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Understanding the Mathematical Language

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Abstract

In this action research study of my calculus classroom consisting of only 12th grade students, I investigated activities that would affect a student's understanding of mathematical language. The goal in examining these activities in a systematic way was to see if a student's deeper understanding of math terms and symbols resulted in a better understanding of the mathematical concepts being taught. I discovered that some students will rise to the challenge of understanding mathematics more deeply, and some will not. In the process of expecting more from students, the frustration level of both the students and the teacher increased. As a result of this research, I plan to see what other activities will enhance the understanding of mathematical language.

High school students tend to expect that the information that they need to learn is only the information that is presented to them in class. Every semester I check a math book out to each student in my calculus class. The students do not find this to be offensive or awkward, for they have been receiving math textbooks for years from all of their teachers. They may take the book home. The book may sit in their locker. The students even carry the book to my class every day. But, in general, if I want daylight to hit the pages of the text, I must instruct my students on the mechanics of opening the cover. While most students scoff at the idea of having to use their book to answer their questions, I would like to see students discover that the math text book offers a great deal of information. The purpose of this study is to encourage students to become better independent learners by understanding the mathematical language. Independent learners take responsibility for their own learning by trying to find answers when they do not understand. They realize when they don't understand, and they are able to articulate questions to clarify their understanding.

As a calculus teacher, I find that my students rely solely on me for the information they need. I do believe that much of the information that they learn will filter through me, but I should not be the only form of education on which they depend. My students are diligent in writing down what I write on the board. This is a good skill for students to have. But, there is an assumption that if the teacher did not show it, then the student is not responsible for the material. When students were asked what helps them understand math problems the best, over half of the students in my calculus classroom responded similarly by saying, "Just show each problem step by step" (Student Survey). As educators, especially calculus teachers, we understand that there is not enough time to show every detail that can occur. We try to enable our students with strategies to solve problems. I have found that when a lesson is finished, students use their textbook mainly

to note down the problems for the assignment. They do not use the text to answer any questions that they may have. If a question is raised while working on an assignment a student will wait until I am free to ask me the question rather than using other resources.

In an ideal classroom, I would like to see students pose questions from the information that is in their textbooks. When a student formulates a question, I know they are communicating at a level they can understand. I would not only like to have students ask questions after a lesson has been taught, but also have them ask questions before the chapter begins, as well as during the lesson. If questions would arise before or during the lesson, more time could be spent clarifying the concept instead of the teacher just lecturing to introduce a concept. The belief is that if questions are being asked, students are thinking about the topic at hand instead of being bystanders in their learning. In the process of clarifying an idea, I would hope to see students asking directed questions on what they did not understand. These questions could even be over main topics. It is in my ideal classroom that the clarifying component of a chapter will occur earlier than it is at the current time.

Combining the introduction and clarifying time would be very beneficial to teachers in the mathematical teaching community. With all the expectations that are placed on teachers, time is just one of those things in short supply. We can't get enough of it. As a calculus teacher, I found that the new symbolism and terminology are a lot for high school students to digest the first time around. I would prefer that they try to absorb some of this shock outside the time limitations of the class period.

Requiring that a chapter be read outside of class before a lesson could enable student-driven questions. In this research study, it was not expected that the concept was completely understood by the students after reading. However, some of the reading would trigger past

knowledge. As the students read, I expected that they would start with what they knew, and then would try to understand the text that they didn't know. The students would be asked to brainstorm questions they had with the reading. In these questions they needed to specify what they understood and what had become unclear. As a teacher, I assessed the incentive needed for students to actively take part in the reading activity. While a student analysis of the material prior to the lesson was to create questions before the lesson, it is my hope that it also helped the student realize the great examples that are nicely printed in the text. This would be helpful to the student as they worked on assignments.

It is important to teach students how to think for themselves. While elementary school teachers will not expect their students to read outside of class in the same manner that I did, it is important to teach skills that will enable the students to further their knowledge by way of their own power. Teachers need to not only give students tools, such as a textbook, to find answers to their questions, but they also need to teach them how to use these tools. Our students will become adult members of our society in the near future. They may not remember all the details of the mathematics that I have taught them; however, it is my hope that they remember the knowledge that they can continue to gain is endless, if they are willing to find it.

Literature Review

Independent Learners

As a teacher, when I come across a problem of practice within my classroom, I naturally want to fix it. Usually, I want to fix it as soon as possible. With this hastiness, I can solve the problem on the surface, but I always wonder if I got to the root of the problem. I find that I ask myself, "Did other problems arise that could have been taken care of if I would have taken the time to really study the first problem at hand?" The following exemplifies the deeper look that I

took when thinking of one of these daily problems of practice. I struggle with the fact that my calculus students, who are all seniors, do not seem like independent learners. They depend on me, as the teacher, to walk them through every step of their learning process. If I am not available, their mindset seems to be that they can not move forward. Borasi (1994) studied a group of secondary school students through ten instructional 40-minute sessions and a take-home project. He was researching the impact of reviewing mistakes for continual learning. Within my calculus classroom I wanted to emphasize "the importance of students' …ownership in their learning of mathematics"(Borasi, 1994, p. 168).

Understanding the Mathematical Text

It seemed that independent learners will be created if the students understand the mathematical language at hand. A study done by Kathryn Sullivan (1982) showed that even understanding small words such as "is", "ones", "many", "numeral", "or", and "have" in the mathematical context can greatly influence computational scores positively. Meisto (2005) studied several mathematics classrooms. She was researching the experiences that can guide the student to connect more effectively with the text. "If we are really trying to help students read and understand for themselves, we must ask them questions instead of explicitly telling them what the text means." (Metsisto, 2005, chap. 2, para. 27). The concept of mathematical language may have been in the back of my mind this entire time, but now it was brought to center stage.

Siegel, Borasi, and Fonzi (1998) conducted a qualitative study of a geometry class. The class being studied focused on taxi-geometry (where only vertical or horizontal moves are allowed). After the class discussed definitions from the text about taxi-geometry, the teacher asked if a circle could be drawn in taxi-geometry. The class had great discussion over this. They were to refer to the Euclidean (our geometry) definition in their book to keep the conversation

going. The big question for the students became, “Does this definition work in taxi-geometry?” They discussed more, and the teacher helped them come to the conclusion that a circle cannot be drawn in taxi geometry using a definition from Euclidean geometry. Then the students asked the question, “How can we make a definition that works in taxi-geometry?” (Siegel, Borasi, & Fonzi, 1998, p. 385-393). Clearly communication drawn from the text was needed to have such an enriched class period. Not only were the students able to communicate that “Euclidean” circles can not be drawn in taxi-geometry, they went a step further to ask how they could define a circle in taxi-geometry.

There were no other research projects that investigated the effect of focusing on learning the mathematical language from a calculus textbook. This is the first part of my research project. It is in the hands of teachers to help engage the language of mathematics in their students through many ways. There were two specific times in my classroom in which the language of mathematics could be more effectively taught; at the beginning of a concept and at the closure of a concept.

As I focused on successfully teaching mathematics as a language in the beginning of a concept, I kept the following in mind. It has been found that “mathematics texts contain more concepts per sentence and paragraph than any other type of text” (Metsisto 2005, chap. 2, para. 10). Students take more time to absorb a written page from a math text book than a page of text from an English book. I also needed to keep the students motivated in reading text that seemed very difficult to them. I anticipated that the mathematical language would be foreign to them time and time again. I would need to help them distinguish symbols and phrases that they did know from the ones that they did not know. Goetz and Sadoski (1995) conducted a study of graduate students who were randomly assigned to read different passages. They wanted to study

what information the students could recall. They mention that texts may provide "little assistance to students in identifying and grasping important information and may be difficult to understand and boring as well" (Goetz & Sadoski, 1995, p. 500). It was important that the students are not side tracked by little distractions within the reading. Equally important was the students' ability to pick out the main topics in a section.

When I was toward the end of teaching a concept, I wanted to be conscious that students' "recall of the questioned information [is] superior to their recall of the unquestioned information" (Wixson, 1984, p. 429). Another idea that I wanted to keep at the front of my mind as I thought of enhancing the mathematical language toward the end of a lesson was that "mathematical communication is strongly related to problem solving and reasoning" (Neria & Amit, 2004, p. 409). In my action research study I will focus on teaching mathematics as a language (written and verbal) to enable independent learners within my classroom.

Purpose Statement

The purpose of my project is to create independent learners by focusing on deciphering the mathematical language used in the classroom. I want to know if it is worth slowing down a little to address the mathematical language that is used in the textbook. Can a deeper understanding of the concepts occur if more time is spent on understanding the language? Will this aid in teaching new concepts later on? Specifically, I want to know if giving time for absorption of the mathematical language at the beginning a concept and giving time at the end of an idea (chapter) for reflection can add to a student's personal mathematical language.

I will be examining the variables of written and oral mathematical language used by students, in seeking to answer these research questions:

- How do pre-reading activities affect the students' understanding of the mathematical language?

- What impact on student learning can occur if the student takes the time to decipher the mathematical language being used?

- How does reflection upon mistakes affect the students' understanding of the mathematical language?

Method

There were three ways in which I collected data. A survey (see Appendix) was given to the students when the research study was implemented. It was also given shortly prior to the end of the study. Copies of all student homework, quizzes, and tests were made. This was work on which students were normally graded. All of the student work used in this study was work that would have been given even if the study were not in progress. A teacher journal was also used as a form of data collection. I wrote in the journal after the completion of each section. It took about a week and a half to finish a section. During each of the journal entries I wrote utilizing these prompts:

- Did the students seem to do the pre-reading activity?
- Were students responsive in asking questions of the material?
- Should a pre-reading activity be used in this section next year?
    - If "Yes", should anything be changed? (e.g. page numbers)
    - If "No", can anything be changed so a pre-reading activity can occur?
- How much time in class was given to decipher the mathematical language being used?
- Did this time seem productive?
- Did any of the students write or say the new language being used?
- Was mathematical language used in the test corrections? How?
- Is this different from the way the student(s) solved the problem on the test?

The survey consisted of eight statements with which the students could agree or disagree. The first three statements pertained to reading activities that occurred before a lesson had taken place. The next two statements dealt with spending time in class deciphering mathematical notation in the classroom. The final three statements discussed reviewing mistakes through test corrections. After the students agreed or disagreed with these eight statements, they were asked

to make statements of their own. Three prompts were given to the students to aid them in this process. The prompts consisted of the beginning of a sentence, and then the student was asked to end the statement any way he or she wanted. The students were told that they did not have to fill the survey out if they did not want to. All of the students chose to fill out the survey. They were also told that they did not have to write their name on the survey.

Before a section in the chapter was started, I would assign the class to read the section to the best of their ability for their homework assignment. As a part of reading the chapter, the students would be asked to pick out ideas that they understood, as well as symbols or phrases that they found difficult. The next day in class I would have students ask questions about what they didn't understand. When a student asks me a question from the text I not only know that they are using terminology which they understand, but I also know that they are being active participants in their education. As long as they were asking, I would keep answering questions by teaching concepts that pertained to the lesson. When the questions were finished, I would give a "bonus" reading quiz. This quiz would be over the important ideas in the text. The intention of the quiz was to create motivation to read the material and try to formulate ideas for discussion. After the quiz I finished teaching the concepts that were covered through student inquiry.

Students were given a homework quiz for each section in a chapter. The quiz problems would come from homework problems previously assigned to them. Before the homework quiz the students would be given time to ask questions. One class period was left open for questions in each section of the chapter. The class period used for questions was the day after lecture. I also allowed an extra day to ask questions if I felt that it was necessary. The students were not allowed to use any notes on the homework quiz. After a homework quiz was given I would grade and copy it. It would be handed back to the students on the following school day. If students

were not happy with the grade that they had earned, they could retake the quiz. The same type of problems would be given on the retake, but the problems would not be identical to the original quiz. I kept each section of homework quizzes together as I started to organize my data.

During the duration of this research study the students took one test. After the test was taken and graded the students were then asked to correct any of the problems that they made on the test. These test corrections would count as a third of the test grade. In these test corrections they were asked to convince me (the teacher) that they understood how to solve the problem. They could use their book and confer with other students as well as myself. The test corrections could consist of words, mathematical work, drawings, or whatever they felt would convince me they understood the problem. "When dealing specifically with argumentation, it was found that … students prefer arguments written in words" (Neria & Amit, 2004, p. 413). After the test and test corrections were complete, I made a copy of both.

Findings

Pre-reading activities can affect a student's understanding of mathematics. The more that one studies, the more one will learn. However, human as we are, frustration can set in when times are difficult. There is this constant balance of pushing students to do well but keeping the frustration at a workable level. My assertion is that pre-reading activities are only beneficial if the student spends time with the reading material. I also assert that students can pick out the important parts of a section even if they don't completely understand what it means. These things can be done if the student has a strong understanding that they will not understand or remember everything. The goal is to learn as much as one can.

One day during the week of February 21, 2007, the students in my calculus class were asked to read section 4.1 to the best of their ability. They had the understanding a reading quiz

would take place the following day. Before class the next day I spoke with a student. She was excited for the homework quiz. This excerpt was from a teacher journal entry during this week. "She told me plans were cancelled on Sunday so she read more than she usually does" (Teacher Journal Entry). On her 4.1 reading quiz she scored five points, which means she pulled 5 concepts from the reading. The scores of the class ranged from one to six on this reading quiz. It is not easy to pull concepts from a calculus text so this showed me that if time is given to reading, connections can be made.

A student made this comment in the survey given at the beginning of this research study. The worst thing about math is when "trying to read the book when I don't understand at all what it is trying to say." I contend that students can find the pertinent parts of the text even when the symbolism is hard to understand. An example of this was shown the day after the reading assignment over section 4.4 was assigned. I started this class period by asking if there were any questions over the reading. One student raised his hand to ask a question. A teacher journal entry the week of March 12, 2007 reads, "one student asked for further explanation about the "accumulating" that occurs on page 281 if the upper limit of an integral is a variable." For a student to formulate a question of this magnitude shows that he was able to understand some prior pieces of the text. During the week of February 27, 2007 the following teacher journal entry was recorded:

> There was no time given before the 4.2 reading quiz. The reading quiz was given and then picked up. The lesson was started promptly. I asked the students if they could name the two basic rules of integration. One of the students who struggles in this class raised his hand. He answered my question correctly. He was given no instruction before this. This was something that he had read in the textbook and deciphered on his own. He also got this question correct on his reading quiz. Since he cannot use a book or any notes on his reading quiz, I knew it was something that he had retained from his previous reading (Teacher Journal Entry).

This made it very evident to me that not only can any student pull information from the text, it can help the student remember the information. This was from a teacher journal entry taken March 6, 2007:

> I relied on 'this same student' today to remind everyone what the two basic rules of integration are. I am not able to ask him many questions in front of the class, so I jumped at the chance to have him restate these rules again. I was confident that he would remember, and he did (Teacher Journal Entry).

While it is wonderful to see students use their reading skills to pull information from the text, it doesn't always happen. It is very frustrating to the student to have to read the text when they would rather just have the teacher tell them how to do the math. This is from an analytic memo I wrote on March 27, 2007:

> I think that some students may tend to not read the material assigned because they think that they will not understand it. I also believe that they assume that I will go over it anyway, so what is the point?

On the student survey given on May 1, 2007 less than half of the students agreed with this statement: I read about a topic before it was taught in class. The survey confirmed the frustration that I had been feeling at times throughout the semester. The frustration had moved from the students to the teacher. The benefits that can come from a reading assignment were not always felt because there were times that half of the class was ready to be taught, having read the assignment, and the other half was not.

While the first portion of this study focused on creating independent learners through textbook reading, there are other ways students can increase their knowledge of the mathematical language. One of these ways is through understanding the mistakes that are made on quizzes and tests. The best time to learn is after you make mistakes. When a mistake is made, the learning that can follow is very personalized to the one who made the mistake. A student has the

opportunity to start right from where he went wrong and move forward. Evidence of this can be seen in a homework quiz taken by a student in section 4.1. This student took the quiz in this section and missed small details, such as adding "C" and using the correct variable when integrating. She also made mistakes on integrating an acceleration function to a velocity function and then to a position function. On her retake she did not make the small errors she had previously. The errors that she made were diminished. Not only did the re-take quiz benefit this student's grade, I believe that she learned something by looking over her original mistakes.

The following chart shows a summary of the amount of times students in my calculus class reflected formally upon the mistakes they made on homework quizzes in Chapters 4 and 5. The first column identifies each student as a number. The second column shows the amount of re-take quizzes that were successfully completed. In this chart a quiz or re-take quiz is considered successful if the student received a C or better. Since some students did well on many of their original homework quizzes, I thought that it would be important to show how many quizzes a student earned with a grade of C or lower. The third column identifies this by showing how many quizzes were originally unsuccessful in completion. The last column shows the test grade that was associated with these two chapters.

STUDENT	Successful Re-take Quizzes	Unsuccessful Original Quizzes	Test Grade
1	4	2	84
2	1	5	80
3	1	1	92
4	1	1	88
5	1	7	61
6	2	5	64
7	2	3	64

Did a student's formal reflection over their mistakes through a re-take quiz help them in the long run? While the numbers vary quite a bit, one thing is evident. Students 2, 5, 6, and 7 did

not re-take all of their unsuccessful original quizzes. Three out of four of those students did not pass the test. All three of the students who completed a re-take for each one of their unsuccessful original quizzes scored well above a passing grade.

In the survey given at the end of the research study, students were asked to choose a statement that best represented their opinion of the following: I learn something when I do test corrections. Two thirds of the class chose that they agree with the statement. The other third of the class either somewhat agreed or was neutral.

Conclusions

This research study has found that mathematics is a language that needs to be decoded as it is learned. A student will gain knowledge of mathematical words and symbols throughout every part of the learning process. A student's willingness to put forth the effort to increase his or her comprehension is essential. Time must also be given to the student to absorb the information that is used. This process can be difficult, but with cooperation from the students a deeper understanding of mathematical concepts can occur.

During this research study a letter was sent to me from a previous calculus student of mine. She took the same calculus course on which I did this research study. In the letter she mentioned the math course that she was taking in college. She said that it was very difficult to understand her math teacher because of a language barrier. She stated, "I never understood why you made us read our textbook, especially when it was hard to understand. But, I am thankful that you made me do it because I feel that I am much more prepared that many of my classmates" (Student Letter). Enabling students with the understanding of the mathematical language allows students to become self-sufficient in processing mathematical information they encounter. For students to achieve "adequate understanding of the text requires…the intention to

understand, ability to recognize and use the text genre concerned, and adequate time to reflect" (Francis & Hallam, 2000, p. 294). This achievement is the key element that will help students become independent learners throughout life.

Implications

As a result of this study, I would like to find other activities that can enhance the understanding of mathematical language without creating great frustration on both the part of the teacher and the students. Other activities will again include the reading of the text. The only difference is that I plan to have the students read smaller portions of a section. In addition to downsizing the amount of reading, I may also include reading guides that accompany the text that could benefit the students as they become lost in difficult readings. I plan to incorporate reading of the text into the Algebra II and Pre-Calculus as a result of this study. I would like to see a gradual build-up to the expectations that I hold in Calculus. A student's test corrections that display a deep understanding of the problem could be presented to the class.

The activities that I would like to investigate further are not only the ones that occur before the topic is discussed or after it is assessed. I would also like to study activities that occur during the lesson. One could break the reading up and assign it to different groups in the class. Then in individual group discussion they could talk about what they understand and what seems foreign. This activity will show students who struggle with motivation that information can be pulled from the text. It may give them incentive to do their reading homework because they know that others will complete the reading assignment. The groups that read in the classroom could then present what they have found to the entire class. This would once again reiterate the fact that students can pull information from the text. One thing is certain; students in my classroom will be asked in one way or another to take an active role in their education.

There are a few suggestions that I have for other teachers considering an addition of textbook reading within their classroom. As previously stated I would start by having the students read small portions of material. Also, give guiding material to help them stay focused and allow group work to maintain motivation for the reading. As a teacher, I sometimes feel that I do not have time to assign a reading assignment. I would just rather teach the concept instead of having the students use the text as a beneficial resource. However, the rewards that come from seeing a student answer another student's question by referring to the assigned reading in the text are priceless.

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Appendix
Student Survey

1-Agree *2-Somewhat Agree* *3-Neutral* *4-Somewhat Disagree* *5-Disagree*

Please circle the number that best represents your opinion.

1. I read about a topic before it is taught in class.

1 2 3 4 5

2. Writing information down (taking notes) while I read is helpful.

1 2 3 4 5

3. I discuss information assigned to read outside of class.

1 2 3 4 5

4. Time should be given in class to explain symbols and new math words.

1 2 3 4 5

5. Time in class should be focused on how to do a problem.

1 2 3 4 5

6. Test corrections are good because they help raise my grade.

1 2 3 4 5

7. Test corrections are more work than they are worth.

1 2 3 4 5

8. I learn something when I do test corrections.

1 2 3 4 5

Please complete the following statements.

9. To help me better understand a math problem it would be best to:

10. My favorite part of math is:

11. The worst thing about math is: