1975

Temperature, Heat, and Thermodynamics: First Law

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INTRODUCTION

How can we keep track of energy as it is transferred from one system to another? How can we calculate the amount of internal energy - a quantity that seems to be hidden within the very "guts" of matter? Further, what is the difference between temperature and heat, and between heat and work?

This module focuses on the first of two central thermodynamic principles: the conservation of energy, or, as it is sometimes called, the first law of thermodynamics. The second basic principle, which deals with the inevitable increase of a quantity called entropy, is the subject of another module Second Law and Entropy. These two abstract principles, plus a few other concepts and laws and the vocabulary needed for literacy in the field, are the entire content of thermodynamics. The energy and entropy principles form the framework that governs all energy conversions involving heat; they are the touchstones we must rely on as we attempt to create new energy devices, such as solar converters or fusion reactors, to limit the wasteful exploitation of the Earth's resources.

The approach of this module is macroscopic - that is, we shall deal with systems that are approximately of human scale in size and mass (thermometers, blocks of ice, heat engines), and we shall choose observable quantities such as pressure, volume, and temperature to describe the behavior of these systems. The macroscopic approach should be seen as supplementary to the microscopic approach, which regards the behavior of the atoms and molecules as fundamental. This latter framework chooses the molecular velocities, energies, and momenta as the starting point, and values for macroscopic observables are derived from the microscopic picture. The microscopic approach is treated in another module Kinetic Theory of Gases, where the behavior of gases is interpreted in terms of molecular energies and collisions.
STUDY GUIDE: Thermodynamics - First Law

PREREQUISITES

Before you begin this module, you should be able to:

| *Interpret an integral as the area under a curve and evaluate it (needed for Objective 5 of this module) |
| Location of Prerequisite Content |
| Calculus Review |

| *Calculate the work done by a constant or variable force oriented parallel or obliquely to the displacement of a particle (needed for Objective 5 of this module) |
| Work and Energy Module |

| *Calculate the kinetic energy of a particle, given its mass and velocity [specifically, use the formula \( K = \frac{1}{2}mv^2 \)] (needed for Objective 5 of this module) |
| Work and Energy Module |

| *Employ the relationship of power to energy and time \( (E = Pt) \) in order to calculate one of the quantities, given the others (needed for Objectives 3 and 5 of this module) |
| Work and Energy Module |

| *Calculate the amount of gravitational potential energy near the surface of the Earth (specifically, use the formula \( U_g = mgh \)) (needed for Objective 5 of this module) |
| Conservation of Energy Module |

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Temperature and zeroth law - (a) State the zeroth law of thermodynamics and define the terms used; (b) deduce Kelvin temperature from the pressure of a gas thermometer; and (c) convert these temperatures from the Kelvin to the Celsius scale and vice versa.

2. Specific heat capacity - Define specific heat capacity, and solve mixture problems (in which several materials insulated from the environment and originally at different temperatures attain equilibrium).

3. Heat conduction - State the heat-conduction equation, define each term used, and, given a conductor of uniform cross section, find the value of whichever variable is unknown.

4. Latent heat and change of phase - Define latent heat, and solve problems in which a substance changes phase and/or temperature.

5. First law of thermodynamics - State the first law of thermodynamics, define the terms involved (heat, work, internal energy), and calculate these quantities for a system undergoing a given process, possibly presented in the form of a pressure-volume (pV) graph. The calculation may require you to convert kinetic or gravitational potential energy to internal energy.
STUDY GUIDE: Thermodynamics - First Law

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

In contrast with the strictly macroscopic point of view in this module, Bueche explains thermodynamics largely within the framework of kinetic theory. To get an overview, we recommend that you first read fairly quickly through Sections 15.1 through 15.4 in Chapter 15, Sections 16.1 through 16.5 and 16.9 through 16.15 in Chapter 16, and Sections 17.1 through 17.4 in Chapter 17. Keep the macroscopic-microscopic distinction in mind; you will not be expected to learn any aspects of kinetic theory in this module.

For your second reading, we recommend that you proceed sequentially by objective, as listed in the Table. It will also be important to study the listed problems with solutions and to work the assigned problems. The table lists only the sections dealing with macroscopic thermodynamics, and in cases where both

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\(^a\)Illus. = Illustration(s).
microscopic and macroscopic ideas are presented together, you should concentrate on the latter. If you wish to do further reading on macroscopic thermodynamics, an excellent treatment is in University Physics, Chapters 15 to 19.*

For Objective 1, read Section 15.3, the definition of thermodynamic equilibrium on p. 302 and the statement of the zeroth law of thermodynamics in Question 4 (p. 320) of Chapter 17, and Section 16.15. Read General Comment 1. Then work Problems A and G. Bueche defines a system to be in thermal equilibrium if the system is identical with a similar isolated system that has existed unchanged for an infinitely long time (p. 302). There are sound reasons behind such a definition, but it is difficult to apply in practice because of the infinite time required. We shall say that two systems are in thermal equilibrium when they are in thermal contact (for example, separated by a thin sheet of copper) and when macroscopic changes have ceased to occur. It is important to realize that the zeroth law ("two systems, each in thermal equilibrium with a third system, are in equilibrium with each other") is one of the fundamental assumptions of thermodynamics; because of Bueche's emphasis on microscopic ideas, he is unable to give the zeroth law much space.

For Objective 2, read Sections 16.1 through 16.3, General Comment 2, study Illustration 16.1, and work Problems B and H. Then read Section 16.14, study Illustration 16.8, and work Problems C and I for Objective 3. Bueche uses the symbol $\lambda$ for heat conductivity, but we shall use $k$. For Objective 4, read Sections 16.10 and 16.12, study Illustrations 16.3 through 16.5, and work Problems D and J.

Objective 5 is the most important and comprehensive objective in this module. Read Sections 16.5 and 17.1 through 17.4. Then read General Comments 3 to 9. Study Illustration 17.1 and work Problem 1 in Chapter 17. Finally work Problems E, F, K, and L.

In Section 17.1 Bueche states the first law as $\Delta U = \Delta Q - \Delta W$. If you find the combined presence of differentials (infinitesimal changes) and deltas (finite changes) confusing, you may prefer to remember this law in the form

$\Delta U = Q - W$ (finite changes) or $dU = dQ - dW$ (infinitesimal changes).

(The meaning of the slashes through the differentials for $Q$ and $W$ is explained in General Comment 7.)

After you have completed the reading and problems, do the Practice Test; if you are successful, take a Mastery Test. If you need more help, reread the appropriate sections of the text and General Comments, then work the relevant Additional Problems listed in the Table.


SUGGESTED STUDY PROCEDURE

We recommend that you proceed objective by objective, as follows: For Objective 1, read and study Chapter 18, Sections 18-1 to 18-6, and General Comment 1; then study Problem A and work through Problem G.

The thermometer shown in Figure 18-1 may be difficult to understand. The more commonly seen device shown in Figure 1 will also function as a constant-volume gas thermometer; however, it will not give a very accurate reading of the Kelvin temperature (because gases are generally stored at quite high pressure in such cylinders).

Read and study Chapter 19, Sections 19-1 and 19-2 and Example 1 for Objective 2. Then work Problem 5 in the text and Problems B and H in this study guide.

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aEx. = Example(s).
STUDY GUIDE: Thermodynamics - First Law

Your text uses the term "specific heat" to refer to the quantity \( c = \frac{\Delta Q}{\Delta m \Delta T} \); we shall employ "specific heat capacity." Read General Comment 2. Then, for Objective 3, read Section 19-3 carefully; study Example 2 of Chapter 19, and work Problem 11. Finally, work Problems C and I.

Read the third paragraph of Section 19-7 (on pp. 367, 368, starting with "Let us consider the boiling process..."). The result you will need to remember for Objective 4 is that for boiling, \( Q = mL \), where \( Q \) represents the total heat absorbed, \( m \) represents the mass of material vaporized, and \( L \) symbolizes the heat of vaporization. This formula can also be used for all phase changes (vaporization, condensation, melting, freezing, sublimation), and \( L \) is then referred to as the heat of transformation or the latent heat. If you have difficulty with Problems 13, D, and J, read Chapter 16, Section 16-6, "Change of Phase," in University Physics,* and work Problems 16-23 and 16-33.

For Objective 5, read and study Sections 19-4 through 19-7, plus General Comments 3 to 9. Look over Example 3 carefully, and work Problems 27 in Chapter 19 and E, F, K, and L in this study guide. Note: In Figure 19-4 the force \( \mathbf{F} \) is drawn incorrectly. It should be directed upward, since it is the force exerted on the piston by the gas inside the cylinder. With this correction, the work done by the gas is correctly given by \( dW = \mathbf{F} \cdot ds \), as stated in the text.

When you feel that you have mastered all of the objectives, try the Practice Test. If you are successful, take the Mastery Test; if not, study the appropriate sections, examples, and problems once more, and, if necessary, work some of the Additional Problems listed in the Table.

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SUGGESTED STUDY PROCEDURE

The concepts of this module are developed extremely well in your text. Therefore, we recommend that you read the relevant chapters twice; the first time, read Chapters 15, 16, 17, and 19, Sections 19-1 through 19-11 fairly quickly to get an overview. The second time, proceed objective by objective, working the relevant problems (see Table below) as you go.

For Objective 1, read and study Chapter 15, Sections 15-1 to 15-4, General Comment 1, and work Problems A and G in this study guide. Then study Chapter 16, Sections 16-1 through 16-3 and General Comment 2. Work Problems B and H and Problem 16-3 in your text. Study Chapter 17, Section 17-1 for Objective 3, then do Problems 16-1 in the text and C and I in this study guide. Problem 17-3 in the text is similar to C and I but somewhat more challenging. For Objective 4, read Section 16-6 in Chapter 16 and work Problems D and J. To master Objective 5 (First Law), read Sections 19-1 through 19-8, 19-10, and 19-11, plus General Comments 3 to 9. This is the most comprehensive and important objective; you should think through a response to Problem 19-1, and check your answer; then work out Problems E, F, and K, and study solutions provided carefully. Finally, do Problem L. When you feel that you have mastered all of the objectives, try the Practice Test. If you succeed, take a Mastery Test; if not, study the appropriate sections, examples, and problems, and, if necessary, do some of the Additional Problems listed in the table.

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SUGGESTED STUDY PROCEDURE

In contrast with the strictly macroscopic point of view adopted in this module, Weidner and Sells explain classical thermodynamics largely within the framework of kinetic theory. To get an overview, you should quickly read through, but not study, the following sections: Chapter 18, Sections 18-1 to 18-3 and 18-6 to 18-8; Chapter 19, Section 19-4; Chapter 20, Sections 20-1 to 20-3, 20-5 and 20-6. As you read, keep the macroscopic-microscopic distinction in mind; you will not be expected to learn any aspects of kinetic theory in this module, but the overview will be helpful. For further reading, an excellent treatment of macroscopic thermodynamics is in Chapter 16 of University Physics.*

For your second reading, we recommend that you proceed objective by objective, studying the problems with solutions and working the assigned problems listed in the table. For Objective 1, read Sections 18-1 through 18-3 and General Comment 1, and work Problems A and G in this study guide. Then read the last three paragraphs of Section 20-2 [Eq. (20-1)], Section 20-3 and General Comment 2. Study Example 20-1, and work Problems B and H in this study guide for

WEIDNER AND SELLS

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*Ex. = Example(s).

Objective 2. Your text's definition of specific heat capacity \( c = \frac{\Delta U}{m \Delta T} \) differs from the definition we shall use below in solving problems \( c = \frac{\Delta Q}{m \Delta T} \), but for liquids and solids the two definitions differ by a negligible amount.

For Objective 3, read Section 20-6, study Example 20-4, and work Problems C and I. Read and study Section 20-5 up to the last paragraph on p. 417 for Objective 4; the rest of this section and Figures 20-3 and 20-4 should be skimmed. Work through part (a) of Example 20-3, and do Problem 20-9. Study Problem D and work Problem J of this study guide.

For the First law, Objective 5, read Sections 18-6 through 18-8, 19-4, 20-2, and General Comments 3 to 9. As you read, study Example 20-2 and part (b) of Example 20-3, then work Problem 20-1 and check your answer at the back of the text. Finally work Problems E, F, K, and L.

Note: Weidner and Sells state the first law of thermodynamics as follows:
\[ dQ = dU + dW \] (Section 19-4, p. 386). Later they state the alternative form \( \Delta Q = \Delta U + \Delta W \) (Section 20-2, p. 411). We shall use slightly different but equivalent statements in presenting solutions to the problem set: \( \Delta U = Q - W \) and \( dU = \delta Q - \delta W \). The sign conventions for \( U, Q, \) and \( W \) are the same. We also prefer to state the first law with the internal energy, \( U \), on the left and with the heat, \( Q \), and work, \( W \), on the right. The meaning of the slashes through the differentials \( \delta Q \) and \( \delta W \) will be explained in General Comment 7.

When you feel prepared, do the Practice Test; if you are successful, take a Mastery Test. If you need more help, reread the appropriate sections of the text or the General Comments and work the relevant Additional Problems listed in the Table.
GENERAL COMMENTS

1. Pressure

The pressure \( p \) is defined to be the force per unit area. If some gas is enclosed in a cylinder with a leak-proof (though frictionless) piston having an area \( A \) (see Fig. 2), this gas will exert an upward force of magnitude

\[
F_p = pA
\]

on the piston. An oppositely directed force of equal magnitude will be needed to hold the piston in place. In the diagram, this force is represented by \( F \).

2. Heat Units

Your text uses units of calories (or kilocalories) for heat, but we shall consistently employ the joule unit (J), in accordance with the International System:

\[
1 \text{ J} = 1.42 \text{ cal} = 1.42 \times 10^3 \text{ kcal}.
\]

The remaining comments concern conservation of energy, as stated in the first law of thermodynamics, which is the most important and comprehensive concept of this module.

3. Statement of the FIRST LAW OF THERMODYNAMICS

For a given system, which undergoes any process beginning and ending in equilibrium, the heat transferred to the system \( Q \) minus the work done by the system \( W \) is exactly equal to the change in the internal energy function \( U \). In symbols, \( \Delta U = Q - W \).

It is important to be clear about the system under consideration and to define the boundaries of the system carefully. For example, if the system is a gas within a cylinder, the cylinder walls and the piston head form the boundaries. The size of the system or the shape of the boundary can change, as when the piston moves, but in order for the first law to be valid one must keep track of the heat and work crossing the boundary. For example, if the piston moves out, then the system does work \( W \) on the environment, and if a flame is put under the cylinder then heat is transferred to the system.
We shall only consider systems in which no material enters or leaves, or which can be idealized as such. It is possible to add a term to the first law to take into account matter entering or leaving the system, but this is taken up in more advanced courses. Thermodynamics was originally the study of "heat engines"; that is, systems that absorbed heat and performed work. Therefore, heat was naturally considered positive when it entered a system, and work was positive when it left the system. The same conventions are still used today, and this is the reason for the difference in sign between $Q$ and $W$ in the first law.

4. Definition of Work

From your previous study of mechanics (Work and Energy) you may recall that work on a particle is defined as

$$ W = \int \mathbf{F} \cdot d\mathbf{r}, $$

where the integral is carried out along the path taken by the particle. We use the same definition for the work done by a system in thermodynamics. In cases where a system expands or contracts by volume $dV$, we can extend this definition to read

$$ W = \int_{V_i}^{V_f} p \, dV, $$

where $p$ represents the pressure within the system, and the integral is carried out from the initial volume $V_i$ to the final volume $V_f$. Notice that work is positive when the system expands, but this tends to decrease the internal energy function because of the minus sign in the first law. It is also useful to realize that $\int p \, dV$ can be represented as the area under the graph of $p$ versus $V$, and this can sometimes be the best way to calculate $W$ for a given process (see Problems K and L). In addition to changes in the volume of the system, it is important to remember that other types of work can also change the internal energy of a system. For example, electrical and frictional forces can also do work, and this must be included in calculating $\Delta U$. 
Work done on a system has the result of increasing the internal energy function. Thus rubbing your hands together rapidly is associated with a rise in temperature of your palms; compressing gas in a cylinder also results in a temperature rise, and rapid stirring of a water-ice mixture brings about melting of some of the ice.

5. Definition of Heat

There is also a way to increase the internal energy of a system without the performance of work. This method is known as heat transfer or heat flow, and it is associated with a difference in temperature across the boundary separating the system and the environment. For example, one could hold one's hands near a fire; a cylinder of gas could be placed in boiling water; and a water-ice mixture could be held above a house radiator. In summary, heat flow is a transfer of energy that brings about changes in system properties; these changes are exactly the same as would be produced by the performance of work, and they are associated with a difference of temperature across the system boundaries.

In this module you study three topics related to heat transfer: conduction, specific heat capacity, and change of phase. These are basically just ways of accounting for or measuring heat transfer $Q$ across the boundary of a system; thus, they provide a way to put numbers into the first law. For example, for a pot of water on the stove, we can calculate $Q$ in three different ways: (1) The temperature difference $\Delta T$ between the inside and outside of the bottom, plus its thermal conductivity $k$, thickness $L$, and area $A$ lead to a value for $\frac{dQ}{dt} = -kA(\Delta T/L)$. (2) If the water temperature is observed to rise by $\Delta T$, then $Q$ can be found from $Q = mc \Delta T$, where $m$ represents the mass of water in the pot, and $c$ represents the known specific heat capacity of water. (3) If the water is allowed to boil and $M$ kg boils away, then, using $L$ to represent the known heat of vaporization of water, $Q = ML$.

6. Equivalence between Internal and Mechanical (Kinetic and Potential) Energy

You should recall the law of conservation of mechanical energy for conservative forces (Conservation of Energy module), which can be stated as

$$K + U(x, y, z) = E,$$

where $K$ (kinetic energy) = $(1/2)mv^2$, $U$ represents the potential energy function, and $E$ represents the total mechanical energy, which is constant. In particular, for motion of a particle (mass $m$) near the surface of the earth, $U = mg$, assuming that no forces other than gravity are acting. Mechanical energy can be converted into internal energy in various ways, always through the medium of work. If the forces acting are known, then one can calculate the work and the energy transferred from mechanical to internal form. However, it is usually easier to calculate the energy transfer directly, as in the example below.
Example

A block of ice at 0°C with mass 50 kg, slides along a horizontal surface, starting at a speed of 5.4 m/s and finally coming to rest after traveling 28.3 m. Given the coefficient of sliding friction \( \mu \), calculate the heat generated and, assuming that all of the heat is transferred to the ice, the mass of ice melted.

Solution

Rather than calculating the work using the weight, coefficient of friction, and distance traveled, we note that all of the initial kinetic energy is converted into internal energy of the ice. \( K = \frac{1}{2}m_1v^2 \), where \( m_1 = 50 \) kg, and \( v = 5.4 \) m/s. Thus

\[
K = \frac{1}{2}(50 \text{ kg})(5.4 \text{ m/s})^2 = 730 \text{ J.}
\]

Now, \( \Delta U_{\text{ice}} = m_2L \), where \( m_2 \) represents the mass of ice melted, and \( L = 3.3 \times 10^5 \) J/kg, the heat of fusion for water. We set \( \Delta U_{\text{ice}} = K \), or \( m_2L = 730 \) J,

\[
m_2 = \frac{730 \text{ J}}{3.3 \times 10^5 \text{ J/kg}} = 2.10 \times 10^{-3} \text{ kg.}
\]

This is a very small amount of ice melted: In general the amounts of mechanical energy with which we have direct experience (10² to 10³ J) usually produce rather small thermal effects.

7. Differential Form of First Law and Inexact Differentials

The first law can also be written for infinitesimal changes as \( dU = dQ - dW \). The slashes through the differentials for \( Q \) and \( W \) indicate that these are not "exact differentials." This is to guard against any misconception that there might exist functions of the thermodynamic variables \( p, V, T, \) etc., of which \( dQ \) and \( dW \) would be differentials - there are no such functions. On the other hand, \( dU \) is an exact differential, and there does exist a unique function \( U \) that depends only on the state of the system and not on the path by means of which the system is brought to that state. There are other such functions of state, for example, \( p, V, T, \) and the entropy \( S \), which will be pursued further in the module Second Law and Entropy.

8. Thermodynamic Processes

You should be familiar with two terms used to describe particular types of processes:

Adiabatic refers to processes in which \( Q = 0 \). This is usually the result of insulation surrounding the system of interest, but many processes are carried out quickly enough so that appreciable amounts of heat are not transferred to the working substance, and these processes are also referred to as adiabatic. An example of the latter type is the expansion of the burning gases in an automobile engine cylinder. For adiabatic processes, \( \Delta U = -W \).
An isobaric process is carried out at constant pressure. All processes carried out with systems open to the atmosphere are isobaric. For an isobaric process, \( W = \int p \, dV = p \int dV = p(\Delta V) \); thus, if a change of volume occurs as part of an isobaric process, the work is easy to calculate.

9. Pressure-Volume Graphs

One of the most useful tools in the study of a thermodynamic system is a pressure-volume (pV) graph. In addition to compactness and clarity, pV graphs have the helpful feature that work can be represented as an area under the curve.

Example

Draw a pV graph for two sequential processes: (1) Isobaric expansion of a gas in a cylinder from \( V_0 \) to \( 2V_0 \) at \( p_0 \), and (2) heating of the cylinder and gas with the piston fixed; the pressure increases from \( p_0 \) to \( 2p_0 \), and the volume remains constant at \( 2V_0 \).

Solution

The pV graph is shown in Figure 3. Now calculate the work done, \( W_1 \) and \( W_2 \), during each process:

\[
W_1 = \int p \, dV = p \Delta V = p_0(2V_0 - V_0) = p_0V_0.
\]

This is represented by the shaded area in the graph.

\[
W_2 = \int p \, dV, \quad dV = 0,
\]

therefore \( W_2 = 0 \).

Additional Learning Materials

S. Angrist and L. Hepler, Order and Chaos (Basic Books, New York, 1967). This is an interesting and easy-to-read book dealing with the basic content and history of thermodynamics. Many fascinating details are given concerning the experiences and personalities of the early thermodynamicists.
Richard Merrill et al. (Eds.), Energy Primer, Solar, Water, Wind, and Biofuels (Portola Institute, 558 Santa Cruz Ave., Menlo Park, Calif., 1974). This contains the meaty article, "Solar Radiation and Its Uses on Earth," with an appendix on "Methods of Estimating Solar Heater Performance," both by John I. Yellott. This article is probably the best introduction to the field; there are numerous opportunities to employ the concepts covered in this module in making estimates of the feasibility and required dimensions of the systems described. This field – utilization of solar energy – is steadily becoming more exciting, and there is a challenging opportunity here for people with knowledge of thermodynamics to make an important social contribution.

Morton Mott-Smith, The Concept of Energy Simply Explained (Dover, New York, 1934). This is slightly more technical than Angrist and Hepler; it also is historical but gives more attention to the kinds of practical engineering problems that James Watt and other early workers were trying to solve. One gets an excellent view of why and how the theory of thermodynamics grew up. For example, the first pV graphs were "indicator cards" drawn by the moving pistons of Watt's engines.

H. C. Van Ness, Understanding Thermodynamics (McGraw-Hill, New York, 1969). This is a slim, enjoyable, and insightful book. It attempts to show, in a conversational way, the plausibility and usefulness of the basic concepts. Topics covered are the first law, reversibility, heat engines, power plants, the second law, and statistical mechanics.

Mark W. Zemansky, Heat and Thermodynamics (McGraw-Hill, New York, 1968), fifth edition. This is an excellent textbook, written at a somewhat more advanced level than this module.

PROBLEM SET WITH SOLUTIONS

A(1). (a) State the zeroth law of thermodynamics and define "thermal equilibrium" and "system."

(b) Suppose that you wish to identify an unknown liquid by measuring its boiling point accurately. You first use a constant-volume gas thermometer to measure the pressure (p) of the confined gas to be $2.6 \times 10^4 \text{ Pa}^*$ at the triple point of water. Then you bring the same confined gas to equilibrium with the unknown boiling liquid and measure $p = 3.5 \times 10^4 \text{ Pa}$. What is the temperature of vaporization on the Kelvin scale?

(c) What is the temperature of vaporization on the Celsius scale?

(d) What is the unknown liquid?

Solution

(a) The zeroth law states that two systems in thermal equilibrium with a third system are in thermal equilibrium with each other. Thermal equilibrium can be defined as the state in which two systems are in thermal contact (for example, separated by a thin sheet of copper) and in which changes in the systems have ceased. A system is a portion of matter that we separate (in our minds) from

*The pascal is the SI unit for pressure: $1 \text{ Pa} = 1 \text{ N/m}^2$. 
the environment external to it; the properties of the system itself are described by variables (for example, temperature, pressure, volume, mass, internal energy), and the interaction with the environment is described in terms of quantities transmitted across the system boundaries (e.g., heat and work).

(b) Using the definition $t = P$, and recalling that the temperature of the triple point of water is defined to be 273.16 K,

$$\frac{T(K)}{273.16 K} = \frac{35 \text{ Pa}}{26 \text{ Pa}} \implies T = \frac{35}{26}(273.16) = 368 \text{ K} = 370 \text{ K}.$$ 

(c) $T(\circ C) = T(K) - 273.15\circ C = 95\circ C$.

(d) Since water has a boiling temperature at standard pressure of 100$\circ C$, the liquid is probably water. (If the atmospheric pressure were below standard pressure during the experiment, it would explain the observed reduction of the boiling temperature.)

B(2). An aluminum block ($c = 9.1 \times 10^2$ J/kg K) of mass 0.80 kg at a temperature of 275$\circ C$ is dropped into an aluminum calorimeter cup of mass 0.200 kg containing 1.00 kg of water ($c = 4.2 \times 10^3$ J/kg K) at 20$\circ C$. The system is insulated and attains equilibrium at a final temperature $T_f$.

(a) Using the definition of specific heat capacity, set up the $Q_{\text{total}} = 0$ equation.

(b) Solve for $T_f$.

Solution

(a) $Q_{\text{gained}}$ by aluminum = $c_A M_A (T_f - T_A)$,

$Q_{\text{gained}}$ by cup = $c_C M_C (T_f - T_C)$,

$Q_{\text{gained}}$ by water = $c_W M_W (T_f - T_W)$.

If $Q_{\text{total}} = 0$, then $c_A M_A (T_f - T_A) + c_C M_C (T_f - T_C) + c_W M_W (T_f - T_W) = 0$.

(b) $T_f \left( c_A M_A + c_C M_C + c_W M_W \right) = c_A T_A + c_C T_C + c_W T_W$,

$$T_f = \frac{c_A T_A + c_C T_C + c_W T_W}{c_A M_A + c_C M_C + c_W M_W} = \frac{2.878 \times 10^5 \text{ J}}{5.1 \times 10^3 \text{ J/}\circ C} = 56\circ C.$$

Note that heat is actually lost by the aluminum, but this is automatically taken into account by the fact that we wrote all temperature changes as $T(\text{later}) - T(\text{earlier})$ in part (a), and we put all of the heat terms on one side of the equation. If you set up the equation $Q(\text{gained}) = Q(\text{lost})$, as Bueche, Halliday and Resnick, and Weidner and Sells do, then you must be careful to make all $\Delta T$'s positive, that is, $T(\text{higher}) - T(\text{lower})$. 
C(3). A copper rod (length 0.35 m) has a circular cross section with radius $r = 0.050$ m. The two ends are kept at different, fixed temperatures, and the sides are insulated. The energy transfer along the rod is observed to be $8.4 \times 10^2$ W, and the temperature in the middle is $145^\circ$C. For copper the thermal conductivity $k = 3.8 \times 10^2$ W/m K. 

(a) Draw a sketch of the setup.
(b) What are the temperatures of the two ends?

Solution

(a) Note: You are only expected to produce a sketch, similar to Figure 4 with $L = 0.35$ m and $r = 5.0 \times 10^{-2}$ m.

(b) Since the rod has constant cross section, we can put $dT/dx = (T_2 - T_1)/L$, and the heat equation becomes

$$\frac{dQ}{dt} = -kA\frac{T_2 - T_1}{L} \quad \text{or} \quad (T_2 - T_1) = -(\frac{L}{kA})(\frac{dQ}{dt}),$$

and

$$T_2 - T_1 = \frac{0.35 \text{ m}}{(3.8 \times 10^2 \text{ W/m K})(2.50 \times 10^{-3} \text{ m}^2)}(8.4 \times 10^2 \text{ W}) = 98 \text{ K};$$

$$T_2 = 145 + 98/2 = 194^\circ \text{C}; \quad T_1 = 145 - 98/2 = 96^\circ \text{C}.$$

D(4). A block of ice, mass 0.20 kg and temperature 0°C, is dropped into a brass calorimeter cup of mass 0.100 kg containing 0.300 kg water at 80°C. Assuming that no heat is lost to the environment, find $T_F$, the final equilibrium temperature.

$$C_{\text{brass}} = 4.2 \times 10^2 \text{ J/kg K}; \quad L_{\text{ice melting}} = 3.3 \times 10^5 \text{ J/kg};$$

$$C_{\text{water}} = 4.2 \times 10^3 \text{ J/kg K}.$$
(a) Write an expression for $Q_1$, the heat gained by the ice. Be sure to include both the heat involved in melting the ice ($Q = ML$) and in warming the resulting water.

(b) Write an expression for $Q_2$, the heat lost by the cup and the water.

(c) Equate the expressions from (a) and (b), solve for $T_f$, and evaluate.

**Solution**

(a) $Q_1 = M_1L + c_wM_1(T_f - 0)$.

(b) $Q_2 = c_{\text{cup}}M_{\text{cup}}(80 - T_f) + c_wM_w(80 - T_f)$.

(c) $Q_1 = Q_2$, $c_wM_1T_f + c_{\text{cup}}M_{\text{cup}}T_f + c_wM_wT_f = c_{\text{cup}}M_{\text{cup}}80 + c_wM_w80 - M_1L$.

$$T_f = \frac{c_wM_180 + c_{\text{cup}}M_{\text{cup}}80 - M_1L}{c_wM_1 + c_{\text{cup}}M_{\text{cup}} + c_wM_w} = \frac{3.816 \times 10^4}{2.142 \times 10^3} \approx 18^\circ C.$$

E(5). (a) State the first law of thermodynamics, and define all the terms appearing in it.

(b) The density of liquid water at 100°C and 1 atm ($= 10^5$ Pa) is $1.00 \times 10^3$ kg/m$^3$, and the heat of vaporization is $2.30 \times 10^6$ J/kg. The density of water vapor under the same conditions is $0.58$ kg/m$^3$. Considering 1.00 kg of water as the system calculate the change in volume $\Delta V$ during vaporization.

(c) Using $W = \int p \, dV$, calculate the work done by the system on the surrounding atmosphere. Is this work positive or negative?

(d) Set up the first law for this problem; be careful about the signs for $Q$ and $W$, and calculate $\Delta U$.

**Solution**

(a) $\Delta U = Q - W$.

(b) $\Delta V = V_{\text{steam}} - V_{\text{water}} = 1/\rho_s - 1/\rho_w = 1.72$ m$^3$, where $\rho_w$ is the density of water and $\rho_s$ is the density of steam.

(c) $W = \int p \, dV = 0 \Delta V$ (since $p$ is constant), $W = (10^5 \text{ Pa})(1.72 \text{ m}^3) = 1.70 \times 10^5$ J. This is positive work since the system expanded; work was done by the system on the environment.

(d) $U = +2.3 \times 10^6$ J/kg $- (1.70 \times 10^5$ J) $= 2.13 \times 10^6$ J/kg.
F(5). (a) A car (M = 1600 kg) is traveling at 56 mph (25.0 m/s) when it brakes to a halt. The car has four iron brake drums (10.0 kg each) with a specific heat capacity $c = 4.6 \times 10^2$ J/kg K. The brakes are air-cooled, and each drum loses 15000 J to the air during the braking. Assume that the internal energies of the four brake drums increase equally and that the other parts of the car and the road suffer no change in internal energy. What is the change in temperature of the brake drums? (Start from the first law.)

(b) A waterfall is 75 m high. The water has a temperature of 20.0°C above the waterfall and is flowing at 4.0 m/s. Calculate the expected temperature at the bottom. Start from the first law; neglect any transfer of heat to the ground or to the air. Water has a density of $10^3$ kg/m$^3$ and a specific heat of $4.2 \times 10^3$ J/kg K.

Solution

(a) $\Delta U = Q - W$. Considering one brake drum as the system, work is done on the drums, and we can calculate this work by equating it to one-fourth of the original kinetic energy of the car; thus

$$W = -\frac{1}{4}(\frac{1}{2})mv^2 = -\frac{1}{4}(\frac{1}{2})(1600 \text{ kg})(25.0 \text{ m/s})^2 = -1.25 \times 10^5 \text{ J}, \quad Q = -1.50 \times 10^4 \text{ J};$$

therefore from the first law

$$\Delta U = (-1.50 \times 10^4 \text{ J}) - (-1.25 \times 10^5 \text{ J}) = +1.10 \times 10^5 \text{ J}.$$

To find the temperature rise we use the definition of specific heat capacity:

$$T = \frac{Q}{cM} = \frac{1.10 \times 10^5 \text{ J}}{(4.6 \times 10^2 \text{ J/kg})(10.0 \text{ kg})} = 24^\circ \text{C}.$$

(b) $\Delta U = Q - W$. The system is a given mass of water. $Q = 0$, since we are neglecting heat transfer to the ground and air. The work is done on the system by the earth and the surrounding water; therefore $W$ is negative. We can calculate $W$ by equating it with the kinetic plus gravitational potential energy of the water at the top of the waterfall,

$$W = (1/2)mv^2 + mgh.$$

To calculate the temperature rise, we use the definition of specific heat,

$$cmT = (\frac{1}{2})mv^2 + mgh \quad \text{and} \quad T = \frac{(1/2)v^2 + gh}{c} = 0.18^\circ \text{C}.$$
Problems

G(1). (a) On a brisk autumn day (5.0°C) you inflate the tires of your car to a pressure of 44 lb/in.² (2.90 x 10⁵ Pa). (Your tire gauge reads 29 lb/in.² or 1.9 x 10⁵ Pa, but you recall that this is the excess above atmospheric pressure, which is about 15 lb/in.² or 1.00 x 10⁵ Pa.) Assuming that the tires and air inside are in equilibrium with the outside air, what is the temperature of the air inside on the Kelvin scale?

(b) After a fast trip down the Interstate, you again measure the pressure and find that the gauge now reads 42 lb/in.² (2.80 x 10⁵ Pa). What is the temperature (in kelvins) now of the air inside the tires? (Assume that the volume of air remains constant.)

H(2). Figure 5 shows a flow calorimeter. A liquid of density 1200 kg/m³ flows through at the rate of 1.20 x 10⁻⁵ m³/s. The heating element supplies 400 W, and, after steady state is achieved, it is observed that the liquid flowing out is 20.0° warmer than the liquid entering.

(a) Assume that no heat is lost to the environment, and consider the mass of liquid flowing through the system in t seconds to be the system. Set up an equation for Q₁ (the amount of heat gained by this system in time t) in terms of M (the mass of liquid flowing through in time t), c (the specific heat of the liquid), and T (the temperature change of the liquid).

(b) Find Q₂, the amount of heat lost by the heating element in t seconds.

(c) Using the results of (a) and (b), set up the Q₁ = Q₂ equation, solve for the specific heat capacity, and evaluate the resulting expression.

I(3). This problem is illustrative of the way heating systems for buildings are designed. Assume that the building is a cube 10.0 m on a side with walls of wood 0.20 m thick. For wood, k = 8.4 x 10⁻² W/m K. If you wish to keep the inside of the cube at 20°C (68°F) when the outside temperature goes down as low as 0°C (32°F), calculate the required capacity (power) of the heating system as follows:
(a) The area $A$ of one wall of the cube is ____ m$^2$.
(b) Therefore, since the cube has ____ sides, the total surface area $A$ of the building is ____ m$^2$.
(c) Therefore the cube is equivalent to a large, flat conductor of constant cross-sectional area $A$, and length (thickness) $L = ____$ m; the difference in temperature between the two sides is $T_2 - T_1 = ____ ^\circ$C.
(d) Therefore, $dT/dx$ is constant throughout the conductor, and we can put $dT/dx = (T_2 - T_1)/L = ____ ^\circ$C/m.
(e) Using the values above, evaluate the required capacity (power) of the heater, $dQ/dt = ____$ W.
(f) Estimate whether the magnitude of the answer is reasonable.

J(4). If 0.150 kg of ice and 0.200 kg of water in thermal equilibrium are heated to 60°C by being mixed with $M$ kg of steam at 100°C, how much water will be in the final mixture? Use

\begin{align*}
L_{\text{ice melting}} &= 3.30 \times 10^5 \text{ J/kg}, \\
L_{\text{water vaporizing}} &= 2.30 \times 10^6 \text{ J/kg}, \\
\text{and } c_{\text{water}} &= 4.2 \times 10^3 \text{ J/kg K}.
\end{align*}

(a) Write an expression for $Q_1$, the heat gained by the ice-and-water system. Be sure to include the heat involved in both melting the ice and raising the temperature of the melted ice and water.
(b) Write an expression for $Q_2$, the heat lost by the steam. Include both the heat to condense the steam and to lower the temperature of the resulting liquid water.
(c) Equate the expressions from (a) and (b), and find the value of $M$.

K(5). Figure 6 shows an elementary form of steam engine, the Newcomen engine. [You may be interested to know that this engine once represented the vanguard of technology, and it was used for quite some time to pump water out of the mines of England, in spite of its shocking inefficiency (by today's standards). Its success and its defects provided the stimulation for James Watt to invent the modern steam engine.] The engine operates as follows:

Stage 1: Constant volume heating - heat flowing into the boiler changes some of the water to steam, gradually increasing the pressure against the piston. The piston is initially held against the stops by a constant force $F$, a combination of the weight of the piston and the atmospheric pressure on the top of the piston. During this stage, the volume of the system is constant, and the pressure rises from 0 to $p_1$.

Stage 2: Constant pressure expansion - when the boiler pressure reaches $p_1$, then the upward
Figure 6

Figure 7

Figure 8

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force just balances $F$, and the piston rises slowly. The pressure of the system remains constant as the volume of the system increases from $V_1$ to $V_2$.

During Stage 1, the system gains $Q_1 = 2.10 \times 10^5$ J, and during Stage 2, the system gains $Q_2 = 2.90 \times 10^5$ J; also $p_1 = 4.0 \times 10^5$ Pa, $V_1 = 0.100$ m$^3$, and $V_2 = 0.200$ m$^3$.

(a) Draw a pressure-volume graph for Stages 1 and 2.
(b) Which of the following terms do we use to describe Stage 1 and Stage 2: isobaric, adiabatic, or neither?
(c) Give the value of $W_1$, the work done during Stage 1.
(d) Find the value of $\Delta U_1$, the change in internal energy in Stage 1; use the first law of thermodynamics.
(e) Calculate $W_2$, the work done during Stage 2. Justify the sign of the answer, and shade the area on the pV graph corresponding to $W_2$.
(f) Calculate $\Delta U_2$, the change in internal energy during Stage 2. Justify the sign of the answer.
(g) For the complete process, Stage 1 plus Stage 2, give values for $W$, $Q$, and $\Delta U$.

L(5). A cylinder contains a sample of gas confined by a piston as in Figure 7. The cylinder is immersed in a water-ice mixture, and the gas is in thermal equilibrium with pressure $p_A = 1.50 \times 10^5$ Pa and volume $V_A = 0.0140$ m$^3$. We shall refer to this as State A of the system. The gas then undergoes the following sequential changes:

Process 1. The cylinder is removed from the ice-water bath, surrounded with an insulating jacket, and the piston is gradually pushed in. In other words, the gas is compressed adiabatically. The final state of the gas is now State B (see pV graph) with $p_B = 3.6 \times 10^5$ Pa and $V_B = 8.3 \times 10^{-3}$ m$^3$. During this process the gas has done work $W_1$.

Process 2. The temperature of the gas is measured, and the cylinder is placed in a controllable bath at the same temperature. The piston is now allowed to move out gradually, and the temperature of the bath (and thus the gas) is controlled in such a way as to keep $p = -kV + c$, with $k$ and $c$ constant (see Figure 7). This process is continued until the piston is at the same position as in State A, and the gas is said to be in State C, $p_C = 2.40 \times 10^5$ Pa and $V_C = V_A$. During Process 2 the gas does work $W_2$ and gains heat $Q_2$. 
Process 3. The piston is fixed in place; the cylinder is placed back in the ice-water mixture and is allowed to come to thermal equilibrium. During this process the heat flow is $Q_3$, and the work done is $W_3$.

(a) Process 1 and States A and B are shown on the accompanying $pV$ graph (Fig. 8). Show Processes 2 and 3 and State C on this graph.
(b) Calculate $W_2$.
(c) Suppose that during Process 2, the net heat gained from the bath is $Q_2 = +3040 \text{ J}$. Calculate the change in the internal energy function during Process 2, $\Delta U_2$.
(d) Suppose that during Process 3, $5.7 \times 10^{-3} \text{ kg}$ of ice melted in the bath. Using $L = 3.30 \times 10^5 \text{ J/kg}$, calculate $Q_3$.
(e) Find the change in internal energy during Process 3, $\Delta U_3$.
(f) Find the change in internal energy during Process 1, $\Delta U_1$. Hint: Use the results of (c) and (e).
(g) Using the result of (f), find $W_1$.
(h) Combine your results for $W_1$, $W_2$, and $W_3$ to find the total work done during the cycle.
(i) Combine your results for $Q_1$, $Q_2$, and $Q_3$ to find the total heat added during the cycle.
(j) Combine the results for $W_1$, $W_2$, and $W_3$ and $Q_1$, $Q_2$, and $Q_3$ to determine whether, over the entire cycle, work input is being converted to heat output or heat input is converted to work output. How many joules of heat (or work) are converted over the entire cycle?

Solutions

G(1). (a) $T = 278 \text{ K}$; (b) $T = 364 \text{ K}$. If your answer is 271 K, you forgot that the gauge represents the excess over atmospheric pressure.

H(2). (a) $Q_1 = cMT$. We also must remember that

$$M = (1200 \text{ kg/m}^3)(1.20 \times 10^{-5} \text{ m}^3/\text{s})t = 1.44 \times 10^{-2}t \text{ kg}.$$  

(b) $Q_2 = 400t \text{ J}$.  
(c) $Q_1 = Q_2$,  
$$c = 400t/M \Delta T = 400t/(1.44 \times 10^{-2}t \text{ kg})(20.0 \text{ K}) = 1.39 \times 10^3 \text{ J/kg K}.$$  

I(3). (a) $A_s = 100 \text{ m}^2$.  
(b) 6 sides, $A_{\text{total}} = 600 \text{ m}^2$.
(c) $L = 0.200 \text{ m}$.  
(d) $(T_2 - T_1)/L = (20.0^\circ\text{C} - 0.0^\circ\text{C})/0.200 \text{ m} = 100^\circ\text{C/m}$.
(e) $dQ/dt = -kA[(T_2 - T_1)/L] = -(8.4 \times 10^{-2} \text{ W/m K})(600 \text{ m}^2)(100^\circ\text{C/m}) = 5000 \text{ W}$.
(f) It seems reasonable that fifty 100-W bulbs would keep a 10-m cube comfortable inside when the outside temperature is 0°C (32°F).

J(4). (a) \( Q_1 = (0.150)(3.30 \times 10^5) + (4.2 \times 10^3)(0.35)(60) = 1.38 \times 10^5 \) J.

\[ Q_2 = M(2.30 \times 10^6) + (4.2 \times 10^3)M(40) = 2.47 \times 10^6 \text{J}. \]

(c) \( M = (1.38 \times 10^5)/(2.47 \times 10^6) = 0.056 \) kg. Therefore there will be \((0.150 + 0.200 + 0.056)\) kg = 0.41 kg of water in the final mixture.

K(5). (a) The pressure volume graph is shown in Figure 9.

(b) Stage 1 = neither (isochoric); Stage 2 = isobaric.

(c) \( W_1 = 0 \) (because \( \Delta V = 0 \)).

(d) \( \Delta U_1 = Q_1 - W_1 = Q_1 = 2.10 \times 10^5 \) J.

(e) \( W_2 = \int p \, dV = p \, \Delta V = (4.0 \times 10^5 \text{ Pa})(0.200 - 0.100) \text{ m}^3 = 4.0 \times 10^4 \) J.

This is positive because the system expanded and did work on the environment.

(f) \( \Delta U_2 = Q_2 - W_2 = (2.90 \times 10^5 \text{ J}) - (4.0 \times 10^4 \text{ J}) = 2.50 \times 10^5 \text{ J}. \) This is positive; the system gained more energy by heat flow than it lost by doing work.

(g) \( W_{\text{total}} = W_1 + W_2 = 4.0 \times 10^4 \text{ J}. \quad Q_{\text{total}} = Q_1 + Q_2 = 5.0 \times 10^5 \text{ J}. \)

\( \Delta U_{\text{total}} = \Delta U_1 + \Delta U_2 = 4.6 \times 10^5 \text{ J}. \)

Note that if we proceed by means of a different path, say by means of Processes 3 and 4 on the graph (Fig. 9), we would get different values for \( W \) and \( Q \) but the same value for \( \Delta U \).

L(5). (a) See Figure 10. (Paths 2 and 3 are represented by straight lines.)

(b) \( W_2 = \text{area under Path 2} = P_c(V_B - V_A) + (1/2)(P_B - P_C)(V_C - V_B) \)

\[ = (-1.368 \times 10^3 \text{ J}) + (-3.42 \times 10^2 \text{ J}) = +1.71 \times 10^3 \text{ J}. \]

This is positive, as it should be, since the system expands and does work on the environment.

![Figure 9](image-url)
(c) \( \Delta U_2 = Q_2 - W_2 = +3040 \text{ J} - (+1710 \text{ J}) = +1330 \text{ J}. \)

(d) \( Q_3 = \text{M}_{\text{ice}} \text{ melted} L = -(5.7 \times 10^{-3} \text{ kg})(3.30 \times 10^5 \text{ J/kg}) = -1880 \text{ J}. \)

This is negative because heat was lost by the gas.

(e) \( \Delta U_3 = Q_3 - W_3; \ W_3 = 0 \text{ since } V = 0; \ \text{ therefore } \Delta U_3 = Q_3 = -1880 \text{ J}. \)

(f) \( \Delta U_1 = -(\Delta U_2 + \Delta U_3) = -(1330 \text{ J} - 1880 \text{ J}) = +550 \text{ J}. \)

(g) \( \Delta U_1 = Q_1 - W_1, \text{ and since Process 1 is adiabatic, } Q_1 = 0; \ \text{ therefore, } \Delta U_1 = -W_1, \)

and \( W = -550 \text{ J}. \text{ This is negative, as it should be, since the system is contract­}

(h) \( W_{\text{total}} = W_1 + W_2 + W_3 = -550 \text{ J} + 1710 \text{ J} + 0 = +1160 \text{ J}. \)

(i) \( Q_{\text{total}} = Q_1 + Q_2 + Q_3 = 0 + 3040 \text{ J} - 1880 \text{ J} = +1160 \text{ J}. \)

(j) Heat input is being converted to work output. One can also determine this

Figure 10

by glancing at the pV graph; the area under the curve directed toward increasing

V (Process 2) is greater than the area under the curve directed toward decreasing

V (Process 1); therefore, the net work is positive.
PRACTICE TEST

1. (a) State the zeroth law of thermodynamics.
   (b) A tank of oxygen gas is at a pressure of $3.8 \times 10^5$ Pa at 20.0°C. The tank is dropped into a vat of boiling water ($T = 100°C$). Calculate the new pressure of the gas.

2. A "solar house" has storage facilities for $4.0 \times 10^9$ J. Compare the space requirements for this storage on the assumption that (a) the heat is stored in water ($c = 4.0 \times 10^3$ J/kg K) heated from a minimum temperature of 20.0°C to 40°C; or (b) the heat is stored in Glauber salt heated in the same temperature range. Glauber salt has the following properties (approx):
   Specific heat capacity (solid): $2.00 \times 10^3$ J/kg K. Specific heat capacity (liquid): $3.00 \times 10^3$ J/kg K. Density: $1.00 \times 10^3$ kg/m$^3$. Heat of fusion: $2.00 \times 10^5$ J/kg. Melting point: 32°C.

3. A rubber ball ($c = 1.20 \times 10^3$ J/kg K) is dropped from a height of 2.00 m onto the floor, where it bounces for a while and finally comes to rest. Find the rise in temperature of the ball, assuming that half of the original energy of the ball went into internal energy of the ball.

4. The pV diagram in Figure 11 represents several processes that may occur in a certain thermodynamic system. In the process $1 \rightarrow 2 \rightarrow 3$, 90 J of heat enters the system, and 40 J of work is done by the system. In the process $1 \rightarrow 4 \rightarrow 3$, 10.0 J of work is done by the system. In the direct process $3 \rightarrow 1$, the amount of work is 22.0 J.
   (a) How much heat enters or leaves the system in the process $1 \rightarrow 4 \rightarrow 3$?
   (b) How much heat enters or leaves the system during the direct process $3 \rightarrow 1$?
   (c) How much work is done during the cycle $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$?
   (d) If $\Delta U_{12} = 30.0$ J, how much work is done during the process $1 \rightarrow 2$, and how much heat enters or leaves the system?

Figure 11

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1. (a) If two systems are in thermal equilibrium with each other, they are in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.

2. (a) Volume (water) = 48 m$^3$, (b) Volume (salt) = 16 m$^3$.

3. AT = 1.6 x 10^5 Pa.

4. (a) $M_{12} = 0$ (p) $M_{23} = 1$ (c) $M_{31} = 4.0$ (q) $M_{18} = 0.0$ (s) $M_{9} = 1.80°C$.

5. (a) $q = 0.0$ (b) $\Delta T = 3$.

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Practice Test Answers
1. (a) State the zeroth law of thermodynamics and define the term "system."

(b) You connect a spherical stainless-steel container to a manometer, which measures the actual pressure of the gas inside the container. When this container is immersed in water at 0°C, the manometer reads a pressure of \(2.5 \times 10^5\) Pa. But when it is placed in a crucible of molten zinc, the pressure reading quickly rises to \(6.0 \times 10^5\) Pa. What is the temperature of the zinc in kelvins?

2. A hot 0.300-kg copper object (\(c = 3.8 \times 10^2\) J/kg K) is dropped into a perfectly insulated container containing 0.200 kg of water at 20.0°C. This causes the water to boil, 0.0050 kg being converted to steam. The heat of vaporization for water is \(2.30 \times 10^6\) J/kg. Calculate the original temperature of the copper object.

3. A cylinder containing a sample of gas is immersed in a mixture of water and ice. The initial state of the gas is A, with pressure, volume, and temperature \(P_0, V_0,\) and \(T_0\). The piston is very slowly pushed in until the volume is \(V_0/4\) and the pressure has increased. We shall call this State B. Then the piston is very rapidly (i.e., adiabatically) pulled out again to its original position, bringing the gas to State C. \([P_C = (1/2)P_0 \text{ and } T_C = 2T_0]\) The gas is then allowed to interact with the ice-water mixture without further changes, and it returns to State A after a period of time.

(a) Show the three states and processes on a pV diagram, and justify your drawing qualitatively.

(b) Make a table showing the sign (+, -, or 0) of the heat transferred and the work done by the gas during each process.

(c) Has the device converted heat to work, or work to heat? If \(2.00 \times 10^4\) J has been converted, calculate the mass of water frozen or ice melted. The heat of fusion of water is approximately \(3.30 \times 10^5\) J/kg.
1. (a) State the zeroth law of thermodynamics and define the term "system."
   (b) You connect a spherical stainless-steel container to a manometer, which measures the actual pressure of the gas inside the container. When this container is immersed in water at 0°C, the manometer reads a pressure of $2.5 \times 10^5$ Pa. But when it is placed in a crucible of molten zinc, the pressure reading quickly rises to $6.0 \times 10^5$ Pa. What is the temperature of the zinc in kelvins?

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   (a) Show the three states and processes on a pV diagram, and justify your drawing qualitatively.
   (b) Make a table showing the sign (+, -, or 0) of the heat transferred and the work done by the gas during each process.
   (c) Has the device converted heat to work, or work to heat? If $2.00 \times 10^4$ J has been converted, calculate the mass of water frozen or ice melted. The heat of fusion of water is approximately $3.30 \times 10^5$ J/kg.
1. You blow up and tie off a balloon from a bottle of gas at $2.00 \times 10^5$ Pa pressure and 25.0°C. Assume that the balloon comes to equilibrium at this temperature and pressure. Then you heat the balloon until it bursts at 60°C. What is the bursting pressure?

2. The copper bar of square cross section ($s = 0.050$ m) in Figure 1 with $L = 1.00$ m has one end immersed in an insulated water bath at 100°C and at atmospheric pressure and the other end connected to a constant-temperature reservoir at 400°C. How long does it take to vaporize 0.300 kg of the water? The thermal conductivity for copper is $4.5 \times 10^2$ W/m K, and the heat of vaporization of water is $2.30 \times 10^6$ J/kg.

3. (a) State the first law of thermodynamics.

(b) Define the following in one or two sentences: system, equilibrium, work, heat, internal energy.

(c) When water freezes at atmospheric pressure, its volume increases by about 10%. Is $\Delta U$ for this process larger or smaller in magnitude than the heat of fusion $3.30 \times 10^5$ J/kg)? By how much? You may be asked to justify your answer in terms of the first law. (1 atm = $1.00 \times 10^5$ Pa; $\rho_{water} = 10^3$ kg/m$^3$.)
1. You bring the probe of a constant-volume gas thermometer into equilibrium with a boiling water bath at 92°C. You measure the pressure to be $1.60 \times 10^5$ Pa. Then you bring the probe to equilibrium with a liquid-solid mixture of an unknown substance. The pressure is then measured to be $0.60 \times 10^5$ Pa. What is the melting point (in kelvins) of the unknown substance?

2. A copper kettle with a base area of $10^{-2}$ m$^2$ evaporates water at a rate of $4.6 \times 10^{-3}$ kg/s. What is the temperature gradient $\Delta T/\Delta x$ across the bottom of the kettle? ($L_{\text{vap}} = 2.30 \times 10^5$ J/kg; $k_{\text{copper}} = 4.5 \times 10^2$ W/m K.)

3. Grapes are placed in a juice press at Point 1 in Figure 1. As the top is slowly depressed, the volume decreases and the average pressure exerted on the sides of the juice press increases, until Point 2 is reached. The grapes burst, and the juice runs at approximately constant pressure until Point 3. Since only skins and seeds are left, further increase in pressure (to Point 4) makes no significant change in volume. $Q_{12} = -10.0$ J of heat flows out during process 1 $\rightarrow$ 2; there is no heat flow during process 2 $\rightarrow$ 3, but juice flows out, carrying away an internal energy $\Delta U_j = 200$ J; there is no heat flow in process 3 $\rightarrow$ 4; $V_1 = 3.00 \times 10^{-2}$ m$^3$, $V_2 = 2.00 \times 10^{-2}$ m$^3$, $V_3 = 1.00 \times 10^{-2}$ m$^3$, and $p_2 = 4.0 \times 10^4$ Pa. (Note: Pressure here is taken relative to atmospheric pressure present.)

(a) How much work is done in the complete process 1 $\rightarrow$ 2 $\rightarrow$ 3 $\rightarrow$ 4?

(b) What is the change in internal energy of the grapes in the juice press, $U_4 - U_1$, during the complete process?

Figure 1
1. State the zeroth law of thermodynamics. Define briefly a system, internal energy and adiabatic.

2. Two blocks of 100 kg each are brought into contact. One block is made of Cu and is initially at 200°C. The material of the second block is unknown but it is initially at 0°C. If the final temperature of the combination is 75°C, what is the specific heat of the unknown material? (For copper c = 3.8 x 10^2 J/kg°C).

3. A waterfall is 10.0 m high. Above the falls the water is flowing at 3 m/s. Below the falls the flow is negligible. If the water is 20.0°C above the falls calculate the final temperature of the water.
Mastery Test Form E

Name ___________________________ Tutor ___________________________

Thermal conductivity of copper $K = 3.8 \times 10^2$ watt/m$^0$K
Latent heat of Ice $L = 3.3 \times 10^5$ J/kg
Heat of vaporization of water $= 2.3 \times 10^6$ J/kg
Specific heat of copper, $c = 3.8 \times 10^2$ J/kg $^0$K
radius of the earth $= 6.4 \times 10^6$ m.

1. A rod of copper is 0.1 meters long and has a cross section of 1 cm$^2$. If one end is kept at 200°C and the other end is pressed into a block of ice, how much ice is melted each second?

2. 0.100 kg of water at 100°C is vaporized at 1 atmosphere. The resulting steam at 100°C occupies a volume of 0.18 m$^3$ at 1 atmosphere.
   
   a) calculate the work done by the steam.
   
   b) calculate the change in the internal energy of the system.
   
   c) Define equilibrium, system, adiabatic. State the zeroth law of thermodynamics.
TEMPERATURE, HEAT, AND THERMODYNAMICS

Mastery Test Form E

Name ___________________________ Tutor ___________________________

Date _______________

1. a) State the first law of thermodynamics.
   b) Define briefly: system, equilibrium, work, heat, internal energy.

2. A hot 500 kg Cu block (c = 3.8 x 10^2 J/kg °K) is dropped into an insulated container containing 500 kg of water at 20°C. This causes 2.0 kg of the water to be converted to steam and escape. The final temperature of the remainder is 95°C. If the heat of vaporization is 2.30 x 10^6 J/kg calculate the original temperature of the copper object.

3. Make a PV graph for (a) an isobaric expansion from Vo to 2Vo at Po followed by (b) an increase in pressure to 2 Po at constant volume. Calculate the total work done in these processes.
1. A 2000 kg automobile traveling at 30 m/s strikes a patch of ice. The driver panics and locks his brakes. Assume the heat of fusion of the ice is $L = 3.3 \times 10^5$ J/kg. Calculate the mass of ice melted by the time the car comes to rest. (Assume the temperature of the ice is 0°C and that the temperature of the water from the melted ice is 0°C.)

2. Define briefly adiabatic, isothermal, internal energy, and heat.

3. During a certain thermodynamic process using an ideal gas starting at 0°C and pressure $P = 1.5 \times 10^5$ Pa and volume 0.014 m$^3$ (State A), the gas is compressed adiabatically until the pressure is $3.6 \times 10^5$ Pa and the volume is $8.3 \times 10^{-3}$ m$^3$ (State B). The gas now undergoes an isobaric expansion until the volume = 0.014 m$^3$ (State C). The gas pressure is then reduced at constant pressure to reach State A.

a) Draw a PV graph of the process.

b) Calculate the work done during the isobaric expansion.

c) Calculate the work done in going from state C back to State A.
1. **What To Look For:** The zeroth-law statement should be fairly close to the one given below. There are various possibilities for the definition of system.

   Solution: (a) Two systems in thermal equilibrium with a third are in thermal equilibrium with each other. A system is a portion of matter that we separate in our minds from the environment external to it.

   (b) \[ T = \left( \frac{P_T}{P_{273\ K}} \right) (273\ K) = \left[ (6.0 \times 10^5)/(2.50 \times 10^5) \right] (273) = 655\ K. \]

2. **What To Look For:** The student should start with \( Q = 0 \) or \( Q_{\text{lost}} = Q_{\text{gained}} \).

   Solution: \[ Q_{\text{lost}} = Q_{\text{gained}}, \quad c_M c_M \Delta T_C = c_w M \Delta T_w + M_v \nabla \]

   \[ T_{\text{final}} = \frac{c_w M \Delta T_w - ML_{\text{vap}}}{c_M} + 100^\circ C \]

   \[ = \frac{(4.2 \times 10^3 \ J/kg K)(0.200 \ kg)(80 \ K) - (0.0050 \ kg)(2.30 \times 10^6 \ J/kg)}{(3.8 \times 10^2 \ J/kg K)(0.300 \ kg)} + 100^\circ C = 790^\circ C. \]

3. **What To Look For:** (a) The diagram should be a "triangle" with B higher than A and C and with AC a vertical straight line. The students should not be penalized if they draw straight lines or different types of curves for AB and BC. (c) Answer should include 3 parts: work to heat, ice melted, and the amount.

   ![Figure 14](image-url)
Solution: (a) See Figure 14. The process $A \rightarrow B$ is a compression; therefore the line must go to the left, and up because the problem states that the pressure rises. The line for $B \rightarrow C$ must go down and to the right because the problem states that the pressure $p_c = (1/2)p_0$, and it is an expansion. Line $CA$ must be vertical because the piston stays fixed.

(b) | AB | BC | CA |
---|----|----|----|
heat | - | 0 | - |
work | - | + | 0 |

(c) Since the area under $AB$ is negative and greater than the area under $BC$, which is positive, we expect that work is converted to heat.

$$M(\text{melted}) = \frac{2.00 \times 10^4 \text{ J}}{3.30 \times 10^5 \text{ J/kg}} = 6.1 \times 10^{-2} \text{ kg.}$$
MASTERY TEST GRADING KEY - Form B

1. **What To Look For:** Student should convert temperatures first to Kelvin scale. If not, mark incorrect and discuss definition of temperature in terms of pressure. Show how to convert from degrees Celsius to kelvins.

   **Solution:**
   \[
   p_{\text{burst}} = \frac{p}{25.0 ^\circ C} T_{\text{burst}} = \frac{(2.00 \times 10^5 \text{ Pa})(333 \text{ K})}{298 \text{ K}} = 2.23 \times 10^5 \text{ Pa}.
   \]

2. **What To Look For:** Student should use both conduction formula and latent-heat formula.

   **Solution:**
   \[
   \frac{dQ}{dt} = \frac{Q}{t} \quad \text{because of steady state.}
   \]
   \[
   \frac{dt}{dx} = \frac{\Delta T}{\delta} \quad \text{because of uniform cross section.}
   \]
   \[
   \frac{Q}{t} = -kA(\Delta T/\delta); \quad Q = ML,
   \]
   \[
   t = \frac{ML}{kA \Delta T} = \frac{(0.300 \text{ kg})(2.30 \times 10^6 \text{ J/kg})(1.00 \text{ m})}{(4.5 \times 10^2 \text{ W/m K})(0.050 \text{ m})^2(300 \text{ K})} = 2.0 \times 10^3 \text{ s}.
   \]

3. **What To Look For:** (a) Student statements do not need to be this comprehensive, but the underlined ideas should be stated explicitly, or at least alluded to. (c) Student should calculate work. Sign is important, as well as number. Student could say that \( \Delta U \) is more negative than \( Q \), therefore \( \Delta U < Q \), but make sure this is properly explained and that \( Q \) and \( \Delta U \) have the proper signs.

   **Solution:** (a) For a system that undergoes any process beginning and ending in equilibrium, the heat transferred to the system \( Q \) minus the work done by the system \( W \) is exactly equal to the change in the internal energy function \( U \).

   \[
   \Delta U = Q - W.
   \]

   (b) A system is a portion of matter that we isolate, in our minds, from the environment. A system is in thermal equilibrium when it is in thermal contact with its surroundings and when macroscopic changes have ceased to occur. \( Q \) represents heat transfer, which is the transfer of energy to a system, bringing about the same changes as if work were done, but without the performance of work; energy transfer by means of heat is associated with a temperature difference across the system boundary. Work, \( W = \int F \cdot d\vec{r} \), is the action of a force over a distance, with at least a small component of the force directed along the displacement; the most common type of work in thermodynamics
is $\int p \, dV$, due to the expansion or contraction of a system. $\Delta U$ is defined by the first law as equal to $Q - W$; $\Delta U$ is a state function that depends only on the internal system properties and not on the path between two states.

(c) $\Delta V = (0.100)(M_{\text{water}}/\rho_{\text{water}})$,

$$W = p \Delta V = \frac{(1.00 \times 10^5 \text{ Pa})(0.100)(1.00 \text{ kg})}{10^3 \text{ kg/m}^3} = +10.0 \text{ J.}$$

Since $\Delta U = Q - W$, $\Delta U$ is larger in magnitude than the heat of fusion by 10 J.
MASTERY TEST GRADING KEY - Form C

1. **Solution:** For a constant-volume gas thermometer:

   \[ T \propto p. \] Therefore
   \[ T = \frac{(92°C + 273.15)(0.60 \times 10^5 \text{ Pa})}{1.60 \times 10^5 \text{ Pa}} = 137 \text{ K}. \]

2. **Solution:** \( A = 10^{-2} \text{ m}^2, \ \Delta m/\Delta t = 4.6 \times 10^{-3} \text{ kg/s}, \)

   \( \text{L}_{\text{vap}} = 2.30 \times 10^6 \text{ m}, \ \Delta Q/\Delta t = kA(\Delta T/\Delta x) \)

   \[ \frac{\Delta T}{\Delta x} = \frac{\Delta Q/\Delta t}{kA} = \frac{(2.30 \times 10^6)(4.6 \times 10^{-3})}{(4.5 \times 10^2)(10^{-2})} = 2.35 \times 10^3 \text{ K/m}. \]

3. **Solution:** (a)

   \[ \Delta Q_{12} = -10.0 \text{ J} \quad 1 \rightarrow 2 \]
   \[ \Delta Q = 0 \quad 2 \rightarrow 3 \quad \Delta U = 200 \text{ J}. \]
   \[ \Delta Q = 0 \quad 3 \rightarrow 4 \]

   \( V_1 = 3.00 \times 10^{-2} \text{ m}^3, \quad V_2 = 2.00 \times 10^{-2} \text{ m}^3, \)

   \( V_3 = 1.00 \times 10^{-2} \text{ m}^3, \quad p_2 = 4.0 \times 10^4 \text{ N/m}^2. \)

   Work = area = \( (1/2)(V_2 - V_1)(p_2) + p_2(V_3 - V_2) \)

   \[ = (1/2)(1.00 \times 10^{-2})(4.0 \times 10^4) + (4.0 \times 10^4)(1.00 \times 10^{-2}) \]

   \[ = 2.00 \times 10^2 + 4.0 \times 10^2 = 6.0 \times 10^2 \]

   Work = 600 J.

   (b) Process 1 \( \rightarrow \) 2: \( \Delta Q = -10.0 \text{ J}, \ \Delta W = +200 \text{ J}, \)

   \[ \Delta U_{12} = \Delta Q - \Delta W = -10 + 200 = +190 \text{ J}. \]

   Process 2 \( \rightarrow \) 3: \( \Delta Q = 0, \ \Delta W = -400 \text{ J}, \ \Delta U_{23} = \Delta Q - \Delta W = 0 + 400 = +400 \text{ J}. \)

   Process 3 \( \rightarrow \) 4: \( \Delta Q = 0, \ \Delta W = 0, \ \Delta U_{34} = 0. \ \Delta U_j = 200 \text{ J} \) (juice lost).

   Therefore, \( \Delta U_{14} = +190 + 400 - 200 = +390 \text{ J}. \)