Impacts of Static Pressure Reset on VAV System Air Leakage, Fan Power and Thermal Energy
——Part I: Theoretical Model and Simulation

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Abstract
As for a variable air volume (VAV) system, the air duct static pressure is a typical control variable maintained by modulating supply fan speed. The static pressure equals to the summation of the duct pressure loss downstream of the sensor to the terminal box and box inlet static pressure. Typically, the air duct static pressure is set as a constant set point based on the system design information and sensor location. However, under partial load conditions, the terminal box dampers have to be closed more since the required airflow is less than the design airflow which directly leads to much less pressure loss. Thus the static pressure set point should be reset lower in order to reduce fan power, avoid noise at terminal box dampers and box damper malfunction due to excessive pressure. Different static pressure reset schedules are reviewed and compared, considering the influence of outside air temperature on the building load, availability of the VAV box damper positions, the airflow ratio based static pressure reset has also applicable advantages over the existing constant static pressure set point and two typical reset methods. This paper presents the theoretical models to express the impacts of static pressure reset on fan airflow, fan head, air leakage, fan power and thermal energy for both pressure independent and pressure dependent boxes. The impacts are also demonstrated using the parametric analysis and numerical results to show the benefits of the static pressure reset including reducing fan power, cooling energy and heating energy.

1. INTRODUCTION
For variable air volume system, it is typical to modulate supply fan speed to maintain the duct static pressure. Traditionally, the static pressure set point is fixed which is set to permit proper air distribution under design load [1]. This set point is the summation of the total pressure loss along the whole duct downstream and the terminal box pressure required by the manufacture under design condition. However, under partial load conditions, the terminal box dampers have to be closed more since the required airflow is less than the design airflow. According to the affinity law, the pressure loss is proportional to the square of the airflow ratio at the similarity point. So the constant static pressure set point could lead to more fan power consumption due to higher fan head and more actual airflow.

The method of supply fan control with static pressure can bring great benefits in reduction of fan power. Liu has demonstrated the fan power savings models by comparing constant static pressure set point and static pressure reset. It is also demonstrated that without static pressure reset, the design minimum airflow for pressure dependent box can not be achieved due to higher pressure before the terminal box dampers [2]. Liu et al studied the impact of low static pressure in dual-duct system on fan energy consumption [3]. Wu et.al presented the fan power saving models for both pressure independent boxes and pressure dependent boxes. Also a case study is used to demonstrate the benefits of integrated static pressure reset with fan airflow station in dual-duct VAV system control [4].

Besides the theoretical research, fan power savings due to static pressure reset is demonstrated by experiments and case studies. Significant energy savings and improved indoor comfort conditions have been measured and presented by Claridge et al [5]. Liu et al presented the impacts of VFD and static pressure reduction on energy consumption [6-8]. The impact of static air pressure on the fan power was recognized by Warren and Norford [9]. The static air pressure reset schedule was investigated by Rose and Kopko [10].
Even though the fan power saving potential of reduced static pressure has been widely acknowledged, no theoretical mathematical model has been developed to analyze the impacts of static pressure reset on thermal energy savings. Also, the impacts of static pressure reset on the air leakage reduction have not been demonstrated yet. Typical duct air leakage is summarized in Table 1 according to ASHRAE handbook [11].

This paper presents the theoretical models to express the impacts of fan airflow ratio based static pressure reset on air leakage, fan power and thermal energy for both pressure independent and pressure dependent boxes. Air leakage is considered as an important factor which contributes to the fan power and thermal energy savings. The impacts are also demonstrated using the numerical results to show the benefits of the static pressure reset including reducing fan power, cooling energy and heating energy.

Table 1 Leakage as Percentage of Airflow a,b

<table>
<thead>
<tr>
<th>Leakage</th>
<th>System Cfm per ft² Duct surface*</th>
<th>Static Pressure, in. of water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>48</td>
<td>2.5</td>
<td>12</td>
</tr>
<tr>
<td>24</td>
<td>2.5</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>2.5</td>
<td>12</td>
</tr>
</tbody>
</table>

Theoretical power and thermal energy model is given by (1)

\[ E_{fan} = \frac{C \times \alpha_m \times CFM_{f,d} \times H_f}{\eta_f} \]  (1)

Where, \( C \) = Conversion factor, 1/8507; \( CFM_{f,d} \) = Design Fan air flow; \( \eta_f \) = Fan efficiency; \( \alpha_m \) is ratio of total air flow in main duct at partial load to the total fan air flow at design condition, and is calculated by:

\[ \alpha_m = \left( \frac{CFM_{t,main} + CFM_{t,down} + CFM_{box}}{CFM_{f,d}} \right) \]

And \( H_f \) = Fan head, in. water, expressed by (2)

\[ H_f = \left( \frac{p_{x,d} + \alpha_m^2}{\alpha_{box}^2} \left( H_{f,d} - P_{s,d} \right), \right. \]

Define fan power consumption without static pressure reset as the base case, and with static pressure reset as improved case. The fan power saving ratio is the ratio of fan power consumption difference between base \( E_{fan,b} \) and improved case \( E_{fan,i} \) to the fan power consumption at design condition \( E_{fan,d} \).

\[ \Delta E_{fan} = \frac{E_{fan,b} - E_{fan,i}}{E_{fan,d}} \]  (3)

2.1 Fan Power Savings Model

The fan power calculation is to determine the numerical saving achieved by static pressure reset. In this section, two basic types of terminal box, pressure dependent and pressure independent box are considered separately. The model also examines the effect of pressure reset on terminal box minimum air flow and the resulting saving for pressure dependent box.

The typical schematic diagram of a single duct air handler unit with supply fan speed controlled by the duct static pressure is shown in Figure 1. The upstream duct of the pressure sensor is main duct, and assume no terminal box branch before the sensor.

In this kind of system, supply fan speed modulates to maintain static pressure set point. Terminal box is consisted of a modulation damper and a reheat coil.

Figure 1 Typical System with Supply Fan Speed Controlled by Duct Static Pressure

The fan power consumption is given by (1)

\[ E_{fan} = \frac{C \times \alpha_m \times CFM_{f,d} \times H_f}{\eta_f} \]  (1)
Assuming constant fan efficiency, integrate equation (1), (2) and (3), the fan power saving ratio is given by (4).

\[
\Delta \phi_{\text{fan}} = \begin{cases} 
(\alpha_{mb} - \alpha_m)^2 \phi + (\alpha_{mb} - \alpha_m) (1 - \phi) & \text{iff } \beta < \beta_{min} \\
(\alpha_{min} - \alpha_{min})^2 \phi + (\alpha_{min} - \alpha_{min}) (1 - \phi) & \text{iff } \beta_{min} < \beta < \beta'_{min} \\
(\alpha_{min} - \alpha_{min})^2 \phi + (\alpha_{min} - \alpha_{min}) (1 - \phi) & \text{iff } \beta < \beta'_{min} 
\end{cases}
\]

(4)

Where, \( \phi = \frac{P_{s,d}}{H_{f,d}} \); \( \alpha_{mb} \) and \( \alpha_{m,i} \) = air flow ratios in main duct in base and improved case respectively, and refer to Appendix I for detailed deduction ; \( \alpha_{box} = \frac{\text{CFM}_{box}}{\text{CFM}_{box,design}} \), which is determined only by building load ratio \( \alpha_{m,mb} \) and \( \alpha_{m,mi} \) are air flow ratio in main duct when minimum air flow ratio is reached in base and improved case respectively.

2.1.2 Pressure dependent box

When pressure dependent boxes are used, if the static pressure set point remains constant, the actual minimum air flow ratio \( \alpha'_{min,box} \) is higher than the design minimum air flow ratio \( \alpha_{min,box} \) due to excessive pressure on terminal damper, while if optimal static pressure reset is used, the design minimum air flow ratio can be achieved, and therefore, fan power consumption in the latter case can be reduced due to reduced air flow. See Fig. 2 for relationship of design, actual minimum air flow and the corresponding building load ratio. This section will numerically study the fan power saving when load ratio is decreasing from design to zero.

\[
\Delta \phi_{\text{fan}} = \begin{cases} 
(\alpha_{mb} - \alpha_m)^2 \phi + (\alpha_{mb} - \alpha_m) (1 - \phi) & \text{iff } \beta < \beta_{min} \\
(\alpha_{min} - \alpha_{min})^2 \phi + (\alpha_{min} - \alpha_{min}) (1 - \phi) & \text{iff } \beta_{min} < \beta < \beta'_{min} \\
(\alpha_{min} - \alpha_{min})^2 \phi + (\alpha_{min} - \alpha_{min}) (1 - \phi) & \text{iff } \beta < \beta'_{min} 
\end{cases}
\]

(5)

### Table 2: Relationship of building load, terminal air flow ratio and main duct air flow ratio

<table>
<thead>
<tr>
<th>Building Load Ratio</th>
<th>( \beta &lt; \beta_{min} )</th>
<th>( \beta_{min} &lt; \beta &lt; \beta'_{min} )</th>
<th>( \beta'_{min} &lt; \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Terminal box air flow ratio</td>
<td>Base case ( \alpha_{box,b} )</td>
<td>( \alpha'<em>{min,box} = \frac{\text{CFM}</em>{min,box}}{\text{CFM}_{box,design}} )</td>
<td>( \alpha_{box} = \frac{\text{CFM}<em>{box}}{\text{CFM}</em>{box,design}} )</td>
</tr>
<tr>
<td>Improved case ( \alpha_{box,i} )</td>
<td>( \alpha_{min,box} = \frac{\text{CFM}<em>{min,box}}{\text{CFM}</em>{box,design}} )</td>
<td>( \alpha_{box} = \frac{\text{CFM}<em>{box}}{\text{CFM}</em>{box,design}} )</td>
<td>( \alpha_{box} = \frac{\text{CFM}<em>{box}}{\text{CFM}</em>{box,design}} )</td>
</tr>
<tr>
<td>Air flow ratio in main duct</td>
<td>Base case ( \alpha_{m,b} )</td>
<td>( \alpha'<em>{min,m} = \frac{\text{CFM}</em>{min,m}}{\text{CFM}_{fan,design}} )</td>
<td>( \alpha_{m,b} = \frac{\text{CFM}<em>{m,b}}{\text{CFM}</em>{fan,design}} )</td>
</tr>
<tr>
<td>Improved case ( \alpha_{m,i} )</td>
<td>( \alpha_{m,mi} = \frac{\text{CFM}<em>{min,design}}{\text{CFM}</em>{fan,design}} )</td>
<td>( \alpha_{m,i} = \frac{\text{CFM}<em>{m,i}}{\text{CFM}</em>{fan,design}} )</td>
<td>( \alpha_{m,i} = \frac{\text{CFM}<em>{m,i}}{\text{CFM}</em>{fan,design}} )</td>
</tr>
</tbody>
</table>

2.2 Thermal Energy Saving Model

Cooling and heating energy saving can be achieved with static pressure reset. This section develops the numerical models to calculate the savings. The saving models developed here are based on system with open plenum for air return. The air flow chart is shown in Fig. 3. The impact of radiant and convective load from lighting and transformers, conduction load from roofs, or glazing are neglected when calculate temperature in ceiling return air plenum.
Figure 3 Flow Chart for Typical HVAC System

### 2.2.1 Cooling Energy Saving

Cooling energy required will reduce due to reduced fan air flow and reduced fan power. For simplicity, only the first reason is considered when calculate reduction in cooling energy.

The model considers using economizer in the system. When outside air temperature is not suitable for free cooling, it is defined as non-economizer mode, and mechanical cooling has to be used. When outside air is suitable for free cooling, it is defined as economizer mode, which can be subdivided into partial free cooling mode and total free cooling mode. Therefore, the cooling energy saving model have to consider two modes: 1. partial free cooling mode; 2. non-economizer mode. The building load ratio at which the switchover happens is defined as \( \beta_c \).

The cooling energy can be calculated by (6)

\[
E_c = 60 \times \rho \times CFM_m (h_{ma} - h_{sa})
\]

Where, \( h_{ma} \) and \( h_{sa} \) are mixed air and supply air enthalpy respectively.

Define base case is the case without static pressure reset, and improved case is with static pressure reset. The energy saving ratio is the ratio of cooling consumption difference between the two cases to design cooling consumption.

\[
\Delta \Phi_c = \frac{E_{c,b} - E_{c,i}}{E_{c,d}}
\]

In partial free cooling mode, for pressure independent box, the energy saving ratio can be calculated as equation (8), and for pressure dependent box, equation (9) can be used.

\[
\Delta \Phi_c = \left\{ \begin{array}{ll}
\left( \alpha_{m,\min,i} - \alpha_{m,\min,j} \right) \times (h_{ma} - h_{sa}) & \text{if} \beta_c \leq \beta_{\min}\vspace{2mm} \\
(h_{ra} - h_{sa,d}) & \text{if} \beta_c > \beta_{\min}
\end{array} \right.
\]

(8)

In non-economizer mode, outside air intake is maintained at the minimum ratio, and mixed with return air. The return air in air handler unit (AHU) is actually mixture of return air from building and air leaking in supply duct, as is shown in Fig 3. Based on this analysis, mixed air enthalpy in AHU can be expressed by equation (10).

\[
h_{ma} = \alpha_m \times \alpha_{\min,oa} \times h_{oa} + (1 - \alpha_m) \times (\alpha_m \times h_{oa} + \alpha_{\min,oa} \times (h_{rn} - h_{oa}))
\]

(10)

Where, \( \alpha_{\min,oa} = \text{minimum outdoor air ratio} = \frac{CFM_{\min,oa}}{CFM_m} \)

The energy saving ratio for pressure independent box can be calculated by (11), and (12) for pressure dependent box.

\[
\Delta \Phi_c = \left\{ \begin{array}{ll}
\left( \alpha_{m,b} (h_{ma} - h_{rn}) - \alpha_{m,i} (h_{ma} - h_{rn}) \times \frac{h_{ra} - h_{ma}}{h_{ra} - h_{rn,d}} \right) & \text{if} \beta_c > \beta_{\min}\vspace{2mm} \\
(h_{rn} - h_{rn,d}) & \text{if} \beta_c \leq \beta_{\min}
\end{array} \right.
\]

(11)

\[
\Delta \Phi_c = \left\{ \begin{array}{ll}
\left( \alpha_{m,\min,b} (h_{ma} - h_{rn}) - \alpha_{m,\min,i} (h_{ma} - h_{rn}) \times \frac{h_{rn} - h_{rn,d}}{h_{rn} - h_{rn,d}} \right) & \text{if} \beta_c > \beta_{\min}\vspace{2mm} \\
(h_{rn} - h_{rn,d}) & \text{if} \beta_c \leq \beta_{\min}
\end{array} \right.
\]

(12)

Where, \( h_{ma}, h_{rn} \) can be obtained by substituting \( \alpha_m \) in (10) by \( \alpha_{m,b} \) and \( \alpha_{m,i} \) respectively.

\( h_{ma} \) can be obtained by substituting \( \alpha_{\min,box} \) by \( \alpha_{\min,box} \) and \( \alpha_m \) by \( \alpha_{m,\min,b} \) and \( \alpha_{m,\min,i} \) respectively in (10).

### 2.2.2 Heating Energy Saving

Heating energy saving can be achieved when required air flow ratio is lower than actual minimum air flow ratio. The heating energy consumption saving comes out from two parts: 1) since the leakage cools down return plenum above the ceiling, this will result in lower room load ratio that reheat energy has to increase for compensation; 2) For pressure dependent box, higher actual minimum air flow...
increase reheat energy; for pressure independent box, there is no saving in this part.

Saving by Reduced Leakage

The increased heating is equal to transferred heat, which can be calculated by:

\[ E_{h,1} = Q_p = U \times A \times (T_{room} - T_p) \]  

\[ E_{h,1} = \frac{\rho \cdot C_F \cdot M_{box}}{A} \]  

Where, \( T_p \) = Temperature in above the ceiling = \( T_{sa} + \frac{\alpha_{box,min} \cdot (1 - \lambda) \cdot (T_{room} - T_{sa})}{\alpha_{m,min}} \).

\[ T_{room} = \text{room air temperature} \]

The heating energy saving ratios are given by (14) and (15) for pressure independent and dependent box respectively:

\[ \Delta E_{h,1} = \begin{cases} 0, & \text{if} \beta_{min} < \beta \\ \varepsilon \cdot 1 \cdot \left( 1 - \lambda \right) \cdot \left( \alpha_{box, min} \cdot \left( 1 - \lambda \right) \cdot \left( \alpha_{box, min} - \alpha_{box, min,b} \right) \right), & \text{if} \beta_{min} < \beta_{min}' \leq \beta \\ \varepsilon \cdot 1 \cdot \left( 1 - \lambda \right) \cdot \left( \alpha_{box, min} \cdot \left( 1 - \lambda \right) \cdot \left( \alpha_{box, min} - \alpha_{box, min,b} \right) \right) \beta_{min}' < \beta \end{cases} \]

\[ (14) \]

\[ \Delta E_{h,1} = \begin{cases} 0, & \text{if} \beta_{min} < \beta \\ \varepsilon \cdot 1 \cdot \left( 1 - \lambda \right) \cdot \left( \alpha_{box, min} \cdot \left( 1 - \lambda \right) \cdot \left( \alpha_{box, min} - \alpha_{box, min,b} \right) \right), & \text{if} \beta_{min} < \beta_{min}' \leq \beta \\ \varepsilon \cdot 1 \cdot \left( 1 - \lambda \right) \cdot \left( \alpha_{box, min} \cdot \left( 1 - \lambda \right) \cdot \left( \alpha_{box, min} - \alpha_{box, min,b} \right) \right) \beta_{min}' < \beta \end{cases} \]

\[ (15) \]

\[ \Delta E_{h,2} = \begin{cases} \alpha_{box, min} - \alpha_{box} \cdot \frac{1}{1 - \lambda}, & \text{if} \beta_{min} < \beta \leq \beta_{min}' \\ \alpha_{box, min} - \alpha_{box, min,b} \cdot \frac{1}{1 - \lambda}, & \text{if} \beta < \beta_{min} \end{cases} \]

\[ (17) \]

\[ \Delta E_{h,2} = \Delta E_{h,1} + \Delta E_{h,2} \]

\[ (19) \]

3. PARAMETRIC ANALYSIS

3.1 Fan Power Savings

Before working on the parametric analysis, several assumptions have been set up based on the general single duct VAV system either with pressure independent boxes (PIB) or pressure dependent boxes (PDB).

Impact of pressure ratio \( \phi \)

The parameter assumptions are shown in Table 3. Fig.4 (a) and (b) show the potential fan power savings with the static pressure reset for the pressure independent box and pressure dependent box respectively. Pressure ratio is defined as \( \frac{P_{s,d}}{H_{f,d}} \), which physically reflects the location of the pressure sensor if the design value is ideally chosen. As can be seen, with the increasing of \( \phi \), the fan power saving ratio also increases, and it remains constant when building load ratio is less than the designed minimum airflow ratio. Compared to PIB, PDB achieve the same saving before actual minimal building load ratio is reached, and could be much higher after that.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \alpha_{min,box} )</th>
<th>( \lambda )</th>
<th>( x )</th>
<th>( y )</th>
<th>( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.7</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Table3: Parameter assumptions

Figure .4 (a) Impact of \( \phi \) for PIB
Impact of Minimal Air Flow Ratio ($\alpha_{min}$)

The parameter assumptions are shown in Table 4. Figure 5 (a) and (b) show the impact of $\alpha_{min}$ on the fan power savings for the pressure independent box and pressure dependent box respectively. When $\alpha_{min}$ equals the design airflow ratio, there is no fan power savings. The largest fan power saving occurs when the $\alpha_{min}$ is about 55% of the design air flow.

Table 4 Parameter assumptions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\phi$</th>
<th>$\lambda$</th>
<th>$x$</th>
<th>$y$</th>
<th>$\omega$ (PDB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.5</td>
<td>0.1</td>
<td>0.3</td>
<td>0.7</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Impact of Leakage Ratio $\lambda$

The parameter assumptions are shown in Table 5. Figure 6 shows that the higher the system leakage, the higher the fan power savings. The potential of fan power savings with the static reset is the lowest if the system is ideal without leakage.

Table 5: Parameter assumptions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\alpha_{min,box}$</th>
<th>$x$</th>
<th>$y$</th>
<th>$\phi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.3</td>
<td>0.3</td>
<td>0.7</td>
<td>0.5</td>
</tr>
</tbody>
</table>
3.2 Cooling Energy Savings

3.2.1 Cooling Energy Savings during Partial Free Cooling Mode

Impact of hoa

The parameter assumptions are shown in Table 6. Figure 7 shows that the higher the outside air enthalpy, the more the cooling energy savings during partial free cooling mode.

Table 6 Parameter assumptions

<table>
<thead>
<tr>
<th>(\alpha_{\text{min,box}})</th>
<th>(\lambda)</th>
<th>(x)</th>
<th>(y)</th>
<th>hsa</th>
<th>hra</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.7</td>
<td>22Btu/lbma</td>
<td>26Btu/lbma</td>
</tr>
</tbody>
</table>

Impact of Leakage Ratio \(\lambda\)

The parameter assumptions are shown in Table 8. Figure 9 shows the relation between the potential cooling energy savings and the outside air enthalpy. For non-economizer mode, when the corresponding building load ratio is below the design minimum air flow ratio, the maximum potential cooling energy savings can be achieved. Also, Figure 8 shows that the higher the hoa, the more the cooling energy savings during non-economizer mode.

Table 8 Parameter assumptions

<table>
<thead>
<tr>
<th>(\alpha_{\text{min,box}})</th>
<th>(\alpha_{\text{min,oa}})</th>
<th>(\lambda)</th>
<th>(x)</th>
<th>(\varphi)</th>
<th>hsa</th>
<th>hra</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>0.3</td>
<td>0.5</td>
<td>22</td>
<td>26</td>
</tr>
</tbody>
</table>
Figure 9 (a) Impact of hoa for PIB

Figure 10 (b) Impact of λ on cooling savings for PIB

Impact of Minimal Outside Air Intake Ratio

Figure 11 shows the relationship between the potential cooling energy savings and the minimum outside air ratio, which is usually determined by building function. The maximum cooling energy savings has been achieved when the building load ratio is below the level of the designed minimum air flow ratio. Building with higher the minimum outside air intake has more the cooling energy savings during non-economizer mode.

Table 10 Parameter assumptions

<table>
<thead>
<tr>
<th>αmin,box</th>
<th>αmin,oa</th>
<th>φ</th>
<th>λ</th>
<th>x</th>
<th>Hsa</th>
<th>hra</th>
<th>Hoa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.2</td>
<td>0.5</td>
<td>0.3</td>
<td>22</td>
<td>26</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

Figure 10(a) Impact of λ on cooling savings for PIB

Figure 11 (a) Impact of Min outside Air Ratio for PIB
3.3 Heating Energy Savings

3.3.1 Pressure Independent Box

Impact of Leakage Ratio $\lambda$

The parameter assumptions are shown in Table 11. Figure 12 shows that the higher the total duct leakage ratio, the more the heating energy savings. No savings exist if duct work has no leakage at ideal conditions.

Table 11 Parameter assumptions

<table>
<thead>
<tr>
<th>$\alpha_{min,box}$</th>
<th>$\phi$</th>
<th>$x$</th>
<th>$y$</th>
<th>$\epsilon$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>0.3</td>
<td>0.7</td>
<td>0.04</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figure 12 Impact of $\lambda$ on Heating for PIB

Impact of design minimal air flow $\alpha_{min,box}$

The parameter assumptions are shown in Table 12. Figure 13 shows that the impact of $\alpha_{min,box}$ on the heating energy savings. When $\alpha_{min,box}$ equals the design airflow ratio, there is no heating energy savings. The largest heating energy saving occurs when the $\alpha_{min,box}$ is about 50% to 60% of the design airflow.

Table 12 Parameter assumption

<table>
<thead>
<tr>
<th>$\alpha_{min,box}$</th>
<th>$\lambda$</th>
<th>$x$</th>
<th>$y$</th>
<th>$\epsilon$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.7</td>
<td>0.04</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figure 13 Impact of $\alpha_{min,box}$ for PIB

3.3.2 Pressure Dependent Box

Impact of Leakage Ratio $\lambda$

The parameter assumptions are shown in Table 13. Figure 14 shows that the change of the total duct leakage ratio does not have much impact on the heating energy saving.

Table 13 Parameter Assumptions

<table>
<thead>
<tr>
<th>$\alpha_{min,box}$</th>
<th>$\phi$</th>
<th>$x$</th>
<th>$\epsilon$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>0.5</td>
<td>0.3</td>
<td>0.04</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figure 14 Impact of $\lambda$ on Heating for PDB

Impact of design minimal air flow $\alpha_{min,box}$

The parameter assumptions are shown in Table 14. Figure 15 shows that the impact of $\alpha_{min,box}$ on the heating energy savings. When $\alpha_{min,box}$ equals the design airflow ratio, there is no heating energy...
savings. The largest heating energy saving occurs when the $\alpha_{\text{min,box}}$ is about 50% to 60% of the design air flow.

Table 14 Parameter assumptions

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\varphi$</th>
<th>$\lambda$</th>
<th>$\chi$</th>
<th>$\epsilon$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.5</td>
<td>0.1</td>
<td>0.3</td>
<td>0.04</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figure 15 Impact of $\alpha_{\text{min,box}}$ on Heating for PDB

Impact of Airflow/ Area Ratio $\zeta$

The parameter assumptions are shown in Table 15. Figure 16 show that $\zeta$ does not have much impact on the heating energy savings.

Table 15 Parameter assumptions

<table>
<thead>
<tr>
<th>$\varphi$</th>
<th>$\alpha_{\text{min,box}}$</th>
<th>$\lambda$</th>
<th>$\chi$</th>
<th>$\epsilon$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.3</td>
<td>0.1</td>
<td>0.3</td>
<td>0.04</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Figure 16 Impact of $\zeta$ on Heating for PDB

4. CONCLUSIONS

The theoretical fan power saving and thermal energy saving model by implementing static pressure reset are presented in this paper. For pressure independent box, the fan power saving is due to reduced fan head, reduced air leakage, and for pressure dependent box the saving also due to the realization of design minimum air flow. In the aspect of thermal energy consumption, the saving comes from reduced duct leakage and for pressure dependent box also comes from reduced minimum air flow. The influences of different parameters on the saving have been studied.

In the aspect of fan power saving, the numerical analysis results show that for system which has higher excessive static pressure design set point, more saving can be achieved by pressure reset, and under the simulation assumption, an average of 15% of fan power saving can be achieved with ideal system with no leakage, and 50% more saving can be achieved for system with 30% leakage at design condition. Also, the largest fan power saving occurs when the terminal box minimum air flow is about 55% of the design air flow for both Pressure dependent and pressure independent box. For pressure dependent box, highest saving happens when load ratio is lower than design minimum air flow ratio, and this is because of the greatly reduced minimum air flow.

Cooling energy consumption has been mostly impacted by system leakage ratio. For building which has higher fresh air requirement, it can expect higher cooling energy saving. And in both partial free cooling and non-economizer mode, the higher the outside air enthalpy, the more cooling energy saving.

Heating energy saving happens only when required air flow ratio is lower than design minimum air ratio for pressure independent box or lower than actual minimum flow ratio for pressure dependent box. The system leakage ratio has great impact on heating energy saving for pressure independent box, but slight influence on pressure dependent box. Terminal box with minimal air flow ratio at 50% to 60% has the largest heating energy saving.

In the second part of this paper, a simulated air handling unit (AHU) system in Omaha, NE is used to demonstrate the energy savings performance in one typical climate year.

NOMENCLATURE

$E_{\text{fan}} = \text{fan power}$

$E_c = \text{Cooling Energy}$

$E_h = \text{Heating Energy}$

$CFM = \text{air flow rate, ft}^3/\text{min}$
ACKNOWLEDGEMENTS

This paper is initiated from homework from our graduate class. The enthusiastic teamwork and collaboration of the authors is greatly appreciated. Special thanks are also extended to Lixia Wu who is dedicated to organizing conduct of this research and this paper.

REFERENCES

Appendix I Deduction of Air Flow Ratio in Main Duct

(A) Without Static Pressure Reset

When the building load ratio is $\beta$, the corresponding air flow ratio through terminal box is $\alpha_{box}$, defined as the corresponding air flow rate over design terminal box air flow rate.

$$CFM_f = CFM_{f,main} + CFM_{f,down} + \alpha_{box}CFM_{box,d}$$ (I-1)

According to ASHRAE handbook the air leakage can be calculated by

$$CFM_l = \delta \times \Delta p_s^N$$ (I-2)

Assuming $\Delta p_s$ is the average of duct static pressure, $N=0.5$, building pressure is zero, then the leakage at downstream duct can be expressed by

$$\frac{(p_{s,d} + 0)/2}{(p_{s,d} + 0)/2} = CFM_{f,down} = 1$$ (I-3)

Therefore, the air leakage at the downstream duct is the same with design condition.

The ratio of total air flow in main duct at partial load to the total air flow in main duct at design condition can be expressed as:

$$\alpha_m = \frac{CFM_{f,main} + CFM_{f,down,d} + \alpha_{box}CFM_{box,d}}{CFM_{f,d}}$$ (I-4)

The leakage at main duct can be expressed by

$$\frac{(p_{s,d} + H_f)/2}{(p_{s,d} + H_f)/2} = CFM_{f,main,d} = \frac{CFM_{f,main}}{\lambda x CFM_{f,d}}$$ (I-5)

Where,

$$x = \frac{CFM_{f,main,d}}{CFM_{f,d}} = \text{ratio of air leakage in main duct to the total air leakage at design condition}$$

$$y = \frac{CFM_{f,down,d}}{CFM_{f,d}} = \text{ratio of leakage in downstream to the total air leakage at design condition}$$

$$\lambda = \frac{CFM_{f,d}}{CFM_{f,d}} = \text{ratio of total air leakage to total fan air flow at design condition}$$

$H_f =$ the fan head at partial load without static pressure reset, which can be calculated

$$H_f = p_{s,d} + \alpha_m^2 (H_{f,d} - P_{s,d})$$ (I-6)

Introduce $\phi = \frac{P_{s,d}}{H_{f,d}}$, we have

$$\lambda \sqrt{\frac{2\phi + \alpha_m^2 (1-\phi)}{(1+\phi)/2}} = \frac{CFM_{f,main}}{CFM_{f,d}}$$ (I-7)

Introduce $A = y\lambda + \alpha_{box} (1-\lambda)$, we have

$$(1-x^2\lambda^2 \frac{1-\phi}{1+\phi})\alpha_m^2 - 2A\alpha_m + A^2 - 2\lambda^2\lambda^2\frac{\phi}{1+\phi} = 0$$ (I-8)

Therefore, $\alpha_m$ the ratio of total air flow in main duct at partial load to the total air flow in main duct at design condition can be calculated as:

$$\alpha_m = \frac{X + \sqrt{Y^2 - 4XZ}}{2X}$$ (I-9)

Where, $X = (1-x^2\lambda^2 \frac{1-\phi}{1+\phi})$ , $Y = 2A = 2\times[y\lambda + \alpha_{box} (1-\lambda)]$ , $Z = A^2 - 2\lambda^2\lambda^2\frac{\phi}{1+\phi}$

(B) With Static Pressure Reset

In ideal condition, when the building load ratio is $\beta$, the corresponding air flow ratio through terminal box is $\alpha_{box}$ the static pressure will be reset to

$$P_s = \alpha_{box}^2 P_{s,d}$$ (I-10)

The leakage at downstream duct can be expressed by

$$\frac{(\alpha_{box}P_{s,d} + 0)/2}{(P_{s,d} + 0)/2} = \frac{CFM'_{f,down}}{CFM_{f,main}}$$ (I-11)

The ratio of total air flow in main duct at partial load to the total air flow in main duct at design condition can be expressed as:

$$\alpha_m = \frac{CFM_{f,main} + \alpha_{box}CFM_{f,down,d} + \alpha_{box}CFM_{box,d}}{CFM_{f,d}}$$ (I-12)

The fan head at partial load with static pressure reset, which can be calculated

$$H_f = \alpha_{box}^2 P_{s,d} + \alpha_m^2 (H_{f,d} - P_{s,d})$$ (I-13)

Using the similar way as without static pressure set point reset, and introduce $A' = \alpha_{box}(1-x\lambda)$, $A' = \alpha_{box}(1-\lambda)$

$$\alpha_m = \frac{Y' + \sqrt{Y'^2 - 4X'Z'}}{2X'}$$ (I-15)

Where, $X' = (1-x^2\lambda^2 \frac{1-\phi}{1+\phi})$ , $Y' = 2A' = 2\times[\alpha_{box} (1-\lambda)]$ , $Z' = A'^2 - 2\lambda^2\lambda^2\frac{\phi}{1+\phi}$

Therefore, $\alpha_m$ can be calculated as:

$$\alpha_m = \frac{Y + \sqrt{Y^2 - 4XZ}}{2X}$$ (I-16)
Appendix II Actual Minimal Air Flow Deduction for Pressure Dependent Box

Without static pressure reset, the actual minimum air flow ratio in the down stream of pressure sensor can be expressed as

$$\alpha'_{\text{min,down}} = \frac{\text{CFM}_{\text{min,f,down}} + \text{CFM}_{\text{min,box}}'}{\text{CFM}_{\text{f,down,d}} + \text{CFM}_{\text{box,d}}} \quad (\text{II-1})$$

Equation (1) can be further simplified with aspect to flow ratio:

$$\alpha'_{\text{min,down}} = \frac{y\lambda + (1 - \lambda)\alpha'_{\text{min,box}}}{y\lambda + (1 - \lambda)} \quad (\text{II-2})$$

The actual static pressure before the box can be expressed as

$$P_{\text{box}} = P_s - \left(\alpha'_{\text{min,down}}\right)^2 \times (P_s - P_{\text{box,d}}) \quad (\text{II-3})$$

we have

$$P_{\text{box}} = P_s \left[\frac{y\lambda + (1 - \lambda)\alpha'_{\text{min,box}}}{y\lambda + (1 - \lambda)}\right]^2 \times (P_s - P_{\text{box,d}})$$

(II-4)

The ratio of actual terminal minimum air flow ratio to design terminal minimum air flow ratio can be expressed by

$$\frac{\alpha'_{\text{min,box}}}{\alpha_{\text{min,box}}} = \sqrt{\frac{P_{\text{box}}}{P_{\text{box,d}}}} \quad (\text{II-5})$$

We have

$$\frac{\alpha'_{\text{min,box}}}{\alpha_{\text{min,box}}} = \sqrt{\frac{P_s \left[\frac{y\lambda + (1 - \lambda)\alpha'_{\text{min,box}}}{y\lambda + (1 - \lambda)}\right]^2 \times (P_s - P_{\text{box,d}})}{P_{\text{box,d}}}}$$

(II-6)

The actual minimum terminal air flow can be calculated by:

$$\alpha'_{\text{min,box}} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad (\text{II-7})$$

Where,

$$a = \left[(y\lambda + 1 - \lambda)^2 + (\omega - 1)\times (1 - \lambda)^2 \times \alpha_{\text{min,box}}^2\right], \text{ in}$$

which $\omega = \frac{P_{s,d}}{P_{\text{box,d}}}$

$$b = 2y\lambda(\omega - 1)(1 - \lambda)\alpha_{\text{min,box}}^2$$

$$c = \left[2y\lambda\omega(1 - \lambda) + \omega(1 - \lambda)^2 + y^2\lambda^2\right]\alpha_{\text{min,box}}^2$$

When the terminal air flow is $\alpha'_{\text{min,box}}$, the total air flow ratio in the main duct is:

$$\alpha'_{\text{min}} = \frac{Y'_{\text{min}} + \sqrt{Y'_{\text{min}}^2 - 4X'_{\text{min}}Z'_{\text{min}}}}{2X'_{\text{min}}} \quad (\text{II-8})$$

Where, $X'_{\text{min}} = (1 - x^2\lambda^2\frac{1 - \phi}{1 + \phi})$

$$Y'_{\text{min}} = 2\alpha'_{\text{min}}\frac{2\alpha'_{\text{min}}}{2\alpha'_{\text{min}} + \alpha'_{\text{min,box}}(1 - \lambda)}$$

$$Z'_{\text{min}} = (\alpha'_{\text{min}})^2 - \frac{2x^2\lambda^2\phi}{1 + \phi}$$