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Optimization of Orthogonal Polyphase Spreading Sequences for Wireless Data Applications

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***Abstract*- In this paper, we propose a simple but efficient method for optimizing correlation properties of polyphase spreading sequences for asynchronous DS CDMA applications. The proposed method can be used to minimize the mean square value of aperiodic crosscorrelation or the mean square value of aperiodic autocorrelation, the maximum value of aperiodic crosscorrelation functions, merit factor or other properties of the sequence set. The important feature of the method is that when applied to orthogonal sequences, it modifies correlation properties of the sequence set, while preserves their orthogonality for perfect synchronization**

I. INTRODUCTION

Walsh-Hadamard bipolar spreading sequences are generally used for channel separation in direct sequence code division multiple access (DS CDMA) systems, e.g. [1]. They are easy to generate, and orthogonal [2] in the case of perfect synchronization. However, the crosscorrelation between two Walsh-Hadamard sequences can rise considerably in magnitude if there is a non-zero delay shift between them [3]. Unfortunately, this is very often the case for up-link (mobile to base station) transmission, due to the differences in the corresponding propagation delays. As a result, significant multi-access interference (MAI) [4] occurs which needs to be combated either by complicated multi-user detection algorithms [5], or reduction in bandwidth utilization.

Another possible solution to this problem can be use of orthogonal complex valued polyphase spreading sequences, like those proposed in [6], which for some values of their parameters can exhibit a reasonable compromise between autocorrelation and crosscorrelation functions. However, in most cases the choice of the parameters is not simple. In addition, improving one of the characteristics is usually associated with a significant worsening of the others [7].

In the paper, we propose a method to optimize correlation properties of polyphase sequences which allows to use standard optimization techniques, like the Nelder-Meade simplex search [8] being implemented in several mathematical software packages, e.g. MATLAB. By using a standard optimization technique, one can choose the penalty function in a way, which takes to account all the important correlation characteristics. The numerical example shows application of the method to optimize properties of the orthogonal sequence set of the length 31 from the family of sequences proposed in [6]. The results show that significant changes in sequence characteristics can be achieved.

The paper is organized as follows. In Section II, we introduce the method used later to optimize correlation characteristics of the spreading sequences. Section III introduces optimization criteria, which can be used for DS CDMA applications. The numerical example of optimization applied to orthogonal polyphase sequences is given in Section IV and Section V concludes the paper.

II. MODIFICATION METHOD

Sets of spreading sequences used for DS CDMA applications can be represented by MxN matrices $\mathbf{S}_{MN}$, where M is the number of sequences in the set and N is the sequence length. The sequences are referred to as orthogonal sequences if, and only if the matrix $\mathbf{S}_{MN}$ is orthogonal, i.e.

$$\mathbf{S}_{MN}\mathbf{S}_{MN}^{T} = k\mathbf{I}_{M} \tag{1}$$

where k is a constant, $\mathbf{S}_{MN}^{T}$ is the transposed matrix $\mathbf{S}_{MN}$, and $\mathbf{I}_M$ is an MxM unity matrix.

There are a few families of orthogonal spreading sequences proposed in literature, e.g. [2], [6], [9], [10], and [11]. Out of them, the most commonly applied are Walsh-Hadamard sequences. Some of the proposed sequence families are designed in a parametric way, which allows for manipulation of parameters to change the desired correlation characteristics. However, those changes are usually of a limited magnitude, and very often while improving the crosscorrelation functions, a significant worsening of the autocorrelation functions is experienced, e.g. [7].

Here, we propose to modify correlation properties of the set of orthogonal spreading sequences by multiplying the matrix $\mathbf{S}_{MN}$ by another orthogonal $N \times N$ matrix $\mathbf{D}_N$. Hence, the new set of spreading sequences is represented by a matrix $\mathbf{W}_{MN}$

$$\mathbf{W}_{MN} = \mathbf{S}_{MN}\mathbf{D}_{N}\,. \tag{2}$$

The matrix $\mathbf{W}_{MN}$ is also orthogonal, since:

$$\mathbf{W}_{MN}\mathbf{W}_{MN}^T = \mathbf{S}_{MN}\mathbf{D}_N(\mathbf{S}_{MN}\mathbf{D}_N)^T = \mathbf{S}_{MN}\mathbf{D}_N\mathbf{D}_N^T\mathbf{S}_{MN}^T \quad (3)$$

and because of the orthogonality of matrix $\mathbf{D}_N$, we have

$$\mathbf{D}_N\mathbf{D}_N^T = c\mathbf{I}_N \quad (4)$$

where c is a real constant. Substituting (4) into (3) yields

$$\mathbf{W}_{MN}\mathbf{W}_{MN}^T = c\mathbf{S}_{MN}\mathbf{I}_N\mathbf{S}_{MN}^T = c\mathbf{S}_{MN}\mathbf{S}_{MN}^T = kc\mathbf{I}_M . \quad (5)$$

In addition, if $c = 1$, then the sequences represented by the matrix $\mathbf{W}_{MN}$ are not only orthogonal, but possess the same normalization as the original sequences represented by the matrix $\mathbf{S}_{MN}$. However, other correlation properties of the sequences defined by $\mathbf{W}_{MN}$ can be significantly different to those of the original sequences.

To this point, it is not clear how to chose the matrix $\mathbf{D}_N$ to achieve the desired properties of the sequences defined by the $\mathbf{W}_{MN}$. In addition, there are only a few known methods to construct the orthogonal matrices, such as those used for the Hadamard matrices [2]. However, another simple class of orthogonal matrices are diagonal matrices with their elements $d_{m,n}$ fulfilling the condition:

$$|d_{m,n}| = \begin{cases} 0 & \text{for} \quad m \neq n \\ c & \text{for} \quad m = n \end{cases}; \quad m, n = 1, \ldots, N \quad (6)$$

To preserve the normalization of the sequences, the elements of $\mathbf{D}_N$, being in general complex numbers, must be of the form:

$$d_{m,n} = \begin{cases} 0 & \text{for} \quad m \neq n \\ \exp(j\phi_m) & \text{for} \quad m = n \end{cases}; \quad m, n = 1, \ldots, N \quad (7)$$

where the phase coefficients ϕ_m; $m = 1, 2, \ldots, N$, are real numbers taking their values from the interval $[0, 2\pi)$, and $j^2 = -1$. The values of ϕ_m; $m = 1, 2, \ldots, N$, can be optimized to achieve the desired correlation and/or spectral properties, e.g. minimum out-off-phase autocorrelation or minimal value of peaks in aperiodic crosscorrelation functions.

III. Optimization Criteria

In order to compare different sets of spreading sequences, we need a quantitative measure for the judgment. Therefore, we introduce here some useful criteria, which can be used for such a purpose. They are based on correlation functions of the set of sequences, since both the level of multiaccess interference and synchronization amiability depend on the crosscorrelations between the sequences and the autocorrelation functions of the sequences, respectively. There are, however, several specific correlation functions that can be used to characterize a given set of the spreading sequences [4], [7], [12].

One of the first detailed investigations of the asynchronous DS CDMA system performance was published in 1969 by Anderson and Wintz [13]. They clearly demonstrated in their paper the need for considering the aperiodic crosscorrelation properties of the spreading sequences. Since that time, many additional results have been obtained (e.g. [4] and [14]), which helped to clarify the role of aperiodic correlation in asynchronous DS CDMA systems.

For general polyphase sequences $\{s_n^{(i)}\}$ and $\{s_n^{(l)}\}$ of length N, the discrete aperiodic correlation function is defined as [12]:

$$c_{i,k}(\tau) = \begin{cases} \frac{1}{N}\sum_{n=0}^{N-1-\tau} s_n^{(i)}[s_{n+\tau}^{(l)}]^*, & 0 \le \tau \le N-1 \\ \frac{1}{N}\sum_{n=0}^{N-1+\tau} s_{n-\tau}^{(i)}[s_n^{(l)}]^*, & 1-N \le \tau < 0 \\ 0, & |\tau| \ge N \end{cases} \quad (8)$$

where $[\bullet]^*$ denotes a complex conjugate operation. When $\{s_n^{(i)}\} = \{s_n^{(l)}\}$, (8) defines the discrete aperiodic autocorrelation function.

In DS CDMA systems, we want to have the maximum values of aperiodic crosscorrelation functions and the maximum values of out-of-phase aperiodic autocorrelation functions as small as possible, while the merit factor as great as possible for all of the sequences used.

The bit error rate (BER) in a multiple access environment depends on the modulation technique used, demodulation algorithm, and the signal-to-noise power ratio (SNR) available at the receiver. Pursley [4] showed that in case of a BPSK asynchronous DS CDMA system, it is possible to express the average SNR at the receiver output of a correlator receiver of the ith user as a function of the average interference parameter (AIP) for the other K users of the system, and the power of white Gaussian noise present in the channel. The SNR for ith user, denoted as SNR_i, can be expressed in the form:

$$\mathrm{SNR}_i = \left(\frac{N_0}{2E_b} + \frac{1}{6N^3}\sum_{\substack{k=1 \\ k \neq i}}^{K} \rho_{k,i} \right)^{-0.5} \quad (9)$$

where E_b is the bit energy, N_0 is the one-sided Gaussian noise power spectral density, and $\rho_{k,i}$ is the AIP, defined for a pair of sequences as

$$\rho_{k,i} = 2\mu_{k,i}(0) + \mathrm{Re}\{\mu_{k,i}(1)\} \qquad (10)$$

The crosscorrelation parameters $\mu_{k,i}(\tau)$ are defined by:

$$\mu_{k,i} = N^2 \sum_{n=1-N}^{N-1} c_{k,i}(n)[c_{k,i}(n+\tau)]^* \qquad (11)$$

However, following the derivation in [15], $\rho_{k,i}$ for polyphase sequences may be well approximated as:

$$\rho_{k,i} \approx 2N^2 \sum_{n=1-N}^{N-1} |c_{k,i}(n)|^2 \qquad (12)$$

In order to evaluate the performance of a whole set of M spreading sequences a, the average mean-square value of crosscorrelation for all sequences in the set, denoted by R_{CC}, was introduced by Oppermann and Vucetic [7] as a measure of the set crosscorrelation performance:

$$R_{CC} = \frac{1}{M(M-1)} \sum_{i=1}^{M} \sum_{\substack{k=1 \\ k \neq i}}^{M} \sum_{\tau=1-N}^{N-1} |c_{i,k}(\tau)|^2 \qquad (13)$$

A similar measure, denoted by R_{AC} was introduced in [7] for comparing the autocorrelation performance:

$$R_{AC} = \frac{1}{M} \sum_{i=1}^{M} \sum_{\substack{\tau=1-N \\ \tau \neq 0}}^{N-1} |c_{i,i}(\tau)|^2 \qquad (14)$$

The measure defined by (14) allows for comparison of the autocorrelation properties of the set of spreading sequences on the same basis as the crosscorrelation properties.

For DS CDMA applications we want both parameters R_{CC} and R_{AC} to be as low as possible [7]. Because these parameters characterize the whole sets of spreading sequences, it is convenient to use them as the optimization criteria in design of new sequence sets. Therefore, we will use them for optimizing the values of the phase coefficients ϕ_m; m = 1, 2, ..., N, in the considered numerical examples. We will also look into the maximum value of aperiodic crosscorrelation functions since this parameter is very important when the worst-case scenario is considered.

IV. OPTIMIZATION OF ORTHOGONAL POLYPHASE SEQUENCES

Oppermann and Vucetic introduced in [7] a new family of polyphase spreading sequences. The elements $u_n^{(k)}$ of these sequences $\{u_n^{(k)}\}$ are given by:

$$u_n^{(k)} = (-1)^{kn} \exp\left[\frac{j\pi(n^m k^p + k^s)}{N}\right], \qquad 1 \leq n \leq N \quad (15)$$

where k can take integer values being relatively prime to N such that $1 \leq k < N$, and the parameters m, p, s can take any real values. They showed there that depending on the choice of the parameters m, p, and s the sequences could have a wide range of the correlation properties. However, no clear method for selecting the appropriate values of the parameters depending on the desired correlation characteristics was given in [7]. Later in [6], Oppermann showed that the sequences defined by (16) were orthogonal if p = 1 and m is a positive nonzero integer.

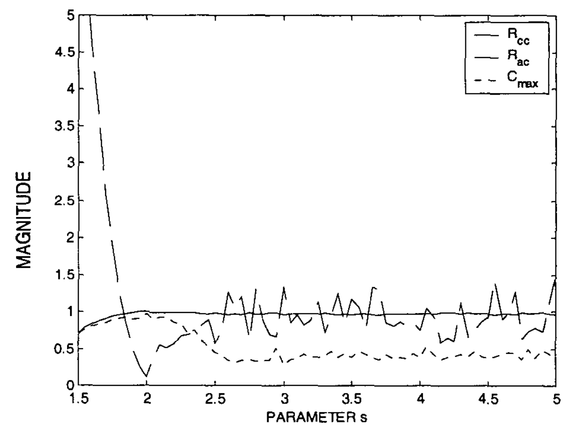

Figure 1: Plots of the values of R_{CC}, R_{AC}, and $C_{\max}$ as functions of the parameter s for the sequences $\{u_n^{(k)}\}$, with N = 31, p = 1, m = 1, and k = 1, 2, ..., 30.

In this section, we apply the developed method to optimize the properties of the spreading sequence set belonging to the family defined by (15), with N = 31, p = 1, and m = 1. Since N is a prime number, k can take any nonzero integer value lower than 31, i.e. k = 1, 2, ..., 30, and the maximum number of sequences in the set is 30. To select the appropriate value for s, we plotted in Fig. 1 the values of R_{CC}, R_{AC} and the value of the maximum peak in all aperiodic crosscorrelation functions $C_{\max}$ as the functions of s. From the plots we choose s = 2.5, for which R_{CC} = 0.9803, R_{AC} = 0.5713, and $C_{\max}$ = 0.4546.

To illustrate the modification method, we first applied it to minimize the value of R_{CC} for this set of sequences. The optimization was performed using the standard '*fmin*' function of MATLAB [8] with the optimized function being $R_{CC}(\mathbf{\Phi})$, where

$$\mathbf{\Phi} = [\phi_m; \quad m = 1,2,\ldots,31] \qquad (16)$$

and the phase coefficients ϕ_m; $m = 1, 2, \ldots, 31$, being used to define the elements of the modification matrix $\mathbf{D}_N$ (see Eqn.(7)).

The function $R_{CC}(\mathbf{\Phi})$ is very irregular and may have several local minima. Therefore, depending on the starting point, different local minima can be reached. To illustrate this, we selected randomly 6 vectors $\mathbf{\Phi}_1, \mathbf{\Phi}_2, \ldots, \mathbf{\Phi}_6$, and performed the optimization for each of them chosen as a starting point. The results of the obtained values of R_{CC}, R_{AC} and C_{max} are given in Table 1.

Table 1: Values of R_{CC}, R_{AC} and C_{max} obtained for the sequences optimized to achieve minimum of R_{CC}.

	R_{CC}	R_{AC}	C_{max}
$\mathbf{\Phi}_{1,opt}$	0.3262	18.1835	0.3262
$\mathbf{\Phi}_{2,opt}$	0.3199	18.9537	0.3199
$\mathbf{\Phi}_{3,opt}$	0.3774	17.6342	0.3334
$\mathbf{\Phi}_{4,opt}$	0.3583	18.3313	0.3233
$\mathbf{\Phi}_{5,opt}$	0.3990	17.8249	0.3561
$\mathbf{\Phi}_{6,opt}$	0.4314	16.8953	0.3872

Next we repeat the procedure, this time optimizing the sequences to obtain minimum value of R_{AC}, and finally to achieve the minimum value of C_{max}. The results of R_{CC}, R_{AC} and C_{max} are given in Table 2 and Table 3, respectively.

Table 2: Values of R_{CC}, R_{AC} and C_{max} obtained for the sequences optimized to achieve minimum of R_{AC}.

	R_{CC}	R_{AC}	C_{max}
$\mathbf{\Phi}_{1,opt}$	0.9965	0.1006	0.5310
$\mathbf{\Phi}_{2,opt}$	0.9974	0.0749	0.3795
$\mathbf{\Phi}_{3,opt}$	0.9969	0.0910	0.3774
$\mathbf{\Phi}_{4,opt}$	0.9963	0.1081	0.3583
$\mathbf{\Phi}_{5,opt}$	0.9974	0.0762	0.3990
$\mathbf{\Phi}_{6,opt}$	0.9972	0.0824	0.4314

Table 3: Values of R_{CC}, R_{AC} and C_{max} obtained for the sequences optimized to achieve minimum of C_{max}.

	R_{CC}	R_{AC}	C_{max}
$\mathbf{\Phi}_{1,opt}$	0.9743	0.7475	0.2817
$\mathbf{\Phi}_{2,opt}$	0.9592	1.1846	0.2871
$\mathbf{\Phi}_{3,opt}$	0.9763	0.6872	0.2855
$\mathbf{\Phi}_{4,opt}$	0.9748	0.7323	0.2933
$\mathbf{\Phi}_{5,opt}$	0.9720	0.8141	0.2781
$\mathbf{\Phi}_{6,opt}$	0.9692	0.8940	0.2730

Comparison of the results listed in Table 1, Table 2, and Table 3, indicates that the best compromise amongst the values of R_{CC}, R_{AC} and C_{max} were obtained while minimizing the value of C_{max}.

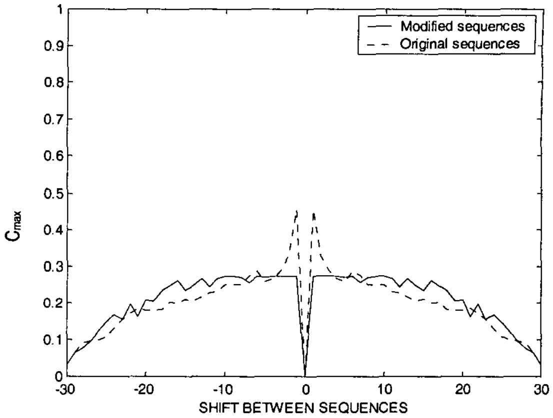

Figure 2: Plots of the maximum peaks in the crosscorrelation functions versus the relative shift between the sequences, $C_{max}(\tau)$.

To show that the optimized sequences are still orthogonal, in Fig. 2, we plotted the function $C_{max}(\tau)$ for the example sequences $\{w_n^{(k)}\}$ obtained from the original sequences $\{u_n^{(k)}\}$. For the comparison, we plotted there also the function $C_{max}(\tau)$ for the original sequences $\{u_n^{(k)}\}$ using a dashed line. It is clearly visible that both sets of sequences are orthogonal, and the values of $C_{max}(\tau)$ are significantly lower for the new sequence set than for the original one around zero, which corresponds to the point of perfect synchronization.

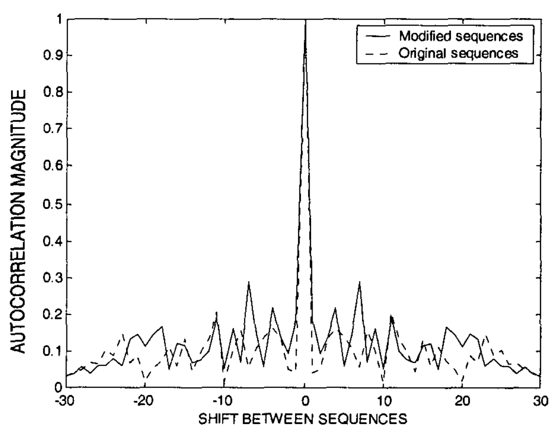

Figure 3: Plots of the maximum magnitudes of the autocorrelation functions, $A_{max}(\tau)$.

To compare the synchronization amiability of the original sequences $\{u_n^{(k)}\}$ and these new sequences $\{w_n^{(k)}\}$, we plotted the maximum magnitudes of the autocorrelation functions $A_{max}(\tau)$

$$A_{max}(\tau) = \max_i c_{i,i}(\tau), \qquad i = 1,2,...,31 \qquad (17)$$

for both sets of sequences in Figure 3. It is possible to notice that the maxima in the off-peak autocorrelation are slightly higher for the set of sequences $\{w_n^{(k)}\}$ than for the set of sequences $\{u_n^{(k)}\}$. However, in both cases the peak at zero shift, corresponding to the perfect synchronization, is very significant.

V. CONCLUSIONS

In the paper we presented a simple method to modify orthogonal spreading sequences to improve their correlation properties for asynchronous applications, while maintaining their orthogonality for perfect synchronization. The method leads, in general, to the complex polyphase sequences but can also be used to obtain real bipolar sequences. In the case of polyphase sequences, the phase coefficients can be optimized to achieve the required correlation/spectral properties of the whole set of sequences. The presented numerical example illustrated how different correlation characteristics can be successfully modified or even optimized for the set of polyphase orthogonal sequences. Of course, different optimization criteria can be used depending on the particular application.

REFERENCES

[1] R.Steele: "Introduction to Digital Cellular Radio," in R.Steele and L.Hanzo (eds), "Mobile Radio Communications, " 2nd ed., IEEE Press, New York, 1999.

[2] H.F. Harmuth: "Transmission of Information by Orthogonal Functions," Springer-Verlag, Berlin, 1970.

[3] B.J.Wysocki, T.A.Wysocki: "Orthogonal Binary Sequences with Wide Range of Correlation Properties," ISCTA'01, Ambleside, Lake District, U.K.

[4] M.B.Pursley: "Performance Evaluation for Phase-Coded Spread-Spectrum Multiple-Access Communication – Part I: System Analysis," IEEE Trans. on Commun., vol. COM-25, pp. 795-799, 1977.

[5] A.Duel-Hallen, J.Holtzman, and Z.Zvonar: "Multiuser Detection for CDMA Systems," IEEE Personal Communications, April 1995, pp. 46-58.

[6] I.Oppermann: "Orthogonal Complex-Valued Spreading Sequences with a Wide Range of Correlation Properties," IEEE Trans. on Commun., vol. COM-45, pp.1379-1380, 1997.

[7] I.Oppermann and B.S.Vucetic: "Complex spreading sequences with a wide range of correlation properties," IEEE Trans. on Commun., vol. COM-45, pp.365-375, 1997.

[8] MATLAB: "*Reference Guide*," The Math Works, 1996.

[9] A.W.Lam and S.Tantaratana: "Theory and applications of spread-spectrum systems," IEEE/EAB Self-Study Course, IEEE Inc., Piscataway, 1994.

[10] B.J.Wysocki: "*Signal Formats for Code Division Multiple Access Wireless Networks,*" PhD Thesis, Curtin University of Technology, Perth, Western Australia, 1999.

[11] J.E.Hershey, G.J.Saulnier and N.Al-Dhahir: "New Hadamard basis," Electronics Letters, Vol.32, No.5, Feb. 1996, pp.429-430.

[12] P.Fan and M.Darnell: "*Sequence Design for Communications Applications,*" John Wiley & Sons, New York, 1996.

[13] D.R.Anderson and P.A.Wintz: "Analysis of a spread-spectrum multiple-access system with a hard limiter," IEEE Trans. on Commun. Techn., vol. COM-17, pp.285-290, 1969.

[14] J.L.Massey and J.J.Uhran: "Sub-baud coding," Proc. of the Thirteenth Annual Allerton Conference on Circuit and System Theory, pp.539-547, October 1975.

[15] K.H.Kärkkäinen: "Mean-square cross-correlation as a performance measure for spreading code families," Proc. IEEE 2nd Int. Symp. on Spread Spectrum Techniques and Applications (ISSSTA'92), pp.147-150.