A STUDY OF DISTRIBUTED BEAMFORMING IN COGNITIVE RADIO NETWORKS

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A STUDY OF DISTRIBUTED BEAMFORMING IN COGNITIVE RADIO NETWORKS

by

Omi Sunuwar

A THESIS

Presented to the Faculty of

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A STUDY OF DISTRIBUTED BEAMFORMING IN COGNITIVE RADIO NETWORKS

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University of Nebraska, 2013

Advisor: Yaoqing (Lamar) Yang

With the rapid increase in new wireless technologies and services, the unlicensed frequency band have been substantially overcrowded while the licensed band are reportedly under-utilized. This spectrum scarcity has brought up the concept of cognitive radio networks. By applying distributed cooperation between users, cognitive radio can direct beam towards the intended receiver and suppress interference in unintended directions which can tackle the problem of spectrum scarcity.

In this MS thesis, first beamforming in uniform array is investigated. The effect of position displacement of antenna elements from ideal uniform array shows that there is an increase in sidelobe level. Least squares method is used for the correction of error due to position displacement. The power consumption of centralized and distributed approach to beamforming is compared. The phase-only distributed beamforming method is investigated for uniformly distributed cognitive nodes with phase synchronization errors. The proposed PODB method calculates weights by adjusting the phase of the carrier signal to form a beam towards the intended receiver. The simulation results on the average beampattern and the complementary cumulative distribution function of the PODB method bring some insights to the distributed beamforming in cognitive radio networks.
Dedicated to my late father, Tek Bahadur Sunuwar, my mother Naralaxmi Sunuwar

and my family
Acknowledgement

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<th>Description</th>
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<tbody>
<tr>
<td>TDOA</td>
<td>Time direction of arrival</td>
</tr>
<tr>
<td>DOA</td>
<td>Direction of arrival</td>
</tr>
<tr>
<td>AOA</td>
<td>Angle of arrival</td>
</tr>
<tr>
<td>DB</td>
<td>Distributed beamforming</td>
</tr>
<tr>
<td>LU</td>
<td>Licensed user</td>
</tr>
<tr>
<td>SU</td>
<td>Secondary user</td>
</tr>
<tr>
<td>PU</td>
<td>Primary user</td>
</tr>
<tr>
<td>WSN</td>
<td>Wireless sensor network</td>
</tr>
<tr>
<td>CRN</td>
<td>Cognitive radio network</td>
</tr>
<tr>
<td>PODB</td>
<td>Phase-only distributed beamforming</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of service</td>
</tr>
<tr>
<td>LS</td>
<td>Least squares</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
</tr>
<tr>
<td>NTIA</td>
<td>National Telecommunication and Information Admin.</td>
</tr>
<tr>
<td>BWRC</td>
<td>Berkeley Wireless Research Center</td>
</tr>
<tr>
<td>DSA</td>
<td>Dynamic spectrum access</td>
</tr>
<tr>
<td>SDR</td>
<td>Software-defined radio</td>
</tr>
<tr>
<td>JTRS</td>
<td>Joint tactical radio system</td>
</tr>
<tr>
<td>DARPA</td>
<td>Defense Advanced Research Projects Agency</td>
</tr>
<tr>
<td>Acronym</td>
<td>Definition</td>
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<td>---------</td>
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<tr>
<td>NPRM</td>
<td>Notice of Proposed Rulemaking</td>
</tr>
<tr>
<td>MSE</td>
<td>Mean squared error</td>
</tr>
<tr>
<td>TDOA</td>
<td>Time difference of arrival</td>
</tr>
<tr>
<td>HPBW</td>
<td>Half power beamwidth</td>
</tr>
<tr>
<td>RF</td>
<td>Radio frequency</td>
</tr>
<tr>
<td>CCDF</td>
<td>Complementary cumulative distribution function</td>
</tr>
<tr>
<td>NOI</td>
<td>Notice of Enquiry</td>
</tr>
<tr>
<td>BS</td>
<td>Base station</td>
</tr>
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</table>
CHAPTER 1. INTRODUCTION AND BACKGROUND THEORY

1.1 Frequency spectrum allocation and usage

The use of radio spectrum is controlled by Federal Communications Commission (FCC). FCC assigns the frequency band to licensed users, known as primary users (PUs), on a long term basis. Figure 1.1 shows the National Telecommunication and Information Administration's (NTIA) chart of radio spectrum allocation. In 2002 FCC reported that there is widespread belief that radio spectrum use in the US is either crowded or becoming very crowded [1]. Contrary to popular belief, there are vast temporal and geographical variations in the usage of allocated spectrum with utilization ranging from 15% to 85% in the bands below 3 GHz [2]. Figure 1.2 shows the spectrum utilization in the frequency range 0 to 6 GHz. From the spectrum usage chart it is clear that the vast majority of spectrum is underutilized. The FCC’s “exclusive rights” policy, states that if a licensed system is not transmitting, its spectrum remains off-limits to other users. But because of underutilization of spectrum in the face of spectrum scarcity, FCC has been considering more comprehensive and flexible uses of the available spectrum. This has brought up the idea of cognitive radios. Cognitive radios allow unlicensed users called secondary users (SUs) to transmit on frequencies which are available without causing harmful interference to licensed users called primary users (PUs) and vacate the frequency band if primary user reclaims its right of spectrum usage. Cognitive radios use the principle of dynamic spectrum access (DSA), which allows secondary users (SUs) to access the licensed radio spectrum. To tackle the problem of spectrum scarcity distributed cooperation between cognitive users has also been discussed lately. By using distributed beamforming cognitive users can collaboratively direct signal
towards the intended receiver and minimize interference to licensed users. Thus, cognitive radios can increase the spectrum utilization and consequently spectrum efficiency.

![Figure 1: The NTIA’s spectrum allocation chart](image1.png)

Figure 1.1: The NTIA’s spectrum allocation chart [3]

![Figure 1.2: Spectrum usage chart](image2.png)

Figure 1.2: Spectrum usage chart [4]
1.2 Software-defined radio (SDR)

Software-defined radio (SDR) was first introduced by Joseph Mitola in 1991. Software-defined radio is defined as “communications technologies that rely on processing power and sophisticated network management, instead of raw transmission power, to prevent interference” [5]. According to FCC a software-defined radio as “a radio that includes a transmitter in which the operating parameters of frequency range, modulation type or maximum output power (either radiated or conducted), or the circumstances under which the transmitter operates in accordance with Commission rules, can be altered by making a change in software without making any changes to hardware components that affect the radio frequency emissions” [6]. SDR is capable of shifting inflexible hardware components of the radio to software quickly. Thus it can operate in significant portions of RF bands and air interface modes through software [7]. A SDR transceiver comprises of all the layers of a communication system from the Physical layer to Application layer [8]. For example: a dual-mode cell phone. A dual mode cell phone switches between two hardware-defined modes. Depending upon the strength of signal it can switch between digital and analog transmission mode. The main objective of SDR is to configure radio as software running on top of a flexible hardware interface. SourceForge’s Open SDR and the GNU Radio project, for example, attempt to “get a wide band ADC as close to the antenna as is convenient, get the samples into something we can program, and then grind on them in software” [9]. This requires cognitive capability to make decisions on how to switch between different modes depending upon the environment and user interface commands. In presence of cognitive
layer a SDR can behave like a cognitive radio and can adapt itself to the outside surrounding. Thus SDRs serve as a platform for development of cognitive radios.

1.3 History of SDR and CR

The term Software-defined radio was coined by Mitola in 1991. Software-defined radios have their origins in the defense sector from late 1970s in both U.S. and Europe. Walter Tuttlebee described a VLF radio that used an ADC and an 8085 microprocessor. One of the first public software radio initiatives was a US army and US Department of Defense project named Speak Easy in 1992. The goal of this project was to use programmable processing to emulate more than 10 existing military radios, operating in frequency bands between 2 and 2000 MHz [10]. The main objective of this project was to easily incorporate new coding and modulation standards in the future, so that military communications can keep pace with advances in coding and modulation techniques. The second phase of this project was launched in 1995. The goal of this phase was to get a more quickly reconfigurable architecture with bridging capability between different radio protocols. In 1997 US military took initiative to expand SDRs by the creation of the Joint Tactical Radio System (JTRS) [11] program to develop an open architecture of cutting edge radio waveform technology that allows multiple radio types (e.g. handheld, aircraft, maritime) to communicate with each other.

The term Cognitive radios was coined by Mitola in 1999. In 2002, a report by FCC in FCC Spectrum Task Force report [1] reported that the unlicensed band are being overcrowded while the licensed spectrum was highly under-utilized. New technologies started being considered which would facilitate the design of more intelligent and
universal radios to the spectrum scarcity problem. The same year a UK professor presented a report on the possibility of selling bandwidth to the user depending on their requirement. So in December of 2002, FCC issued a Notice of Enquiry (NOI) to see if TV channel bands can be made available to unlicensed users. In 2003, FCC formed a set of rules and proposed interference temperature model to keep interference caused by unlicensed user to primary users under control. Meanwhile Defense Advanced Research Projects Agency (DARPA) launched the Next Generation (XG) program sponsored by DARPA’s Strategic Technology Office. The main objective of this project was to develop both the enabling technologies and system concepts to dynamically redistribute allocated spectrum in order to provide improvements in assured military communications. The project supported a full range of worldwide deployments. In 2004, FCC published Notice of Proposed Rulemaking (NRPM) which showed possibility of allowing secondary users to use licensed spectrum. Three bands i.e. 6525 to 6700 MHz, 12.75 to 13.15 GHz and 13.2125-13.25 GHz were then opened for unlicensed users, which allowed secondary users with cognitive capability to transmit six times more. In August 2006, the Shared Spectrum Company (SSC) and DARPA demonstrated, for the first time, a six node network of XG radios capable of using spectrum over a wide range of frequencies, i.e. 225-600 MHz, on a secondary basis [12]. Other major contributions in the development of cognitive radios include spectrum pooling system by Professor Timo A Weiss from Karlsruhe University Germany), OFDM based Cognitive radios by professor Ian F Akyildiz et al from GIT (USA). In January 2010 first call over a CR network was made in university of Oulu using CRAMNET (Cognitive Radio Assisted Mobile Ad Hoc Network). Various forums and organizations have been established since
then which are working towards the development of SDR and cognitive radios. These forums hold technical conferences and seminar on a regular basis to promote new developments in the field. In addition to all these contributions a plethora of research has been going on in the field of SDR and cognitive radios.
Chapter 2. Overview of Cognitive Radios

2.1 Definition: What is a cognitive radio?

A cognitive radio is an “intelligent radio” which is able to monitor, sense and detect its surrounding environment and dynamically reconfigure its operating parameters to best match those conditions. It uses the technique of dynamic spectrum access (DSA) that enables next generation communication networks to opportunistically access the licensed spectrum without causing harmful interference to primary users and vacate the frequency band if primary user claims its spectrum usage right. Mitola coined the term cognitive radio and defined it as [Mitola_99]: “A radio that employs model based reasoning to achieve a specified level of competence in radio-related domains.” In [13] Haykin defined cognitive radio as “An intelligent wireless communication system that is aware of its environment and uses the methodology of understanding-by-building to learn from the environment and adapt to statistical variations in the input stimuli, with two primary objectives in mind:

- highly reliable communications whenever and wherever needed;
- efficient utilization of the radio spectrum.

Cognitive radio differs from conventional radio devices in that a cognitive radio can equip users with cognitive capability and re-configurability [14]. Cognitive capability refers to the ability to sense and gather information from the surrounding environment, such as information about transmission frequency, bandwidth, power, modulation, etc. and re-configurability refers to the ability to rapidly adapt the operational parameters according to the sensed information in order to achieve the optimal performance [14]. Cognitive radio can be described by six key words: awareness, intelligence, learning,
adaptivity, reliability and efficiency [13]. Thus cognitive radio increase spectrum utilization and consequently spectrum efficiency.

2.2 Cognitive cycle

A typical duty cycle of CR, as shown in figure 2.1, includes detecting spectrum white space, selecting the best frequency bands, coordinating spectrum access with other users and vacating the frequency when a primary user appears [14]. Thus a cognitive cycle can be divided into following three parts:

- spectrum sensing and analysis;
- spectrum management and handoff;
- spectrum allocation and sharing.

Figure 2.1: Cognitive cycle [14]
2.2.1 Spectrum sensing and analysis

Due to underutilization of radio spectrum there is formation of white spaces in frequency band called spectrum holes. A spectrum hole is a band of frequencies assigned to a primary user, but, at a particular time and specific geographic location, the band is not being utilized by that user [13]. As shown in figure 2.1, through spectrum sensing and analysis a cognitive radio can detect spectrum holes and utilize it by allowing unlicensed users to transmit without causing harmful interference to licensed users. And when the primary user comes back and reclaims its right to utilize the frequency band, through sensing cognitive users can continue to transmit without causing harmful interference to primary.

2.2.2 Spectrum management and handoff

Spectrum management and handoff enables secondary users to choose the best frequency band and hop among multiple bands according to the time varying channel characteristics to meet various Quality of Service (QoS) requirements [15]. That is when the primary user comes back and reclaims its right to utilize the frequency band, the secondary user can hop to other available frequencies according to the noise and interference levels, path loss, channel error rate, holding time etc [14].

2.2.3 Spectrum allocation and sharing
To improve spectrum utilization and spectrum efficiency, cognitive radio utilizes the technique called dynamic spectrum access (DSA) which allows unlicensed user to use available licensed spectrum. In dynamic spectrum access, a secondary user may share the spectrum resources with primary users, other secondary users, or both [14]. Due to this spectrum efficiency increases greatly. Secondary users can transmit in the licensed spectrum while there are no primary users by detecting spectrum holes. They can also transmit while there are primary users in the band by sharing licensed spectrum efficiently. But when primary users are also present in the same licensed frequency band, the secondary users should keep their interference temperature under a certain required threshold so that no harmful interference is caused to the primary users. And when multiple secondary users share a frequency band, their access should be coordinated to alleviate collisions and interference [14].

2.3 Cognitive radio network architecture

Figure 2.2 shows the network architecture of cognitive radio. By using dynamic spectrum access technique an unlicensed user can transmit in the licensed spectrum. The two main network components of cognitive radio network architecture are primary users and secondary users. Hence the network architecture of cognitive radio consists of primary network and secondary network [14]. The secondary network consists of only secondary users. There may be a secondary base station but there is no primary base station in the secondary network. The opportunistic frequency access in the secondary network is coordinated by a secondary base station. If several secondary networks share one common spectrum band, their spectrum usage may be coordinate by a central network entity, called
spectrum broker which collects operation information from each secondary network, and allocates the network resources to achieve efficient and fair spectrum sharing [16].

The primary network consists of primary users and primary base station. Primary users have right to operate in licensed band and if the network architecture is an infrastructure they are controlled through primary base stations [16]. Primary users have the license to use the assigned spectrum so the cognitive users are not allowed to cause hindrance in their transmission. Secondary users or cognitive users do not have license to access licensed bands but they can opportunistically access both licensed and unlicensed bands by use of various dynamic spectrum access techniques but they should maintain their interference temperature under certain threshold temperature. If a secondary network share a licensed spectrum band with a primary network, besides detecting the spectrum white space and utilizing the best spectrum band, the secondary network is required to immediately detect the presence of a primary user and direct the secondary transmission to another available band so as to avoid interfering with primary transmission [16]. The cognitive radio network architecture can be classified into Infrastructure, Ad-hoc and Mesh Architecture.

2.3.1 Infrastructure Architecture

In infrastructure architecture, a base station (BS) is used to control the spectrum access. Each user communicates to base station in one hop fashion. The base station directly communicates with each user in the network and controls the medium access and the cognitive users. The base station can use one or multiple protocols to fulfill the demands of different users.
2.3.2 Ad-hoc architecture

In ad-hoc architecture, there is no base station i.e. no infrastructure support. Information is shared directly between cognitive users by using existing communication protocols or dynamically accessing spectrum holes.

2.3.3 Mesh architecture

Mesh architecture is the combination of infrastructure and ad-hoc. Here the users can communicate directly with the base station or by using other users as multi-hop relay nodes.
2.4 Cognitive Radio Applications

Since cognitive radios can sense, monitor and detect surrounding environment and change its operating parameters to best match the operating conditions they are used to increase the spectrum efficiency and support higher bandwidth services [14]. Thus applications are often included in the definition of cognitive radio. Similarly due to cognitive capabilities the burden of centralized spectrum management is greatly reduces.

In today’s world where there is plethora of wireless devices and technologies cognitive radio has many applications. Some of the applications are listed below:

• Improving spectrum utilization & efficiency

• Improving link reliability

• Less expensive radios

• Advanced network topologies

• Enhancing SDR techniques

• Automated radio resource management.
CHAPTER 3. BEAMFORMING IN UNIFORM ARRAYS

3.1 Principles of beamforming

In the 1960s, beamforming was introduced in order to remove unwanted noise from military SONAR and RADAR systems. Since then it has been studied in many areas such as seismology, communications, imaging, geophysical exploration, and acoustic source localization etc. Signals propagating in space or three dimension by nature encounter the presence of noise and interference. In many applications, there is a need to separate the multiple sources or extract a source of interest while minimizing undesired interfering signals and noise [17]. If the signals are separated in frequency band temporally they can be separated from harmful effects of noise and interference. But if they occupy the same temporal frequency band then we have to use some kind of separating mechanism to avoid the effect of noise and interference. Since in many situations the desired signal and the interfering signal originate from different locations in space or three dimension, the signals can be separated. By exploiting the spatial separation between the desired signal and the interfering signal, beamformers can be used to separate the signals from the interference.

A beamformer is basically a signal processor with spatial filtering capability. The goal of beamforming or interference cancellation is to isolate the signal of the desired user from the interference and noise. Beamforming technique can separate sources with overlapping frequency content that originate at different spatial locations [18]. In beamforming, the amplitude and phase of each antenna element are adjusted to direct beam towards intended direction and minimize interference in other directions. The combined relative amplitude $I_k$ and phase shift $\theta_k$ for each antenna is called a “complex weight” and is represented by a
complex constant $w_k$ for the $k^{th}$ antenna [19]. Thus by the process of beamforming the direction of beampattern, beamwidth of the mainlobe and the characteristics of the sidelobes are controlled according to the requirements. Some of the uses of beamforming are signal detection, controlling direction of arrival (DOA) of signal, and maximizing signal which is corrupted by noise, competing sources, and reverberation in the intended direction [17].

3.2 Transmit beamforming

A beamformer for a radio transmitter applies the complex weight to the transmit signal (shifts the phase and sets the amplitude) for each element of the antenna array [19].

Figure 3.1: Transmit beamforming [19]
3.3 Receive beamforming

A beamformer for radio reception applies the complex weight to the signal from each antenna element, then sums all of the signals into one that has the desired directional pattern [19]. Output signals of different elements of an antenna array are combined to form a beam towards the intended direction and minimize interference in other directions. The process of receive beamforming involves two steps: synchronization and weight-and-sum. In synchronization, the output of each antenna element is delayed by proper amount of time so that the signal components coming from the desired directions are synchronized. If the direction of the receiver is not known in advance, the time difference of arrival (TDOA) is estimated from the array measurements using a time-delay estimation technique. In weight-and-sum, the signals received from the antenna array are assigned proper weights and added together to form beampattern in the desired direction.

Figure 3.2: Receive beamforming [19]
3.2 Basic terminologies

Some terminologies used to understand beamforming in uniform arrays are described below:

a) **Radiation pattern**: The radiation pattern of an antenna is the relative distribution of the radiated power as a function of direction in space and is given by:

\[ P(\theta, \phi) = f(\theta, \phi) \times F(\theta, \phi) \]

Where, \( P(\theta, \phi) = \) the radiation pattern or beampattern;
\( f(\theta, \phi) = \) the element pattern;
\( F(\theta, \phi) = \) the array factor;
\( \theta = \) elevation angle; \( \phi = \) azimuth angle.

b) **Array factor** \( F(\theta, \phi)\): Array factor is the far-field radiation pattern of an array of isotropic elements.

c) **Mainlobe**: The main lobe of a radiation pattern is the lobe containing the direction in which the radiation power is maximum.

d) **Sidelobes**: Sidelobes are lobes of a radiation pattern which do not constitute the mainlobe. These sidelobes are usually radiation in undesired direction.

e) **Beamwidth**: The beamwidth of an antenna is the angular width of the mainlobe.

f) **Half Power Beamwidth (HPBW) or 3 dB beamwidth**: The HPBW is the angular separation in which the magnitude of the radiation pattern is reduced by 50% (or -3 dB) from the peak of the beam.
Figure 3.3: Array element of 10 element array with $d = 0.5\lambda$

Figure 3.4: Array factor of 10 element array with $d = 0.5\lambda$
Figure 3.5: Array pattern of 10 element array with $d = 0.5\lambda$

3.3 Antenna arrays

An antenna array is a configuration of individual radiating elements arranged in space and used to produce a directional radiation pattern. Arrays can be arranged in various geometrical configurations: line, circle, plane etc. each of which yields a different radiation pattern. Linear arrays being the most common type of array are antennas arranged along a straight line while circular arrays are arranged in a circle. Arrays usually employ identical antenna elements. The radiating pattern of the array depends on the configuration, the distance between the elements, the amplitude and phase excitation of the elements, and also the radiation pattern of individual elements. A uniform antenna
array is defined by uniformly-spaced identical elements of equal magnitude with a linearly progressive phase from element to element.

3.3.1 Beamforming in uniform linear array

Let us consider a linear array with $M$ isotropic radiating elements denoted by $x = \{x_1, x_2, \ldots, x_m\}$ separated by a distance of $d = \lambda/2$ as shown in figure 3.6. Let the reference node be located at the origin. Let the distance between the source and the array...
be very large so that the source lies far field of the radiation pattern. Let the plane wave \( s(t) \) from the source arrives an angle \( \theta_a \) with respect to the array axis (i.e. x axis).

\[ z\text{-axis} \]

\[ \text{source} \]

\[ s(t) \]

\[ x_{z} \cos \theta_{\infty} \]

\[ 1 \quad 2 \quad 3 \quad m \quad M \]

\[ x_2 \]

\[ x_3 \]

\[ x_m \]

\[ \theta_{\infty} \]

Figure 3.7: A linear array with \( M \) elements

The signal arrives at \( (m + 1)^{th} \) array element \( t_m \) seconds earlier than the \( m^{th} \) element. The time difference \( t_m \) is given by [20]:

\[ t_m(\theta_{\infty}) = \frac{x_m \cos \theta_{\infty}}{c} \]  \hspace{1cm} (3.1)

Where, \( c \) is the speed of light.
Let \( w = (w_1, w_2, \ldots, w_m) \) be the complex weights assigned to antenna elements, \( m = 0, 1, 2, \ldots, M \). Then the array factor for any arbitrary angle \( \theta_a \) is given as follows:

\[
F(\theta_a) = \sum_{m=1}^{M} w_m^* e^{j\omega m (\theta_a)}
\]

(3.2)

Where, \( w_m = I_m e^{j\omega m \theta_{a0}} \)

\( I_m \) = the magnitude of complex weights;

\( e^{j\omega m \theta_{a0}} \) = phase of the complex weights.

We know, \( \beta = \frac{2\pi}{\lambda} \) where, \( \beta = \) wave number. Putting value of \( \beta \) in equation (3.2), we get

\[
w_m = I_m e^{j\beta x_m \cos \theta_{a0}}
\]

\[
F(\theta_a) = \sum_{m=1}^{M} w_m^* e^{j\beta x_m \cos \theta_{a0}}
\]

(3.3)

The array response is maximum when the antenna is steered in the direction of the signal source i.e. \( \theta_a = \theta_{a0} \) and is given as follows:

\[
F(\theta_{a0}) = \sum_{m=1}^{M} I_m e^{-j\beta x_m \cos \theta_{a0}} e^{j\beta x_m \cos \theta_{a0}}
\]

\[
= \sum_{m=1}^{M} I_m = M \text{ if } I_m = 1, \forall m
\]

(3.4)

Now the steering vector is given as follows:

\[
d(\theta_{a0}) = [1, e^{j\beta x_2 \cos \theta_{a0}}, e^{j\beta x_3 \cos \theta_{a0}}, \ldots, e^{j\beta x_M \cos \theta_{a0}}]^T
\]

(3.5)

The array response then becomes:

\[
F(\theta_{a0}) = w^H d(\theta_{a0})
\]

(3.6)

The normalized power gain \( P \) is defined as [21]:
\[ P(\theta_0) = \frac{|F(\theta_0)|^2}{\max |F(\theta_0)|^2} \] (3.7)

### 3.3.2 Beamforming in uniform planar array

When antenna elements are placed in forming a two-dimensional plane they form planar array. Planar array provide higher directivity, symmetrical beams and low sidelobe levels than individual antenna elements. Let us consider a \( M \times N \) planar array. Let each element in the array is separated in both directions by a distance, \( d = \lambda / 2 \). the array is located in the far field of the beampattern. The plane wave \( s(t) \) arrives at an elevation angle of \( \theta_0 \) and an azimuth angle of \( \phi_0 \). The \((m, n)^{th}\) element receives the signal earlier by \( t_{mn} \) seconds compared to the reference element at the origin and is given by [20]:

\[ t_{mn} = \frac{x_{mn}\sin\theta_0\cos\phi_0 + y_{mn}\sin\theta_0\sin\phi_0}{c} \] (3.8)

Now the array factor \( F(\theta, \phi) \) for any arbitrary angle \((\theta, \phi)\) is given as below:

\[ F(\theta, \phi) = \sum_{n=1}^{N} \sum_{m=1}^{M} w_{mn}^* e^{j\beta(x_{mn}\sin\theta_0\cos\phi_0 + y_{mn}\sin\theta_0\sin\phi_0)} \] (3.9)

Where the complex weights for each element \((m, n)\) is given as follows:

\[ w_{mn} = l_{mn} e^{j\beta(x_{mn}\sin\theta_0\cos\phi_0 + y_{mn}\sin\theta_0\sin\phi_0)} \] (3.10)

Now, the array factor in matrix form is given as below:

\[ F(\theta, \phi) = w^H d(\theta, \phi) \] (3.11)

Where, \( w \) is a \( MN \times 1 \) vector and \( d(\theta, \phi) \) is a \( MN \times 1 \) steering vector given as follows:
The weights $w_{mn}$ are chosen to maximize the array response $F(\theta, \phi)$ in the desired direction $(\theta_0, \phi_0)$. The array response is maximum when $\theta = \theta_0$ and $\phi = \phi_0$ and is given as follows:

$$F(\theta_0, \phi_0) = MN \quad if \quad I_{mn} = 1, \forall m, n$$

(3.13)

Now the beampattern of the array is given as:

$$P(\theta_0) = \frac{|F(\theta_0)|^2}{\max_{\theta_0}|F(\theta_0)|^2}$$

(3.14)

### 3.4 Effect of position displacement of antenna elements from uniform array

Some assumptions used to model displacement of antenna elements from uniform array is given as below:

1. The nodes are randomly distributed and there is displacement in both x and y directions.

2. The antenna are displaced from uniform array uniformly with a minimum value of 0 and a maximum value of $\lambda/2 \times a$. Here, $'a'$ is the maximum percentage error.

3. The best line of fit for randomly distributed nodes is the array axes. And the deviations from the uniform array are calculated using these array axes (x axis & y axis), as axis of reference.
In wireless networks, nodes are randomly distributed so there is position displacement of antenna elements from a spaced uniform array. The random displacement of antenna elements causes phase errors and mismatch which degrades the performance of beamformers. Let us assume that there are no amplitude errors, i.e., the magnitude of all the weights $w_{mn}$ is $l = l_{mn}$. Then, for an $M \times N$ array, the increase in the sidelobe level ($\Delta_s$), with respect to the main lobe is given by [20]:

$$\Delta_s = \frac{1}{MN}(e^{\sigma_{\Delta\phi}^2} - 1)$$

(3.14)

Where, $\sigma_{\Delta\phi}^2$ = the variance of the phase error $\Delta\phi$ which follows a Gaussian distribution.

The fractional loss in the main lobe gain due to phase errors is given by [15]
\[
\frac{P}{P_0} = e^{-\sigma^2 \Delta \phi}
\]  

(3.15)

Where, \(P\) = mainlobe power gain with phase error;

\(P_0\) = mainlobe power gain without phase error.

### 3.5 Least squares method

Least squares method in beamforming implementation minimizes the mean square error (MSE) between the desired response and the actual response in such a way that the resulting response approximates the desired response. Let us consider \(n\) randomly distributed nodes in a plane. Let the azimuth angle be \(\phi_0\) and the angle of arrival (AOA) i.e. elevation angles be defined as \(\theta = [\theta_1, \theta_2, \ldots, \theta_m]\) over \(m\) angles for \(m \times 1\) linear array. The array response for the angle of arrival AOA, \(\theta\) is given by:

\[
F(\theta) = w^H d(\theta)
\]

(3.16)

Let the desired response \(F_d(\theta)\) is defined over \(m\) number of angles \(\theta_i\) for \(i = 1, \ldots, m\). The array response \(F(\theta)\) of \(n\) antenna elements for these \(m\) angles is given by:

\[
F(\theta)^H = w^H D(\theta)
\]

(3.17)

Where \(F(\theta) = [F(\theta_1), F(\theta_2), \ldots, F(\theta_m)]^T\)

\(w = [w_1, w_2, \ldots, w_n]^T\)

\(D(\theta) = [d(\theta_1), d(\theta_2), \ldots, d(\theta_m)]\) is a \(m \times n\) steering matrix.
Or, \( D(\theta) = \begin{bmatrix} e^{j\beta x_1 \sin \theta_1} & e^{j\beta x_1 \sin \theta_2} & \cdots & e^{j\beta x_1 \sin \theta_m} \\ e^{j\beta x_2 \sin \theta_1} & e^{j\beta x_2 \sin \theta_2} & \cdots & e^{j\beta x_2 \sin \theta_m} \\ \vdots & \vdots & \ddots & \vdots \\ e^{j\beta x_n \sin \theta_1} & e^{j\beta x_n \sin \theta_2} & \cdots & e^{j\beta x_n \sin \theta_m} \end{bmatrix} \)

By using the least squares method the weights \( w \) is calculated in such a way that the actual response \( F(\theta) \) is approximately equal to the desired response. The weight coefficients are chosen to minimize the mean squared error (MSE) between the desired response \( F_d(\theta) \) and the actual response \( F(\theta) \)

\[ \varepsilon = \min_w |F(\theta) - F_d(\theta)|^2 \]

Or, \( \varepsilon = \min_w |D(\theta)^H w - F_d(\theta)|^2 \) \hfill (3.18)

Let us suppose \( m > n \), the problem becomes an over determined LS problem. And the solution to the LS problem in equation (3.18) is given by:

\[ w = D^+(\theta) F_d(\theta) \] \hfill (3.19)

Where, \( D^+(\theta) \) is the pseudo inverse of \( D(\theta)^H \) and is defined as below:

\[ D^+(\theta) = \{D(\theta)D(\theta)^H\}^{-1}D(\theta) \] \hfill (3.20)

Now using the Fourier analysis, the desired array function is be expanded in a Fourier series. Truncating this Fourier series, results in an array with a finite number of elements. Such an array is optimum in the sense that no other array with the same number of elements can approximate the desired array function with lower mean squared error (MSE) [22].
3.6 Simulation results:

Figure 3.9: A $10 \times 1$ uniform linear sensor array showing position errors

Figure 3.9 shows the displacement of antenna elements from uniform $10 \times 1$ linear array. The simulation is taken 50 times. The angle of elevation is $\theta = 30^\circ$ and the azimuth angle is $\phi = 45^\circ$. The position displacements are considered to be uniformly distributed between 0 and 20% of the distance between two antenna elements i.e. $\frac{\lambda}{2}$ in both x and y directions.
Figure 3.10: Average beampattern of a $10 \times 1$ uniform linear array with position displacements

Figure 3.10 shows the average beampattern of a $10 \times 1$ uniform linear array compared with the average beampattern of array with uniformly distributed antenna position displacement, averaged over 50 simulation runs. The elevation angle is $\theta = 30^\circ$ and the azimuth angle is $\phi = 45^\circ$. The position displacement are considered to be uniformly distributed between 0 and 20% of the distance between two antenna elements i.e. $\frac{\lambda}{2}$ in both x and y directions. The results show that there is increase in sidelobe level due to position displacement of antenna elements from uniform linear array.
Figure 3.11: Average beampattern for a $10 \times 1$ uniform linear array with $\theta = 30^\circ$, $\phi = 45^\circ$.

Figure 3.11 shows the average beampattern of least squares method, ideal linear array and the beampattern with position displacement. The result shows that least squares method corrects increase in sidelobe level due to uniformly distributed position displacements in antenna elements and the average beampattern of LS method approximates same as ideal uniform linear array. The average is taken over 50 simulation runs. The position errors are uniformly distributed between 0 and 20% of the distance between two antenna elements i.e. $\frac{\lambda}{2}$, in both x and y directions.
3.7 Summary

In this chapter, beamforming and basic terminologies used to describe the process of beamforming was introduced. Transmit and receive beamforming were illustrated with figures to get better understanding. Then we discussed about beamforming in uniform linear and planar array. The effect of position displacement of antenna elements form ideal uniform array was investigated. The simulation results show that there is increase in sidelobe due to position displacement of antenna elements. The least squares method which minimizes the mean squared error between the actual and desired response is used for correction of position displacement. The simulation result show that average beampattern using least squares method approximates that of ideal linear array.
CHAPTER 4. CENTRALIZED vs DISTRIBUTED APPROACH TO BEAMFORMING

There are two ways of implementing a beamformer in wireless networks: Centralized and distributed. Both type of beamforming implementations are discussed in this literature.

4.1 Centralized approach

In the centralized approach, there is a central node called cluster head which does all required computations. The cluster head collects all information such as the steering vector $d(\theta, \phi)$, the direction of the desired signal $(\theta_0, \phi_0)$, the desired response $F_d(\theta)$ and the direction of potential interferences, and calculates the weight vector. The cluster head then constructs the steering matrix using the relative position of the nodes and the set of angles and calculates the weight vector by solving the LS problem of equation (3.18) [22]. The calculated weight is then transmitted to all nodes in the cluster. Since all the computation is done by the cluster head, failure of cluster head requires the whole cluster to re-converge and the LS problem has to be solved from the beginning which is a waste of valuable processing and communication resources. Also if there is a topology change caused by leaving or joining of nodes the cluster has to re-converge and the LS problem to be solved from the beginning in this case too. This shows that centralized approach lacks robustness and scalability. Thus centralized implementation of a beamformer is not optimal since all the computation burden is carried by a single node but it serves as baseline for the analysis of distributed solutions [23].
### 4.1.1 Power consumption analysis

The power consumption can be divided into two components: a component related to the computational burden and a component related to the communication burden [23]. The computational power is the power required for calculating weight vector, steering matrix etc. while the communication power is the power required to transmit the weight vector to all nodes in the cluster. The computational power is proportional to the number of operations executed while the communication power is proportional to the size and number of messages transmitted [23]. For a $m \times n$ steering matrix, the number of floating point operations for a centralized solution of the least squares problem is given by [24]

$$N_{flops} = 2n^2 \left(m - \frac{n}{3}\right) + mn + n^2 \tag{4.1}$$

The resulting computational power is given by [28]

$$P_p = \left[2n^2 \left(m - \frac{n}{3}\right) + mn + n^2\right] \times P_{flop} \tag{4.2}$$

Where, $P_{flop}$ is the power required to execute one floating point operation.

For $n$ nodes and each data element containing $b$ bits, the total number of bits to be transmitted is $N_b = 2nb$. Twice because of the two way communication.

Thus the communication power required is given by:

$$P_c = 2nb \times P_b \tag{4.3}$$

Where $P_b$ is the power required to transmit a single bit. The total power consumption is the sum of two powers and is given as below:

$$P_{cent} = \left[2n^2 \left(m - \frac{n}{3}\right) + mn + n^2\right] \times P_{flop} + 2nb \times P_b \tag{4.4}$$
4.2 Distributed approach to beamforming

In distributed approach, the steering vector is stored in a distributed manner across all nodes in the cluster. And the weight vector is also calculated in a distributed manner by all the nodes. Since the weight vector is calculated across all nodes it has to be communicated to all other nodes throughout the cluster which increases the communication power requirement. This is the tradeoff of the distributed approach compared to centralized approach. But the centralized approach to beamforming is not optimal power consumption of two distributive approach are discussed below. The first is solution of LS problem is based on QR factorization using Householder transformation. The second is an energy efficient distributed approach based on parallel method of solving the LS problem. Both algorithms distribute the load among all nodes effectively. Power consumption requirements of both algorithms are discussed in [23]

4.2.1 Distributive LS approach based on QR factorization using Householder transformation

A distributed beamforming algorithm based on the QR factorization is performed using Householder transformations is proposed in [22]. As described in [23], the proposed algorithm stores the steering matrix in a distributive manner across all nodes in the cluster. Each node then calculates the Householder transformation in a single column. The result is broadcasted to all other nodes within the cluster. Using the Householder transformation, the nodes locally update the desired response and use back substitution to solve for the weights [23]. Each node then broadcasts the updated weights. Since no redundant calculations are executed the computation cost is equivalent to that of the centralized
solution. However the tradeoff of the distributed approach is the increase in communication power requirement.

The distributive solution to LS problem as described in [23] is as follows. Let us consider a $m \times n$ steering matrix. Then the Householder transformation matrix $H_k$ can be represented by a $(m - k + 1) \times 1$ vector $v_k$ and a scalar $\beta_k$. Sensor node 1 will transmit the $H_1$ matrix through $m$ data elements in $v_1$ and one data element to represent the scalar $\beta_1$. Similarly, sensor node 2 will transmit $H_2$ through $m - 1$ data elements in $v_2$ and the one data element to represent the scalar $\beta_2$. In general, sensor node $i$ will transmit $(m - i + 1)$ data elements to represent $H_i$. Thus, the total power consumption during the decomposition phase is given by:

$$P_{QR} = (m + 2 - \frac{n}{2})(n - 1)b \times P_b$$  \hspace{1cm} (4.5)

During the back substitution phase, each node transmits two data elements to represent its weight and position so the associated power is given by [23]:

$$P_{BS} = 2(n - 1)b \times P_b$$  \hspace{1cm} (4.6)

The total communication power for distributed solution is given by:

$$P_c = (m + 4 - \frac{n}{2})(n - 1)b \times P_b$$  \hspace{1cm} (4.7)

The total power consumption is given by the sum of computational and communication power which is as given below:

$$P_{dist1} = \left[2n^2 \left(m - \frac{n}{3}\right) + mn + n^2\right] \times P_{flop} + (m + 4 - \frac{n}{2})(n - 1)b \times P_b$$  \hspace{1cm} (4.8)

This total power is plotted as a function of number of sensors and number of approximation angles in the beampattern and compared to the centralized approach to beamforming.
4.2.2 Distributive parallel method of solving LS problem

The distributed parallel method of solving LS problem of equation (3.18) is based on [25].

Expanding the LS problem given in equation (3.18), we get

$$\varepsilon = \min_w |D_1 w_1 + D_2 w_2 + \cdots + D_n w_n - F_d (\theta) |^2$$

Where, $D_i = i^{th}$ column of the steering matrix

$w_i = i^{th}$ element of the weight vector, $i \in (1,2,\ldots,n)$

The distributed algorithm as described in [23] is as follows. Let us suppose $w^k$ is an approximation to the solution $w$ after $k$ iterations and its element are $w_i^{(k)}$, $i \in (1,2,\ldots,n)$. If arbitrarily all the elements have been updated through $k + 1$ iterations while the remaining $x_1, x_2, \ldots, x_n$ have been updated in $k$ equations, equation (4.9) can be written as

$$\varepsilon = \min_w \left| \sum_{j=1}^{i-1} D_j w_j^{(k+1)} + D_i w_i^{(k+1)} + \sum_{j=i+1}^{n} D_j w_j^{(k)} - F_d (\theta) \right|^2$$

(4.10)

Now the least squares problem can be transformed into

$$\varepsilon = \min_{s_i^{(k)}} \left| D_i s_i^{(k)} + r^{(k,i-1)} \right|^2$$

(4.11)

Where $s_i^{(k)}$ is the weight correction and is given by:

$$s_i^{(k)} = w_i^{(k+1)} - w_i^{(k)}$$

(4.12)

And $r^{(k,i-1)}$ is defined as the residual given by

$$r^{(k,i-1)} = \sum_{j=1}^{i-1} D_i w_j^{(k+1)} + \sum_{j=i}^{n} D_i w_j^{(k)} - F_d (\theta)$$

(4.13)
The solution of (3.18) is equivalent to solving the local sub problems of (4.11). $D_i$ is known locally to sensor node $i$, $s_i^{(k)}$ is the locally computed correction and the residual $r^{(k,i-1)}$ calculated at sensor node $i-1$ can be transmitted to sensor node $i$.

The parallel way of solving LS method is described as follows in [23]. The initial estimation of the weight is $w_1^{(0)}$ and the residual $r^{(0)}$. Node $i$ solves equation (4.11) for $s_i^{(1)}$ and updates its own weight $w_i^{(1)}$. Then node $i$ calculates the residual $r_i^{(1)}$ using equation (4.13) and forwards it to node $i + 1$. Node $i + 1$ then updates its solution $w_{i+1}^{(1)}$ and so on until convergence. The residual is a $m \times 1$ vector and requires the transmission of $m$ elements. If the position of sensor nodes are known throughout the array, the nodes can construct the columns of the steering matrix and sensor node $i$ does not need to send the entire residual $r^{(k,i)}$ but only the scalar correction $s_i^{(k)}$. The remaining nodes can reconstruct residual using following equation:

$$r^{(k,i)} = r^{(k,i-1)} + D_i^{(k)}$$ (4.14)

Hence for each iteration only one scalar element has to be transmitted and the remaining computations can be done locally. The number of floating point operations per node is given by:

$$N_{flop} = [2\left(m - \frac{1}{3}\right) + k(3m + 1)]$$ (4.15)

The total processing power is given by:

$$P_p = [2\left(m - \frac{1}{3}\right) + k(3m + 1)]n \times P_{flop}$$ (4.16)

The number of data bits to be transmitted is $N_b = (k + 1)nb$ so the communication power consumption is given by:
\[ P_c = (k + 1)nb \times P_b \] (4.17)

The sum of two powers give the total power consumption.

\[ P_{dist} = n\left[ 2\left( m - \frac{1}{3}\right) + k(3m + 1) \right] \times P_{flp} + (k + 1)nb \times P_b \] (4.18)

4.3 Simulation results:

Figure 4.1 to 4.4 show the results of comparison between total power requirement of solving the LS problem by both centralized and distributed approach.

![Simulation results graph](image)

Figure 4.1: Normalized power requirement as a function of number of angles in a beampattern approximation for distributed and centralized approach
Figure 4.1 shows the comparison of total power requirement between centralized approach and distributed approach based on QR factorization using Householder transformation. It shows the normalized power requirement for both centralized and distributed approach as a function of number of angles in beampattern approximation. The total power consumption is normalized using $P_{f_{top}}$. The parameters used for simulation are as follows: $m = 20$, $b = 32$ and $P_b = 200P_{f_{top}}$.

Figure 4.2: Normalized power requirement as a function of number of sensor for distributed and centralized approach.
Figure 4.2 shows the comparison of total power requirement between centralized approach and distributed approach based on QR factorization using Householder transformation. It shows the normalized power requirement for both centralized and distributed approach as a function of number of sensors. The total power consumption is normalized using $P_{flop}$. The parameters used for simulation are as follows: $m = 20$, $b = 32$ and $P_b = 200P_{flop}$. The result show that as the number of sensor increases the total power required for both approach increases exponentially.

![Figure 4.2: Comparison of power requirement](image)

Figure 4.3: Normalized power requirement as a function of number of angles in a beampattern approximation
Figure 4.3 shows the comparison of total power requirement between centralized approach and distributed approach based on parallel method of solving LS problem of equation (3.18). It shows the normalized power requirement for both centralized and distributed approach as a function of number of angles in beampattern approximation. The total power consumption is normalized using $P_{flp}$. The parameters used for simulation are as follows: $m = 20$, $b = 32$ and $P_b = 200P_{flp}$.

Figure 4.4: Normalized power requirement as a function of number of sensors for distributed and centralized approach
Figure 4.4 shows the comparison of total power requirement between centralized approach and distributed approach based on parallel method of solving LS problem of equation (3.18) It shows the normalized power requirement for both centralized and distributed approach as a function of number of sensors. The total power consumption is normalized using $P_{\text{flop}}$. The parameters used for simulation are as follows: $m = 20$, $b = 32$ and $P_b = 200P_{\text{flop}}$. The result show that as the number of sensor increases the total power required for both approach increases exponentially.

From the above four simulation results it is observed that by using parallel way of solving LS problem the total power consumption is approximately 63% less than that required by distributed approach with QR factorization using Householder transformation. Also the power consumption is relatively constant function of the number of angles in approximation. Hence we can conclude that the parallel approach is energy efficient.

### 4.4 Summary

In this chapter, the comparison of power consumption between the centralized approach and two distributed approach to beamforming is performed. The simulations results show that the power required for distributed approach is more than that of centralized approach due to the increase in communication power requirement. Another conclusion from this chapter is that the parallel method of solving the LS problem of equation (3.18) is an energy efficient method.
CHAPTER 5. DISTRIBUTED BEAMFORMING IN COGNITIVE RADIOS WITH IMPERFECT PHASE SYNCHRONIZATION

5.1 Distributed Beamforming in cognitive radio networks

Recently introduced key technology for tackling the challenges of practical implementation of CR network is distributed cooperation between users [26]. By applying beamforming technique, CR can direct beam towards intended receiver and suppress interference in unintended directions to improve the network performance. In distributed beamforming each distributed user is equipped with a single antenna and a number of such users collaboratively transmit the signal by adjusting the carrier phase of each transmitter such that the interference caused by the cognitive users to the primary user is reduced [27, 28]. It is shown in [29] by using virtual array of N antennas, an N-fold power gain can be achieved in comparison to single antenna transmission. Using beamforming in cognitive radio can increase the range of the communication link, since the signal beam is concentrated only to the desired direction so that no energy is wasted in other directions. Using the spatial filtering nature of the beamforming, cognitive users can collaboratively form a beam toward the intended receiver and minimize interference to undesired directions. This increases the spectrum utilization and enables cognitive users to transmit without causing harmful interference to primary users. Other benefits of using beamforming technique can be reduction in delay spread and multipath fading, co-channel interference reduction, etc.

A novel phase-only DB (PODB) for CR networks has been proposed in [30] which can successfully direct the beam towards the intended receiver and null towards the licensed users. In this method only the phase of the transmitted signal is adjusted, the
magnitude being unity for all the cognitive users. PODB method also prolongs the lifetime of CR networks due to effective battery consumption at cognitive radio users and thus are called green cognitive radio networks [30]. However to implement maximum beamforming gain there should be accurate carrier frequency and phase synchronisation of all the cognitive users and intended receiver. Because of the distributed implementation each cognitive users have separate RF signals supplied by separate local oscillators so the phase synchronization poses a significant challenge.

5.2 System Model

Figure 5.1: System model with uniformly distributed CR users within a disk [30]
The system diagram is shown in figure 5.1. The geometrical configuration of the cognitive radio network and the receiver including licensed users is shown in figure. As shown in figure 5.1 $N$ cognitive users are uniformly distributed on a disk of radius $R$ centered at $O$. Let us denote the location of CR users is $(r_k, \psi_k)$ which is given in polar coordinates. Similarly let us denote the receiver is given in spherical coordinates by $(A, \theta_0, \phi_0)$ where its distance from origin $A \gg R$. Here the angle $\theta_0 \in (0, \pi)$ is the elevation angle and $\phi_0 \in (-\pi, \pi)$ is the azimuth angle. Now let us assume there are $M$ licensed users given in polar coordinates by $(A_m, \phi_m)$. Some of the necessary assumptions for the analysis are listed below:

**Assumptions:**

1. The cognitive nodes are uniformly distributed on a disk of radius $R$
2. Each cognitive node is equipped with a single isotropic antenna.
3. The communication channel between the cognitive users and the receiver is purely line of sight and there no multi path fading or shadowing.
4. The receiver and licensed users are located in the far-field of the beampattern.

The distance between the $k^{th}$ node and the receiver is given by:

$$d_k (\theta, \phi) = \sqrt{A^2 + r_k^2 - 2r_k A \sin \theta \cos (\phi - \psi_k)} \quad (5.1)$$

Since the receiver is assumed to be in the far-filed of the radiation pattern i.e. $A \gg r_k$, equation (5.1) can be approximated as:
\[ d_k (\theta, \phi) \approx A - r_k \sin \theta \cos(\phi - \psi_k) \] (5.2)

The initial phase of the cognitive user at the \( k^{th} \) node is:

\[ \beta_k = -\frac{2\pi}{\lambda} d_k = -\frac{2\pi}{\lambda} [A - r_k \sin \theta \cos(\phi - \psi_k)] \] (5.3)

The relative phase at \( k^{th} \) cognitive user with elevation angle \( \theta_0 \) and azimuth angle \( \phi_0 \) is

\[ \alpha_{0,k} = \frac{2\pi}{\lambda} [A - r_k \sin \theta_0 \cos(\phi_0 - \psi_k)] \] (5.4)

Let the elevation angle \( \theta = \theta_0 = \frac{\pi}{2} \)

Thus, the array factor is given by

\[
F(\phi) = \sum_{k=1}^{N} e^{j \frac{2\pi}{\lambda} r_k [\cos(\phi - \psi_k) - \cos(\phi_0 - \psi_k)]} w_k \\
= \sum_{k=1}^{N} e^{-j \frac{4\pi}{\lambda} r_k [\sin(\frac{\phi - \phi_0}{2}) \sin(\frac{\phi + \phi_0 - 2\psi_k}{2})]} w_k \] (5.5)

Thus the far-field beampattern is obtained as:

\[ P(\phi) \approx \frac{1}{N} |F(\phi)|^2 \] (5.6)

Therefore for realizing maximum beamforming gain carrier phase synchronization is necessary. Many of the literatures have assumed that there is perfect phase synchronization between all cognitive nodes. But in case of distributed network, users have separate RF signals supplied by separate local oscillators resulting in phase synchronization errors [31].

The receiver transmits a beacon to all the cognitive users periodically which is given by:

\[ s(t) = A_c \cos(2\pi f_c t + \psi_c) \] (5.7)
Where $A_c$ is the carrier amplitude, $f_c$ is the carrier frequency and $\psi_c$ is the carrier phase.

The received signal at $k_{th}$ user is given by:

$$r_k(t) = A_c \cos(2\pi f_c t + \psi_c + \alpha_{0,k}) + n(t)$$  \hspace{1cm} (5.8)

Where $n(t)$ is the additive white Gaussian noise.

To reduce the effect of noise and phase offset, each user’s oscillator uses the received signal $r_k(t)$ corrupted with noise to synchronize its phase with phase-locked loop (PLL) \[10\]. The variance of the VCO output phase of the PLL is given by \[31\]

$$\sigma_{\gamma_k}^2 = \frac{N_0/B}{A_c^2}$$ \hspace{1cm} (5.9)

Where $N_0$ is noise spectral density and $B$ is bandwidth of PLL.

Also,

$$\sigma_{\gamma_k}^2 = \frac{1}{\rho_L}$$ \hspace{1cm} (5.10)

Where $\rho_L$ is the loop SNR.

Equation (5.10) shows that the variance of output phase is inversely proportional to loop SNR. Thus phase errors can be described in terms of loop SNR.

The pdf of the phase offset is given by \[32\]

$$f(\gamma_k) = \frac{e^{\rho_L \cos(\gamma_k)}}{2\pi I_0(\rho_L)}$$ \hspace{1cm} (5.11)

Where $I_0(\bullet)$ is the zeroth-order modified Bessel function of first kind.

The array factor with phase offset is given by \[29\]

$$\bar{F}^o(\phi) = \sum_{k=1}^{N} e^{-j\frac{4\pi}{\lambda} r_k \left[ \sin\left( \frac{\phi - \phi_0}{2} \right) \sin\left( \frac{\phi + \phi_0 - 2\psi_k}{2} \right) \right] + j\gamma_k} w_k$$ \hspace{1cm} (5.12)
Where, $\gamma_k$ is the phase offset at $k^{th}$ node

The beampattern with phase offset is given by:

$$\bar{P}_e(\phi) = \left| \frac{1}{N} \tilde{F}_e(\phi) \right|^2 \quad (5.13)$$

The PODB method is used to find $w = [w_1 w_2 \ldots w_k]^T$ by adjusting the phase of each user, which satisfies:

$$\max_w |\tilde{F}_e(\phi)|$$

subject to $|w_k| = 1 \quad (5.14)$

### 5.3 PODB method for imperfect phase synchronization

Phase-only DB (PODB) for CR networks has been proposed in [30] which can successfully direct the beam towards the intended receiver and null towards the licensed users. In this method only the phase of the transmitted signal is adjusted, the magnitude being unity for all the cognitive users. PODB method also prolongs the lifetime of CR networks due to effective battery consumption at CR users and thus called green cognitive radio networks [30]. The objective of PODB method is to find the weight $w_k$ of each user so that the beampattern is maximized in the direction of receiver given we only have the information of the receiver location $(A, \theta_0, \phi_0)$ and the cognitive users $(r_k, \psi_k)$. To find the solution of equation (5.14) let us assume that $w_k$ is given by:

$$w_k = e^{j\mu_k} \quad (5.15)$$

Let $\mu = [\mu_1 \mu_2 \ldots \mu_k]^T \in \mathbb{R}^{K \times 1}$ and is given by:
\[\mu_m = \left[ e^{-j\frac{4\pi}{\lambda} r_1 \sin\left(\frac{\phi_m - \phi_0 - \psi_1}{2}\right) \sin\left(\frac{\phi_m - \phi_0}{2}\right)} \right. \left. \ldots, e^{-j\frac{4\pi}{\lambda} r_1 \sin\left(\frac{\phi_m - \phi_0 - \psi_k}{2}\right) \sin\left(\frac{\phi_m - \phi_0}{2}\right)} \right] \]

\[= x_m + jy_m\]

Where \(x_m = \text{Re}[\mu_m]\) \quad \(y_m = \text{Im}[\mu_m]\)

The relative phase with \(m^{th}\) node at the \(n^{th}\) user is given by:

\[\alpha_{m,n} = 4\pi \frac{R}{\lambda} \sin\left(\frac{\phi_m - \phi_n}{2}\right)\]  

(5.16)

Let us define the following variables

\[X = [x_1 \ x_2 \ \ldots \ x_M]\]

\[Y = [y_1 \ y_2 \ \ldots \ y_M]\]

\[\Delta = XY\]

\[\Gamma = \Delta^H \Delta\]

\[c = [c_1 \ c_2 \ \ldots \ c_M]\]

Where \(c_m = \frac{2J_1(\alpha_{m,0})}{\alpha_{m,0}}\)

The calculation of \(\Gamma\) depends only upon the prior knowledge of \((r_k, \psi_k), (A, \theta_0, \phi_0), \phi_m\)

\(\text{where } m = 1, 2, \ldots M\) and is given by [4]

\[
\lim_{k \to \infty} \frac{1}{\Gamma} P1 \Gamma E \left[ \frac{\Gamma}{K} \right] = \begin{bmatrix} \Gamma_1^{M \times M} & 0 \\ 0 & \Gamma_2^{M \times M} \end{bmatrix}
\]  

(5.17)

\[
\Gamma_1^{(m,n)} = \begin{cases} 
\frac{J_1(2\alpha_{m,0})}{2\alpha_{m,0}} + \frac{1}{2}, & m = n = 1, 2, \ldots, M \\
\frac{J_1(\alpha_{m,n})}{\alpha_{m,n}} + \frac{J_1(\alpha_{n,m})}{\alpha_{n,m}}, & m \neq n = 1, 2, \ldots, M
\end{cases}
\]  

(5.18)
\[(\Gamma_2)_{m,n} = \begin{cases} \frac{1}{2} - \frac{j_1(2\alpha_{m,0})}{2\alpha_{m,0}}, & m = n = 1,2, \ldots, M \\ \frac{j_1(\alpha_{m,n})}{\alpha_{m,n}} - \frac{j_1(\alpha_{m,n})}{\overline{\alpha}_{m,n}}, & m \neq n = 1,2, \ldots, M \end{cases} \] (5.19)

Let \( c\Gamma_2^{-1} = [q_1 q_2 \ldots q_M] \)

Now the value of \( \mu \) is given by [30]

\[ \mu^H = c\Gamma_2^{-1}Y^H \] (5.20)

From (5.15) the weight of the \( k^{th} \) user is given by:

\[ w_k = e^{j\mu_k} = e^{(\sum_{m=1}^{M} j\alpha_m(y_m)_k)} \] (5.21)

Hence by substituting the value of \( w_k \) in equation (5.13) gives the array factor of the uniformly distributed cognitive users with imperfect phase synchronization. Here the array factor is related with with loop SNR of PLL and applied PODB method to adjust errors due imperfect phase. The beampattern is given by:

\[ \bar{P}^e(\phi) = \left| \frac{1}{N} \tilde{F}^e(\phi) \right|^2 \] (5.22)
CHAPTER 6. SIMULATION RESULTS

In this chapter simulation results are presented to show the statistical distribution of the mainlobe and sidelobe power levels. We perform 10,000 Monte Carlo trials to obtain the average beampattern using equation (5.13). The radius of the disk is normalized by wavelength and is chosen to be \( \frac{R}{\lambda} = 2 \), azimuth angle \( \phi_0 = 0^\circ \) and the elevation angle \( \theta = \theta_0 = \frac{\pi}{2} \). The cognitive users are generated using uniform distribution and their number is chosen to be 4, 7, 16, 100 and 256. The loop SNR of VCO output of PLL is considered 2dB, 3dB and 10 dB. Two licensed users are considered to be situated at \( \phi_m = 20^\circ, 30^\circ \).

Figure 6.1: Average beampattern with phase offset for \( N = 100, \; \frac{R}{\lambda} = 2 \)
Figure 6.1 shows the average beampattern of POdB method with imperfect phase. The number of users $N = 100$ for this case. For $\phi = 0^\circ$ the average gain is 0 dB for loop SNR 10dB but the gain decreases slightly for loop SNR 2dB and 3dB. Similarly, there is a slight drift in mainlobe peak location due to imperfect phase. For $\phi \approx 20^\circ, 30^\circ$ i.e. the sidelobe power level is close to $10 \log_{10}(1/N) \approx -20dB$.

Figure 6.2: Average beampattern with phase offset for $N = 256, \frac{R}{\lambda} = 2$

Figure 6.2 shows the average beampattern of POdB method with imperfect phase for $N = 256$ users and $\frac{R}{\lambda} = 2$. For $\phi = 0^\circ$ the average gain is 0 dB for loop SNR 10dB but the gain decreases slightly for loop SNR 2dB and 3dB. Similarly, there is a slight drift in
mainlobe peak location due to phase offset. For $\phi \approx 20^\circ, 30^\circ$ the average power is close to $10 \log_{10} (1/N) \approx -24 dB$.

Figure 6.3: Average beampattern with phase offset for $\frac{R}{\lambda} = 2$, loop SNR = 10 dB

Figure 6.3 shows average beampattern with phase offset for loop SNR = 10 dB and smaller values of $N$ i.e. 4, 7, 16 and $n = 100$. The sidelobe power level of $10 \log_{10} (1/N)$ holds true for this case too. For loop SNR 10 dB we observe that the phase synchronization error has less impact and the average gain is 0 dB for $\phi = 0^\circ$ i.e. in the direction of mainlobe. The simulation result shows that there is a slight drift in mainlobe peak location due to phase synchronization error so that the beampattern is not symmetric.
Figure 6.4: CCDF of beampattern at $\phi = 0^\circ$ for loop SNR $= 10$ dB

Figure 6.4 shows cumulative complementary distribution function (CCDF) of the PODB with phase offset for smaller values of $N \ i.e. \ 4, 7$ and 16. CCDF shows the time percentage of the beampattern equal to or greater than certain power level. We take 10,000 Monte Carlo simulations for the calculation of CCDF. The CCDF is taken at $\phi = 0^\circ$ for loop SNR $= 10$ $dB$. Since $\phi = 0^\circ$ the CCDF shows the time percentage distribution of instantaneous power at mainlobe peak location for $N \ i.e. \ 4, 7$ and 16.
Figure 6.5 shows cumulative complementary distribution function (CCDF) of the PODB with phase offset for smaller values of $N \ i.e. \ 4, 7$ and 16. CCDF shows the time percentage of the beampattern equal to or greater than certain power level. We take 10,000 Monte Carlo simulations for the calculation of CCDF. The CCDF is taken at 3 $dB$ points for loop SNR = 10 $dB$. Since the CCDF is taken at 3 $dB$ points the simulation shows the time percentage distribution of instantaneous power at points 3 $dB$ less than the mainlobe peak location for $N \ i.e. \ 4, 7$ and 16.
CHAPTER 7. CONCLUSION

In this MS thesis, distributed beamforming in cognitive radios is studied. Beamforming in uniform arrays is discussed and the effect of position displacement of antenna elements from ideal uniform array is investigated. The simulation result shows that there is an increase in sidelobe level due to position deviation of antenna elements from ideal uniform array. Least square algorithm is used for the correction of error due to position displacement of antenna elements. The least squares algorithm minimizes the mean squared error between the actual response and the desired response by calculating weights in such a way that the actual response is as close as possible to the desired response. The simulation result show that the beampattern averaged over 50 simulation runs approximates that of ideal uniform linear array. Then power consumption of centralized approach to beamforming and two distributed approach are compared. The simulation results show the comparison between the centralized approach and the parallel way of solving LS problem and also the comparison between centralized approach and distributed approach based on QR factorization using Householder transformation. It is observed that by using parallel way of solving LS problem the total power consumption is approximately 63% less than that required by distributed approach based QR factorization using Householder transformation. Also the power consumption is relatively constant function of the number of angles approximated which proves that the parallel approach is energy efficient.

As illustrated by the results distributed approach has many benefits compared to centralized approach. A system model with uniformly distributed cognitive users is used to study distributed beamforming in cognitive radios. But because of the distributed nature
of the users phase synchronization poses a significant problem. The effect of imperfect phase in statistical distribution of the beampattern using PODB method is investigated. The simulation results of average beampattern show that there is some effect in the mainlobe power level but no effect in the sidelobe power level due to phase offset. The simulation results of complementary distribution function shows the distribution of power levels at $\phi = 0^\circ$ and 3 dB points. The results provide some insights and can be useful in distributed beamforming for cognitive radios.
REFERENCES


