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Flight Path of Least Time

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A problem which one might assume to be of considerable practical interest, but certainly a problem to arouse the curiosity of the undergraduate student, is the problem of determining the most economical flight path of an airplane in the presence of known winds. In general one must assume that the wind system between source and destination is a complex one involving circulation about several centers with winds of variable speed and direction. The geodesic between source and destination is clearly not the flight path of least time in the presence of winds, nor is a cursory examination of a weather map sufficient to establish the best route.

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A PROBLEM which one might assume to be of considerable practical interest, but certainly a problem to arouse the curiosity of the undergraduate student, is the problem of determining the most economical flight path of an airplane in the presence of known winds. In general one must assume that the wind system between source and destination is a complex one involving circulation about several centers with winds of variable speed and direction. The geodesic between source and destination is clearly not the flight path of least time in the presence of winds, nor is a cursory examination of a weather map sufficient to establish the best route.

A solution to the problem can be obtained from the Huyghens' construction, to any desired precision. For pedagogic purposes this problem and its solution provide an interesting connection between optics and kinematics.

For simplicity we shall consider motion over a plane surface in which the wind pattern is independent of altitude. If the windspeed at the point of departure is W_0 and the airspeed of the airplane is A, the velocity of the airplane with respect to ground is given by the vector sum of W_0 and A, and the locus of all points which can be reached by the airplane in unit time by maintaining a fixed heading is given (to a first approximation) by a circle of radius A centered at the head of the W_0 vector. The locus of points which can be reached at the end of two unit time intervals by maintaining two fixed (but possibly different) headings will be given by the "wave front" which is the envelope of all circles obtained by a second construction identical to the first, except that points on the circumference of the first circular "wave front" are considered as departure points and local wind velocities W_1 are used instead of W_0. Continuing in this way a family of "wave fronts" may be generated which represent the loci of points which can be reached by the airplane by n successive fixed headings in n unit intervals of time. The process is concluded when the desired destination is contained within the nth "wave front." The desired flight path is found by retracing one's steps. The desired trajectory is not orthogonal to the "wave front." Thus the problem of finding the flight path of least time from a given point of departure to all possible destinations is simultaneously solved for one particular airspeed and wind pattern.