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Toni Scusa
Yuma, CO

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Five Processes of Mathematical Thinking

Toni Scusa
Yuma, CO *

Math in the Middle Institute Partnership
Action Research Project Report

in partial fulfillment of the MA Degree
Department of Teaching, Learning, and Teacher Education
University of Nebraska-Lincoln
July 2008

* I began the program as a fifth grade teacher at Paxton, Nebraska but have since moved to Colorado
Five Processes of Mathematical Thinking

Abstract

My research project was to investigate key processes of mathematical thinking in my seventh grade mathematics classroom. Its purpose was to see whether I could affect the quality of student mathematical thinking and solution writing by teaching students five key processes of mathematical thinking I had identified, and by providing students with opportunities to evaluate sample student solutions using traits describing these processes. Every two weeks, students attempted solutions for a given problem and rated their work according to the specific characteristics identified as key to mathematical thinking. Every other week I gave the class sample student work at varied proficiency levels to rate according to a rubric and they discussed or defended their decisions. I found that student reasoning, whether written or oral, did improve over time as we emphasized these processes, although the change was slow and in small steps. Student engagement was also affected by the time we spent working in large or small group activities. The change, however, did not occur without an investment of substantial effort and time on my part and the students’. Learning about specific processes to emulate, model and then use to evaluate another’s work is an in-depth task that does not happen quickly or easily.
INTRODUCTION

My problem of practice was to pursue the idea of traits of good mathematical thinking based on the five process standards. I teach at a school with a 70% Hispanic population with about half of the population qualifying for free or reduced lunch and a high mobility rate. Last year there was about a 26% gap between those considered minority or low income who achieved a proficient or advanced rating on their Colorado State Assessment and those students classified as white or non poverty level who were proficient or advanced in Math. Since I began participating in the Math in the Middle Institute, I had been trying weekly problem solving activities in my classroom chosen to focus on developing good mathematical Habits of the Mind and had been toying with the idea that I needed to model for students how to approach the problem solving of a Habits-of-Mind type problem. The students I have in seventh and eighth grade had poor reasoning skills overall, and needed to develop good problem solving behaviors and familiarity with different formats of representation. When given a problem solving activity, most students truly had no idea of where to start.

I have been an elementary teacher for more than 20 years and during that time have had to become familiar with Six Traits Writing (now called Six Traits Plus One and developed by Northwest Regional Educational Laboratory). With that program, students are taught the key traits of good writing: Voice, Word Choice, Fluency, Organization, Ideas, Presentation and Conventions. Students are taught to look for these traits in other students’ writing and then to evaluate them in their own writing using a set of pre-made rubrics. I wondered if there might be a list of key traits for mathematical thinking, similar to those identified for writing. Could the processes of mathematical thinking be taught? Was this focus missing from my teaching? How could I model the kind of process that I had seen as a Math in the Middle participant so that my
students could experience the same kind of learning? I wanted my students to be exposed to the kind of problem solving activities we had in Math in the Middle and to experience the struggle of figuring out ways to reason, prove and solve as we had. What would be the best way to achieve this in the middle school classroom?

I decided to develop a rubric of those traits or particulars for mathematical reasoning similar to what is available presently for teaching and assessing Six Traits Writing. I then used this list of characteristics and rubrics to work on more difficult problems focused on mathematical habits of thinking with my class. We used the rubric as a class to discuss and assess student sample answers and in the evaluation of sample individual work. In the process I modeled what it takes to make good mathematical thinking. I provided ideas of how a solution could be changed to improve the attempt.

I had not assigned many harder problems yet in the year that would qualify as a problem specifically chosen to help students work on mathematical habits of the mind. Frankly the students I had were not in a place where they could do even the simplest type of word problem. Their biggest challenge was they had no idea of even where to begin. I felt as if they had somewhere learned it was the ANSWER that was the most important part, and everything else was static. I wanted students to see that the struggle has value. I wanted them to be able to work at a more independent level and not have as their key strategy - ask Ms. Scusa for help.

I wanted to have a classroom with students of all ability levels who would have the confidence to try something new or different. They would look forward to challenging mathematical problems. They would want to learn new and better strategies and would be anxious to hear from others about alternative strategies. They would “see” the big picture and understand the value in attempting to solve problems, but I needed to make it achievable. We
would have a lot of work in groups and lots of discussion time. I would need to use many examples and be consistent in modeling, since this type of Math work would not be something they were used to.

I came up with the idea of focusing on specific processes of mathematical thinking—practices I had decided my students needed to be successful mathematics students. I decided to try to teach these practices to students, and I figured out the characteristics that each of these embodied. They were based on five key areas 1) Representation, 2) Reasoning and Proof, 3) Communication, 4) Problem Solving, and 5) Connections. If these look familiar, it is because they are the five process standards from the National Council of Teachers of Mathematics (NCTM, 2000). It was my thinking that each of these process standards from NCTM had a specific set of behaviors that one could use to characterize each. My task was to make such a list for each of these five process standards and develop problem solving activities that afforded students opportunities to work on each.

My research project focused on teaching the students these specific processes and what they looked like. We spent time learning about each process and its identifying traits. We spent time evaluating student work based on this list, discussing the work’s merits, and then worked on improving our own abilities to achieve good mathematical solutions with these characteristics in mind.

**PROBLEM STATEMENT**

**Problem of Practice**

Many teachers in my Math in the Middle sessions had commented on the thinking ability of their students and the students’ abilities to apply what they knew to a problem solving situation. Members of my cohort talked about the difference between a product skill, such as
knowing multiplication facts, and a process skill, such as providing reasoning and proof. From this discussion, we decided that we would like to know more about teaching the second type of skill in the classroom.

Reading TIMSS (the Third International Mathematics and Science Study) and other studies comparing U.S. classrooms with classrooms in other countries, told my cohort members and me what we knew in our hearts already—we wanted to provide in our classrooms something beyond providing practice on rote skills and memorization. Many of us did not know how to begin. I wondered if I could come up with a list of specific behaviors for each of the five process standards and then create some sort of a system or “vehicle” that could be used to teach these process skills in the mathematics classroom. Could it be done in such a way that it could be replicated from year to year with consistency? What kind of support would my students need? How would I go about making it easy to understand and imitate?

I hoped that teaching specific strategies of problem solving to my students would not only increase student confidence as they learned to work problems and we worked through the steps of reasoning, but would also require higher level thinking and have real world applications. I believed students who reasoned and solved problems were much better equipped to function in today’s society than those who did not have this practice.

I also believed establishing clear cut behaviors of mathematical thinking, modeling the process and learning to evaluate sample work helped to equate students of differing levels of ability as ALL learn the steps to better reasoning and problem solving skills. I thought there must be a better way to improve problem solving and reasoning than by merely providing more practice doing problem solving and reasoning. I asked myself, “What happens in the real world?” If I were a coach who wanted better basketball players, for example, I would break basketball
down into a set of skills I wanted my players to learn—dribbling, shooting, passing, defending etc… and then we would practice and practice. I would help players by evaluating weaknesses and strengths myself and help them to assess themselves. I would teach them what to do in different situations to use those skills. I would not just put out some basketballs and then say to the group, “Go, get better at basketball.” Just providing the basketballs helps those who are already skilled by providing the time for them to get better, but it does not help the ones who need to address specific lack of skills. Not dividing problem solving into a set of skills we could practice would be the same for math class. More practice without attention to improving skill would only help those who already were skilled. I did not want a classroom where the strong got stronger and the rest did not have a clue of how to become better. The gap between those with mathematical skill and those who did not have this ability would only widen.

As a mathematics teacher, it was important for me to identify and clearly communicate the expectations I had for the classroom. Creating a list of traits and a rubric helped me state and communicate my expectations with respect to high quality reasoning for ALL students. My students had multiple opportunities to discuss what makes good reasoning and were able to view reasoning through modeling techniques. By incorporating all of these various strategies centered on the traits of good mathematical reasoning, I believed good mathematical thinkers would emerge in my classroom.

**Research Questions**

The purpose of my research then, was to determine if one could teach mathematical thinking in a systematic manner. I taught my students the five process standards (the “Processes of a Mathematical Thinker”) and saw what would happen when mathematical thinking was taught in a structured way that showed students how to evaluate their work and the work of
others. I examined the use of a rubric to identify the characteristics of mathematical thinking and whether effective rubric use would influence the quality of student reasoning and engagement in problem solving situations. I sought to answer the following research questions:

- What will happen to the quality of student written reasoning when students use a rubric to evaluate their work?
- What will happen to the level of student engagement in small group discussions when using the five traits of mathematical thinking to solve problems?
- What will happen to the quality of student oral explanations of solutions when using the traits of a mathematical thinker to guide student solutions?
- What will happen to my teaching when I specifically set aside time to teach traits of mathematical thinking and deliberately spend time on mathematical discussion and reasoning?

**LITERATURE REVIEW**

I looked at the literature for current trends and research. What did it say? Could one teach mathematical thinking? What were the keys to mathematical thinking? I believed that if I determined the answers to questions like these, I could use problem solving to teach and develop successful mathematics students.

In the research examined within the United States and other countries, problem solving was being used as both a means and an end result. In the past ten to twenty years, the trend in problem solving had been similar. It had been to concentrate on developing mathematic skill and not just arithmetic skill by developing or emphasizing problem solving. It was a difference I had heard before amongst colleagues - - teaching math could be divided into two realms - - teaching
students process and product, or in other words, the teaching of arithmetic vs. teaching of mathematics. I wondered if it was possible to do both.

All articles I researched for the six countries of the United States, China, Singapore, Australia, Japan, and Portugal mentioned problem solving as a major focus of the country’s mathematics emphasis with concentration on development of higher level thinking skills for at least the past ten years, if not longer. The difference from country to country was in the curriculum and textbooks used, and in the degree of influence the teacher had over the change.

The emphasis on problem solving has meant in some cases a change in teaching strategies, classroom atmosphere, and/or a change in the role of the student and teacher. In any case, it has meant using problem solving in the classroom to achieve problem solving success. It has meant using problem solving to teach mathematics while at the same time helping students learn how to problem solve. This emphasis has been as much about the process as about the product. The big question seems to have been—how does one teach the process of problem solving?

I wondered as I read what the research literature would suggest. Was there a need to create in students certain procedures in order for them to be successful mathematicians? Would I be teaching students the qualities of good mathematical thinking by using these? Could these processes be grouped? What key identifiers could be listed under each? Would promoting these skills also promote higher level thinking and reasoning?

When I identified this list, I planned to focus my research project on teaching these significant processes and reaching higher level thinking skills by helping students learn about solving problems and the traits of a mathematical thinker while actively problem solving. By using what characteristics I could find in common amongst the literature, I thought I could teach
the keys to mathematical thinking and use these characteristics to create a rubric of the traits to evaluate example student work and to evaluate our own class work. The key questions I tried to look for in the research literature were 1) What are the keys to successful mathematical thinking? 2) How does a student become a good mathematician? 3) What are the traits necessary in order to demonstrate proficiency in mathematics? And 4) Could these traits be lumped together in some way under specific processes of mathematical thinking?

**Problem Solving as a Process**

Several researchers noted using problem solving as a process in order to promote higher level thinking and reasoning. Many mention some common skills in problem solving I wanted to pursue. According to Segurado (2002) good problem solvers are confident in their abilities.

It is possible to provide students of this school level a mathematical experience of doing investigations. Students are able to approach the tasks and move in the direction of becoming confident in their abilities, of enlarging their ability to solve and formulate problems and of communicating and reasoning mathematically. (p. 72)

Costa and Kallick (2000) say those good at problem solving are risk takers. Students who practice what they call responsible risk taking show a willingness to try out new strategies or techniques and are willing to test new hypotheses with an attitude of “What’s the worst thing that can happen? We’ll only be wrong?” Costa and Kallick also list persistence among the skills of those good at the problem solving process. They say in order to be successful problem solvers, students must not give up when encountering a difficult problem, even if they are not used to such struggle.

Persistent students have systematic methods of analyzing a problem. They know how to begin, what steps must be performed, and what data need to be generated and collected. They also know when their theory or idea must be rejected so they can try another…If the strategy is not working, they back up and try another. (p. 22)
The development of good problem solving techniques takes time, however. Ponte (2007) cites some examples in Portugal in which problem solving or mathematical investigations were used in a school setting. Having no one right answer seemed to generate some insecurity for students. According to Ponte, as time went on, the activity improved in quality, and, with teacher support and continuation of the work, student confidence in their abilities grew. The voiced “unpleasantness” by some that the activities required high personal perseverance lessened.

Allowing students to struggle and develop persistence is not always easy for teachers either. Ben-Hur (2006) believes that among teachers are two camps of thought when it comes to allowing students to “struggle.” One camp seeks to take the shortcut of teaching key words, algorithms and other tricks that work for given types of problems. He believes that this shelters students from the uncertain nature of problem solving. The other camp of teachers seeks ways to enhance reflective practices thus provoking students through use of cognitive dissonance. Wood (2001) says, “In order to create these situations for mathematical learning in classrooms, teachers must resist their natural inclination to tell students information, make the task simpler, or step in and do part of the task” (p. 116). Therefore, to develop students who are persistent problem solvers who take risks, teachers need to exhibit those qualities as well.

Fan and Zhu (2007) talk about a framework for problem solving modified from Polya’s problem-solving model and published in a syllabus by the Ministry of Education in 1990. Its list includes developing a plan, carrying out the plan and/or modifying the plan if necessary and ending with seeking alternative solutions and checking for reasonableness. Students good at problem solving do all of these things.

Costa and Kallick (2000) say that as students increase in their problem solving ability, they become more flexible in their thinking. They consider, express or paraphrase other points of
view, can state several ways of solving the same problem, and evaluate the merits of more than one course of action. Students who have this habit of mind in place become systems thinkers. They analyze and scrutinize parts, but also shift their perspective to the big picture.

The Australian Mathematics Education Program (AMEP), established by the Curriculum Development Centre (CDC), in its first national statement of basic mathematical skills and concepts (CDC, 1982) states,

Problem solving is the process of applying previously acquired knowledge in new and unfamiliar situations. Being able to use mathematics to solve problems is a major reason for studying mathematics at school. Students should have adequate practice in developing a variety of problem solving strategies so they have confidence in their use. (p. 3)

Good problem solvers do just that. When given an unfamiliar problem, they know what to do and can switch strategies because they have an unofficial list of problem solving strategies to call upon.

Successful problem solvers are agile users of what Schoenfeld (1994) calls the tools and logic of mathematics. That ability is improved through the solving of “good problems.” Schoenfeld defines a good problem:

Good problems can introduce students to fundamental ideas and to the importance of mathematical reasoning and proof. Good problems can serve as starting points for serious explorations and generalizations. Their solutions can motivate students to value the processes of mathematical modeling and abstraction and develop students’ competence with the tools and logic of mathematics. (p. 60)

So, to be good at problem solving a student must exhibit the following: 1) show confidence in solving problems; 2) demonstrate persistence when encountering a difficult problem and refuses to give up; 3) when given an unfamiliar problem, knows what to do and can switch strategies if one is not working; and 4) has an unofficial list of problem solving strategies to call upon when solving problems.
The Process of Reasoning and Proof

Problem solving requires more than listing or summarizing an answer solution. In order to help students think mathematically, they must be given opportunities to conjecture, test these conjectures and prove or reason. This is the process of reasoning and proof. It is what some other countries call a mathematical investigation that promotes learning mathematics with understanding. Wood (2001) states,

Learning mathematics with understanding is thought to occur best in situations in which children are expected to problem solve, reason, and communicate their ideas and thinking to others. Moreover, it is thought that situations of confusion and clash of ideas in which students are allowed to struggle to resolution are precisely the settings that promote learning with understanding. (p. 116)

Wood sees the heart of reform as a transformation in the ways students learn and teachers teach mathematics and that the ways of learning and teaching result in students knowing a different kind of school mathematics. One of its byproducts is a mathematics student who can reason. A student who is good at reasoning can adequately explain his or her thinking and do more than just list the procedure or summarize the answer.

A student who possesses good reasoning can use data to make, test, or argue a conjecture. According to Diezmann, Watters and English (2001), a student with good reasoning is able to speculate, test ideas and defend or argue them through contextualized problem solving tasks. Segurado (1998) talks about a study of sixth grade students who had initial difficulties with investigation activities but notes that the performance of the pupils evolved during the study, citing improvement in their capacity to observe, conjecture, test and justify, as well as communicate mathematically.

Ponte (2007) says these mathematical investigations should begin with a question that is very general or from a set of little structured information from which one seeks to formulate a
more precise question and produces a number of conjectures along the way. One tests these conjectures, and in the process forms new questions or validates the first line of thinking. He says problem solving investigations call for abilities that are beyond computation and memorization and require higher order abilities related to communication, critical spirit, modeling, data analysis, logical deduction and metacognition. Such learning of mathematics is active learning, not passive. Ponte says the student is called to be an active participant in such a problem. He or she is called on to be a mathematician, think for himself, evaluate decisions and the work done.

Problem solving is a situation in which the role of the student and teacher might change. Schoenfeld (2007) calls it a highly productive learning environment where students are encouraged to take on intellectual problems, students are given authority in addressing such problems, students are accountable, and students have adequate resources to do all of the above. Wood (2001) states,

Mathematical reasoning best develops in classes that have highly interactive situations and in which teachers make possible all students’ active participation in the interaction and discourse. (p. 112)

Some believe mathematical reasoning requires direct instruction. Students who are unfamiliar with reasoning and problem solving processes need direct instruction in how to reason. Ben-Hur (2006) says that students who perform poorly need to learn how to process mathematics and that they need instruction that targets the problem solving processes they fail to do efficiently and that this instruction is too often absent.

A student who is good at mathematical reasoning uses a variety of reasoning methods and proof and listens to others’ mathematical thinking. This is determined, in part, by the classroom teacher and the classroom atmosphere. Yeo and Zhu (2005) recommend that classroom teachers
try to establish a communicating environment for interaction that encourages students to verify, question, criticize, and assess others’ arguments.

Students in tune with the characteristics of good reasoning ask good questions. Costa and Kallick (2000) say these students link a sequence of questions to test hypotheses, guide data searches, clarify outcomes or illuminate poor reasoning. They see the significance and power of good questioning and that it can lead to better understanding.

In summary, those students successful at mathematical reasoning and proof can: 1) use data to make, test, or argue a conjecture; 2) adequately explain the reasons behind his or her mathematical thinking and can do more than just explain the procedure or summarize the answer; 3) use a variety of reasoning methods and proof; and 4) listen to others’ mathematical thinking.

**The Communication Process**

Problem solving and good mathematical reasoning are probably two of the most important characteristics of a successful mathematical thinker. Another that is probably equally important is mathematical communication. What makes a student a good communicator mathematically? After 23 years in the classroom, I knew what it did not entail. A student who is poor at communicating cannot explain his or her thinking. He or she does not have the ability to justify with examples and does not see feedback as important.

Students who are successful at mathematical communication, however, seek clarification. It happens as part of that communicating environment that Yeo and Zhu (2005) alluded to, that allows for interaction and enables students to question, criticize, and clarify. It is part of a community of learners Engle and Conant (2002) call sense-making communities—highly productive learning environments that can either support or inhibit the sense-making inclinations in students. Ponte (2007) says it is in this struggle for explanation that clarification happens. The
more that students are asked to do these kind of tasks, the more their approximation of what makes a good mathematical thinker (and therefore what makes a good communicator) will improve.

Costa and Kallick (2000) state that those who are successful at mathematical communication understand that it is okay to struggle and to let others know when one is struggling. They also mention that when others come up with new ways to solve a problem, good communicators ask for an explanation or try to figure why that makes sense. They hear beyond the words said to the mathematical meaning and can consider other ways to solve. They explain

They demonstrate their understanding and empathy for another person’s idea by paraphrasing it accurately, building upon it, clarifying it, or giving an example of it. We know students are listening to and internalizing others’ ideas and feelings. After paraphrasing another person’s idea, a student may probe, clarify, or pose questions that extend the idea further: ‘I’m not sure I understand. Can you explain what you mean by . . .’ (p.23-24)

The ability to explain what one is thinking mathematically and clarify one’s thinking and the thinking of others will result in not only in an increase in understanding, but in the ability to take risks. This however, depends on the classroom atmosphere. Wood (2001) says the classroom needs to be an atmosphere of acceptance for all views that is not threatening and yet is challenging to the students allowing them to struggle when appropriate.

A student who is successful at math communication 1) is able to explain his/her thinking clearly and concisely; 2) seeks clarification; 3) realizes it is okay to struggle in math and make mistakes; and 4) when others come up with new ideas, asks them to explain or tries to figure why that makes sense.

**The Process of Representation**

Being able to get a clear idea of what a student is thinking is often difficult unless a good explanation and representation of the solution is provided. Clarke, Goos and Morony (2007) say
that developing an appropriate visual representation of the information in a problem is crucial to successful problem solving. This is another identifying characteristic of successful mathematical thinking. Students need practice, however, in presenting and defending their answers and repeated chances to show what they are thinking and how the problem was solved, if they are to improve at this skill.

A successful math thinker has a variety of representation strategies in his/her repertoire that he/she can call upon when needed. The Agenda for Action (NCTM, 1980) made as one of its eight recommendations that problem solving should be expanded to include “a broad range of strategies, processes, and modes of presentation that encompass the full potential of mathematical applications” (p. 2). Those good at the process of representation have an unofficial list of ways to present the problem and its solution that expresses thinking in a variety of ways for example: words, drawings or pictures, charts or graphs, as well as written explanations. Costa and Kallick (2000) say these kinds of thinkers use representation to help show exactly what he or she was thinking when figuring out a problem and arriving at a solution. When confronted with a problem, students who are good at representation suggest strategies for gathering data or for solving the problem that may incorporate more than one method. Students who have found this success can list the steps needed to solve a problem and can tell where they are in the sequence. When asked to explain their solution, they can give their conclusion and describe the reasoning process that brought them there. They can move easily from one kind of representation to another and know the right or appropriate representation to use and when to use it.

A successful math student good at the process of representation: 1) has an unofficial list of ways to represent a problem and its solution; 2) uses a range of representation in expressing my thinking, (for instance- - words, drawings or pictures, charts or other graphs); 3) uses
representation(s) to help others know exactly what he or she was thinking, how he or she figured it out, and how the problem was solved; and 4) can move easily from one kind of representation to another and knows the right or appropriate representation to use and when to use it.

**Making Connections As A Process Skill**

Problem Solving, Reasoning and Proof, Communication, and Representation all lead to making better connections between mathematical problems and/or concepts. Schoenfeld (2007) calls it sense-making and says that what is reflected in the current standards based curricula is an understanding that a successful mathematical thinker can develop conceptual understanding in the context of solving problems. According to Ben-Hur (2006),

> Meaningless action can only reproduce, copy, or imitate other actions. It does not result in transfer to other than identical situations. The meaningless repetition, copying and imitation that are typical in mindless practice (and lack of thinking) render students unable to know what to do with standardized test items that fall outside those drills practiced. Meaningful learning results in conceptualization. (p. 32)

Successful mathematical thinking means noticing how ideas are related. Costa and Kallick (2000) say it is making higher level connections that allows the student to draw forth a mathematical event and apply it to a new context in a way that connects familiar ideas with new concepts or skills. Ben-Hur (2006) states,

> When it appears that students have grasped a new concept, the teacher must direct them to apply the new concept consistently to new situations. New applications shape and reinforce the new concepts. Adding variations to the concept helps the learner to reach a greater generalization of the concept and to embrace a wider set of possible applications. (p. 35)

In China, this is done by teaching with variation in which a series of related problems are presented to students. Cai and Nie (2007) say the use of variations is not only an instructional approach, but also an effective way to solve mathematical problems.
Making good connections means seeing how mathematical concepts are connected to others and to the real world. Abrantes et al. (1999) cite an initiative from Portugal called Project Mathematics For All developed in 1990. They say that investigation activities in the curriculum stimulate a holistic way of thinking that goes beyond application of knowledge or procedures in isolation and implies the connection of ideas from different areas of mathematics. When asked to make these higher level connections between concepts, however, students can struggle. Ponte (2007) warns that these opportunities for students to consolidate their knowledge and undertake new learning may highlight weak points in their thinking that may need to be addressed.

Costa and Kallick (2000) describe students good at making mathematical connections as students who like to know when others think of a solution strategy in a different way. They say these students are able to build upon, and consider the merits of another’s ideas. They reflect the desire to understand how others are thinking and to keep making sense out of the problem or text.

Therefore, a student who is successful at making mathematical connections: 1) likes to see how mathematical ideas are related; 2) connects new problems to old by asking, “Where have I seen a problem like this before?”; 3) likes to see how mathematical ideas or concepts are connected to other subjects and the real world; 4) can easily connect familiar ideas to new concepts or skills; and 5) likes to know when others think of a solution strategy in a different way.

What did this mean for my classroom? There is a difference between having a list of mathematical strategies to choose from and knowing when to use one or the other and having the decision made for you. Making these connections is about seeing relationships and increasing the level of learning but it takes time.
Ponte (2007) warns that a change to a curriculum that asks students to make conjectures, and then postulate about them, defending and/or debating is very different from simple recall of facts, figures and procedures. He notes that this change means students need time to understand. Ben-Hur (2006) says that there needs to be varied and balanced attention of instructional time spent on exercising and drilling procedural skills and time spent on discussion of concepts and that these concepts cannot simply be passed from one person to another by talk. “Teachers must not assume that meaning is transported from a speaker to a listener as if the language is fixed somewhere outside its users.” (p. 34). He says that it is necessary to guide students’ reasoning toward the accepted view through carefully thought out/guided questions, and by engaging student in self-evaluation, and reflection.

Ponte (2007) attributes some of the difficulty to an initial conception by the students of their role and the teacher’s role, the belief that there is always only one right answer and that it is the teacher who establishes the validity. This began to change as time went on, but it changed slowly. It is realized by researchers that “developing students’ ability in a higher level in solving challenging mathematics problems could take a longer time than expected” (Fan & Zhu, 2007, p. 499).

The research says it is important to allow time to discuss what students are learning and to think about thinking, thereby making mathematical connections. This metacognition is seen by Lester (1994) as the driving force behind problem solving and its influence on cognitive behavior as well as student beliefs and attitudes. Lester is quick to caution the degree of influence of metacognition, however, is not known for sure. It is generally accepted though, that “teaching students to be more aware of their cognitions and better monitors of their problem-solving
In order for students to improve in problem solving, they need to learn what it is that makes for good problem solving, or in other words what makes for good mathematical thinking. Clarke, Goos and Morony (2007) call this working mathematically and refer to the metacognition as cognitive engagement. Ben-Hur (2006) calls it concept-rich instruction, which he says is founded on two key principles of 1) learning new concepts reflects a cognitive process and 2) process involves reflective thinking which is greatly facilitated through mediated learning.

So, what did this mean for my research project? It became the reason I wanted to investigate using problem solving to practice good mathematical thinking. I wanted to see if time spent practicing, discussing and evaluating sample work could be used to promote a deeper, higher level of thinking. Lovitt and Clarke (1988) promoted using problem solving as the most effective way to teach. It was seen as a teaching methodology that involves teaching through applications and modeling through which students learn by grappling with real world problems. That is what I hoped to do—use problem solving to solve non-routine problems, develop good problem solving habits and representation, learn more about problem solving strategies at the same time, and think about as well as discuss, these experiences thereby promoting communication and mathematical connections as well.

I would use the lists I had noted for each of the 5 process standards of 1) Problem Solving, 2) Reasoning and Proof, 3) Communication, 4) Representation, and 5) Connections to develop rubrics I could use with my students to teach for and develop the characteristics of each of these processes.
I wanted to focus on real-life problem solving situations that would ask students to apply mathematic skill and yet also have real meaning for them. I knew that this would be a difficult task for some because it wasn’t the kind of mathematics they were used to and it might not be apparent to students why the struggle was necessary. Would they be able to trust me in that respect?

**PURPOSE STATEMENT**

My study was to determine if focusing on the key traits of a mathematical thinker namely: Communication, Representation, Reasoning and Proof, Problem Solving and Connections and learning about the characteristics I had come up with for each of these five processes would improve my students’ mathematical thinking. Based on the literature review, the following master list of processes would be used to create rubrics for student use (see Figure 1).

I investigated whether these items could be taught to students using a systematic, organized approach with carefully selected problems and by providing students with specific support structures to help them learn how to model their mathematical thinking. I provided rubrics or checklists for students to use, used group work, spent time modeling the thinking or metacognition involved, and specifically chose examples of student work that exemplified the good, fair and poor aspects of a solution and the reasons why.

I wanted to understand whether and how mathematical thinking could be taught in ways similar to how teachers try to use Six Traits Plus One to teach better writing.
### Characteristics of the Five Processes of a Mathematical Thinker

<table>
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<tr>
<th>Process</th>
<th>Description</th>
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| Process 1 | **Connections**  
A student who is successful at making mathematical connections - - |
|          | - likes to see how mathematical ideas are related.                         |
|          | - connects new problems to old by asking, “Where have I seen a problem like this before?” |
|          | - likes to see how mathematical ideas or concepts are connected to other subjects and the real world. |
|          | - can easily connect familiar ideas to new concepts or skills.              |
|          | - likes to know when others think of a solution strategy in a different way. |
| Process 2 | **Representation** - A student who is successful at representation - - |
|          | - has an unofficial list of ways to represent a problem and its solution.   |
|          | - uses a range of representation in expressing my thinking, (words, drawings or pictures, charts or other graphs . . . ) |
|          | - uses representation(s) to help others know exactly what he/she was thinking, how he/she figured it out, and how the problem was solved. |
|          | - can move easily from one kind of representation to another and knows the right or appropriate representation to use and when to use it. |
| Process 3 | **Communication** - A student who is successful at communicating mathematically - - |
|          | - is able to explain his/her thinking clearly and concisely.                |
|          | - seeks clarification.                                                     |
|          | - realizes it is okay to struggle in math and make mistakes.               |
|          | - when others come up with new ideas, asks them to explain or tries to figure why that makes sense |
| Process 4 | **Reasoning and Proof** - A student who is successful at reasoning and proof - - |
|          | - Can use data to make, test, or argue a conjecture.                       |
|          | - Can adequately explain the reasons behind his/her mathematical thinking and can do more than just explain the procedure or summarize the answer. |
|          | - Uses a variety of reasoning methods and proof.                           |
|          | - Listens to others mathematical thinking.                                 |
| Process 5 | **Problem Solving** - A student who is a successful problem solver - - |
|          | - shows confidence is solving problems.                                   |
|          | - demonstrates persistence when encountering a difficult problem and does not give up. |
|          | - when given an unfamiliar problem, knows what to do and can switch strategies if one is not working. |
|          | - has an unofficial list of problem solving strategies to call upon when solving problems. |

**Figure 1** Master List of Processes of a Mathematical Thinker

**METHOD**

I started by choosing five problems from the notebook of sample problems my principal had given me. It was a book of Exemplar Problems my school had purchased to use in the classroom. Mine was problems for grades 5-8 with concepts students were expected to learn sometime that year. These exemplars came with sample student answers at four proficiency levels. I wanted to choose problems that would have some application to things the students
would learn in seventh grade and also interest them. Next I planned how to collect data and
lastly, how to present the problems to the students and gather information.

I gave individual folders to the students to hold their work. Each folder had a pocket for
the students to put their problem and any work they did and another for parents that would hold
any information I sent home. I created a personal letter to parents explaining the procedure, a
calendar, and a sample evaluation. I divided the research period into five 2-week sessions. Each
problem was allotted two weeks from start to finish. The first week students were given the
problem and asked to come up with individual ideas pertaining to the solution. We handed out
the problems on Monday and discussed on Friday. After Friday’s discussion, students revised
their first drafts. Parents knew these were handed out on a Monday and that students had to have
an initial guess at the solution by Friday. On Fridays, I asked for volunteers to give us their ideas.
I also asked for any questions the students had or clarified any concerns warranted. Problem 1
was the Lawn Mower Problem. It related to area and perimeter.

The second week of Problem One I taught a specific process. I decided to teach Problem
Solving each week along with one of the four other processes. I thought Problem Solving was
the hardest to evaluate. Most of what happens with this process needs to be observed when it
happens or is internal and difficult to identify, discuss and assess and so I wanted students to
have as much experience with Problem Solving as possible. For example in Problem One, I
taught students about Problem Solving and Representation. We spent time looking at the
characteristics of these two traits and during that second week, we also looked at sample student
work and rated their solutions according to the rubrics I created for that purpose. Time was spent
discussing what made a good solution, what the problem solving process looked like, and what
made for good representation. Students were invited to revise their drafts of a solution and resubmit by the following Monday.

The following week, we started over again with Problem Two. It was called Fair Game and involved probability. Students were given the problem on Monday (when I collected their final solutions from Problem One) and had until Friday to come up with an idea for a solution. Problem Two’s second week I revisited the process of Problem Solving and taught about a new one—Reasoning and Proof (See Appendix B-1 and 2 for the sample list and rubric used in class).

Each week I used the posters and checklists I had made to teach about the processes of good mathematical thinking. Students also used these when evaluating the student work examples for each of the problems. At the end of the second week of each problem, the students filled out a learning log over that particular problem. The problems, the order in which they were covered and the processes taught follows (See Figure 2).

<table>
<thead>
<tr>
<th>Problem</th>
<th>Mathematical Topic</th>
<th>Processes Taught</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1 – Lawn Mower Problem</td>
<td>Perimeter and Area</td>
<td>Problem Solving &amp; Representation</td>
</tr>
<tr>
<td>#2 – Fair Game?</td>
<td>Probability</td>
<td>Problem Solving &amp; Reasoning / Proof</td>
</tr>
<tr>
<td># 3 – Cake Decorating Dilemma</td>
<td>Pascal’s Triangle, Patterns</td>
<td>Problem Solving &amp; Communication</td>
</tr>
<tr>
<td>#4 – Babbling Brook</td>
<td>Patterns &amp; Formulas</td>
<td>Problem Solving &amp; Connections</td>
</tr>
<tr>
<td>#5 – House of Cards</td>
<td>Pascal’s Triangle, Patterns &amp; Formulas</td>
<td>All 5</td>
</tr>
</tbody>
</table>

See Appendix A-1 and A-2 for the five problems and corresponding learning logs

Figure 2      Problem and Topic Timeline

I had planned to focus on a new process/list of characteristics each time we started a new problem so that by the end of the ten weeks, students would be familiar with all five methods. During that time, I thought that we could evaluate each other’s work and our own work for the
traits we were learning about but that was asking students to assimilate too much information in too short of a time frame. It was asking a lot to require students to learn about a trait, internalize that information and then apply it so quickly.

The first time we tried to rate each other’s oral observations I could tell I was asking way too much and way too quickly. The students were having difficulty remembering what we had discussed and were either acting very confused or going down the rubric choosing the highest score without any thought whatsoever. Some also kept coming to me to have me define what some of the words on the rubric meant. I knew then that the vocabulary was not student friendly and needed to be modified and I needed the time to modify them. I wanted my students to not only have the skills internalized to make correct decisions but also to take the time and thought in evaluating oral explanations for it to have any real meaning. I made the executive decision to delay evaluating our own work using the rubrics and would revisit after Problem 3.

The second time we tried oral observations was during the second week of Problem 3. I split the students into four color coded teams. I chose a team leader for each group based on their overall work ethic and level of cooperation. We discussed particular jobs for each group member and concentrated on two traits only – Representation and Communication—two I felt that students could rate easily according to the rubric because these two processes are particularly easy to observe or identify according to our list of characteristics. Each group presented and when finished, rated the other groups as a team. I rated each group as well and tallied the results. Each group got an evaluation sheet from me with my comments and the scores from the other groups for the two traits of Communication and Representation. The whole process went much more smoothly this time.
The next time we worked on oral reasoning evaluation, we split into new color coded groups with new team leaders for Problem 5/Week 10. After a brief discussion on what made a good team leader, the old leaders picked new group leaders and then new groups were chosen. This time after presentations of solutions, teams rated themselves. I also rated each group and tallied the scores. We rated groups on four traits: Representation, Communication, Reasoning and Proof, & Problem Solving. We worked with Connections the least, so we did not rate this trait. I felt the students were really getting the hang of it! By the end of the project, we were learning to evaluate when in small groups but were a long way from doing so individually.

These were my research questions and the data collection instruments I used for each:

(Figure 3)

<table>
<thead>
<tr>
<th>Question</th>
<th>Instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td>What will happen to the quality of student written reasoning when students use a rubric to evaluate their work?</td>
<td>Administration of a pre-, mid- and post- problem set. (See Appendix A-1 for sample problems and C-1 for scores)</td>
</tr>
<tr>
<td>What will happen to the level of student engagement in small group discussions when using the five traits of mathematical thinking to solve problems?</td>
<td>Individual Interviews (See Appendix A-3 for sample interviews)</td>
</tr>
<tr>
<td>What will happen to the quality of student oral explanations of solutions when using the traits of a mathematical thinker to guide student solutions?</td>
<td>Journal and/or anecdotal records of class worked problems (See Appendix A-6 for sample teacher journal)</td>
</tr>
<tr>
<td>What will happen to my teaching when I specifically set aside time to teach traits of mathematical thinking and deliberately spend time on mathematical discussion and reasoning.</td>
<td>Journal and/or anecdotal records. (See Appendix A-6 for sample teacher journal)</td>
</tr>
</tbody>
</table>

Figure 3  Question and Instrument Table
FINDINGS

What will happen to my teaching when I specifically set aside time to teach traits of mathematical thinking and deliberately spend time on mathematical discussion and reasoning?

Just as it took time for students to develop familiarity with the traits and how to use, it took time to develop skill in teaching process skills. The more that I concentrated on this, the more adaptations and changes I made in the process and the more honed my teaching skills became. The change, however, was happening slowly.

My teacher journals showed that the time devoted to discussing and modeling a process skill was not something I was used to doing. In my journals, I noted that even though I thought I had a well laid out plan and purpose and structure and felt well prepared to teach what I had planned, it was frustrating to put myself out there on a limb so to speak and try something new (something I had not been taught before Math in the Middle classes). In my journal I wrote,

It is hard for me as a veteran teacher to do something so new that makes me feel like a beginning teacher all over again. No one has ever taught me HOW to teach mathematical thinking. Although I think this is the approach for MIM, it is hard for me to teach it to students. I feel like I am constantly unprepared. I am also always stressed about the time issue. I have limited amount of time to spend on this. I have limited amount of time to spend teaching my PELS (Power Essential Learnings). I have my other Math class and papers to check. This taking the time to journal and data collect is also difficult. (Teacher Journal, Week of January 21, 2008)

My journals showed that what I had originally planned needed modification. I decided almost right away that the rubrics I planned to use were not going to work. It was very slow
going the first time we looked at them. I was forever explaining what the words meant. I thought they could be made to be more student friendly. I state in a journal entry,

The time spent on the rubrics already tells me that I am going to have to change the vocabulary. I spent a lot of time explaining and re-explaining the same words over and over. Next time we try to use, we will do as a class. I will read, translate, and then they can mark as we go along. Didn’t realize I was so far off on the wording. . . When the vocabulary is too difficult, (think of it as the vehicle I am using) the journey is going to be long and difficult and slow going. (Teacher Journal, Week of January 21, 2008)

In another I stated,

I was going to try to have the students rate each other using the rubrics for the traits we’ve discussed and rate each other’s presentations (moving to peer and self evaluation eventually) but they are still so ‘new’ to the process of presenting, I am going to hold off on this for a bit although I am writing down notes about the quality of their explanations myself. Also the rubrics I intended to use are still way off in vocabulary and right now would only frustrate them more (Teacher Journal, Week of January 28, 2008).

I needed to learn to be patient and allow students time to internalize it all.

Facilitating this process and allowing students to experience and arrive at a place of knowing is difficult and depends on time constraints. I noted that the first time we tried to evaluate oral observations of each other, it just did not work for a variety of reasons. The following are journal entries over this time period. I decided students needed something to help them because applying what they were learning to their own work was proving difficult.

Transferring process skills. Hmmm. What have I learned about transferring? I think the biggest thing I’ve learned is that it is not as easy even for my high ability kids to do this
with unfamiliar concepts and the traits are new to them. (Teacher Journal, Week of February 4, 2008)

I was asking students to digest a lot of information in a short amount of time. We have not tried again to evaluate oral explanations of each other or self. I plan to get back to, but it seems a good idea to wait since everything else is happening so slowly. Also what with the vocabulary issues, I really need to stop and think about how to change. Do the students need a checklist of some sort when they are working on trait work? (Teacher Journal, Week of February 4, 2008)

I had originally planned to spend two weeks on each of the five processes I had identified—solve the problem, discuss possible solutions and eventually rate our oral explanations and a second week to look at sample student answers and rate according to the rubrics for the processes learned so far. Two weeks for each of the five identified meant a total of ten weeks to learn, use the trait and learn to evaluate in other’s work and our own. I found, however, that it was asking the students to move too quickly. By the end of the research project, we had really only covered three traits in any great detail. (See Appendix A-7 for a sample of the oral rubric used and A-8 for teacher page used for evaluation).

What will happen to the quality of student written reasoning when students use a rubric to evaluate their work?

It took time for students to move through the process of knowing about mathematical traits, understanding what they were and then applying what they knew to evaluate their own work and the work of others. Although this change was slow, it DID happen.
Change in what students knew about the traits was evident according to their learning log answers. Students were asked on their learning logs for each problem to identify the traits focused on for that particular problem and to list reasons why they thought so. In an analysis of student learning log responses, I looked for the number of processes correctly identified that also included an adequate explanation and split into three key groups—those who could identify and explain more than two mathematical processes correctly, those who could correctly identify and justify at least one or two processes, and those who did not have any process identified or could not explain correctly. A comparison of processes identified and adequate reasons given is listed in Figure 4.

Although change happened, it was slow and only for those items students had become familiar with and were comfortable using. In looking over the learning logs for the first problem (the Lawn Mower problem) and comparing it to the answers for each successive problem, students were more likely to correctly identify the traits used and to give adequate reasons for their thinking. By the third problem, three times as many students were able to correctly identify three to five traits and provide acceptable explanations than did the first week we tried it. Also,

<table>
<thead>
<tr>
<th>Problem Name</th>
<th>Lawn Mower</th>
<th>Fair Game</th>
<th>Cake Decorating</th>
<th>Babbling Brook</th>
<th>House of Cards</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Processes Identified</strong></td>
<td>Problem 1</td>
<td>Problem 2</td>
<td>Problem 3</td>
<td>Problem 4</td>
<td>Problem 5</td>
</tr>
<tr>
<td>More than 2 processes w/ acceptable explanation</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>At least 1 or 2 with an acceptable explanation</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>No explanation given or no processes identified</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 4  Key Processes Identification
student answers showed that the traits they knew and could adequately explain were the ones we’ve been discussing specifically Representation and Communication and Reasoning/Proof. (See Appendix C-5 for Learning Log Scores by student)

One of the data collection instruments was to compare pre-, mid- and post- problem scores. The scores were considered to be of either high, medium or low quality. Using a rubric to evaluate the quality of problem solutions, I rated student work and split into three groups—high scores of three, medium scores of two and low scores of one—according to their overall performance on the mathematical processes we had learned about by that time (see Appendix A-9 for rubric used for teacher evaluation).

<table>
<thead>
<tr>
<th>Student Solution Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Problem #1</strong></td>
</tr>
<tr>
<td>Lawn Mower Challenge</td>
</tr>
<tr>
<td>Hi</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

Scores for the problems we worked showed some improvement. Seven out of 15 students received a high or medium score on the first problem we did. On the third problem scored, 13 received a high or medium score and five a low score. By the fifth problem, 13 received a high or medium score versus two who had a low score (See Appendix C-1 for all five problem scores listed by student). Student solutions showed an application of what was learned and an increase in familiarity with the processes each time we tried a new problem. Although not evident for all students, scores showed that some students improved and could apply some of the new learning to their work.

Comparison of the quality of solution answers for pre-, mid- and post problems indicated some growth. Although small, this was easier to see when comparing the first problem attempted
and the last problem discussed. While not apparent for all students, more students came “on board” each time we started a new two-week session. These changes included more detail in the response, clear organization and/or structure, better or more complete representation, and increased precision. Seven of the problems turned in on the Problem Two (Fair Game Dilemma) had a clear format or structure to them including an introduction or restatement of the problem, a diagram or drawing and explanation of how the answer was arrived at, a conclusion or answer to the problem and possible summary. By the fifth problem (House of Cards) ten student solutions had a specific format or organization to them and five were partially specific. These solutions included clearly labeled sections: a title, an introduction or restatement of the problem, and a solution section with diagram or chart and explanation. Five of these also included a summary or reflection at the end. Many student answers improved the more we practiced. Several students continued to include much more detail and were more precise in their explanations. Ellie’s answer for Fair Game (Problem 2/Week 3):

Is it a fair game or not? To figure out the answer to this question, I made a simple table. On the table, I drew dice that numbered one through six. I drew two sets of dice. One set of dice was vertical (sic) and the other set was horizontal. Then, I added up the numbers on the dice. For example, one and one was two, one and two was three . . . Since the problem had to do with odd and even numbers, I decided to find out how many odd and even numbers there were. First, I found out how many numbers there were all together. There was 36. So that meant that there was 6 numbers . . I found out that there were 18 even numbers and 18 odd numbers. So the answer to the question is that it is a fair game.

This was part of the same person’s answer on Problem 1/Week 1:

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1 All names are pseudonyms.
To try and find out how many trips Randy took, I subtracted four from 80 and 40, then multiplied. I got an answer of 2,736. I subtracted four again from 76 and 36, then multiplied. I got 2,304. Then I subtracted four from 72 and 32 and multiplied again…I then got 1600. Next I counted the trips he had made and I got 3 ¾ trips.

Ellie’s explanation was much clearer in the second attempt than in the first. She seemed to do a better job of taking the reader through the process of what she was thinking at the time she solved the problem on the second try. Her first problem attempt seems to be a list of computational steps without an explanation of why she did the things she did and there does not seem to be any overall goals in mind (See Appendix D-1 for all of Ellie’s work).

In looking at the students whose solutions included a conclusion or reflection on the fifth and last problem and comparing to their first attempt, I could see more depth and detail in their last problem’s answers. Although Fred’s work on the first problem was fairly high, his answer on the last problem showed improvement in that he demonstrated the answer solution in three ways. The questions asked for the number of cards in a house of cards that is five levels high, ten levels high and \( n \) levels high. He used a picture to explain how to figure for three and five levels, then made a \( t \) chart to show how to figure the answer for up to ten and then generalized to the formula to show for \( n \) levels. He went on to prove this formula by working through it for the number ten to justify his work on the chart. By showing part of his solution in three different forms, Fred showed that he not only understood how to solve in more than one way, (and could generalize for \( n \) levels) but also that he had an idea of what makes for good representation (See Appendix D-1 for Fred’s work).

Fifia’s work on the first problem/week 1 showed a drawing and explanation of her work. The problem asked if two people are mowing a lawn that is 40 by 80 feet and each wanted to
mow half (and the lawn mower mowed a path that is two feet wide), how many trips would each person take? Her explanation started out:

First on grid paper, I made my lawn 80 ft. by 40 ft. I labeled it and now I am going to get to 1600 feet. Randy is going to make his first trip. So I went around and found how many feet, and then I timesed [multiplied] it by 2 because the mower mows 2 feet at a time.

Then I had to add all the numbers together…

Fifia’s last problem/week 9 started with her own title, “Castle of Cards” and had Roman Numeral Sections - - an introduction, a solution, an answer, a reflection and a second page with her charts and diagram. Her answer included:

I used a chart for some of the easier levels (1-5) My chart showed how one side went up by 1 while the other went up by 3. You just had to add up all the numbers before it to get the total amount because the chart only shows the number of cards on that level. . . . I used my formula when we went to bigger numbers so I didn’t have to add so much. I also drew a drawing…

Fifia’s second example shows a much better understanding of solution organization. Her explanation also included more of the process she used to solve and WHY she chose to proceed as she did (See Appendix D-1 for Fifia’s work).

What will happen to the level of student engagement in small group discussions when using the five traits of mathematical thinking to solve problems?

Students liked the time in class spent working on the problem solving packets especially when we worked in groups. This time together discussing and critiquing sample work increased their interest level and participation. The answers to the small group interview questions showed that students liked working with others because they got exposed to other viewpoints. During group interviews when asked what they liked about working on the problems as a group. Sevie
responded, “You have many ideas to choose from.” Trey agreed. “Yes. It’s not just one. You can get different ideas of how to do not just how you did.” Eithia answered, “I like having to hear other people’s opinions on what the answer is.” This was from a student with many learning problems who is on medication and does not work well with others most of the time. Ted replied, “You get more explanation from a group” (See Appendix D-2 for interview questions and lists of responses).

Responses to individual interview questions indicated that working together was a way for students to help each other “arrive” at solution answers that were reasonable and thorough. In the individual interviews, 17 of the students interviewed said working on the problem solving packets together as a group helped them get involved in learning and that they preferred it at least part of the time to working individually. That was evenly represented by students who were high ability as well as average and low. Fifia said, “You can tell them your ideas and they can tell you theirs.” Forman replied, “Yes because if I do not know something I can get help.” When asked what he liked about working on the packets as a group versus as an individual, Fred answered, “I like working as a group to get the main idea and as an individual or small groups you can do your own work or get into it farther.” Xavier said “Yes, because they can help me and I can understand it more.” Fortran replied, “Yes, because we’re talking and discussing the problems.” Sithe answered, “Working together- that makes me do the work with (a) group” (See Appendix D-3 for questions and answers).

On the pre and post survey students indicated they liked working on the problem solving packets together in class and that they thought it important to listen to one another’s thinking. For the pre survey students took during the first week of research, 13 out of 19 answered strongly
agree or agree to the question, “I like to work on problem solving in math class.” Fourteen out of 21 agreed or strongly agreed on the post survey (see Fig 6).

Eighteen students rated listening to others’ mathematical thinking as important to them (strongly agree = 10; agree = 8; Total 19 students answered) on the pre survey. Eighteen still felt the same way on the post survey.

Figure 6 Ques One

Eighteen students rated listening to others’ mathematical thinking as important to them (strongly agree = 10; agree = 8; Total 19 students answered) on the pre survey. Eighteen still felt the same way on the post survey.

Figure 7 Ques 6 Answers
Fifteen students said they like to know when others think of a solution strategy in a different way. (15 answered strongly agree or agree, 2 = neither agree or disagree). Eighteen answered strongly agree or agree to this question on the post survey. (See Appendix C-2 for tallies of results of Pre and Post Survey Questions and and C-3 for a comparison of answers).

More students indicated problem solving work as the time they were most involved on the post survey than on the pre survey. Three had identified problem solving work in the classroom as the time they were most involved on the pre survey. Ten said problem solving time on the post survey. On the post survey six of the ten students who had said homework was the time they were most involved on the pre survey, had changed their answers to during problem solving (See Appendix C-2).
During which part of math class do you feel the most involved?

Figure 9  Student Engagement

What will happen to the quality of student oral explanations of solutions when using the traits of a mathematical thinker to guide student solutions?

Students need more time to work on developing familiarity with the traits of a mathematical thinker and feel more comfortable with using the information, before they can apply this knowledge to rate each other’s oral explanations. My journals showed that knowing about a mathematical trait and applying it by evaluating someone else’s work (or one’s own work) is a higher level thinking skill that required time for the student to become familiar with the trait’s characteristics and lots of practice using in order to use for evaluation purposes. I mentioned,

Students seem to understand we will eventually get to different approaches (and the right answers) and what high quality work looks like. (Teena says “mine’s not like that, is that okay?”) They realize others tried the same wrong path first like putting candies in the middle or trying to use color when counting the # of lines. (on the Cake Decorating
problem) I’m still concerned that some are parroting and not understanding.. (Teacher Journal, Week of February 18, 2008)

And in another journal entry, I noted the same kind of thing,

I think the transference of this trait work and its application is going to involve a lot of time and emphasis and may not happen as easily as I first thought. Silly me. I thought I would teach them about the 5 traits, show them the rubrics, give them some examples, that we’d discuss and then presto! Chango! It would magically appear in their work. I am constantly reminded of Bloom and his taxonomy. These kids may have the knowledge now or at least more of it but comprehending it and applying it are stages of learning that need to be moved through and each student is going to have to move through this (some slower than others) at his or her own pace. (Teacher Journal, Week of January 28, 2008)

In another entry I wrote, “I chose this problem because we have been working on perimeter and area and I thought it would make more sense. I am realizing that taking a computational skill and transferring into a PROCESS skill is difficult” (Teacher Journal, Week of January 21, 2008).

Initial work on rating of oral reasoning skill (on Problem One), was asking students to apply a newly learned skill too quickly. When students are not ready, this does not work well. The first time we tried to rate each other’s work was on the first problem, the Lawn Mower Problem. It kind of turned out like the answers on the pre-survey. Students thought their work was good enough and did not need revision. I mentioned at that time that I wondered if it was because students thought revising would mean more work. Also, even considering the difficulty we had working through the rubrics because of the vocabulary, students took as little time (and as little thought) as possible to complete and most everyone gave most everyone else perfect
scores of five on Representation and Problem Solving. I made the executive decision to toss these and revise the rubrics. They simply were not ready and needed more experience before we could try again.

For the third problem, we revisited the evaluation of oral explanations. I divided students into color coded groups. Each had a team leader and they were given the revised (more student friendly) rubrics for Representation, Communication and the newest one we have been working on - - Reasoning and Proof. I observed each group and scored on Representation and Communication and students scored each other. This has been the first time we have tried it again since that first week. We spent a whole period preparing the presentations and a whole another day (we have block schedule) presenting and scoring each other. Three of the four groups got a total of 200 points or above on Representation (1 had 192) out of 225 total points possible. Communication scores were not as good but better. Out of 175 total points, their scores ranged from 131 to 154. Almost everyone on the teams presented or played a part in the preparations except for a few new students and students who were absent the previous day (See Appendix C-7 for oral reasoning averages for color coded groups on the Cake Decorating Problem).

Teams’ Representation strategies showed some attempts at a variety of methods to solve and/or represent the answer including the use of a t chart, a drawing or diagram, a written explanation of the process and the solution as well as the broader application to a formula generalization. This variety of representation was not present for all groups, however.

As far as Communication was concerned, the work showed a need for ALL members to participate equally and learn how to function as a group. Overall, communication seemed segmented and lacking. Some group members made an attempt to explain or adequately
represent their solution but it was not cohesively presented in such a way to demonstrate the trait of good mathematical communication.

We tried again in different color coded groups for the fifth and last problem. Again each team had a leader. Students were given the rubrics for Representation, Communication, Problem Solving, and Reasoning and Proof. I observed each group and rated them. Students also were asked to evaluate themselves. This was the second time we have tried evaluating oral observations. We spent a period preparing the presentations and another day presenting and scoring. I compared the scores of Representation, Communication because that is what we had done on the previous oral scoring. This time all of the four groups got a total of 200 points or above on Representation (the lowest was 202) out of 225 total points possible. Three of the four groups scored 160 or above on Communication. Out of 175 total points, their scores ranged from 141 to 166. Of the 21 students present, only one person did not play an active part in the preparations.

Presentations were much more organized and well rehearsed this time. The second round had more overall participation from group members, and members seemed to do a better job of not just telling what the solution was but representing how it was they got to the solution. All groups represented their solution strategies in a variety of ways including t charts, drawings and/or diagrams, written explanations of the process, the solution and a generalization to a formula. The team work also showed more willingness to “go beyond” what was expected to make their team approach different or unique and showed much more thought. Some groups included a restatement of the problem at the beginning, a summary at the end and a reflection of what other problems compared or an example of a harder version (See Appendix C-7 for oral
reasoning averages for color coded groups on the House of Cards Problem and D-1 for sample team work).

**CONCLUSIONS**

Mathematical reasoning was a complicated skill. It took lots of practice to become familiar with the concepts. Before one could apply it to his or her work or evaluate it in someone else’s work, time was essential to be able to walk through the process and not only learn about reasoning, but understand it. Mathematical reasoning was harder for those less proficient in the arithmetic part of mathematics and took longer to develop. It is as if they were concentrating so hard on the individual parts, that they could not look up and see the big picture. I imagined it as a new dance step I had taught them and now as they practiced, their head was down, and they were looking at the footprints and were busy putting one foot in front of the other. For some students, the why of the mathematical work we were doing, and the answer produced are just disconnected steps in a process they had long given up understanding. One could also see that in their initial learning log entries. It was as if they had decided, “If you tell me to add, I’ll add, but if you tell me I need to subtract, then I’ll do that.”

Meaning was so important and so clearly tied to mathematical understanding. Written and oral explanation was difficult for seventh graders to put into words. Sometimes the meaning behind the mathematical operations was unclear. There was not always agreement between what we had discussed and done (and what they had put down) and what they wrote. Even my advanced students found it hard to explain why they got what they got. Again, this transfer or internalizing of what we learned and then applying it was a complicated and time consuming process.
It was very different to get students to think about math THE PROCESS and not math THE PRODUCT. Along the way, I had many “So, what’s the answer?” and “Am I right?” This change in thinking took work and did not change overnight. It was especially surprising to me however, because it was from some of my brightest students that some of the questions came. I continued to model as much of the thinking and the process of problem solving as possible to give students an insight into what was involved and asked them to do the same. I highly valued time to discuss and learn from each other in the classroom and tried to use that time by asking higher level questions of my students.

I believed that these students were demonstrating an overemphasis on the answer and an under-emphasis on the “how” and the “why.” This led them into concentrating on writing down the answers without supplying the work, or copying from someone else’s paper. By going over the solutions the second week and concentrating on what made a good solution, I think I steered my students around that issue.

I also think I saw students who did not see the need to be “involved” in the process. They viewed grades and homework assignments as something that was done to them, instead of something they needed to do for themselves. Getting them more involved by incorporating more small group or whole group activities helped tremendously.

I came to believe the following to be true: 1) I needed to have faith in teaching process skills and the resolve to spend the time on it, although difficult and time consuming; 2) modeling the process and the thinking was key to help students learn how to do the process; 3) students need “visual reminders” or checklists to help light their journey; and 4) although slow and sometimes hard to see, growth and, therefore, positive change happened.
IMPLICATIONS

So, what does this mean for me? I believe that I have established a case for more in-depth study of problem solving within my mathematics classroom. The research discusses using problem solving as a most effective way to teach. It was seen as a methodology that involves teaching through modeling and applications through which students learn while trying to figure out real world problems. That is what I hope to continue to do—use problem solving to solve non-routine problems, develop good problem solving habits and representation, learn more about problem solving strategies in the process, and think about as well as discuss these experiences thereby promoting communication and mathematical connections as well.

Through careful and considerate continued research, it is my hope that I can continue to explore the process of problem solving and provide some answers to what makes a good mathematical thinker. The quotation may be 62 years old, but it is as true today as it was when Polya (1945) said it,

Thus a teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking. (p. V)

According to all that I have read and done, there are several major issues I would like to keep in mind in the future. First I need to use criteria when choosing problems. I believe I need to have some kind of idea of what makes a good problem and need to keep that in mind when choosing the problems for my class to work on. Second, I need to use a rubric to “teach” problem
solving characteristics or key traits. Because some students are not good problem solvers, I will need to continue to take time to address the skills of problem solving. I plan to do this by continuing to use a rubric with key traits of mathematical thinking that is according to the five process standards of Reasoning and Proof, Communication, Connections, Representation and Problem Solving. I will continue to use this next year to evaluate example student work as a class and on the off week to evaluate our own work. I plan to keep this kind of instruction as part of my curriculum.

Another item I would like to keep in mind is to involve metacognition and discussion of what is a “good answer” using the key characteristics of each of the processes. The research tells me it is important to allow time to discuss what we are learning and to think about “thinking.” I plan for my students to learn more about problem solving in the context of solving problems. We will continue to work on process skill while learning other skills like the surface area of a prism or volume or probability of an outcome. I believe that the only way for students--all students--to improve in problem solving, is to learn about the characteristics of good problem solving, or in other words what makes for good mathematical thinking.

Also, I need to allow for cognitive dissonance and use variation when possible. I plan to continue to provide opportunities for concept-rich instruction that asks the student to elevate thinking beyond the evidence to make a generalization. New applications force the students to shape and reshape their conceptual knowledge. These new applications will also ask them to create wider generalizations which will serve a greater set of possible applications.

Fifth, I need to create a proper classroom atmosphere. I know it is my responsibility to arrange my classroom in such a way that my students are able to become part of a community of learners who respect each others’ ideas and can work together to reflect, think out difficult
problems, and analyze their work. I want that to happen in an atmosphere where students feel safe and uninhibited. I also need to have the proper amount of awareness. I want my students to learn new mathematic skill but I also want them to experience their learning in such a way that it has real meaning for them. I want my students to be able to demonstrate these newfound skills when asked to whether that is on a test or a state achievement assessment or in real-life.

It is true what I mentioned Ben-Hur (2006) saying before. Just because I tell students something does not mean they have learned it. I need to provide meaningful learning that is a balance between practice of procedural skills and discussion of concepts that allows students chances to practice and apply what they’ve learned. Ben-Hur also said that students’ reasoning needs to be guided toward the accepted view through carefully thought out/guided questions, and by engaging student in self-evaluation, and reflection. (Ben-Hur, 2006, p. 34).

And so lastly, I need to remember to allow for practice, practice and more practice and the time to develop problem solving skills. That change will not happen in my classroom immediately and it may not be easy to get students to see that the change has value.
REFERENCES


*Theory Into Practice, 40*(2), 110-117.

Problem 1
Randy and his sister agreed to work together to mow the family’s 40 foot by 80 foot lawn. Randy said he would go first and mow half the lawn, and then his sister could take over and finish the job.

Determine how many trips around the lawn Randy must make in order to mow half of the grass if the mower cuts a path two feet wide.

Problem 2
Fair Game? A few students want me to play a game with them. They will give me a dime for each odd sum I roll with 2 die. I have to give them a dime for each even sum they roll with two die. I think I’m going to get cheated!

\[
\text{_odd} + \text{_odd} = \text{odd}
\]

Are the chances of me getting an odd sum as good as the chances that they will get an even sum? Should I play this game with the students? Using as much mathematical language and representation as you can, show me that this is or is not a fair game.

Problem 3
Cake Decorating Dilemma
My niece, Andrea, was decorating a cake for her daughter’s birthday. Her daughter was four (4) and so she put four candies along the circumference of the top of the cake and connected all of the candies to each other with colored icing—it looked quite pretty!

View of the Top of the Cake

\[
\begin{align*}
\text{Age} &= 4 \\
\text{Candies} &= 4 \\
\text{Lines} &= 6
\end{align*}
\]

I began to think about what the cake would look like if her daughter was 3 years old, or what it would look like next year. In fact, I would like to know how many lines I would have to draw to connect the candies on a cake for any age.

Using some sort of way to keep track, can you identify a pattern, and tell me how to figure the number of lines of colored icing it would take to connect the candies on a cake for any aged person?
Problem #4

Down the Babbling River

Some family and friends have asked you to plan an end of the summer getaway. A rafting company has agreed to take your group down the Babbling River. (It’s actually a little smaller than a river, more like a brook.) The rafting company has given you specific details as to how much weight each raft can hold.

- A raft can safely carry the weight of 24 babies.
- The weight of 12 babies is exactly equal to the weight of 4 teenagers.
- The weight of 6 teenagers is equal to the weight of 3 adults.

Question: What is the fewest number of rafts needed for a trip with 11 adults, 5 teenagers, and 21 babies? Note: for this task only, supervision of the babies is NOT necessary.

Problem #5

House of Cards

This is a house of cards that is 3 stories high.
- The top story is made of 2 cards leaned against each other, with one card as its base.
- The next story is made of 4 cards (2 sets of 2 cards leaning against each other), with 2 cards as its base.

The following story is about a house of cards with 6 cards (3 sets of 2 cards leaning against each other), with 3 cards as its base.

Question:
- Determine the total number of cards needed to build a house of cards that is 5 stories high.
- Determine the total number of cards needed to build a house of cards that is 10 stories high.
- Determine the total number of cards needed to build a house of cards that is N stories high.
Student Learning Log Questions

Research Questions that pertain:
- What will happen to the quality of student written reasoning when students use a rubric to evaluate their work?
- What will happen to the quality of student oral explanations of solutions when using the traits of a mathematical thinker to guide student solutions?

Student: 
Class: 
Date: 

Learning Log Questions:

1. Briefly describe the problem you were given to work on this week.
2. Briefly describe your solution to the problem.
3. Of the five traits of mathematical thinking listed below, what traits do you think were the focus of the math problem this week? Why?
   - Representation
   - Problem Solving
   - Reasoning and Proof
   - Communication
   - Connections

4. On a scale of 1 to 5 rate your understanding of the 5 traits of Mathematical Thinking. Then explain why you think so.
   
   1-Very Strong  2-Strong  3-Average  4-Weak  5-Very Weak

5. On a scale of 1 to 5 rate the problem’s relationship to the 5 traits of Mathematical Thinking. Then explain why you think so.
   
   1-Very Strong  2-Strong  3-Average  4-Weak  5-Very Weak

6. What could you have done differently in solving this week’s problem?
7. What did you learn?
Individual Student Interview Questions (to be administered orally)

**Research Question that pertains:**
- What will happen to the level of student engagement in small group discussions when using the five traits of mathematical thinking to solve problems?

**Student (or students):**

**Class:**

**Date:**

1. **What makes math easy or difficult for you?**

2. **What could teachers do to help students with math?**

3. **On average, how would you rate your involvement in math class? Why?**
   (1 being ‘not involved’ and a 4 being ‘very involved’)

4. **What helps to get you involved in math class?**

5. **Does working on the problem solving packets together as a group, help get you involved in your learning? If so, why do you think that is?**

6. **How do you participate in the group activity (the problem solving packets groupwork)?**

7. **What do you think about working on the problem solving packets as a group vs. as an individual or a small group?**
8. What do you like about working on the problem solving packets as a group vs. as an individual or a small group?

9. What do you dislike about working on the problem solving packets as a group vs. as an individual or a small group?

10. Are you confident in your math ability? Why?

11. Are you confident in your math ability when working on the Exemplar problems in the problem solving packets? Why do you think that is?

12. Is there anything else I should know about you to better understand your problem solving in math or your general math experience?

13. These are the 5 process standards for Math: (the Key Traits of Mathematical Thinking)
   Representation
   Problem Solving
   Communication
   Reasoning and Proof and
   Connections
   Which of these 5 traits help you most to solve problems? How?

14. How has your work on problem solving changed this year (if at all)?
15. Rate your overall work on solving problems from 1 to 5.
   1-Very Strong  2-Strong  3-Average  4-Weak  5-Very Weak
Small Group Interview Questions

Research Question that pertains:
- What will happen to the level of student engagement in small group discussions when using the five traits of mathematical thinking to solve problems?

Students: Class:
Date:

Interview Questions:
1. Tell me about the problem solving packets that we are doing as a class.

2. What do you think you are learning about mathematical thinking? (if anything)

3. What do you like about working on the problems as a group?

4. What is it that you do not like about working on the problems as a group?

5. Do you think the time spent has been beneficial? If so, why?
6. Do you think the time spent discussing and working on these weekly problems has changed the way you think about solving a problem or providing a solution? If yes, how would you describe the change? If no, why not?

7. What would you change about the weekly setup? (if anything)

8. Do the Five Traits of Mathematical Thinking help you solve problems?

9. Are you more involved in problem solving with your team when you do or do not use the Five Traits? Why do you think that is?

10. What advice can you give me for next year when I use the Five Traits of Mathematical Thinking?
Student Pre/Post Survey Questions

Research Questions that pertain:

- What will happen to the level of student engagement in small group discussions when using the five traits of mathematical thinking to solve problems?

Student: ___________________________  
Class: ___________________________  
Date: ___________________________

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<thead>
<tr>
<th></th>
<th>Strongly Agree</th>
<th>Agree</th>
<th>Neither Agree or Disagree</th>
<th>Disagree</th>
<th>Strongly Disagree</th>
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<tbody>
<tr>
<td>I like to work on problem solving in math class.</td>
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<td>I ask questions of others when I problem solve.</td>
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<td>When I can’t find a solution right away to a difficult problem,</td>
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<td>I give up.</td>
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<td>Others ask questions of me when we problem solve.</td>
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<td>I can adequately explain the reasons behind my mathematical</td>
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<td>thinking and do more than just explain the procedure or</td>
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<td>summarize the answer.</td>
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<td>Listening to others’ mathematical thinking is important.</td>
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<td>I am able to express my mathematical ideas without fear of</td>
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<td>ridicule.</td>
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<td>I am able to explain my thinking clearly and concisely.</td>
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<td>I seek clarification when I do not understand.</td>
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<td>When others come up with an idea I didn’t think of or that I</td>
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<td>am unsure of, I ask them to explain, or try to figure out why</td>
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<td>that makes sense.</td>
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<td>I like to know when others think of a solution strategy in a</td>
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<td>different way.</td>
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<td>Connecting new concepts or skills to familiar ideas is helpful</td>
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<td>to me.</td>
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</table>

During which part of math class do you feel most involved?  
(Warm-up, checking homework, the lesson, working on homework, problem solving... )

Why do you feel more involved during this time?
Teacher Journal Prompt Guidelines for my Research Project: A-6

Research Questions to focus on:
• What will happen to the quality of student written reasoning when students use a rubric to evaluate their work?
• What will happen to the level of student engagement in small group discussions when using the five traits of mathematical thinking to solve problems?
• What will happen to the quality of student oral explanations of solutions when using the traits of a mathematical thinker to guide student solutions?

Reflection Questions:
1. How does each of the two incidents I wrote about relate to my research questions (Teaching the Key Traits of Mathematical Thinking by using a rubric)
2. What went really well this week, related to my problem of practice (the 5 Traits)?
3. What changes have I seen in my students this week as we work on their problem solving packets?
4. What did I learn this week about support structure for mathematical thinking?
5. What did I learn this week about transferring the process skills into problem solving activities?

Guidelines for me to follow:
• Each day, I will take 60 seconds and jot down notes of possible things I could write about, related to my research questions (i.e., a student, a math problem, a conversation).
• I will be journaling weekly, and will set aside a specific journaling day – Fridays. Each week on my journaling day, I will choose 1-2 of the possibilities I noted, and write about them in my journal. This writing should be part description of the event and part reflection on why I chose this event, how it relates to my research question(s), and what it means to me.
• For the reflective questions, the following will help me reflect on my topic/problem of practice/research
  o How the two incidents I wrote about relate to my research questions (see above)
  o What went really well this week with the problem solving activity? What didn’t? Why?
  o What changes have I seen in my students this week as we work on their problem solving packets?
  o What did I learn this week about my research project?
  o What will I do differently next time?
• I will aim to write for approximately 30 minutes per week; 15 minutes on describing the 1-2 events, and then 15 minutes writing the reflection.
Teacher Journal Template

Week of ______________________

Problem Solving Activity for the Week: ________________________________________

• How does each of the two incidents I wrote about relate to my research questions (Teaching the Key Traits of Mathematical Thinking by using a rubric)?
• What went really well this week, related to my problem of practice (the 5 Traits)?
• What changes have I seen in my students this week as we work on their problem solving packets?
• What did I learn this week about support structure for mathematical thinking?
• What did I learn this week about transferring the process skills into problem solving activities?

Possible Journal Topics:
Monday

________________________________________________________________________

Tuesday

________________________________________________________________________

Wednesday

________________________________________________________________________

Thursday

________________________________________________________________________

Friday

________________________________________________________________________

Details of Two Events:
Part description of the event and part reflection on why you chose this event, how it relates to your research questions, and what it means to you.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
Research Questions to focus on:
- How does each of the two incidents I wrote about relate to my research questions (Teaching the Key Traits of Mathematical Thinking by using a rubric)
- What went really well this week, related to my problem of practice (the 5 Traits)?
- What changes have I seen in my students this week as we work on their problem solving packets?
- What did I learn this week about support structure for mathematical thinking?
- What did I learn this week about transferring the process skills into problem solving activities?

Reflection Questions:
1. How does each of the two incidents I wrote about relate to my research questions?

   Support Structures:

   Transferring the process skills into better mathematical thinking in the classroom:

2. What went really well this week, related to my problem of practice (the 5 traits)

3. What did I learn this week about support structure for the 5 traits and using the rubric?

4. What did I learn this week about transferring 5 Trait Work into classroom work?

5. With respect to oral explanations by the students, how are the students’ work assignments changing (if at all)? What do I attribute any differences to? What still needs work?
### Sample ORAL EXPLANATION RUBRICS by Color Group

#### Student: Green Group

<table>
<thead>
<tr>
<th>Problem: Rasning and Proof</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
<th>Score 5</th>
<th>Tot.</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>This student shows he/she can use data to make, test, or argue an idea or point of view.</td>
<td>0</td>
<td>0</td>
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<td>This student shows he/she can adequately explain the reasons behind his/her mathematical thinking.</td>
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<td>This student’s mathematical arguments are elegant.</td>
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<td>This student shows he/she knows the difference between inductive and deductive reasoning.</td>
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<td>Others would say this student uses a variety of reasoning methods and proof.</td>
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<td>I think this student shows he/she knows exactly what makes for good reasoning and proof.</td>
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<td>This student shows that listening to others’ mathematical thinking is important.</td>
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#### Student: Blue Group

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<tr>
<th>Problem: Rasning and Proof</th>
<th>Score 1</th>
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<tr>
<td>This student shows he/she can use data to make, test, or argue an idea or point of view.</td>
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<td>I think this student shows he/she knows exactly what makes for good reasoning and proof.</td>
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#### Student: Red Group

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<thead>
<tr>
<th>Problem: Rasning and Proof</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
<th>Score 5</th>
<th>Tot.</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>This student shows he/she can use data to make, test, or argue an idea or point of view.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This student shows he/she can adequately explain the reasons behind his/her mathematical thinking.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This student’s mathematical arguments are elegant.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This student shows he/she knows the difference between inductive and deductive reasoning.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Others would say this student uses a variety of reasoning methods and proof.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I think this student shows he/she knows exactly what makes for good reasoning and proof.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This student shows that listening to others’ mathematical thinking is important.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Student: Yellow Group

<table>
<thead>
<tr>
<th>Problem: Rasning and Proof</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
<th>Score 5</th>
<th>Tot.</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>This student shows he/she can use data to make, test, or argue an idea or point of view.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This student shows he/she can adequately explain the reasons behind his/her mathematical thinking.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This student’s mathematical arguments are elegant.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This student shows he/she knows the difference between inductive and deductive reasoning.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Others would say this student uses a variety of reasoning methods and proof.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I think this student shows he/she knows exactly what makes for good reasoning and proof.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>This student shows that listening to others’ mathematical thinking is important.</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Traits of Good Mathematical Thinking

- This student shows he/she can use data to make, test, or argue an idea or point of view.
- This student shows he/she can adequately explain the reasons behind his/her mathematical thinking. He/she does more than just explain the procedure or summarize the answer.
- This student’s mathematical arguments are elegant.
- This student shows he/she knows the difference between inductive and deductive reasoning and he/she can use either method.
- Others would say this student uses a variety of reasoning methods and proof.
- I think this student shows he/she knows exactly what makes for good reasoning and proof.
- This student shows that listening to others’ mathematical thinking is important.
### Traits of Good Mathematical Thinking

#### Sample Teacher Evaluation Page for Oral Presentations

**Student:**

<table>
<thead>
<tr>
<th>Representation</th>
<th>Score 1</th>
<th>Score 2</th>
<th>Score 3</th>
<th>Score 4</th>
<th>Score 5</th>
<th>Total</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>- I think this student has a list of ways to represent a problem and its solution.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- He/she uses lots of representation in expressing thinking, (words, drawings, charts or other graphs)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- This student shows he/she understands the meaning of important forms of representation.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- I think this student shows he/she can use representation to solve real world problems.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- This student shows he/she knows there’s more than one way to represent a mathematical answer.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Others can tell from the representation(s) used exactly what he/she was thinking, what he/she was trying to figure out, &amp; how the problem was solved.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- I believe this student shows he/she can move easily from one kind of representation to another.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- I believe this student knows the right or appropriate representation to use and when to use it.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- This student has a strong sense of what good representation looks like, can list the ways or can give examples of the ways to use representation.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Problem:**

**Things to work on:**

**What I liked:**

**Comments/Questions/Concerns:**
<table>
<thead>
<tr>
<th>Representation</th>
<th>Hi</th>
<th>Detailed representation to show thinking. Analyze and/or work through problem. Representation used to analyze, extend thinking, clarify and interpret information.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Medium</td>
<td>Representation used to somewhat show solution or communicate problem solving attempt.</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>No representation present or if present does not communicate what happened or connect to the solution presented.</td>
</tr>
<tr>
<td>Reasoning/Proof</td>
<td>Hi</td>
<td>Deductive arguments made with good mathematical basis. Evidence used to support systematic approach.</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>Some reasoning and evidence present. Shows attempt to use to support approach.</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>Incorrect reasoning shown or no reasoning present.</td>
</tr>
<tr>
<td>Communication</td>
<td>Hi</td>
<td>Communication approach shows organization. Is coherent and sequenced. Uses precise mathematical language and symbol notation to communicate ideas.</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>Communication of an approach or ideas present is limited. Not as clear or organized as it could be.</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>Little or no communication of an approach. Ideas unclear and communication if present is vague.</td>
</tr>
<tr>
<td>Problem Solving</td>
<td>Hi</td>
<td>Shows strategies and progress toward a solution. Adjusts, changes or alternatives considered. Evidence shown of analyzing the situation. Uses prior knowledge and application to the particular problem.</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>Some strategies present and progress toward a solution. Partially correct strategy chosen or correct strategy but only solves part of the problem.</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>Strategy is present does not lead to a solution. Little or no evidence of connecting with prior tasks and application.</td>
</tr>
<tr>
<td>Connections</td>
<td>Hi</td>
<td>Connections and observations used to extend. Builds on what is given to go beyond what’s asked in the problem itself.</td>
</tr>
<tr>
<td></td>
<td>Medium</td>
<td>Some connection to interests and experiences but the connection and/or observation is limited.</td>
</tr>
<tr>
<td></td>
<td>Low</td>
<td>No attempt at connection is made or the connection and application is severely lacking or does not make sense.</td>
</tr>
</tbody>
</table>
Connections

- I like to see how mathematical ideas are related.
- When I start a new problem, I try to ask myself, “Where have I seen a problem like this before?”
- I like to see how mathematical ideas or concepts I am learning are connected to science, social studies, language arts and other subjects.
- I see connections between problems I try to solve and the real world.
- I easily connect familiar ideas to new concepts or skills.
- I like to know when others think of a solution strategy in a different way.
### Sample Rubric of Connections Characteristics

<table>
<thead>
<tr>
<th>Connections</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shows recognition of a connection between mathematical ideas.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Understand how mathematical concepts are interrelated and build on one another</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Builds new understanding on previous knowledge</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Makes relations and connections to other subjects or real world applications</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Uses developed mathematical connections to achieve understanding</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>New ideas are generated from the old.</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Recognizes and applies mathematics in context</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>
## Traits of Good Mathematical Thinking

### APPENDIX C - Data and Results

#### Student Scores on State Assessment

<table>
<thead>
<tr>
<th>Student</th>
<th>Prob Scores</th>
<th>Totals Ability Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hi = A</td>
<td>Med = B and Low = C</td>
<td></td>
</tr>
</tbody>
</table>

#### Last Year Challenge & Dilemma Prob

<table>
<thead>
<tr>
<th>No</th>
<th>L Mower Fr. Game Cake Décor Babbling House</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>C   A   C   C   B</td>
</tr>
<tr>
<td>2</td>
<td>A   A   A   A   A</td>
</tr>
<tr>
<td>3</td>
<td>no score</td>
</tr>
<tr>
<td>4</td>
<td>A   A   A   A   A</td>
</tr>
<tr>
<td>5</td>
<td>A   C   A   A   A</td>
</tr>
<tr>
<td>6</td>
<td>no score</td>
</tr>
<tr>
<td>7</td>
<td>C   B   A   C   B</td>
</tr>
<tr>
<td>8</td>
<td>C   C   C   C   no score</td>
</tr>
<tr>
<td>9</td>
<td>C   C   C   C   no score</td>
</tr>
<tr>
<td>10</td>
<td>no score</td>
</tr>
<tr>
<td>11</td>
<td>no score</td>
</tr>
<tr>
<td>12</td>
<td>no score</td>
</tr>
<tr>
<td>13</td>
<td>no score</td>
</tr>
<tr>
<td>14</td>
<td>no score</td>
</tr>
<tr>
<td>15</td>
<td>A   A   A   A   A</td>
</tr>
<tr>
<td>16</td>
<td>no score</td>
</tr>
<tr>
<td>17</td>
<td>C   A   B   B   A</td>
</tr>
<tr>
<td>18</td>
<td>no score</td>
</tr>
<tr>
<td>19</td>
<td>C   C   A   no score</td>
</tr>
<tr>
<td>20</td>
<td>C   A   C   C   A</td>
</tr>
<tr>
<td>21</td>
<td>B   B   A   A   A</td>
</tr>
<tr>
<td>22</td>
<td>B   moved</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>L Log Score</th>
<th>Hi</th>
<th>Med</th>
<th>Low</th>
<th>No Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob 1</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Prob 2</td>
<td>9</td>
<td>3</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>Prob 3</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Prob 4</td>
<td>6</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Prob 5</td>
<td>7</td>
<td>6</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>
**Question on the Pre/Post Survey:**

*During which part of math class do you feel most involved?*

*Why do you feel more involved during this time?*

<table>
<thead>
<tr>
<th></th>
<th>C-2</th>
</tr>
</thead>
</table>

On the Pre Survey only 3 mentioned Problem Solving Work on the Traits as the time they were most involved. Ten students said homework was when they were the most involved. On the Post Survey ten mentioned Problem Solving Work on the Traits as the time they were most involved. Eleven still mentioned homework.

<table>
<thead>
<tr>
<th>PRE SURVEY ANSWERS</th>
<th>POST SURVEY ANSWERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doce- Identified on the pre survey that she was most involved during homework.</td>
<td>Doce -Identified on the post survey that she was most involved during problem solving.</td>
</tr>
<tr>
<td>Trey- Identified the regular lesson and warm-up as the time he was most involved (pre survey.)</td>
<td>Trey- Identified problem solving as the time he was most involved (post survey.)</td>
</tr>
<tr>
<td>Forman- Identified homework as the time he was most involved (pre survey.)</td>
<td>Forman- Identified problem solving packets as the time he was most involved (post survey.)</td>
</tr>
<tr>
<td>Fred- Identified homework time as when he was most involved on the pre survey.</td>
<td>Fred- Said problem solving in groups was the time he was most involved on the post survey.</td>
</tr>
<tr>
<td>Nancy- Identified checking homework as the time when she was most involved. (pre survey)</td>
<td>Nancy- Said group activities like problem solving are the times she is most involved.</td>
</tr>
<tr>
<td>Fifia- Said problem solving work and also homework are the times she is most involved. (pre survey)</td>
<td>Fifia- Identified the problem solving warmup activities and the lesson are the times she is most involved. (post survey)</td>
</tr>
</tbody>
</table>

**When asked on the post survey to explain why they are more involved during that time, students answered:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Doce- “I feel more involved during this time (problem solving) because almost everyone can share ideas of how to solve problems.”</td>
<td>Trey- “Discussing—We all present a part.”</td>
</tr>
<tr>
<td>Forman- “Problem solving because I can ask questions if I’m not.”</td>
<td>Fred- “Problem solving is the part I’m most involved. I feel more involved because people ask me questions and I know most of the answers.”</td>
</tr>
<tr>
<td>Nancy- “When we do things together or in a group. When I’m by myself I do not understand but when I’m with others it helps me understand.”</td>
<td>Fifia- “We all get to ask questions and tell about our ideas.”</td>
</tr>
<tr>
<td>Vince- “I like doing homework and problem solving.”</td>
<td>Vanessa- “Group Problem Solving because I can express my ideas.”</td>
</tr>
<tr>
<td>Una- “I feel more involved because I work with others.”</td>
<td>Sevie- “When we do problem solve because we all need to work to figure out the answer.”</td>
</tr>
<tr>
<td>Pre/Post Survey Question</td>
<td>Strongly Agree</td>
</tr>
<tr>
<td>--------------------------</td>
<td>----------------</td>
</tr>
<tr>
<td>Pre/Post Survey Question #1</td>
<td>10 2 3 1</td>
</tr>
<tr>
<td>Pre/Post Survey Question #2</td>
<td>3 10 2 4 1</td>
</tr>
<tr>
<td>Pre/Post Survey Question #3</td>
<td>10 7 1 0</td>
</tr>
<tr>
<td>Pre/Post Survey Question #4</td>
<td>6 10 2 4 1</td>
</tr>
<tr>
<td>Pre/Post Survey Question #5</td>
<td>10 7 1 0</td>
</tr>
<tr>
<td>Pre/Post Survey Question #6</td>
<td>6 10 2 4 1</td>
</tr>
<tr>
<td>Pre/Post Survey Question #7</td>
<td>10 8 1 0 0</td>
</tr>
<tr>
<td>Pre/Post Survey Question #8</td>
<td>10 8 1 0 0</td>
</tr>
<tr>
<td>Pre/Post Survey Question #9</td>
<td>8 10 4 2 0</td>
</tr>
<tr>
<td>Pre/Post Survey Question #10</td>
<td>10 8 1 0 0</td>
</tr>
<tr>
<td>Pre/Post Survey Question #11</td>
<td>5 7 2 4 1</td>
</tr>
<tr>
<td>Pre/Post Survey Question #12</td>
<td>8 8 3 0 0</td>
</tr>
</tbody>
</table>
### Traits of Good Mathematical Thinking

#### APPENDIX C- Data and Results

<table>
<thead>
<tr>
<th>Trait</th>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
<th>Problem 4</th>
<th>Problem 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Solving</td>
<td>17</td>
<td>18</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>Representation</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>Communication</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Reasoning/Proof</td>
<td>4</td>
<td>6</td>
<td>12</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>Connections</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

Traits were taught in the order above (always problem solving) and then first Representation, then Communication . . .

<table>
<thead>
<tr>
<th>Trait</th>
<th>Problem 1</th>
<th>Problem 2</th>
<th>Problem 3</th>
<th>Problem 4</th>
<th>Problem 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>TOT TRAITS ID'D on L LOG by prob</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>More than 1 or 2 w/ acceptable explan</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>At least 1/2 w/ acceptable explan.</td>
<td>11</td>
<td>10</td>
<td>9</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>No explan. or no trait id'd</td>
<td>6</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>
## APPENDIX C - Data and Results

### Totals Ability Level

2 Students were identified Advanced Ability accor to last yr's state assessment

7 were identified as Lo Ability in Math

and 3 were identified as Below Lo Ability or Very Low

<table>
<thead>
<tr>
<th>Student Scores</th>
<th>L LOG SCORES</th>
<th>Totals Ability Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>Ability Level</td>
<td></td>
</tr>
<tr>
<td>Med = B and Low = C</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hi = A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ability Level</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Last Year Challenge Dilemma Prob of Cards</th>
<th>Totals Ability Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob 1</td>
<td>Hi 5  Med 5  Low 9  No Score 2</td>
</tr>
<tr>
<td>Prob 2</td>
<td>Hi 6  Med 7  Low 6  No Score 3</td>
</tr>
<tr>
<td>Prob 3</td>
<td>Hi 8  Med 4  Low 7  No Score 3</td>
</tr>
<tr>
<td>Prob 4</td>
<td>Hi 6  Med 4  Low 9  No Score 2</td>
</tr>
<tr>
<td>Prob 5</td>
<td>Hi 5  Med 6  Low 10  No Score 0</td>
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<table>
<thead>
<tr>
<th>Mower Fr. Game Cake Decor Babbling House</th>
<th>Totals Ability Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Med = B and Low = C</td>
<td></td>
</tr>
<tr>
<td>Hi = A</td>
<td></td>
</tr>
<tr>
<td>Ability Level</td>
<td></td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>L Log Score</th>
<th>Hi</th>
<th>Med</th>
<th>Low</th>
<th>No Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob 1</td>
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<td>5</td>
<td>9</td>
<td>2</td>
</tr>
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<td>Prob 2</td>
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<td>3</td>
</tr>
<tr>
<td>Prob 3</td>
<td>8</td>
<td>4</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Prob 4</td>
<td>6</td>
<td>4</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td>Prob 5</td>
<td>5</td>
<td>6</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>Question on the Learning Log:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Of the five traits of mathematical thinking listed below, what traits do you think were the focus of the math problem this week? Why?</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** For Problem #1 we were just learning Problem Solving and Representation. By the third problem, we had talked about Problem Solving, Representation and Communication and were beginning work on Reasoning/Proof. By the fifth problem we had ‘covered’ all of the traits although Connections was last. I chose to look at problem solving identification because it was the trait we’ve spent the most time on.

<table>
<thead>
<tr>
<th>Lawn Mower Problem (#1)</th>
<th>Cake Decorating Problem (#3)</th>
<th>Babbling Brook (#4)</th>
<th>House of Cards (#5)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Overall:</strong> Five of 17 students identifying problem solving as one of the focuses did not provide any reasons.</td>
<td><strong>Overall:</strong> One of the 16 students identifying problem solving as one of the focuses did not provide any reasons.</td>
<td><strong>Overall:</strong> All 16 students identifying problem solving provided adequate reasons.</td>
<td><strong>Overall:</strong> Eighteen students identified problem solving. Sixteen had justifiable reasons. Two of the 18 did not.</td>
</tr>
</tbody>
</table>

**Ellie’s** reasoning for picking Problem Solving on Problem 1:
- “To give us a challenge.”
**Ellie’s** reasons for Problem Solving:
- “Because we have to tell how we got our answer we had to solve a problem. We have to show our work.”

**Trey’s** reason for Problem Solving:
- No reason given.
**Trey’s** reason for Problem Solving:
- “It has a lot of problem solving.”

**Teena’s** list of traits used on the first problem:
- “None of them. I kinda did good on problem solving.”
**Teena’s** reasons for identifying Problem Solving:
- “You had to figure out how many lines for every year (a new age).”

**Vanessa’s** reasons for Problem Solving:
- “Because you need to solve the problem.”
**Vanessa** -- Problem Solving:
- “Problem solving because to solve the problem.”

**Nina’s** reasons for identifying Problem Solving:
- “Because we were trying to figure out the answer.”
**Nina** -- Problem Solving:
- “Because you’re trying to figure out the answer to how many lines go onto a cake.”

**Tom’s** reasons--Problem Solving:
- “We had to solve it.”
**Tom** -- Problem Solving:
- “What idea you use for solving.”

**Sevie’s** reasons--Problem Solving:
- “Because it focuses on finding out how much half of the lawn is then they can mow the lawn.”
**Sevie** -- Problem Solving:
- “It focused on how few rafts you could use.”

Ellie’s reasoning for picking Problem Solving on Problem 1--
- “To give us a challenge.”
Ellie’s reasons for Problem Solving--
- “Because we have to tell how we got our answer we had to solve a problem. We have to show our work.”
Ellie -- Problem Solving--
- “We had to solve a challenging problem.”

Ellie -- Problem Solving--
- “We had a challenging problem to solve.”

Ellie--Problem Solving
- “We had to solve a challenging problem.”

Ellie--Problem Solving
- “We had a challenging problem to solve.”

Trey--Problem Solving
- “It had a lot of work analyzing.”
# Trey --Problem Solving
- “We had to use words.”

Teena--Problem Solving
- “You had to solve the problem.”

Teena--Problem Solving
- “You had to solve the problem.”

Vanessa -- Problem Solving
- “We had to solve the problem.”

Vanessa -- Problem Solving
- “We had to solve the problem.”

Nina-- Problem Solving
- “We were trying to find an answer.”

Nina-- Problem Solving
- “We were solving a problem.”

Tom-- Problem Solving
- “We had to solve the problem.”

Sevie --Problem Solving
- “You had to find the number of cards in 5 and 10 stories.”
### Traits of Good Mathematical Thinking

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>I think this student has a list of ways to represent a problem and its solution.</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>He/she uses lots of representation in expressing thinking, (words, drawings, charts or other graphs)</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>This student shows he/she understands the meaning of important forms of representation.</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>I think this student shows he/she can use representation to solve real world problems.</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>This students shows he/she knows there’s more than one way to represent a mathematical answer.</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Others can tell from the representation(s) used exactly what he/she was thinking, what he/she was trying to figure out, &amp; how the problem was solved.</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>I believe this student shows he/she can move easily from one kind of representation to another.</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>4</td>
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<tr>
<td>I believe this student knows the right or appropriate representation to use and when to use it.</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>This student has a strong sense of what good representation looks like, can list the ways or can give examples of the ways to use representation.</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
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<tbody>
<tr>
<td>This student expresses his/her mathematical ideas and shows he/she is not afraid of ridicule.</td>
<td>5</td>
<td>4</td>
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<tr>
<td>This student shows he/she is able to explain his/her thinking clearly and concisely.</td>
<td>5</td>
<td>4</td>
<td>4</td>
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<tr>
<td>I believe that when he/she doesn’t understand, this student shows he/she asks questions for clarification.</td>
<td>4</td>
<td>4</td>
<td>3</td>
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<tr>
<td>This student demonstrates a sense of what makes good mathematical communication.</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>4</td>
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<tr>
<td>This student shows he/she believes it is okay to struggle in math and make mistakes.</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>When others come up with an idea he/she didn’t think of or is unsure of, this student asks to explain, or tries to figure why that makes sense.</td>
<td>4</td>
<td>4</td>
<td>4</td>
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<tr>
<td>I think this student believes he/she has a good idea of what making good communication is all about.</td>
<td>4</td>
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<td>4</td>
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<thead>
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<td>5</td>
<td>5</td>
<td>5</td>
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<tr>
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<td>5</td>
<td>5</td>
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<td>5</td>
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<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
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<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Others can tell from the representation(s) used exactly what he/she was thinking, what he/she was trying to figure out, &amp; how the problem was solved.</td>
<td>4</td>
<td>5</td>
<td>5</td>
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<td>5</td>
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### APPENDIX C- Data and Results

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<tr>
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<td>Communication</td>
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(Continued)
Ellie’s Work Problem 1 Part 1

Ellie’s Work Problem 2 Part 1

Ellie’s Work Problem 1 Part 2

Ellie’s Work Problem 3 Part 1

Ellie’s Work Problem 3 Part 2
Fred’s Work 1

Traits of Good Mathematical Thinking 76

Fred’s Work 2
Fifia’s Problem 1

The original lawn that Randy and his sister have to mow is 40 feet by 80 feet. The total of the yard is 3200 square feet. Randy and his sister each have to mow half of the lawn, so each has to mow 1600 square feet. The question asked how many times Randy has to mow if the mower cuts 2 feet wide.

First, on grid paper, I made my lawn 80 by 40. I labeled it and now I am going to get to 1600 feet. Randy
is going to make 1600 feet. I used a grid and counted how many feet, and then I counted by 2 because the mower mows 2 feet at a time. Then I had to add all the numbers together. For trip one there was 464 square feet. I did the same thing for trip 2. Only the numbers were less because I had already taken some feet off know the first trip. The total for trip 2 was 432. I had to take these 2 trips and add them together to get a total of 968. Randy had not yet mowed all of this half. So I did another trip. For trip number 3, I did the same thing. But I noticed that there was a pattern. The length or width decreases by 4 every time. Also, the answers for the length and width decrease by 4. And the total for the trips decrease by 3.

I used this pattern for trip 3, and got a total of 400 square feet. I added 468 and the total of trip 142. 2896 + 400 = 2096 square feet. The answer still has to mow off the lawn 1000. I had to go another trip. I finished this 4 and broke it up and it was 358.2 feet. I then added trip 1.234 and added got 160. That was more than half the lawn by 64 feet. Then I had to go back. At that point, I showed this by coloring in that half of the lawn.

Randy took 3.95 trips or 3.94 trips.
CASTEL OF CARDS

The problem was to figure out how many cards were used to make the number of stories. I had to figure out the number of cards for 5 stories + 10 stories. Then for any number of stories.

If I figured the problem to my answer by using a formula that I got from a lecture. \(3\ln(n+1)\) I also used a chart.

For some of the easier levels (1-8), my chart showed how one side went up by 1 while the other went up by 3. You just had to add up all the numbers before it to get the total amount because the chart only shows the number of cards on that level.

So I used my formula when we went to bigger numbers so I didn’t have to add so much. I also drew a drawing.

If I figure out that you can find the # of cards for any story by using the formula \(3\ln(n+1)\) it will work for any #. 2.

I thought that this problem required a little thinking to find the formula. I also thought it was easier.

This problem is easier to other problems we have done.

Fifia’s Problem 5
House of Cards

In the problem House of Cards we want to know, if you build a house of cards 5, 10, or 20 stories high, how many cards will you need.

First, we decided to make a Pascal's Triangle. We made it 10 levels. We got some numbers, 1, 3, 6, 10, etc. We found the pattern, if you take the level (n), you get the number of cards, the formula is 1, 3, 6, 10, 15, 21.

Cake Decorating

By Jaila, Riley, Ricardo, and Addie

<table>
<thead>
<tr>
<th>Age</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td># of lines</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>21</td>
<td>28</td>
<td>36</td>
</tr>
</tbody>
</table>

Pascal's Triangle

How many lines are on a cake for any age?

First, we decided to make a drawing and a chart. Our drawing is showing how many lines are connecting the candles. We discovered the formula for the Pascal triangle is $\frac{n(n+1)}{2}$.

We figured this out according to the chart. So the answer is $\frac{n(n+1)}{2}$.

Sherelle, Morgan, Kaityn, Marcus, Mike
Determine the total number of cards to build a house with five and ten stories high. It also said to determine the total number of cards that is 'N' stories high. (n= formula) The formula for five stories is 3[(n(n+1))]. It is the same formula for ten stories. Our explanation is that we made a Pascal's Triangle and a T-Chart to show how many cards are in that story. The fifth story needed 45 cards and ten stories needed 165 cards.
Group Interview – Example Answers

Appendix D-2

Question 1
Tell me about the problem solving packets that we are doing as a class.

Forman “The problems so far were Lawn Mower Challenge and Fair Game.”

Una “We have red, white and blue folders to put our work in and sometimes notes to go to mom and dad.”

Eithia “I like them. The first one was about mowing lawns. The second one is called Fair Game. It’s about who’s going to win and who’s going to lose.”

Ted “At first it was pretty hard. Now it’s getting better. (when asked why) We have more examples.”

Vanessa agrees with Ted “Yes. Sample explanations.”

Teena “It’s still hard for me.”

Trey “Some of them are challenging.”

Sevie “Mmm. Not so bad once you get the hang of it.”

Fred “They are not what they seem at first.”

Question 2
What do you think you are learning about mathematical thinking? (if anything)

Fred “About representation - the charts and graphs - and explaining reasoning.”

Sevie It (they) makes you think.”

Trey “Some problems (in Math) are just adding or subtracting. These you have to really think about.”

Vanessa “How to explain your reasoning.”

Teena “How to prove your work.”

Ted “It gives me more explanation.”

Oches (We are) “Learning how to solve problems on my own.”

Fifia “Problem Solving and the other 5 traits (when asked what other traits she can remember she lists . . . ) Representation, Communication and Problem Solving.”
Xavier “Working on how to work in a group.”

Fortran “Learning to figure out harder problems.”

Tom “How to explain.”

Ellie “How to represent our problems.”

Question 3

*What do you like about working on the problems as a group?*

Ted “It’s easier for me. (Teena interrupts. “You get different ideas.) You get more explanation from a group.”

Vanessa (you) “Get other ideas.”

Fred “I like the fact that you can see other’s ideas and how they figure.”

Sevie “You have many ideas to choose from.”

Trey “Yes. Not just one. You can see different ideas of how to do not just how you did.”

Eithia “I like having to hear other people’s opinions on what the answer is.”

Forman “You can get help. It can get you involved.”

Fifia “I had help at home.” (Scusa note: all 3 interviewed at this session had family at home help them on the problems.) It gets the family involved.”

Tom “You get more ideas.”

Ellie “You get to see how people think.”

Nina “Knowing how to solve the problem.”

Question 4

*What is it that you do not like about working on the problems as a group?*

Forman “It takes time.”

Una “It’s hard to explain how you get an answer.”

Eithia “I do not like when I have to change my answer.”

Trey “People do not participate sometimes.”
Sevie “Yeah, they’re not trying to learn.”

Vanessa “Some (people) take over. They think they’re right.”

Ted “Or they do not do their work.”

Teena “We fight—what to do when you disagree.”

Fortran, Sithe and Oches – having to explain (language differences)

**Question 5**

*Do you think the time spent has been beneficial? If so, why?*

Teena “Yes. I explain myself more.”

Vanessa “I am better at proving my work.”

Trey “Yes because it helps you to think more and learn how to solve. You think deeper into the problem because you know you’re going to have to explain what you’re thinking.”

Eithia “I think it’s good to hear. You’re going to learn how to work around stuff.”

Fifia “Yes because we’re learning more. It will ‘keep.’ We do not just take it in and get rid of it, like some of the other things we do. Forman and Una agree.

Forman “It will help us next year.”

Tom “Yes, we’ve learning how to do different problems.”

We’re learning a different way of thinking.”

Nina (interrupts Una) “ . . . and different ways to solve problems.”

**Question 6**

*Do you think the time spent discussing and working on these weekly problems has changed the way you think about solving a problem or providing a solution? If yes, how would you describe the change? If no, why not?*

Forman “I can explain it better. I know how to tell.”

Fifia “That it’s not about getting the answer—it’s about showing your work.”

Eithia “No, because my old way is probably better.”

Sevie “Normally it’s just the answer, but these it’s more than that.”
Fred “Its more complex. It’s a formula maybe or a pattern.”

Teena “Each week they look a little bit easier.”

Ted “You learn from your mistakes on the first try.”

Vanessa “After the student examples.”

Ellie “It changed (for me.) I can show my work. I can explain my thoughts without just telling what I did.”

**Question 7**

*What would you change about the weekly setup? (if anything)*

Fred “Nothing. I like that we give our initial reaction - - you know, first get an idea, then work in group all the way to a final draft. You get several chances to do.”

Trey “You can check answers and fix before you turn it in.”

Sevie (alludes to the last answer she gave) “Like I said, it’s not about the answer. It’s about how you show it.”

Teena “Explain more.”

Vanessa “I liked the presentations this week when we got into groups.” (Scusa note: we started evaluation of oral explanations again and divided into color coded groups each with a team leader and did in small groups. When asked about the evaluation part as groups all 3 interviewed said it was okay. - - They are still not very enthusiastic about it.)

Eithia “I wouldn’t probably. (change anything)”

Forman “Nothing.” (Scusa note: See Ques 9 for #15’s answer about groups on their level.)

Fortran “More time in class to work on and more sharing (presentations). Explain more.”

**Question 8**

*Do the Five Traits of Mathematical Thinking help you solve problems?*

Fifia “Yes, I think so. We know about presenting now. (Scusa note: representation not presenting) and how to do it right.”

Forman “We know what to include.” (Scusa note: all 3 being interviewed agree that it’s the traits we’ve gone over more so than the others that they are familiar with and feel comfortable with.)

Ted “Reasoning and Proof helps me.”
Vanessa “Representation –the charts and graphs. How you show it.”

Sevie “Representation. It’s there or not there. Fred agrees. Fred “With something like communication it’s hard for some to tell you what they’re thinking but with representation you can see.”

Tom “They are all about showing your answer.”

Question 9

Are you more involved in problem solving with your team when you do or do not use the Five Traits? Why do you think that is?

Fred “When you use. We work together. It’s (the 5 traits) sort of like a checklist. It makes you more organized.”

Sevie “I like doing in groups (the color coded teams) especially if each has a part.”

Ted When you use. Vanessa interrupts. “Without it, it would be confusing. You wouldn’t know how to change it to make it (the work) right.”

Eithia “Yes, I’m going to learn more by doing it that way.”

Forman “When we do - - we know what we need to have to get a good grade.

Ellie “In the teams we can express ideas.”

Fifia “It sometimes it depends on the team. We can do that more with our friends or sometimes it helps to be in groups on our level cuz it’s harder when I’m trying to explain and they do not understand me.”

Ellie “When we use. We have to explain more.”

Nina “When we use. We get an idea of how to answer.”

Question 10

What advice can you give me for next year when I use the Five Traits of Mathematical Thinking?

Fred “Do it like you are.”

Sevie “I like doing it as a class after the individual part and you get that second chance to finish and make it better.” (Scusa note: before handing in to be graded)

Fifia “I like it when you go over it with us. I had no idea what to do for the dice problem (Scusa note: the Fair Game problem) but once you went over some of it and we discussed I
knew how to get started.” Una agrees. Forman adds, “Yeah, I know what you mean. I was getting stressed out about it.”

Una “It help to know what it should look like and ideas of different ways to do it.”

Eithia “I think you should be allowed to use a calculator when you’re doing complicated problems.”

Ted “Its pretty good right now.”

Teena “More time to work in (Scusa note: in groups)

Ted, Tina and Vanessa (when asked about groups said they do not mind it when I choose the groups as long as I change them and mix up the groups from time to time.)
Individual Interview – Example Answers

QUESTION 1

What makes math easy or difficult for you?
Forman “Math is easy when we go over the work together.”

Fifia “It makes it easy if we learn about the lesson first, and then do papers on it.”

Tom “Explaining what you know.”

Nina “Explaining what we’re going to be doing before we do it.”

Tracy “It’s a little bit of hard because I do not understand because I do not speak English”

(Scusa note: speak English)

QUESTION 2

What could teachers do to help students with math?

QUESTION 3

On average, how would you rate your involvement in math class? Why?
(1 being ‘not involved’ and a 4 being ‘very involved’)
Fifia “A 4 - - we do a lot of things aloud and together.”

Oches “A 3—some questions leave me hanging.”

Sevie “A 4- - I listen and I pay attention to how you do the problems.”

Xavier “A 4- - I help my friends.”

Fortran “A 4- - I’m kind of involved in math class.”

Ellie “A 4- - because I am involved in all discussions.”

QUESTION 4

What helps to get you involved in math class?
Forman “Doing a lot of things in partner.”

Trey “The class working together.”

Fortran “What helps me is to do is when we do it in the whole class.”

Ellie “When we have group discussions.”

Nina “Working together as a class.”
QUESTION 5

Does working on the problem solving packets together as a group, help get you involved in your learning? If so, why do you think that is?

Fifia “Yes you can tell them your ideas and they can tell you theirs.”

Sevie “Yes it does help (Scusa note: group) because instead of getting it wrong, you go over it and explain it together.”

Oches “Different kids can have different ideas.”

Forman “Yes because if I do not know something I can get help.”

Vanessa “Yes because people can explain their reasoning.”

Fred “…you get to hear other people’s thoughts and feelings on the answer.”

Ted “because it helps you when people are explaining while working. They give you more reasonings.”

Vince “In a group you can get different ideas.”

Una “Yes because you are doing stuff for people.”

Ellie “Yes because we get to learn more from different people.”

Xavier “(Yes) because they can help me and I can understand it more.”

Tom “Yes because you hear more ideas.”

Fortran “Yes because we’re talking and discussing the problems.”

Sithe “Working together that makes me do the work with (a) group.”

QUESTION 6

How do you participate in the group activity (the problem solving packets groupwork)?

Teena “I do not really work very comfortable (comfortably) A group work not ok. (large group) An individual I do not understand. A small group is better.”

Xavier “We talk about the problems.”

Tom “By sharing what we think.”

Ellie “I share my ideas by talking and showing my work.”

Tracy “Trying to help my team.”
Una “I do some of it and not let my group do it all.”

QUESTION 7
What do you think about working on the problem solving packets as a group vs. as an individual or a small group?
Sevie “I think working on it as a group is better because you get different ideas on how to do the problem.”

Vanessa “A group is better because I like to hear other people’s ideas.”

Fred “I like working as a group because everyone gets involved and usually everyone has a different way to solve the questions.”

Ted “As a group you get more reasonings and explaining. Individually- you get stuck if you do not remember it.”

Trey “As an individual, working on problem solving packets is a great way to flex my brain.”

Tracy “I think it is good as a small group.”

Nina “Small groups because we share thoughts.”

Ellie “I think solving packets as a group is more effective because we get to know different people’s thinking.”

Tom “I like as a group.”

Fortran “Thinking as a small group.”

Sithe “I think that working as a group sometimes that makes us do this homework.”

Una “It is pretty good.” (as a group)

QUESTION 8
What do you like about working on the problem solving packets as a group vs. as an individual or a small group?
Sevie “I like it because it helps us get better at problem solving.”

Vanessa “I like this because groups help me think differently.”

Ted “I like it as a group more than individual because you get more help and they explain it.”

Trey “You talk to everybody in the class.”
Fifia “I like working as a group to get the main idea and as individual or small groups you can do your own work or get into it farther.”

Teena “A (large) group really does not help. As an individual I get confused. As a small group I kinda (kind of) understand.”

Sithe “I like to work together because they help me.”

Tom “(In) group you get more things to think about but individually you only get your ideas.”

Xavier “I can learn more when my group does the work.”

Ellie “I like that we all share ideas in the group.”

Tracy “I like talking about the problem with the group.”

Una “You are getting involved in it.”

QUESTION 9
What do you dislike about working on the problem solving packets as a group vs. as an individual or a small group?
Una “If you are working with someone and then they just goof around.”

Tracy “I do not like being with girls.”

Nina “People mess around too much.”

Fortran “(I) Do not like individual because you do not talk about the answers.”

QUESTION 10
Are you confident in your math ability? Why?
Ellie “It (Math) is sometimes hard for me.”

Xavier “Yes. I am starting to understand some of the math problems.”

Trey “You get harder questions (Scusa note: in the packets) Because you get an idea.”

QUESTION 11
Are you confident in your math ability when working on the Exemplar problems in the problem solving packets? Why do you think that is?
Fred “Yes because maybe I will get the answer wrong but I get to learn something new.”

Sevie “Yes because I know I can do the problems we are given.”

Vanessa “Yes because it gets my mind working.”
QUESTION 12
Is there anything else I should know about you to better understand your problem solving in math or your general math experience?
Fred “Yeah because sometimes teachers do not push me enough and I get relaxed and do not try as hard.”
Xavier “I like to do problems where you multiply or add and stuff like that.”
Doce “I have trouble with writing about the problem.”

QUESTION 13
These are the 5 process standards for Math: (the Key Traits of Mathematical Thinking)
Representation
Problem Solving
Communication
Reasoning and Proof and
Connections
Which of these 5 traits help you most to solve problems? How?
Representation
Trey “Representation because it is on paper.”
Fred “Representation because I can easily understand graphs, charts etc…”
Vanessa “Representation because you show by charts or drawings.”
Sevie “Representation because graphs help you understand what people are thinking or trying to say.”
Una “Representation because I can show how I think.”
Fortran “Representation cause you do charts.”

Communication
Fafia “Communication helps me the most because I get to hear others’ ideas.”
Forman “Communication because I can ask my friends for help if I do not understand it.”
Xavier “Communication. I tell people the problems and they tell me if it’s right or wrong and how it’s wrong.”
Una “Communication because if people talk to you then you can get their ideas.”

Problem Solving
Doce “Problem Solving helps me because that is when I find methods for solving the problem.”
Nina “Problem Solving. I understand it more.”
QUESTION 14

*How has your work on problem solving changed this year (if at all)?*

Forman “It has changed by me being able to explain my work.”

Doce “I understand problem solving better this year.”

Sevie “It changed because I had troubles at the beginning. Now I understand it more.”

Vanessa “I show my work more.”

Fred “I am able to explain my reasoning a lot better.”

Ted “It’s worked a lot because I learned more ways in solving problems.”

Oches “It has changed ever since I keep on getting help from Ms. Scusa.”

Teena “I explain myself more and I actually proof (prove) my answer.”

Ellie “I can solve problems a little better.”

Xavier “I have gotten better and I know more math problems and how to do them.”

Tom “It got me better at math.”

Nina “Understanding it more.”

QUESTION 15

*Rate your overall work on solving problems from 1 to 5.*

1-Very Strong 2-Strong 3-Average 4-Weak 5-Very Weak