1975

Rotational Dynamics

STUDY GUIDE

ROTATIONAL DYNAMICS

INTRODUCTION

A diver, in making several turns in the air, grabs his knees to achieve a high rate of rotation, and a skater does much the same thing when she goes into a spin with arms and legs extended but brings them in close to her body for the extremely rapid part of this motion. This module considers the physics describing these motions, and those of other rotational systems - starting or stopping a record turntable (or a washing-machine tub), unwinding of winch cord as a bucket is dropped into a well, etc.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*State Newton's second law for linear motion (needed for Objective 2 of this module)	Newton's Laws Module
*Analyze and solve problems involving conservation of energy (needed for Objective 2 of this module)	Conservation of Energy Module
*Relate work and energy (needed for Objective 3 of this module)	Work and Energy Module
*Define the center of mass (needed for Objective 1 of this module)	Impulse and Momentum Module
*Define angular momentum and relate angular momentum to torque for a point-mass particle (needed for Objective 4 of this module)	Rotational Motion Module
*Relate angle, angular velocity, and angular acceleration (needed for Objectives 2 and 3 of this module)	Rotational Motion Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Moment of inertia - (a) Apply the definition of moment of inertia, and the parallel-axis theorem where needed, to calculation of moments of inertia of simple extended bodies (not requiring integration), or demonstrate general understanding of the concept of moment of inertia by ranking several regular and irregular bodies according to their moments of inertia.

   (b) Write down the moments of inertia of a circular hoop (identical to that for an oil drum without ends), a circular disk (identical to that for a solid cylinder), and a long thin rod about an axis through the center of mass, and perpendicular to the plane of hoop or disk, or perpendicular to the length of the rod.

2. Rotation about fixed axis - In cases of rotation about a fixed axis, solve problems using Newton's second law of motion for rotation, or by using conservation of energy.

3. System of objects - For a system of objects rotating about a fixed axis where some of the following quantities are given, find others: moment of inertia, angular momentum, angular velocity, rotational kinetic energy, work, and power.

4. Conservation of angular momentum - For a system of objects rotating about a fixed axis, solve problems where angular momentum is conserved about some axis, but where angular velocity changes because the system changes size or shape; be able to recognize those groups of objects for which angular momentum will be conserved about a given axis.

GENERAL COMMENTS

You have seen conservation of energy in an earlier module (Conservation of Energy), but in this module it is broadened to include cases that have kinetic energy of rotation. Systems to which we may apply conservation of energy are those where only conservative forces act (gravity, spring, etc.) or else those where "other" forces are not doing any work on the system. Forces acting at the fixed axis of rotation of some body, for instance, are not acting through any distance, and thus do no work. When you get to Problems C and H, you will see that they each involve a body rotating about a fixed axis; thus energy conservation applies to their motions.

When you look for conservation of angular momentum, you must find a system of objects on which all torques that come from forces acting THROUGH the "plastic bag" around the system add vectorially to zero. You should study Problem E, which looks carefully at this situation.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

Read Chapter 11, Sections 11.3 to 11.7, for Objectives 1. In particular, study Figure 11.11. Study Problems A and B in the Problem Set, and work Problems F and G. For Objectives 2 and 3, read the text Chapter 12, Sections 12.1 through 12.8. Study Problems B, C, and D and work Problem H. Sections 12.6 to 12.8 also apply to Objective 4. Study Problem E and work Problems I, J, and K. If you need more practice, try the Additional Problems in the text.

Take the Practice Test before trying a Mastery Test.

BUECHE

Objective Number	Readings	Problems with Solutions: Study Guide	Problems with Solutions: Text*	Assigned Problems: Study Guide	Additional Problems
1	Secs. 11.3 to 11.7	A, B		F, G	Chap. 11, Probs. 5, 6, 7
2	Secs. 12.2, 12.8	B, C, E	Illus. 12.8	H, K	Chap. 11, Probs. 12, 13, 14; Chap. 12, Prob. 18
3	Secs. 12.1 to 12.3, 12.7	C, D, E,		H, J, K	Chap. 12, Probs. 1, 3, 4
4	Secs. 12.6 to 12.8	E		I, J	Chap. 12, Probs. 2, 16, 12

*Illus. = Illustration(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Chapter 11, Section 11-4 for background, and then Sections 11-5 through 11-7. Example 5 can be omitted, since this module does not require the calculation of rope tensions. Note also that the text has very few energy-conservation examples, which are called for in Objective 2. To prepare for Objective 1, study Problems A and B, then work Problems F and G. For Objectives 2 through 4, study Problems C, D, and E before working Problems F through K. If you need more practice, try the Additional Problems in the text.

Take the Practice Test before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions	Assigned Problems	Additional Problems
		Study Guide	Study Guide	(Chap. 11)
1	Sec. 11-5	A, B	F, G	Quest.* 4, 6; Probs. 15, 19(b)
2	Sec. 11-6	B, C, E	H, K	17, 18, 23, 46
3	Sec. 11-6	C, D, E	H, J, K	25, 27, 31, 32
4	Sec. 11-7	E	I, J	35, 37, 39, 41, 43, 45

*Quest. = Question(s).

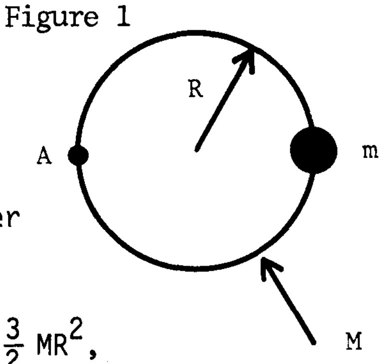
TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

Read Chapter 9, Sections 9-6 and 9-7 for Objectives 1 and 2. You may skim Section 9-7, but should study Section 9-6 more carefully, noting in particular Figure 9-13 on p. 137. Now read Sections 9-8 and 9-9 for Objectives 3 and 4. Study Problems A through E before working Problems F through K. Work some of the Additional Problems from the text if you are unsure of your mastery, before taking the Practice Test. Then try a Mastery Test.

Your text does not cover the parallel-axis theorem, which is called for in Objective 1. Briefly, the theorem states that if you know the moment of inertia I_A about axis A through the center of mass, you can calculate the moment of inertia about an axis B that is parallel to axis A and distant from it by D, using the formula $I_B = I_A + MD^2$, where M is the mass of the object. A text where this is proved and discussed is Fundamentals of Physics,* Chapter 11, Section 11-5. Note that Problem A is a good example of how to use this theorem, as is the following problem.

Figure 1



Problem: Calculate the moment of inertia about axis A of a cylinder (mass M) with a small mass m attached as shown in Figure 1.

Solution: Moments of inertia are additive:

$I_{MA} + I_{mA} = I_A$, $I_m A + m(2R)^2$, and since I for cylinder is the same as that for a disk, $I_{M(\text{about c.m.})} = MR^2/2$.

By the parallel-axis theorem, $I_{MA} = I_M + MR^2$, $I_{MA} = \frac{3}{2} MR^2$,

and the total moment of inertia is $I = (3/2)MR^2 + 4mR^2$.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions Study Guide	Assigned Problems Study Guide	Additional Problems
1	Sec. 9-7	A, B	F, G	9-15, 9-14(a), (b), 9-16, 9-18
2	Sec. 9-6	B, C, E	H, K	9-23, 9-24(a), 9-26(b), 9-34, 9-38, 9-39(a)
3	Secs. 9-8, 9-9	C, D, E	H, J, K	9-24(b), 9-29(b), (c), (d), 9-31, 9-35, 9-37, 9-39(b), (c), (d), 9-41, 9-43, 9-49
4	Sec. 9-9	E	I, J	9-41, 9-43, 9-49, 9-51

*David Halliday and Robert Resnick (Wiley, New York, 1970; revised printing, 1974).

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read Chapter 11, Sections 11-5 through 11-7 and Chapter 12, Sections 12-2 through 12-5. Study Figure 12-1. Note that Examples 12-5 to 12-8 involve more complex situations then called for in the objectives - either due to a moving axis of rotation or because Newton's second law for linear motion is used together with the second law for rotation.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions	Assigned Problems	Additional Problems
		Study Guide	Study Guide	
1	Secs. 12-2, 12-7	A, B	F, G	12-27(a)
2	Secs. 12-3, 12-4	B, C, E	H, K	12-14, 12-17, 12-26, 12-27(b), (c), 12-28
3	Secs. 12-3, 12-4	C, D, E	H, J, K	12-19, 12-27(e), (f)
4	Sec. 11-7	E	I, J	11-14, 11-18, 11-19, 11-23

PROBLEM SET WITH SOLUTIONS

A(1). Calculate the moment of inertia of four equal masses M at the corners of a square of side L as shown in Figure 2, with respect to axis A and also with respect to axis B, both axes perpendicular to the paper.

Solution

Axis A is at the center of mass:

$$I_A = \sum_{i=1}^{i=4} M_i r_{iA}^2 = M(r_{1A}^2 + r_{2A}^2 + r_{3A}^2 + r_{4A}^2),$$

where r_{1A} is the distance from axis A to first mass; r_{2A} is the distance from axis A to second mass, etc. Convince yourself that $r_{1A} = r_{2A} = r_{3A} = r_{4A} = L(\sqrt{2}/2)$, and $I_A = 2ML^2$.

To calculate I about axis B, we can use the parallel-axis theorem, or we can calculate

$$I = \sum_i M_i r_{iB}^2 = M(r_{1B}^2 + r_{2B}^2 + r_{3B}^3 + r_{4B}^2).$$

Using this last method first, we find that $r_{1B} = 0$, $r_{2B} = L$, $r_{3B} = L\sqrt{2}$, $r_{4B} = L$, and we get $I_B = M(0^2 + L^2 + 2L^2 + L^2) = 4ML^2$. Now, using the parallel-axis theorem, we have

$$I_B = I_A + (\text{total mass})(\text{distance between axes A and B})^2,$$

$$I = 2ML^2 + (4M)[L(\sqrt{2}/2)]^2 = 4ML^2,$$

and both methods agree.

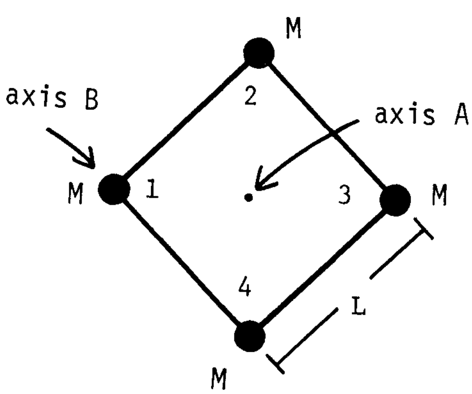


Figure 2

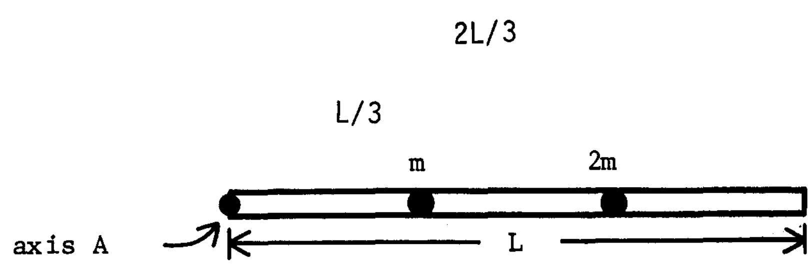


Figure 3

B(1,2). Consider a rod of mass M and length L, frictionlessly pivoted about axis A (out of the page) with masses m and 2m at L/3 and 2L/3 as shown in Figure 3. The moment of inertia of this rod about A is

$$I = ML^2/3 + mL^2 \quad \text{(The student should work this out!).}$$

Calculate the angular acceleration of the rod and masses if just released under gravity.

Solution

Calculate total torques about A:

$$\tau_1 = mg(L/3), \qquad \tau_2 = (2m)g(2L/3), \qquad \tau_3 = (MgL)/2,$$

$$\Sigma\tau_{ext} = +(MgL)/2 + (5/3)mgL = I\alpha = (ML^2/3 + mL^2)\alpha \quad \text{(into the paper),}$$

thus

$$\alpha = \frac{(5/3)mgL + MgL/2}{ML^2/3 + mL^2} = \left(\frac{g}{L}\right)\frac{5/3 + M/2m}{1 + M/3m} \quad \text{(into the paper).}$$

C(2,3). Calculate the angular velocity of the rod in Problem B when it reaches a vertical position, after having rotated through 90°.

Solution

We apply conservation of energy: $K_i + U_i = K_f + U_f$. We take a PE reference height equal to 0 where the rod starts, and we note that the center of mass of the rod will have a height at the end of -L/2, m will have a height -L/3, and 2m will have height -2L/3.

$$K_i = 0, \qquad U_i = 0, \qquad K_f = I\omega^2/2,$$

$$U_f = Mg(-L/2) + mg(-L/3) + 2m(-2L/3),$$

thus

$$\omega^2 = \frac{2}{I}(gL)\left(\frac{M}{2} + \frac{5m}{3}\right) = \frac{2gL}{L^2}\left(\frac{M/2 + (5/3)m}{m + M/3}\right),$$

$$\omega = \text{angular velocity} = \left(\frac{2g}{L}\right)^{1/2}\left(\frac{M/2 + (5/3)m}{m + M/3}\right)^{1/2} \quad \text{(into the paper).}$$

D(3,4). A disk of mass M and radius R rotates frictionlessly with angular speed ω_0 about an axis through its center. At its center is a cricket of mass m. Since the disk is isolated, no torques exist external to the (M + m) system.

(a) Calculate the final angular velocity ω_f, after the cricket jumps from the center to a point R/2 from the center as shown in Figure 4, where he holds fast, rotating with the disk.
(b) Calculate KE before and after. Why is it not the same?

Solution

(a) Angular momentum is conserved, thus

$$L = I_0\omega_0 = I\omega_f, \qquad I_0 = MR^2/2, \qquad I = MR^2/2 + m(R/2)^2$$

and

$$\omega_f = \omega_0\left(\frac{I_0}{I}\right) = \omega_0\left(\frac{MR^2}{MR^2 + m(R^2/2)}\right) = \frac{\omega_0}{1 + m/2M} \qquad \text{(in same direction as } \omega_0\text{).}$$

(b) $K = I\omega^2/2$, $L = I\omega = I_0\omega_0$, thus

$$K_f = L^2/2I, \qquad K_i = L^2/2I_0$$

The L is the same, but I's are different. Thus $K_f < K_i$. We may regard the cricket's landing as an inelastic collision with the disk, in which some energy was converted into heat.

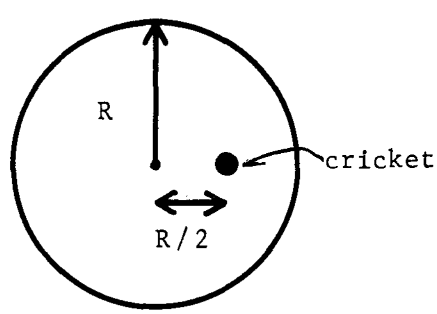


Figure 4

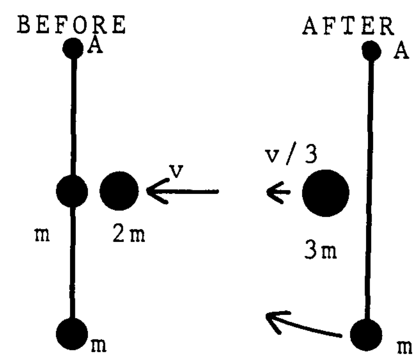


Figure 5

E(2,3,4). A massless rod of length 2L is free to rotate about axis A as shown in Figure 5, with two masses m attached. Along comes mass 2m and "captures" one of the masses as shown. Find the height to which the remaining mass will rise.

Solution

First we notice that angular momentum about A is conserved for the system of (rod + all three masses) before and after the collision. This is because the torques that act on the system (rod + three masses) just before and after the collision are zero about A. There are four forces external to the system: gravity acting on the three masses and the force of the support on the rod at A. Every one of these forces exerts zero torque about point A, because all forces pass through A. Notice that once the collision is over and the (rod +

mass) starts to rotate, there will be a torque from the gravity on the mass, and angular momentum of (rod + mass) will not be conserved after the collision. Conserving angular momentum about A we get

$$2mvL = (3mv/3)L + I\omega, \quad I\omega = mvL = \omega[m(2L)^2], \quad \omega = v/4L.$$

Energy is not conserved in this collision (inelastic collision), as the student may check ($mv^2 \neq mv^2/6 + mv^2/8$). Mechanical energy <u>will</u> be conserved, though, for the (rod + mass) system after the collision. No further collisions are involved, gravity is a conservative force, and the force on the rod at the pivot does no work:

$$K_i + U_i = K_f + U_f, \quad I\omega^2/2 + 0 = 0 + mgh,$$

$$h = \left(\frac{m(4L^2)}{2}\right)\left(\frac{v}{4L}\right)^2\left(\frac{1}{mg}\right) = \frac{v^2}{8g}.$$

<u>Problems</u>

F(1). Calculate the moment of inertia of three masses at the corners of an equilateral triangle of side L about axes A and B, perpendicular to the page, see Figure 6.

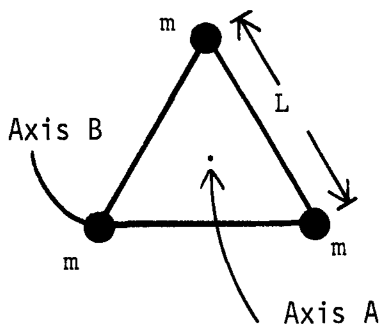


Figure 6

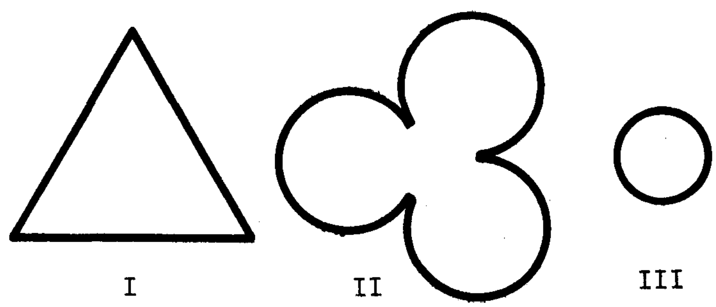


Figure 7

G(1). Put the masses of Figure 7 in order of decreasing moments of inertia about an axis perpendicular to the paper through the center of mass. All systems (I, II and III) have the same mass and uniform density.

H(2,3). Calculate the angular velocity of the disk in Figure 8 in position 2 released from the position 1.

G(1). Put the masses of Figure 7 in order of decreasing moments of inertia about an axis perpendicular to the paper through the center of mass. All systems (I, II and III) have the same mass and uniform density.

H(2,3). The uniform disk of mass m and radius R is free to roll along a vertical surface with no slippage. At point A, a concentrated mass of m is placed on the rim of the disk. If the disk is released from position 1, derive an expression which gives the angular velocity of the disk when in position 2.

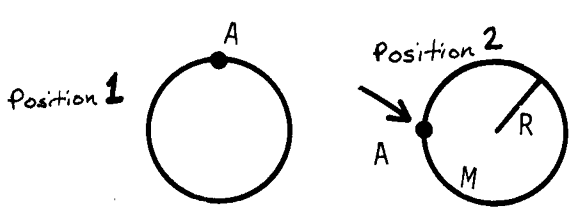


Figure 8

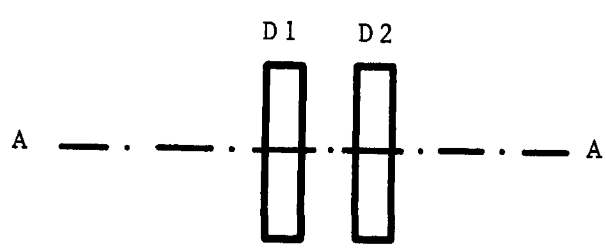


Figure 9

I(4). Consider disk D1 in Figure 9 of mass M and radius R rotating with angular velocity ω_0 about axis AA′, while disk D2 is not in motion. (Its mass is 2M and its radius is R.) Suppose that disks D1 and D2 suddenly become attached without external intervention. Calculate the final common angular velocity of the combined disks.

J(3,4). Suppose two 60-kg skaters are going in a circle, with angular speed 1.00 rad/s, each holding on to a 2.00-m (massless) broom handle. Combined, they are able to "power" themselves at the rate of 400 W. How long will it take them to pull on the broom and get within 1.00 m of each other?

K(2,3). Block A (mass = 10.0 kg) rests on a frictionless surface in Figure 10. The pulley is a disk of mass 20.0 kg, radius 0.50 m, and is free to rotate without friction about an axis at its center. A massless rope that is tied from block A passes without slipping over the pulley, and is tied to block B (mass = 30.0 kg). This system is released from rest. Find the angular velocity of the pulley disk when the block A has moved 4.0 m. Hint: The new element in this problem is the mixing of angular motion of the disk with the linear motion of the blocks. The velocity of block A must also be the velocity of the rope. The rope must have the same speed as the tangential velocity of the pulley where they meet (no slipping); this gives a relation between ω of the pulley and V of the block ($\omega = V/R$). Use energy conservation and solve for ω, after eliminating V.

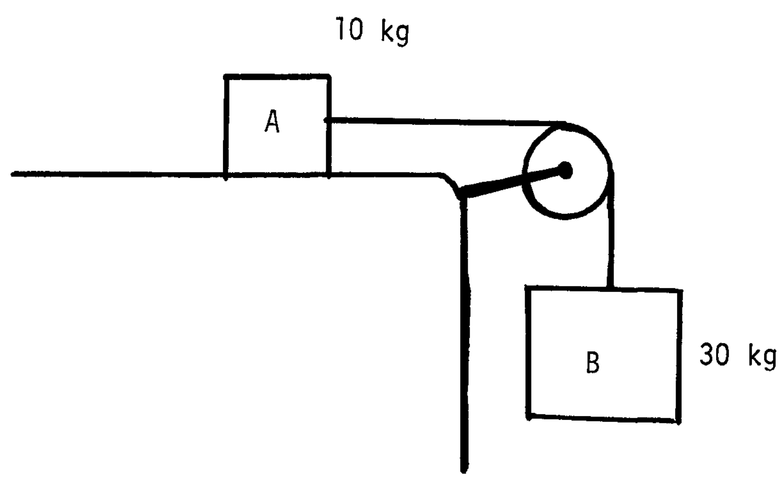


Figure 10

Solutions

F(1). $I_A = ML^2$; $I_B = 2ML^2$.

G(1). In each case, we estimate the position of the center of mass, and judge in which case there is more mass "away from the center": for which one is $\Sigma_i M_i r_i^2$ the largest. In Figure 7, Case II clearly has most mass furthest from the center of mass, and Case III has the least. Therefore, $I_{II} > I_I > I_{III}$.

H(2,3). $\omega = \sqrt{(4/3)(g/R)}$ (into the paper).

I(4). $\omega_f = \omega_0/3$.

J(3,4). Angular momentum is conserved:

$$I_0\omega_0 = I_1\omega_1 = L, \quad I_0 = 2(60\text{ kg})(1\text{ m})^2, \quad I_1 = 2(60\text{ kg})(1/2\text{ m})^2,$$

$$\text{Work needed} = \Delta KE = \frac{I_1\omega_1^2}{2} - \frac{I_0\omega_0^2}{2} = \frac{L^2}{2I_1} - \frac{L^2}{2I_0} = (KE)_0\left(\frac{I_0}{I_1} - 1\right) = \frac{I_0\omega_0^2}{2}\left(\frac{I_0}{I_1} - 1\right)$$

$$= \frac{60\text{ kg} \times 2 \times 1.00\text{ m}^2}{2} \times (1.00\text{ rad/s})^2 \left(\frac{r_0^2}{r_1^2} - 1\right) = (60\text{ J})\left(\frac{1^2}{(1/2)^2} - 1\right)$$

$$= 180\text{ J}.$$

$$t = \frac{\text{work}}{\text{power}} = \frac{180\text{ J}}{400\text{ W}} = \frac{1.80}{4}\text{ s} = 0.45\text{ s}.$$

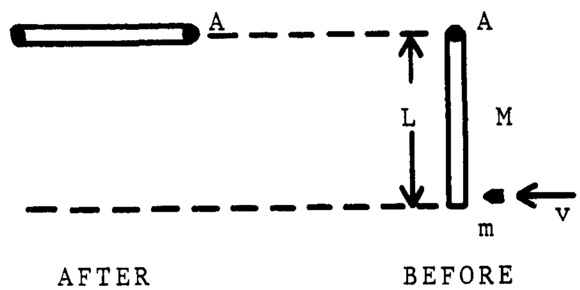
K(2,3). 13.7 rad/s.

PRACTICE TEST

1. A rod of mass M and length L is suspended to rotate freely about axis A coming out of the page. It is struck by a bullet of mass m and speed v, which becomes imbedded in the rod after impact. The rod-plus-bullet has rotated but is brought to rest after 90° of rotation by gravity as shown in Figure 11. (a) What is the angular velocity of the rod-plus-bullet just after the bullet is imbedded? (b) What value must the bullet speed have for this to happen?

2. A uniform disk of radius 0.200 m and mass 5.0 kg is rotating at an angular speed of 2.50 rad/s about a fixed axis through its center. It is brought to rest in 5.0 s by a uniform torque. Find the value of this torque.

Figure 11



Practice Test Answers

1. (a) Conserve angular momentum about axis A: $mvL = I\omega = (ML^2/3 + mL^2)\omega$.

$$\omega = \frac{mvL}{ML^2/3 + mL^2} = \frac{v}{L}\left(\frac{1}{1 + M/3m}\right).$$

(b) Conserve energy after collision:

$$mgh + Mg\frac{h}{2} = \frac{I\omega^2}{2} = \left(\frac{ML^2/3 + mL^2}{2}\right)\left[\frac{v}{L}\left(\frac{1}{1 + M/3m}\right)\right]^2,$$

$$v^2 = 2gh(1 + M/2m)(1 + M/3m), \qquad v = \sqrt{2gh(1 + M/2m)(1 + M/3m)}.$$

2. $\omega - \omega_0 = \alpha t, \qquad \alpha = \frac{2.50\ \text{rad/s}}{5.0\ \text{s}} = 0.50\ \text{rad/s}^2,$

$I = mr^2/2 = (5.0\ \text{kg})(0.200\ \text{m})^2/2 = 0.100\ \text{kg m}^2.$

$\tau = I\alpha = (0.100\ \text{kg m}^2)(0.50\ \text{rad/s}^2) = 0.050\ \text{N m}.$

Date________________________

ROTATIONAL DYNAMICS

pass recycle

Mastery Test Form A

1 2 3 4

Name________________________ Tutor________________________

1. A thin cord is wrapped around a disk of mass 10.0 kg and radius 0.100 m, which is free to rotate about a fixed axis at its center. See Figure 1. The end of this cord is tied to a 10.0-kg mass that is released from rest and travels 0.50 m vertically. At this point, what is the angular velocity of the disk?

2. A merry-go-round of radius 4.0 m and mass 220 kg in the shape of a disk is rotating at 1.00 rad/s. A 70-kg man standing next to the merry-go-round runs directly toward the center at 3.00 m/s, jumps and lands on the edge, turning with the merry-go-round after he lands. What is the angular speed of the merry-go-round after this maneuver?

3. In Figure 2 there are three objects of equal mass and uniform density. Number them 1, 2, 3, assigning 1 to the object with the largest moment of inertia, 2 to the middle, and 3 to the object with the smallest moment of inertia about an axis through the center of mass and perpendicular to the paper.

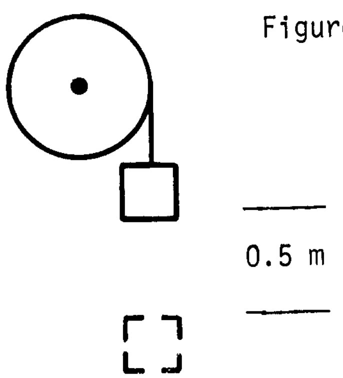


Figure 1

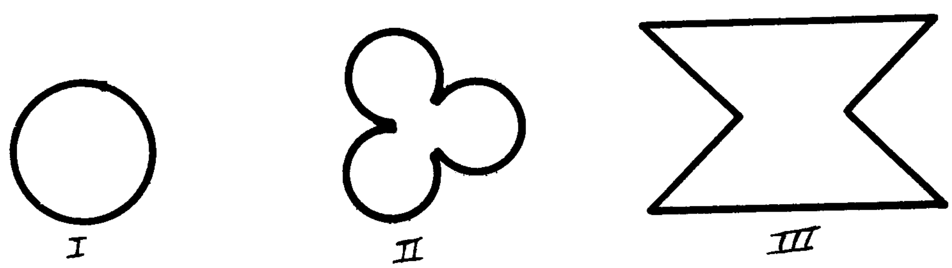


Figure 2

ROTATIONAL DYNAMICS

Mastery Test   Form B

Date______________________

pass   recycle
1   2   3   4

Name______________________________

Tutor______________________

1. (a) While running through the forest at 12.0 m/s chased by a rampaging rhinocerous, Tarzan grabs on to a tree limb dangling by a strong bark strip at its upper end (see Fig. 1). If the limb has a mass of 50 kg, and a length of 5.0 m, what is the angular velocity just after Tarzan grabs on? Treat the limb as a uniform rod. Tarzan's mass is 80 kg.

   (b) How high does Tarzan swing if the bark holds?

2. In Figure 2 there are three objects of equal mass and uniform density. Number them 1, 2, 3, assigning 1 to the object with the largest moment of inertia, 2 to the middle, and 3 to the object with the smallest moment of inertia about an axis through the center of mass and perpendicular to the paper.

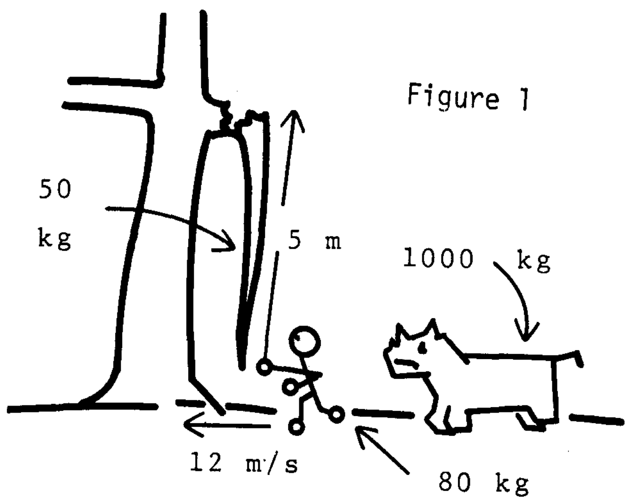
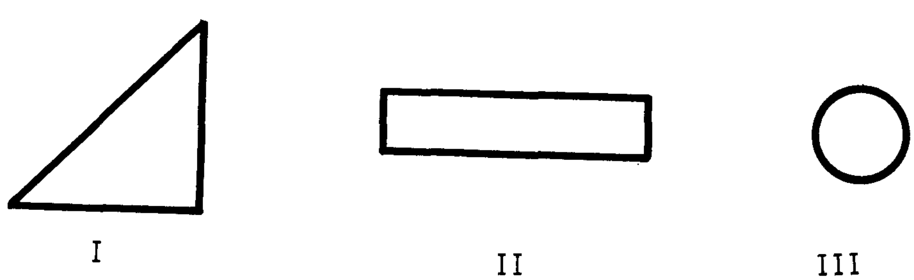


Figure 1

Figure 2



ROTATIONAL DYNAMICS

Mastery Test   Form C

Date____________________

pass   recycle

1   2   3   4

Name______________________________   Tutor____________________

1. Two uniform bars (one bar having mass M and the other having mass 2M) of length L are located side by side and each is pivoted about one end, as shown in Figure 1. The two are initially at rest in a horizontal orientation and released in such a way that they reach the vertical simultaneously.

   (a) Find the angular <u>velocity</u> of each bar immediately before impact.

   (b) Find the angular <u>momentum</u> of each bar immediately before impact.

   (c) After impact, the 2M bar rebounds at one-third of its preimpact angular velocity. What is the angular velocity of the M bar immediately after impact?

   (d) Compare the kinetic energy of the system immediately before collision with the kinetic energy immediately after collision.

2. In Figure 2 there are three objects of <u>equal mass</u> and uniform density. Number them 1, 2, 3, assigning 1 to the object with the largest moment of inertia, 2 to the middle, and 3 to the object with the smallest moment of inertia about an axis through the center of mass and perpendicular to the paper.

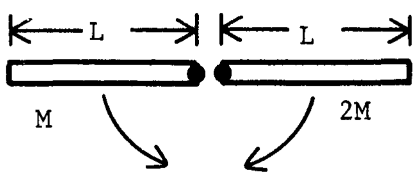


Figure 1

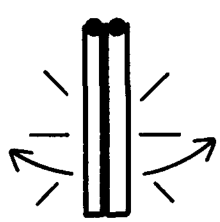
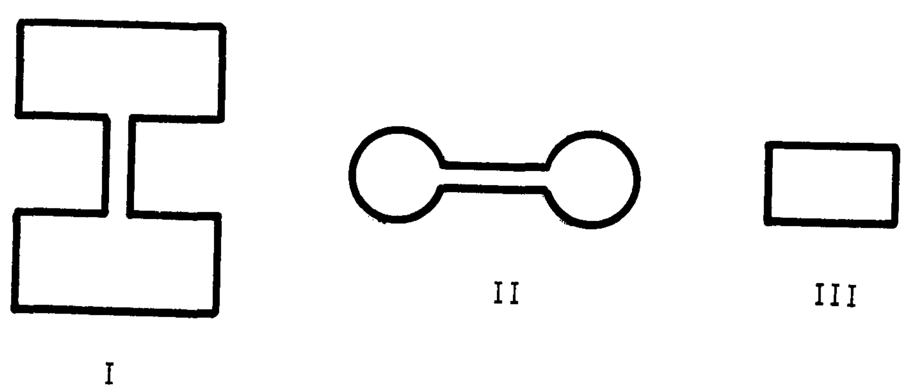


Figure 2



ROTATIONAL DYNAMICS

Date______________________

pass recycle

Mastery Test Form D

1 2 3 4

Name______________________________ Tutor______________________

1. An 80-kg astronaut is in a rotating simulator (mass of 40 kg concentrated at outer rim) with a frictionless center axle. The simulator is rotating at 1.00 rad/s. See Figure 1.

   (a) An alarm bell rings, and the astronaut is told he has 80 s to reach the escape door at the center of the simulator. If his power output is 200 W, will he escape in time? Start by calculating initial and final moments of inertia.

   (b) The motor used to start the simulator spinning delivers a constant torque of 100 N m. What is the angular acceleration? With the astronaut inside, at the outer edge, how long does it take to reach the speed above?

2. In Figure 2 there are three objects of <u>equal mass</u> and uniform density. Number them 1, 2, 3, assigning 1 to the object with the largest moment of inertia, 2 to the middle, and 3 to the object with the smallest moment of inertia about an axis through the center of mass and perpendicular to the paper.

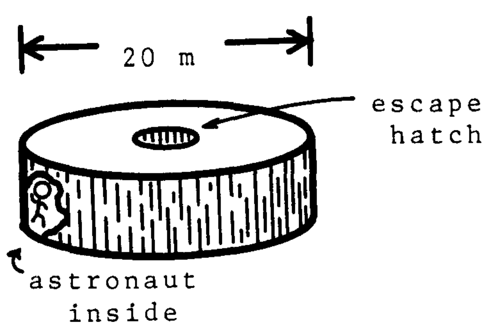
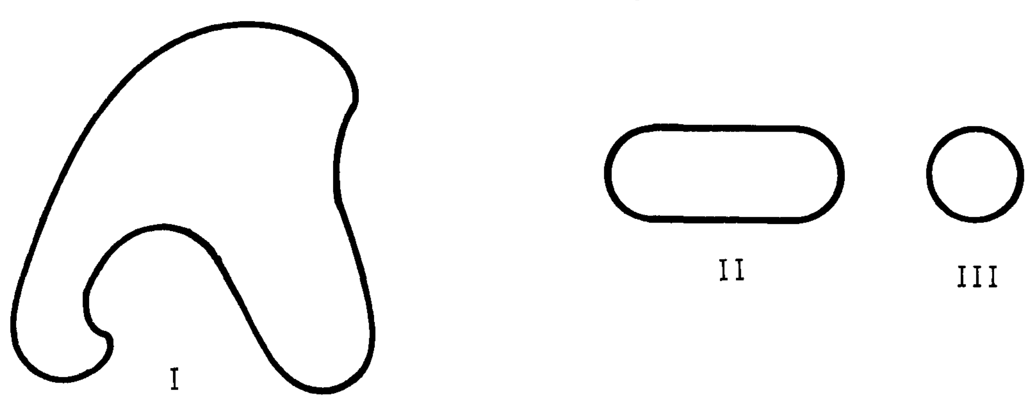


Figure 1

Figure 2



Date________________________

ROTATIONAL DYNAMICS

pass recycle

Mastery Test Form E

1 2 3 4

Name________________________ Tutor________________________

1. A thin cord is passed over a disk to mass 20.0 kg and radius 0.100 m, which is free to rotate about a fixed axis at its center. See Figure 1. One end of this cord is tied to 10.0-kg mass and 5 kg mass is tied to the other. The masses are released from rest and travel 0.50 m vertically. At this point, what is the angular velocity on the disk?

2. A merry-go-round of radius 5.0 m and mass 1000 kg in the shape of a disk is rotating at 1.00 rad/s with a 50-kg man standing at the edge. If he runs directly toward the center at 3.00 m/s. What is the angular speed of the merry-go-round after he reaches the center.

3. In Figure 2 there are three objects of equal mass and uniform density. Number them 1, 2, 3, assigning 1 to the object with the largest moment of inertia, 2 to the middle, and 3 to the object with the smallest moment of inertia about an axis through the center of mass and perpendicular to the paper.

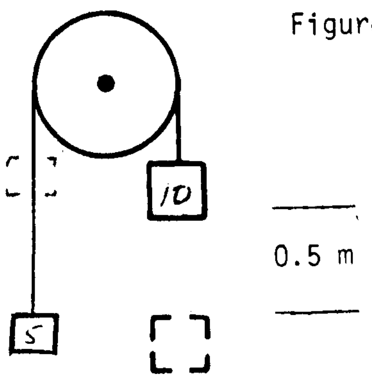


Figure 1

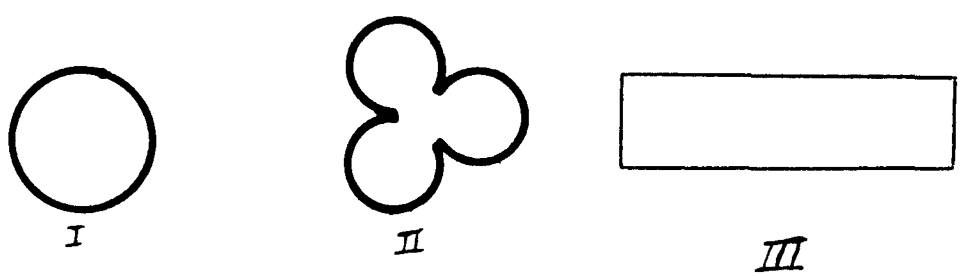


Figure 2

Courtesy of University of Missouri-Rolla

ROTATIONAL DYNAMICS

Mastery Test Form F

Date________________

pass recycle

1 2 3 4

Name______________________

Tutor________________

1. Two children of equal mass, m=40 kg are swinging in adjacent swings. The swing ropes are 5 m long. Jane's swing reaches a maximum height of 2 m above its lowest point. Billy's swing reaches a maximum height of 1 m above its lowest point. Neglect the mass of the swing.

   a) Calculate the angular velocity of each child - swing at the low point.

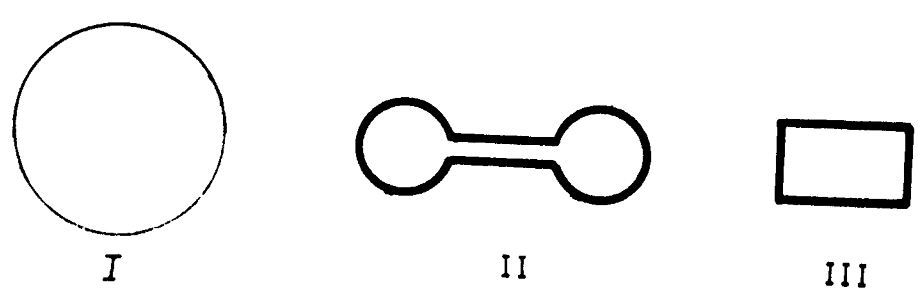
   b) Assume the swings are out of phase and as the children pass through the center point they lock arms. What is the angular momentum of the children after locking arms?

   c) How much energy was lost in the "collision"?

2. A man wishes to start a large disk of 2 m radius and 1000 kg mas turning by pulling on a rope wrapped around the disk. If the rope is sufficiently long with what force must he pull to accelerate the disk to 10 rad/s in 100 seconds.

3. In Figure 1 there are three objects of equal mass and uniform density. Number them 1,2,3, assigning 1 to the object with the largest moment of inertia, 2 to the middle, and 3 to the object with the smallest moment of inertia about an axis through the center of mass perpendicular to the paper.

Figure 1



MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
	1. $(KE + PE)_{start} = (KE + PE)_{end}$. Take reference for gravitational PE at final position of falling mass:

$$0 + mgh = mv^2/2 + I\omega^2/2 + 0,$$

$$I = mR^2/2, \quad \omega = v/R, \quad mgh = (3/4)\,MR^2\omega^2,$$

$$\omega = \left(\frac{4}{3}\,\frac{gh}{R^2}\right)^{1/2} = \left(\frac{4}{3}\,\frac{(9.8\ \text{m/s}^2)(0.50\ \text{m})}{(0.100\ \text{m})^2}\right)^{1/2}$$

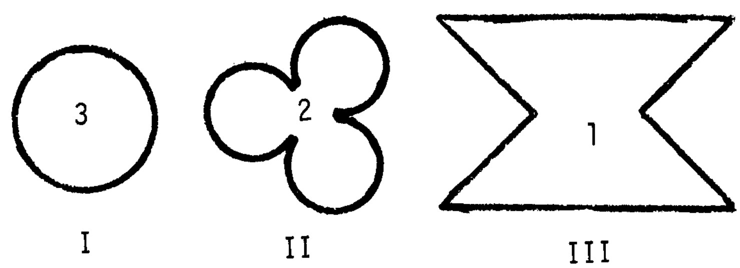
$$= 25.6\ \text{rad/s} \quad \text{(into the paper).}$$

What To Look For	Solutions
2. Note that radial velocity of 3.00 m/s has no effect on the problem.	2. $I_{empty} = MR^2/2$, $I_{with\ man} = MR^2/2 + MR^2$. Conserve angular momentum:

$$\omega_0 R^2/2 = \omega_{final}(MR^2/2 + mR^2),$$

$$\omega_f = \omega_0\left(\frac{1}{1 + 2m/M}\right) = \frac{220}{360} = 0.61\ \text{rad/s}.$$

What To Look For	Solutions
3. Object III is clearly the greatest. Ask student to explain reasoning if II and I are interchanged.	3.



MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
	1. (a) Conserve angular momentum about point where tree limb swings for system of (Tarzan + limb): M_T = Tarzan mass, M_L = mass of limb, L = length of limb.

$M_T V_T L = I\omega$, $I = M_L(L^2/3) + M_T L^2$,

$$\omega = \frac{M_T V_T L}{M_L L^2/3 + M_T L^2} = \frac{V_T}{L}\left(\frac{1}{1 + M_L/3M_T}\right) \simeq 1.47 \text{ rad/s}$$

(b) Tarzan gains height h and tree limb center of mass gains h/2. Conservation of energy gives us

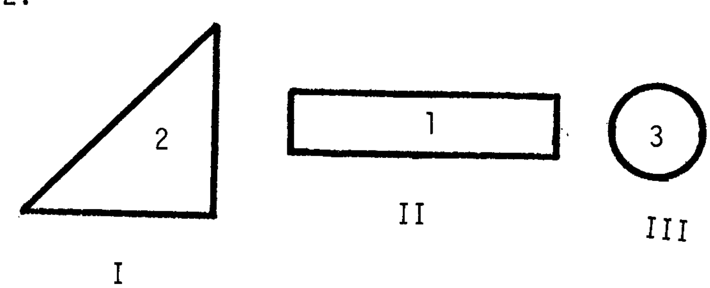
$$M_T gh + M_L g\frac{h}{2} = \frac{I}{2}\left(\frac{M_T V_T L}{I}\right)^2,$$

$$h = \left(\frac{V_T^2}{2g}\right)\frac{1}{(1 + M_L/3M_T)(1 + M_L/2M_T)}$$

$$h = \frac{(12.0 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)(1 + 5.0/24)(1 + 5.0/16.0)} = 4.6 \text{ m.}$$

2. Object III is clearly the smallest. Ask student to explain if Objects I and II are interchanged.

2.



MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions

What To Look For

1. Ask student about $\vec{\omega}$, $\vec{\ell}$ directions, if he doesn't show them.

Solutions

1. (a) $\frac{I\omega^2}{2} = mg(\frac{L}{2}) = (\frac{ML^2}{3})(\frac{\omega^2}{2})$, $\omega_1^2 = 3(\frac{g}{L})$ for each bar.

ω_1 into paper for 2M bar, out of paper for M bar.

(b) Angular momentum = $I\omega$. M bar has angular momentum $(ML^2/3)(3g/L)^{1/2}$ out of the paper and 2M bar has twice this, into the paper.

(c) $\ell_{before} = (\frac{mL^2}{3})(\frac{3g}{L})^{1/2}$,

$\ell_{after} = -(\frac{2}{3})(\frac{ML^2}{3})(\frac{3g}{L})^{1/2} + \ell_{bar}$

$\ell_{before} = \ell_{after}$, thus $\ell_{bar} = (\frac{5}{3})(\frac{ML^2}{3})(\frac{3g}{L})^{1/2}$

$= I_M\omega_M$;

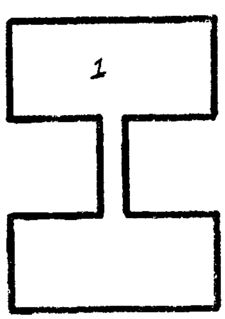
$\omega_M = (5/3)\omega_1$ into the paper.

$$KE_{before} = \frac{I_1\omega_1^2}{2} + \frac{2I_1\omega_1^2}{2} = \frac{3I_1\omega_1^2}{2},$$

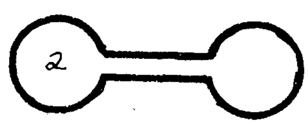
$$KE_{after} = \frac{1}{2}[I_1(\frac{5}{3}\,\omega_1)^2 + 2I_1(\frac{\omega_1}{3})^2] = \frac{27}{18}\,I_1\omega_1^2,$$

Since the kinetic energies are the same, the collision is elastic.

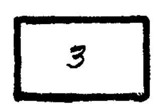
2. I = 1, II = 2, III = 3.



I



II



III

MASTERY TEST GRADING KEY - Form D

What To Look For	Solutions

1. (a) $I_0 = (10 \times 10^2 + 80 \times 10^2)$ kg m^2 = 12 000 kg m^2,

I_1 = 4000 kg m^2.

Angular-momentum conservation gives $I_1\omega_1 = I_0\omega_0 = L$.

ω_0 = 1.00 rad/s, L = 12 000 kg^2 m/s.

$$KE_{final} - KE_{initial} = \frac{L^2}{2I_1} - \frac{L^2}{2I_0} = \frac{L^2}{2I_0}\left(\frac{I_0}{I_1} - 1\right).$$

$$= KE_{initial}\left(\frac{I_0}{I_1} - 1\right).$$

Work needed = ΔKE = (6000 J)(12 000/4000 - 1).

To expend 12 000 J in 80 s we need

12 000 J/80 s = 150 W, thus astronaut will escape.

At 200 W he escapes in 60 s.

(b) $$\alpha = \frac{\tau}{I} = \frac{100 \text{ N m}}{(80 \text{ kg} + 40 \text{ kg})(10.0 \text{ m})^2} = \frac{1}{120} \text{ rad/s}^2,$$

$$\tau = \frac{\omega}{\alpha} = \frac{1.00 \text{ rad/s}}{1/120 \text{ rad/s}^2} = 120 \text{ s}.$$

2. I = 1, II = 2, III = 3.