Educative Supports for Teachers in Middle School Mathematics Curriculum Materials: What is Offered and How is It Expressed?

Lorraine Males

University of Nebraska-Lincoln, lmales2@unl.edu

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Abstract for DBER Group Discussion on 2013-04-04

Presenter, Department(s):
Lorraine Males
Department of Teaching, Learning, and Teacher Education
University of Nebraska-Lincoln

Title:
Educative Supports for Teachers in Middle School Mathematics Curriculum Materials: What is Offered and How is It Expressed?

Abstract:
In this talk I will present the findings from my study of the educative nature of four middle school mathematics curriculum materials. Educative curriculum materials are materials for Grades K-12 that are “intended to promote teacher learning in addition to students” (Davis & Krajcik, 2005, p. 3). I particularly investigated the opportunities for teacher learning embedded within four middle school curricular series in the areas of introduction to variable and geometric transformations. Drawing on the work on educative curriculum materials from science education (Davis & Krajcik, 2005) and Systemic Functional Linguistics (Halliday, 1985), I investigated opportunities to learn by examining the content presented (i.e., subject-matter, pedagogical content, curricula) and certain aspects of the voice (i.e., use of personal pronouns, modality) of teachers’ guides. Results indicated that although all four curricular series included content supports, these supports may be insufficient. In addition, the ways in which authors spoke to teachers in the written text may further hinder teachers’ opportunities to learn. Implications for curriculum development, teacher education, and research will be discussed.
Educative Supports for Teachers in Middle School Mathematics Curriculum Materials: What is Offered and How is it Expressed?

Lorraine M. Males
Mathematics Education
Teaching, Learning, and Teacher Education
Presentation Agenda

• A Bit About Me
• Motivation & Background
• Methods
• Results
• Discussion & Next Steps
• Questions
A Bit About Me

Bay Shore, NY:
Where I grew up and attended K-12 and decided I wanted to be a teacher.

Ithaca, NY:
Received my B.S. in Mathematics & Computer Science with 7-12 Teacher Certification. Did student teaching in Ithaca and HS in Harlem.

Teaching Job #1:
Moanalua High School, Honolulu, HI [Public School] Taught: 9-12

Teaching Job #2:
The Wilbraham & Monson Academy, Wilbraham, MA [Private Boarding] Taught: 6-12

Teaching Job #3:
The Dwight School, Englewood, NJ [Private] Taught: 9

Graduate Work:
Michigan State University East Lansing, MI Ph.D., mathematics education 2007-2011

Assistant Professor:
UNL

Graduate Work:
Teachers College New York, NY Ed.D. Program, 1 semester

Ph.D.:
Michigan State University East Lansing, MI Mathematics Education 2007-2011

### Why I wanted to do this study

Everyone knows that the world's population is increasing. 

<table>
<thead>
<tr>
<th>Year</th>
<th>Estimated Population</th>
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<td>1990</td>
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<tr>
<td>1995</td>
<td>5,730,000,000</td>
</tr>
</tbody>
</table>

### Small World, Isn't It?

**Mathematical Context**

- Applying the derivative
- Defining the derivative
- Defining the derivative at a point

**In Class**

1. **Discussion of Homework 13:**
   **The Growth of the Oil Slick**

   Have spade card students present their work on different parts of the assignment. The concept of an instantaneous rate of change may be harder for students to visualize in this context than it was in the context of speed (for instance, in *The Instant of Impact*).

   Have students sketch (by hand) a graph of the area (as a function of time) in order to get an intuitive sense of the instantaneous growth rate. Have them mark line segments on the graph to show the relevant values. For instance, the graph here indicates the segments needed to find the rate of change in the area for the time interval from $t = 0$ to $t = 2$. (A larger version of this graph, without the fine segments, is included in Appendix B.)
Why the field needs this study

Teacher learning is critical to success (Cohen, Raudenbush, & Ball, 2002; Putnam & Borko, 2000), especially with the implementation of the Common Core Standards (Confrey, 2010; Lappan, McCallum, Kepner, 2010). We know little about the features of written curriculum materials, let alone how these materials may impact teacher learning (Stein, Remillard, & Smith, 2007).

Curriculum materials can and do learn from using curriculum materials (Choppin, 2008; Collopy, 2003; Drake & Sherin, 2009; Lloyd, 2008; Remillard, 2000; Remillard & Bryans, 2004; Schneider, Krajcik, & Blumenfeld, 2005; Schneider & Krajcik, 2002; Schneider, 2006; Van Zoest & Bohl, 2002).
Background – Curriculum and Teachers

Figure 1: Visual representation of the variables that influence the enacted curriculum.

- **Factors that influence OFFICIAL GOALS & CURRICULUM**
  - E.g., Needs of society,
  - Advances in mathematics, understanding of learning, technology;
  - Policies, assessment practices and constraints;
  - Values and beliefs about mathematics goals of education

- **Factors that Influence TEACHER INTENDED CURRICULUM**
  - E.g., knowledge, beliefs, practices, access to resources and support, knowledge of students and local context

- **Factors that Influence INSTRUCTIONAL MATERIALS**
  - E.g., Market forces, professional societies, learning research, curricular goals, developers' vision

- **Designated Curriculum**
  - Curricular Goals
  - Content of Consequential Assessments (curriculum & norm referenced)

- **Instructional Materials**
  - Teacher Intended Curriculum
  - Enacted Curriculum

- **Student Learning**

---

Factors that Influence ENACTED CURRICULUM
- E.g., Teacher and student knowledge, beliefs, practices, access to resources, contextual resources and constraints

Factors that Influence STUDENT LEARNING
- E.g., Prior knowledge, identity, attitude, motivation, home support, comfort

Adopted Selected Incorporated
The **teacher** plays an active role in enacting the curriculum.

“The [the curriculum’s] primary value, its primary indication, is for the teacher, not for the child. It says to the teacher: Such and such are the capacities, the fulfillments, in truth and beauty and behavior, open to these children. Now see to it that day by day the conditions are such that their own activities move inevitably in this direction, toward such culmination of themselves. (Dewey, 1902, p. 39).”

“Its [the curriculum’s] primary value, its primary indication, is for the teacher, not for the child. It says to the teacher: Such and such are the capacities, the fulfillments, in truth and beauty and behavior, open to these children. Now see to it that day by day the conditions are such that their own activities move inevitably in this direction, toward such culmination of themselves. (Dewey, 1902, p. 39).”

The **curriculum** plays a role in teacher learning.

“If it (new curriculum) cannot change, move, perturb, inform teachers, it will have no effect on those they teach. It must first and foremost be a curriculum for teachers. If it has any effect on pupils, it will have it by virtue of having an effect on teachers” (Bruner, 1977, p. xv).
Educative Curriculum Materials

To be educative (serve as a source of learning for teachers), curriculum materials must:
→ be more than just a textbook for students

Content must include
→ *Enactment* Guidance

→ *Rationale* Guidance

Expression must
→ allow for the text to speak through teachers

(Ball & Cohen, 1996; Beyer et al., 2009; Li, 2004; Remillard, Watanabe, 2003)

**Enactment Guidance Example**
"In the case of tables, one has to decide which values of the independent variable should be represented in the table to give most informative results" (CMP, Variables & Patterns, p. 3).

**Rationale Guidance Example**
"Unlike the combination chart, notebook notation can be used to solve problems involving combinations of more than two kinds of items. In addition, notebook notation can be used to record the new combinations. Students should come to appreciate the advantages of notebook notation over guess-and-check strategies and combination charts" (MiC, Comparing Quantities, p. 22B).
Focus of Study - Research Questions

1. What is the relative frequency of educative supports in middle school mathematics curriculum materials for teachers’ subject matter content knowledge, pedagogical content knowledge for mathematics topics and practices, and curricular knowledge?

2. How do middle school mathematics curriculum materials speak to teachers (i.e., what language choices do they make) through the written text in the teachers’ guides?

3. How does opportunity to learn (content and expression) differ for introduction to variable and geometric transformations?
A Note about my Purpose

Curriculum Development is difficult

All of what authors intend does not always make it to the final printed page

My purpose is **NOT** to name the curriculum with the best teacher support, but to **describe opportunities to learn** and to **raise awareness** of how content and expression may influence these opportunities.
METHOD
Sample- Choice of Curricular Series & Content

Curricular Series
• Connected Mathematics Project (CMP)
• Math Connects (Glencoe)
• Mathematics in Context (MiC)
• Transition Mathematics (UCSMP)

Mathematical Content
• Introduction to Variable
• Geometric Transformations
Analytic Framework – Capturing Content

Support for understanding the mathematics curriculum, including how topics relate to each other and the features and philosophies of materials

Codes
- Subject-matter Content
  - E/R - Experiences
  - E - Possible Student Pitfalls

Codes
- E - Using Questions (w/sug answers)
- E - Using Questions (w/o sug answers)
- R - Questions
- E - Helping Students Ask/Answer Own ?s
- E/R - Justification, Reasoning, & Proof
- E/R - Developing Math Terminology
- E/R - Participation Structures

Adapted from Beyer et al. (2009)
Analytic Framework – Capturing Expression

“Voice” (Herbel-Eisenmann, 2007; Morgan, 1996)
→ helps to identify “how speakers shift positions, identities, and alignments toward the words they speak as well as toward one another” (Herbel-Eisenmann, p. 347).

Systemic Functional Linguistics (Halliday, 1985)
→ Strata & Realizations

→ Metafunctions
  → interpersonal
  → ideational
  → textual

Martin & Rose, 2007, p. 5
Analytic Framework – Capturing Expression

Personal Pronouns
→ first person – “I” and “we”
→ second person – “you”

“You”-forms
→ implicit - imperatives
→ explicit - “you” + modal verbs

You should emphasize that the value could be found by solving an equation (UCSMP, Some Imp Geom Ideas, p. 397)

You might suggest that they imagine a point leaving a trail (CMP, KHM, p. 53)

Explain that the function rule describes the relationship between... (Glencoe, Algebra: Functions, p. 47)

"the degree of likelihood, probability, weight, or authority a speaker attaches to the utterance" (Hodge & Kress, 1993) and can be negotiated on a scale that describes "how obliged" you are to act (Martin & Rose, 2007)
Example of coded page (MiC)

Content → SMK
→ PCK-Topics
→ PCK-Practices
→ CK

Expression → imperatives
→ “you”+modal verbs
# Inter-rater Reliability - Content

<table>
<thead>
<tr>
<th>Variable</th>
<th>Transformations</th>
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<tbody>
<tr>
<td>CMP</td>
<td>85</td>
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<tr>
<td>Glencoe</td>
<td>88</td>
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<td>MiC</td>
<td>85</td>
</tr>
<tr>
<td>UCSMP</td>
<td>91</td>
</tr>
</tbody>
</table>
# Inter-rater Reliability - Expression

<table>
<thead>
<tr>
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<th>Personal Pronouns</th>
<th>Imperatives</th>
<th>Modal Verbs</th>
<th>Transformations</th>
<th>Personal Pronouns</th>
<th>Imperatives</th>
<th>Modal Verbs</th>
</tr>
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<td></td>
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<tr>
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<td>96</td>
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<td></td>
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<tr>
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<td>96</td>
<td>100</td>
<td>100</td>
<td>96</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
TOP LEVEL RESULTS
Structure of the Teachers’ Guides

Did not exist in Glencoe and UCSMP

Embedded vs. Non-embedded
Content – Knowledge by Category

Subject Matter Knowledge
Pedagogical Content Knowledge for Topics
Pedagogical Content Knowledge for Practices
Curricular Knowledge

Bar chart showing the comparison of different categories of knowledge for various subjects and topics.
<table>
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<tr>
<th></th>
<th>Variable</th>
<th>Translations</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Enactment Guidance</td>
<td>Rationale Guidance</td>
</tr>
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<td>3%</td>
</tr>
<tr>
<td>Glencoe</td>
<td>99%</td>
<td>1%</td>
</tr>
<tr>
<td>MiC</td>
<td>94%</td>
<td>6%</td>
</tr>
<tr>
<td>UCSMP</td>
<td>94%</td>
<td>6%</td>
</tr>
</tbody>
</table>
## Content - Location of Support

<table>
<thead>
<tr>
<th></th>
<th>Variable</th>
<th></th>
<th>Transformations</th>
<th></th>
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<td>85%</td>
<td>12%</td>
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<tr>
<td>MiC</td>
<td>24%</td>
<td>14%</td>
<td>62%</td>
<td>16%</td>
</tr>
<tr>
<td>UCSMP</td>
<td>19%</td>
<td>N/A</td>
<td>81%</td>
<td>18%</td>
</tr>
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</table>
## Expression – Frequencies of Pronouns

<table>
<thead>
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<th></th>
<th>Transformations</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>we</td>
<td>you</td>
<td>we</td>
<td>you</td>
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<tr>
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<td>25</td>
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<tr>
<td>MiC</td>
<td>0</td>
<td>61</td>
<td>3</td>
<td>129</td>
</tr>
<tr>
<td>UCSMP</td>
<td>20</td>
<td>33</td>
<td>17</td>
<td>38</td>
</tr>
</tbody>
</table>
Clear referents for “we”

“we” refers to curriculum authors

Functions:

1. Make suggestions or recommendations for teachers
   (e.g., "We recommend that...," "We encourage...")

2. State assumptions/choices without rationale
   (e.g., "We introduce...," "We ask students...," "We assume...")

3. State assumptions/choices with rationale (e.g., "We start with tables because...," "We have purposely chosen...because)
Expression – Use of “we”

Unclear referent for “we”

“we” is used in questions or statements, particularly when outlining solutions or procedures

Some possible functions:

→ Bring authors into the community of the discipline of mathematics

→ Bring authors into the mathematical classroom community

B. When $\triangle 2$ is translated angles are preserved, so $\angle W' = \angle W$. This makes $\overline{W'X'}$ parallel to $\overline{W''X''}$. The angle in the gap triangle marked $X'''$ must be congruent to $X'$ (alternate interior angles). We already know that $\overline{W''''X'''}$ is congruent to $\overline{W''X''}$ since translations preserve length. Thus we know that the gap triangle is congruent to the original triangle, by Side-Angle-Side using $\overline{W''X''}$ as the other side.

Method 2:

- Rotate $\triangle 1$ to $\triangle 2$.
- Translate $\triangle 2$ to $\triangle 3$.
- Translate $\triangle 1$ the distance of $\overline{WX}$ to fill the gap.

The question is now, “Can we be sure that the last transformation will exactly fill the gap?” Here are brief sketches of two ways we might reason about this.

A. When $\triangle 2$ is translated to $\triangle 3$ by a distance of $\overline{WV}$, point $X'$ is moved to $X''$. So this distance is congruent to $\overline{WV}$. This gives us the three sides of the gap triangle congruent to the three sides of the original triangle. So $\triangle 1$ will fit the gap.
Expression – “You”-Forms

![Bar chart showing implicit and explicit forms for CMP, Glencoe, MiC, UCSMP. The chart compares variables and transformations across different curricula.]
Expression – Types of Imperatives

<table>
<thead>
<tr>
<th></th>
<th>CMP</th>
<th>Glencoe</th>
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<th>UCSMP</th>
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<tbody>
<tr>
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<td></td>
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</tr>
<tr>
<td>Transformations</td>
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</table>

 CMP Glencoe MiC UCSMP CMP Glencoe MiC UCSMP
# Expression – Types of Imperatives

<table>
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<td>73%</td>
</tr>
<tr>
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<td>31%</td>
<td>69%</td>
<td>17%</td>
<td>83%</td>
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<tr>
<td>MiC</td>
<td>43%</td>
<td>57%</td>
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<tr>
<td>UCSMP</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Expression – Modal Verbs

- CMP
- Glencoe
- MiC
- UCSMP

Variables

- Low
- Median
- High

Transformations
DISCUSSION & NEXT STEPS
Opportunity to Learn?

In general, insufficient opportunities

→ Over 15 of 31 content supports were unobserved or infrequent

→ Absent supports were often *Rationale Guidance* supports (accounted for no more than 6% of support)

→ Language often positioned teachers as agents doing the bidding of the “more knowledgable” curriculum authors
Implications – Curriculum Development

1. Raise awareness about possible impacts of content and expression

2. Speak more “to” rather than “through” teachers
   - Expression – Choose to use less imperatives and more modals verbs
   - Content – Include more rationale

3. Be more strategic about location of support
   - more (particularly SMK) at lesson level
Implications – Teacher Education

1. Recognize the important role of curriculum materials in teacher education

2. Create opportunities for teachers to work with curriculum materials
   - help with “reading” curriculum materials (including pointing out where to find support)
   - compare and critique various curriculum materials
   - plan and enact lessons from materials

3. “Teaching with rationales” – Think about being more explicit in teacher education courses
Implications – Research

1. Recognize the importance of expression in curriculum materials, not just the content
   - Seeking out new frameworks and combining them to get a more nuanced description of opportunity to learn

2. Consider how the philosophy and structure of the curriculum materials impacts analysis
Future Directions for Research and Development

Teacher learning from curriculum use

Opportunities for teacher learning in written curriculum materials

Examining how teachers learn from the opportunities in written mathematics curriculum
Future Directions for Research and Development

Design and research teachers use of electronic curriculum materials
→ more support is possible
→ customizable
→ adaptable
Thank you all for attending this talk and thank you to

I’d like to acknowledge:
Jack Smith, Glenda Lappan, Beth Herbel-Eisenmann, Kristen Bieda, Aladar Horvath, Jillian Cavana, Joanne Phillhower, & The Division of Science and Mathematics Education at Michigan State University
Questions

Lorraine M. Males
Teaching, Learning, & Teacher Education
lmales2@unl.edu
Educative Supports for Teachers in Middle School Mathematics Curriculum Materials: What is Offered and How is it Expressed?

Lorraine M. Males
Mathematics Education, Department of Teaching Learning, and Teacher Education

Focus of Study and Research Questions
To describe opportunities for teacher learning embedded in written middle school mathematics curriculum materials by investigating the content of the teachers’ guides and how this content was expressed. Specifically, I address the following research questions:
1. What is the relative frequency of educative supports in middle school mathematics teachers’ guides for teachers’ subject matter content knowledge, pedagogical content knowledge for mathematics topics and practices, and curricular knowledge?
2. How are supports expressed to teachers in middle school mathematics teachers’ guides?

Data Sources
Teachers’ Guides for the Introduction to Variable and Geometric Transformations units from each of the following:
- Connected Mathematics Project 2 (Lappan, Fey, Fitzgerald, Friel, Phillips, 2006) [CMP]
- Mathematics in Context (Wisconsin Center for Education & the Freudenthal Institute, 2010) [MiC]
- UCSMP’s Transition Mathematics (Viktora, Cheung, Highstone, Capuzzi, Heeres, Metcalf, et al., 2008) [UCSMP].

Results

<table>
<thead>
<tr>
<th>Content Supports</th>
<th>Types of Knowledge</th>
<th>Subject Matter</th>
<th>PCK for Topics</th>
<th>PCK for Practices</th>
<th>Curricular</th>
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</thead>
<tbody>
<tr>
<td>Variable</td>
<td>Rationale</td>
<td>Enactment</td>
<td>Rationale</td>
<td>Enactment</td>
<td>Rationale</td>
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<tr>
<td>MiC</td>
<td>94</td>
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<td>94</td>
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<td>97</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

Expression
Aspects of “Voice” (Herbel-Eisenmann, 2007)
→ helps to identify “how speakers shift positions, identities, and alignments toward the words they speak as well as toward one another” (Herbel-Eisenmann, 2007, p. 347).

Systemic Functional Linguistics (Halliday, 1985)

Metafunctions → interpersonal → ideational → textual

Martin & Rose, 2007, p. 5

Personal Pronouns
→ first person – “I” and “we”
→ second person – “you”

“You”-forms
→ implicit - imperatives → explicit - “you”+ modal verbs

“the degree of likelihood, probability, weight, or authority a speaker attaches to the utterance” (Hodge & Kress, 1993) and can be negotiated on a scale that describes “how obliged” you are to act (Martin & Rose, 2007)

Distribution of “You”-forms

<table>
<thead>
<tr>
<th>Types of Guidance</th>
<th>Variable</th>
<th>Transformations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enactment Rationale</td>
<td>we</td>
<td>you</td>
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<tr>
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<td>61</td>
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<tr>
<td>UCSMP</td>
<td>20</td>
<td>33</td>
</tr>
</tbody>
</table>

You might suggest that they imagine a point leaving a trail (CMP, KHM, p. 53)

You should emphasize that the value could be found by solving an equation (UCSMP, Some Imp Geom Ideas, p. 397)