “short-period type II Cepheids.” The subclass names proposed by Diethelm (1983) may also not be highly appropriate, particularly the confusing designation RRd (PD).

Similarly, the classifications employed in the current work are useful mainly as a shorthand description of various groupings. A definitive classification scheme must await further information.

It is clear that much more work needs to be done on this subject. The conclusion drawn in the present, and earlier, investigations have been based on a rather small sample of type II Cepheids. Further observations are necessary to improve the existing light curves and to obtain additional ones. Of particular interest in this connection would be CCD photometry of type II Cepheids in globular clusters. Spectroscopic and photometric studies of the type II field stars are also imperative to determine metal abundances and space velocities for a broader sample. Stars like KZ Cen, HQ Cra, and V745 Oph are especially important, since they have relatively high elevations above the disk (Harris 1985) and are thus obvious candidates for halo type objects among the II M stars.
Turning to the theoretical side, new hydrodynamic calculations must be now be undertaken with a range of masses and luminosities. The interpretations offered in the present study rely heavily on the runs of the Fourier phases with period ratio as determined from the calculations of HCK. These relationships need to be checked and refined with a more systematic grid of models. In addition, the insensitivity of the Fourier phases to artificial viscosity as found in RR Lyrae models (Simon and Aikawa 1986) needs to be confirmed for the type II Cepheid calculations.

Finally, we note that our present results underline the statement of Harris and Wallerstein (1984) that "in spite of our lack of understanding of the process, the old disk population of our galaxy does produce Cepheids." In our opinion, improving our knowledge of these stars is likely to add much to our picture of stellar evolution and to our understanding of the nature of the Galaxy.

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