1975

Vector Multiplication

Module ________

STUDY GUIDE

VECTOR MULTIPLICATION

INTRODUCTION

How much is A times B? This is a simple question to answer when A and B represent scalars; however, when A and B represent vectors, the answer is not obvious. In fact, on the face of it, one cannot even say whether the result should be a scalar or a vector!

Several different definitions of vector multiplication have been found useful in physics; in this module you will study two types: the scalar and vector products. Just to sharpen your interest, we point out that the vector product has the strange but useful property that $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$!

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Find values for sin θ and cos θ when θ is larger than 90° (needed for Objectives 1 and 2 of this module)	Trigonometry Review
*Transform vectors from polar form (direction and magnitude) to rectangular form (components and unit vectors) and vice versa (needed for Objectives 1 and 2 of this module)	Dimensions and Vector Addition Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Scalar product - Given two vectors (in either polar or rectangular-component form), calculate the scalar (dot) product and the angle between the vectors.

2. Vector product - Given two vectors (in either polar or rectangular-component form), calculate the vector (cross) product.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

The text defines the scalar and vector products briefly in Section 8.1 on page 112, and in Section 11.2 on pages 178 and 179, respectively. You should look over those definitions, read General Comments 1 and 2 carefully, and work out the Problem Set. When you feel ready, try the Practice Test. If necessary reread the General Comments, and rework the appropriate problems in the Problem Set.

BUECHE

Objective Number	Readings	Problems with Solutions Study Guide	Assigned Problems Study Guide
1	Page 112 (last paragraph of Sec. 8.1), General Comment 1	A	C
2	Sec. 11.2, General Comment 2	B	D

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

Read Section 2-4 and General Comments 1 and 2 carefully. There is some overlap, but the text does not show how to calculate dot and cross products for two vectors in unit-vector form. After the readings, work Problems A through D in the Problem Set and try the Practice Test. If you find that you need additional practice, try Problems 35, 37, 39, 45, and 47 in Chapter 2 of the text, for which answers are given.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions Study Guide	Assigned Problems Study Guide	Additional Problems
1	Sec. 2-4, General Comment 1	A	C	Chap. 2: Problems 35, 36, 37, 39, 40, 45, 47
2	Sec. 2-4, General Comment 2	B	D	

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Massachusetts, 1970) fourth edition

SUGGESTED STUDY PROCEDURE

The text defines scalar and vector products briefly on pp. 95 and 139, respectively. You should look over these definitions and read General Comments 1 and 2 below. After working out Problems A through D, compare your results with the solutions given, and try the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions Study Guide	Assigned Problems Study Guide
1	p. 95, General Comment 1	A	C
2	p. 139, General Comment 2	B	D

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973) second edition, Vol. 1

SUGGESTED STUDY PROCEDURE

Read and study Sections 2-6 and 2-7 plus General Comments 1 and 2 below. There is some overlap between the text and the General Comments, but the text does not show how to calculate the cross product of two vectors in unit-vector form, as is done in General Comment 2. After the readings, work Problems A through D in the Problem Set, and try the Practice Test. If you find that you need more practice, try the additional problems in the text listed below, for which answers can be found on p. 8 of the Appendix.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions Study Guide	Assigned Problems Study Guide	Additional Problems
1	Sec. 2-6, General Comment 1	A	C	2-15, 2-21(a)
2	Secs. 2-6,2-7 General Comment 2	B	D	2-17, 2-19, 2-21(b), 2-23

GENERAL COMMENTS

1. Calculation of the Scalar (Dot) Product

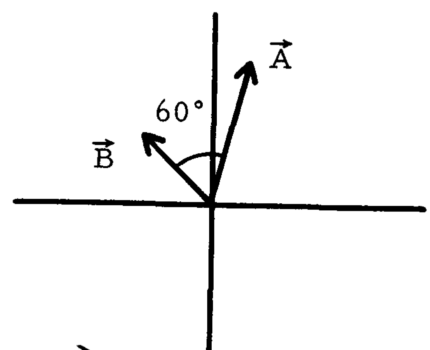
The scalar, or dot, product of two vectors is defined as follows:

$$\vec{A}\cdot\vec{B} = |\vec{A}|\ |\vec{B}| \cos\theta,$$

where θ represents the angle between the two vectors, $|\vec{A}|$ represents the magnitude of $\vec{A}$, and $|\vec{B}|$ represents the magnitude of $\vec{B}$. Thus the scalar product results in a scalar or number, rather than another vector. If the vectors have the same direction, then $\theta = 0$, $\cos\theta = 1$, and the dot product is equal to the product of the magnitudes of the two vectors, $|\vec{A}|\ |\vec{B}|$. However, if the vectors are not parallel, then $\cos\theta < 1$, and the dot product is less than the algebraic product of the magnitudes. In particular, if the vectors are perpendicular to each other, then $\theta = 90°$, $\cos\theta = 0$, and the dot product is zero. The above definition is easy to use when both vectors are expressed in polar form and one can find θ easily. However, when one or both vectors are in rectangular form, θ is not usually known, and one cannot use the above equation.

Example

Given that $\vec{A}$ has magnitude 6.0 m with $\theta_A = 80°$, and $\vec{B}$ has magnitude 3.0 m with $\theta_B = 140°$, calculate $\vec{A}\cdot\vec{B}$.

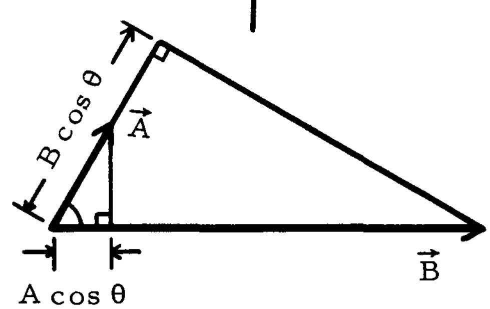


Solution

From the diagram we see that the angle between the vectors is 60°; thus

$$\vec{A}\cdot\vec{B} = (6.0\ \text{m})(3.0\ \text{m}) \cos 60°$$
$$= 9.0\ \text{m}^2.$$

The dot product can also be interpreted as the component of $\vec{A}$ in the direction of $\vec{B}$ multiplied by $|\vec{B}|$, or alternatively, as the component of $\vec{B}$ in the direction of $\vec{A}$, multiplied by $|\vec{A}|$, as in the drawing on the right.



We shall now derive a general formula for the dot product of two vectors written in terms of their rectangular components. To do this, we need to know the dot products of the unit vectors with each other. First, $\hat{i}\cdot\hat{i} = 1$ since a vector is parallel to itself and the magnitude of $\hat{i}$ is 1. Similarly, $\hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$. Furthermore, since $\hat{i}$ is perpendicular to $\hat{j}$, $\hat{i}\cdot\hat{j} = 0$, and, similarly, $\hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$. Collecting these results:

$$\begin{aligned} \hat{i}\cdot\hat{i} &= \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1, \\ \hat{i}\cdot\hat{j} &= \hat{j}\cdot\hat{i} = 0, \\ \hat{j}\cdot\hat{k} &= \hat{k}\cdot\hat{j} = 0, \\ \hat{k}\cdot\hat{i} &= \hat{i}\cdot\hat{k} = 0. \end{aligned}$$

Now suppose we have two vectors, $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$, and we wish to calculate their dot product $\vec{A}\cdot\vec{B}$:

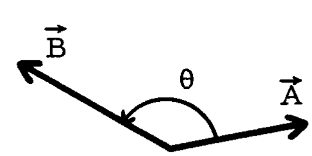
$$\begin{aligned} \vec{A}\cdot\vec{B} &= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\ &= A_xB_x(\hat{i}\cdot\hat{i}) + A_xB_y\overbrace{(\hat{i}\cdot\hat{j})}^{0} + A_xB_z\overbrace{(\hat{i}\cdot\hat{k})}^{0} \\ &\quad + A_yB_x\overbrace{(\hat{j}\cdot\hat{i})}^{0} + A_yB_y(\hat{j}\cdot\hat{j}) + A_yB_z\overbrace{(\hat{j}\cdot\hat{k})}^{0} \\ &\quad + A_zB_x\overbrace{(\hat{k}\cdot\hat{i})}^{0} + A_zB_y\overbrace{(\hat{k}\cdot\hat{j})}^{0} + A_zB_z(\hat{k}\cdot\hat{k}). \end{aligned}$$

Thus,

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z.$$

This formula makes it easy to calculate the dot product of two vectors expressed in terms of the three unit vectors. For two-dimensional vectors, one of the components equals zero, and the above formula reduces to two terms.

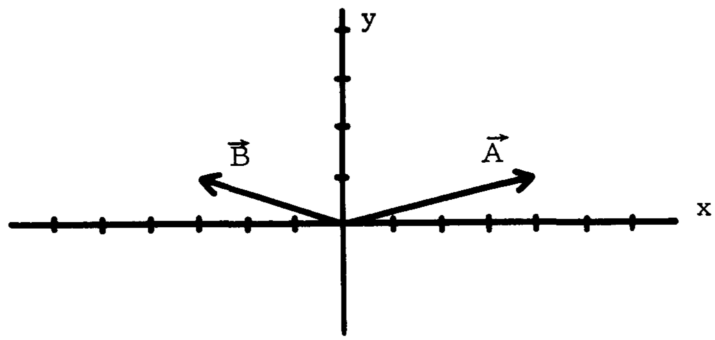
Let us consider the value of the dot product when the angle between the two vectors is greater than 90° (see figure). Using the definition



$|\vec{A}|\ |\vec{B}| \cos\theta$, one sees that $\cos\theta$ is negative, and therefore the dot product itself is negative.

To illustrate this we shall calculate a dot product for two vectors with $\theta > 90°$. Let $\vec{A} = 4\hat{i} + \hat{j}$ and $\vec{B} = -3\hat{i} + \hat{j}$; inspection of the figure below shows that $\theta > 90°$.

$$\begin{aligned}\vec{A}\cdot\vec{B} &= A_xB_x + A_yB_y \\ &= (4)(-3) + (1)(1) \\ &= -12 + 1 \\ &= -11.\end{aligned}$$



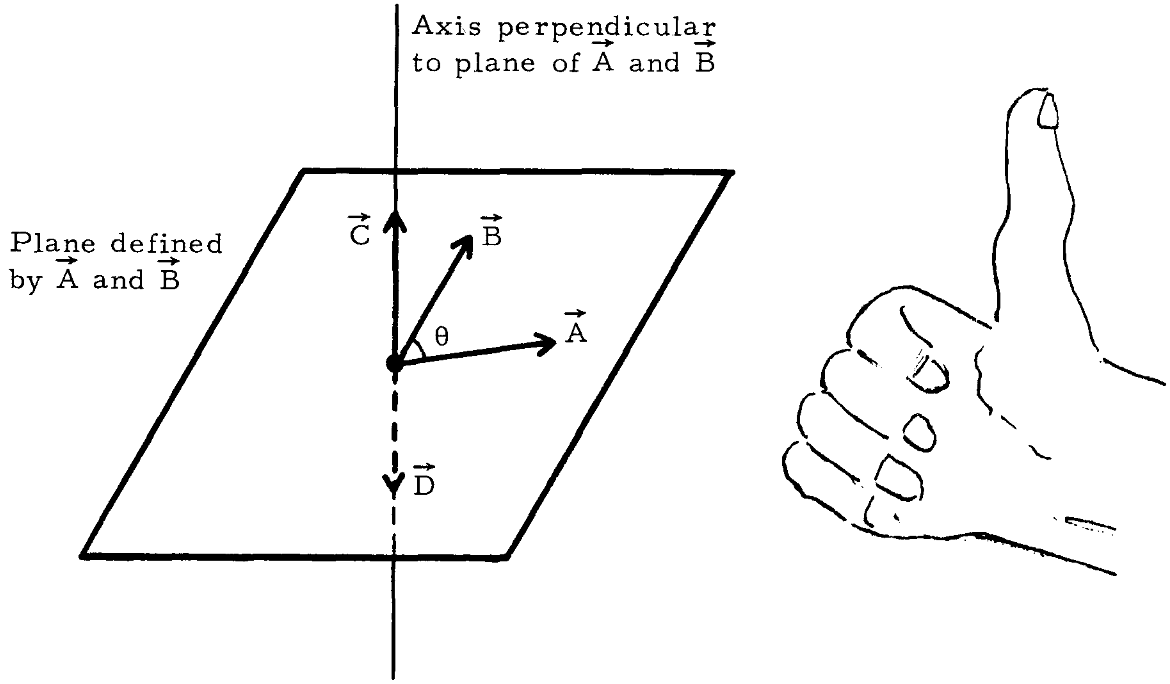
Thus the dot product is negative, as stated above.

<u>Exercise</u>: Transform $\vec{A}$ and $\vec{B}$ to polar form, calculate the dot product from the definition, and check that the result is equal to the value calculated above.

In summary, there are two ways to calculate the dot product: using $|\vec{A}|\ |\vec{B}| \cos\theta$ or $A_xB_x + A_yB_y + A_zB_z$. The first formula is useful when the vectors are expressed in polar form in two dimensions, or whenever you know the magnitudes of the vectors and the angle between them. The second formula is much easier to use when the vectors are expressed in terms of rectangular components.

2. <u>Calculation of the Vector (Cross) Product</u>

The vector product of two vectors is written as $\vec{A} \times \vec{B}$ and is equal to a third vector $\vec{C}$; the magnitude of $\vec{C}$ is defined to be $|\vec{C}| = |\vec{A}|\ |\vec{B}| \sin\theta$, where θ is the smaller angle between $\vec{A}$ and $\vec{B}$. The direction of $\vec{C}$ is defined to be perpendicular to <u>both</u> $\vec{A}$ and $\vec{B}$, or, in other words, along an axis perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$ (see diagram). The direction of $\vec{C}$ along this axis is



specified by the "right-hand rule." You should imagine that the fingers of your right hand are curled around the axis, pushing $\vec{A}$ into $\vec{B}$ through the angle θ. If the thumb is held erect, it will indicate the positive direction of $\vec{A} \times \vec{B}$. Note that there are two possible angles between $\vec{A}$ and $\vec{B}$: θ and $(360° - \theta)$; ambiguity is removed by always choosing the smaller angle, as in the diagram.

Notice that the positive direction of the cross product depends crucially upon the order of $\vec{A}$ and $\vec{B}$ in the product. Using the right-hand rule and the diagram to find the direction of $\vec{D} = \vec{B} \times \vec{A}$ shows that $\vec{D}$ points in the opposite sense to $\vec{C}$. Thus $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$; this is an important property of the cross product. Note that, in contrast to the dot product, the cross product results in a vector; these properties are the origin of the names scalar and vector product.

Example

Evaluate the cross products of the various unit vectors: $\hat{i} \times \hat{i}$, $\hat{i} \times \hat{j}$, $\hat{i} \times \hat{k}$, $\hat{j} \times \hat{i}$, $\hat{j} \times \hat{j}$, $\hat{j} \times \hat{k}$, $\hat{k} \times \hat{i}$, $\hat{k} \times \hat{j}$, and $\hat{k} \times \hat{k}$. (Hint: Remember that $|\vec{A}\times\vec{B}| = |\vec{A}|\ |\vec{B}| \sin \theta$.)

Solution

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0 \ (\sin 0° = 0),$$

$$\hat{i} \times \hat{j} = \hat{k},\ \hat{j} \times \hat{k} = \hat{i},\ \hat{k} \times \hat{i} = \hat{j} \ (\sin 90° = 1, \text{ and right-hand rule}),$$

$$\hat{j} \times \hat{i} = -\hat{k};\ \hat{k} \times \hat{j} = -\hat{i};\ \hat{i} \times \hat{k} = -\hat{j} \ (\sin 90° = 1, \text{ and right-hand rule}).$$

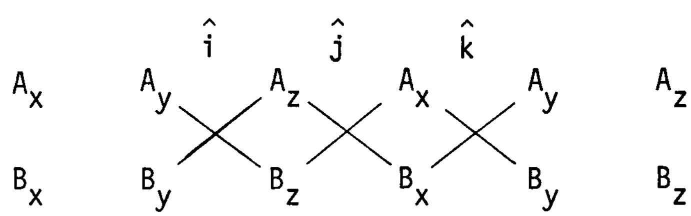
Using these results, we can calculate a general formula for the cross product of two vectors in component form. If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$,

$$\begin{aligned}\vec{A} \times \vec{B} &= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\ &= A_xB_x(\overbrace{\hat{i} \times \hat{i}}^{0}) + A_xB_y(\hat{i} \times \hat{j}) + A_xB_z(\hat{i} \times \hat{k}) \\ &\quad + A_yB_x(\hat{j} \times \hat{i}) + A_yB_y(\overbrace{\hat{j} \times \hat{j}}^{0}) + A_yB_z(\hat{j} \times \hat{k}) \\ &\quad + A_zB_x(\hat{k} \times \hat{i}) + A_zB_y(\hat{k} \times \hat{j}) + A_zB_z(\overbrace{\hat{k} \times \hat{k}}^{0}).\end{aligned}$$

Thus

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}.$$

This result is worth preserving, but it is difficult to remember in this form. A device to assist in remembering the cross product is as follows: Write the three components of the two vectors twice as follows:



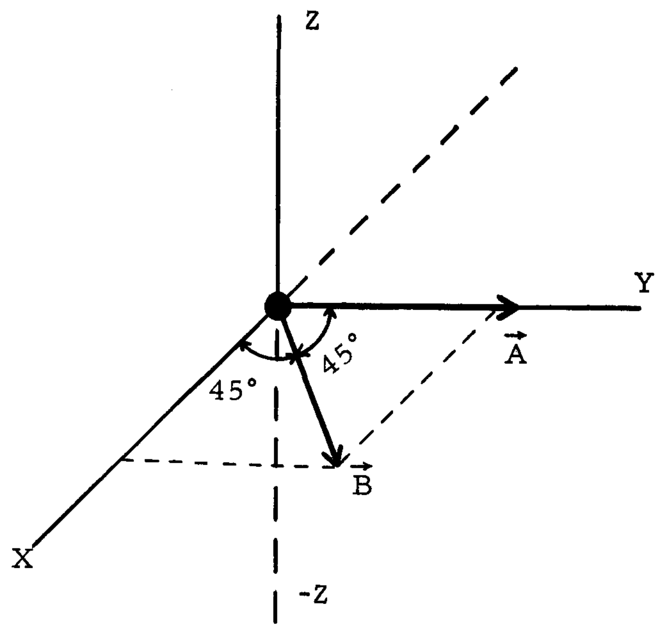
The components of the cross product can then be read from the three ×s. The first × gives the x component $(A_yB_z - A_zB_y)$; the second × gives the y component $(A_zB_x - A_xB_z)$; and the third × gives the z component $(A_xB_y - A_yB_x)$. To summarize,

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{i} + (A_zB_x - A_xB_z)\hat{j} + (A_xB_y - A_yB_x)\hat{k}.$$

This formula provides a way to calculate the cross product of two vectors directly when the original vectors are given in rectangular form. However, when the vectors are given in some other form, one can either transform them to rectangular coordinates, or use the alternative formula for the magnitude, $|\vec{A} \times \vec{B}| = |\vec{A}|\ |\vec{B}| \sin\theta$. With the latter method, one must find the direction of the product vector by use of the right-hand rule and by remembering that it is perpendicular to both $\vec{A}$ and $\vec{B}$.

For example, suppose that we wish to find the cross product of two vectors, both of which are in the xy plane. $\vec{A}$ is 4.2 cm in magnitude, along the y axis, and $\vec{B}$ has magnitude 3.6 cm and points in a direction at 45° to the positive y and positive x axes, as shown in the diagram.

The magnitude of the cross product can be found easily:



$$
\begin{aligned}
|\vec{A} \times \vec{B}| &= |\vec{A}|\ |\vec{B}| \sin\theta \\
&= (4.2)(3.6) \sin (45°) \\
&= (15.1)(0.707) \\
&= 11\ \text{cm}^2.
\end{aligned}
$$

The direction is somewhat more difficult to find; we know that $\vec{A} \times \vec{B}$ must lie along the z axis because this is the only direction perpendicular to both vectors, and the right-hand rule shows that the result must lie along the negative z axis. Therefore,

$$\vec{A} \times \vec{B} = (-11\hat{k})\ \text{cm}^2.$$

As another example, we seek the cross product of

$$\vec{A} = 3\hat{i} + \hat{j} - \hat{k} \qquad \text{and} \qquad \vec{B} = -2\hat{i} - 5\hat{j} + 6\hat{k}.$$

Since these two vectors are expressed in rectangular form, we can calculate

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

$$= [(1)(6) - (-5)(-1)]\hat{i} + [(-1)(-2) - (3)(6)]\hat{j} + [(3)(-5) - (1)(-2)]\hat{k}$$

$$= (1)\hat{i} + (-16)\hat{j} + (-13)\hat{k}.$$

Thus,

$$\vec{A} \times \vec{B} = \hat{i} - 16\hat{j} - 13\hat{k}.$$

PROBLEM SET WITH SOLUTIONS

A(1). Given $\vec{D} = \hat{i} + 3\hat{j} - 2\hat{k}$ and $\vec{E} = 4\hat{i} - 5\hat{j} + \hat{k}$, calculate the following:

(a) $\vec{D}\cdot\vec{E}$;

(b) the angle between $\vec{D}$ and $\vec{E}$.

Solution

(a) $\vec{D}\cdot\vec{E} = D_x E_x + D_y E_y + D_z E_z = [(1)(4) + (3)(-5) + (-2)(1)]m^2 = -13\ m^2$

(b) We wish to find θ; we could use either

$$\vec{D}\cdot\vec{E} = |\vec{D}|\ |\vec{E}| \cos\theta \qquad \text{or} \qquad |\vec{D} \times \vec{E}| = |\vec{D}|\ |\vec{E}| \sin\theta.$$

Using the former, we have

$$\cos\theta = \vec{D}\cdot\vec{E}/|\vec{D}|\ |\vec{E}|\ .$$

From above we have that the numerator $\vec{D}\cdot\vec{E} = -13\ m^2$. Now,

$$|\vec{D}| = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{1 + 9 + 4}\ m = \sqrt{14}\ m = 3.75\ ,$$

and

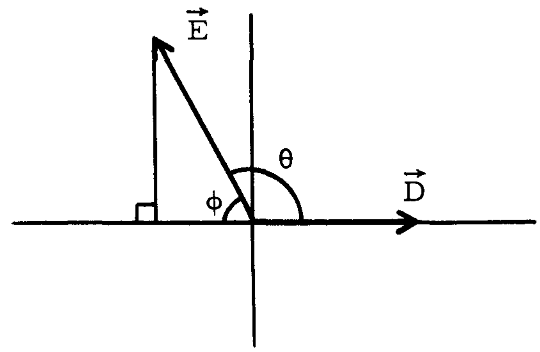
$$|\vec{E}| = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{16 + 25 + 1}\ m = \sqrt{42}\ m = 6.48\ .$$

(Note: We have preserved three significant figures because the calculation is not yet completed.) Therefore,

$$\cos\theta = \frac{-13\ m^2}{(3.75)(6.48)\ m^2} = -0.54.$$

The fact that the dot product (and therefore cos θ) is negative alerts us to the fact that the vectors are at an obtuse angle with each other and θ > 90°, as shown. Looking up 0.54 in a trig table gives $\phi = 58°$. But the angle we want is

$\theta = 180° - \phi = 122°$.



B(2). Given the vectors from Problem A, calculate the following quantities:

(a) $\vec{D} \times \vec{E}$;

(b) $\vec{E} \times \vec{D}$ (show explicitly that $\vec{D} \times \vec{E} = -\vec{E} \times \vec{D}$).

Solution

(a) $\vec{D} \times \vec{E} = (D_yE_z - D_zE_y)\hat{i} + (D_zE_x - D_xE_z)\hat{j} + (D_xE_y - D_yE_x)\hat{k}$

$= [(3)(1) - (-2)(-5)]\hat{i} + [(-2)(4) - (1)(1)]\hat{j} + [(1)(-5) - (3)(4)]\hat{k}$,

$\vec{D} \times \vec{E} = -7\hat{i} - 9\hat{j} - 17\hat{k}$.

(b) $\vec{E} \times \vec{D} = (E_yD_z - E_zD_y)\hat{i} + (E_zD_x - E_xD_z)\hat{j} + (E_xD_y - E_yD_x)\hat{k}$

$= [(-5)(-2) - (1)(3)]\hat{i} + [(1)(1) - (4)(-2)]\hat{j} + [(4)(3) - (-5)(1)]\hat{k}$

$\vec{E} \times \vec{D} = +7\hat{i} + 9\hat{j} + 17\hat{k} = -\vec{D} \times \vec{E}$ by comparison with (a).

Problems

C(1). Given $\vec{A}$ with magnitude 15 m pointing along the positive y axis, and $\vec{B} = (4\hat{i} - 5\hat{k})$ m, calculate the following quantities:

(a) $\vec{A} \cdot \vec{B}$;

(b) the angle between $\vec{A}$ and $\vec{B}$.

D(2). Given the vectors from Problem C, calculate the following quantities:

(a) $\vec{A} \times \vec{B}$;

(b) $\vec{B} \times \vec{A}$ (show explicitly that $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$).

Solutions

C(1). (a) 0; (b) 90°.

D(2). (a) $\vec{A} \times \vec{B} = (-75\hat{i} - 60\hat{k})\ m^2$; (b) $\vec{B} \times \vec{A} = (75\hat{i} + 60\hat{k})\ m^2$.

PRACTICE TEST

1. Suppose $\vec{A}$, a vector in the xy plane, can be written in polar form as (2.4 m, 310°), and $\vec{B} = -1.0\hat{i} - 1.0\hat{j}$. Calculate the following:
   (a) $\vec{A}\cdot\vec{B}$;
   (b) the angle between $\vec{A}$ and $\vec{B}$.

2. A vector $\vec{A}$ in the xy plane of magnitude 0.2 m is directed at an angle of 155° with the positive x axis. $\vec{B} = (-0.4\hat{k})$ m. Calculate $\vec{A} \times \vec{B}$.

Practice Test Answers

1. (a) $\vec{A}\cdot\vec{B} = |\vec{A}|\ |\vec{B}| \cos\theta = (2.4\ m)(\sqrt{2}\ m) \cos 85° = 0.30\ m^2$;
   (b) $\theta = 85°$.

2. $|\vec{A}\times\vec{B}| = |\vec{A}|\ |\vec{B}| \sin\theta = (0.2\ m)(0.4\ m) \sin (90°) = 0.08\ m^2$.
   $\vec{A} \times \vec{B}$ is directed at an angle of 155° + 90° = 245° with the positive x axis.

Date ____________________

VECTOR MULTIPLICATION

Mastery Test   Form A

pass   recycle

1   2

Name________________________________ Tutor________________________

1. Given $\vec{A} = (6.0\hat{i} + 1.0\hat{j} - 2.0\hat{k})$ m and $\vec{B} = (-4.0\hat{i} - 3.0\hat{k})$ m, calculate
   (a) $\vec{A}\cdot\vec{B}$;
   (b) the angle between $\vec{A}$ and $\vec{B}$.

2. Given $\vec{A} = 3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k}$ and $\vec{B} = -1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$, calculate $\vec{A} \times \vec{B}$.

VECTOR MULTIPLICATION pass recycle

Mastery Test Form B 1 2

Name_______________________________ Tutor_________________________

1. A certain vector $\vec{A}$ in the xy plane is 250° counterclockwise from the positive x axis and has magnitude 7.4 cm. Vector $\vec{B}$ has magnitude 5.0 cm and is directed parallel to the z axis. Calculate
   (a) $\vec{A}\cdot\vec{B}$;
   (b) the angle between $\vec{A}$ and $\vec{B}$.

2. For the vectors in Problem 1, calculate $\vec{A} \times \vec{B}$.

Date ____________________

VECTOR MULTIPLICATION

pass recycle

Mastery Test Form C

1 2

Name________________________________ Tutor________________________________

1. Given $\vec{A} = -2.0\hat{j} + \hat{k}$ and $\vec{B} = 4.0\hat{i} - 3.0\hat{k}$, calculate
   (a) $\vec{A}\cdot\vec{B}$;
   (b) the angle between $\vec{A}$ and $\vec{B}$.

2. For the vectors in Problem 1, calculate $\vec{A} \times \vec{B}$.

MASTERY TEST GRADING KEY - Form A

What To Look For	Solutions
1.(a) The answer must be of the proper sign. If the units and/or the number of significant figures in the answer are not correct, remind the student of this, but do not mark it incorrect.	1.(a) $\vec{A}\cdot\vec{B} = [(6.0)(-4.0) + (1.0)(0) + (-2.0)(-3.0)]\ m^2$ $= (-24 + 6.0)\ m^2$ $= -18\ m^2$.
(b) If the student gives $\theta = 56°$, he probably picked the incorrect quadrant, and this should be marked wrong.	(b) $\lvert\vec{A}\rvert = \sqrt{(36 + 1.0 + 4)\ m^2} = \sqrt{41\ m^2}$. $\lvert\vec{B}\rvert = \sqrt{(16 + 9)\ m^2} = \sqrt{25\ m^2} = 5.0\ m$. $\cos\phi = \dfrac{-18\ m^2}{(\sqrt{41\ m^2})(5.0\ m)} = -0.56$. $\phi = 56°$, and $\theta = 124°$.
2. Answer should be in unit-vector form with no units.	2. $\vec{A}\times\vec{B} = (6.0 - 8.0)\hat{i} + (+2.0 - 6.0)\hat{j} + (-12 + 3.0)\hat{k}$ $= -2.0\hat{i} - 4.0\hat{j} - 9.0\hat{k}$.

MASTERY TEST GRADING KEY - Form B

What To Look For	Solutions
1.(a) Student can see directly from statement of problem that $\theta = 90°$ and $\vec{A}\cdot\vec{B} = 0$, but converting to unit vectors and doing complete calculation is also satisfactory, although the easy way should be pointed out.	1.(a) Since vectors are perpendicular, $\vec{A}\cdot\vec{B} = 0$.
(b) See above.	(b) $\theta = 90°$.
2. If result does not have the correct units and number of significant figures, remind the student of this, but do not mark it incorrect. Results can be expressed in either component or polar form; numbers must be correct.	2. $\lvert\vec{A} \times \vec{B}\rvert = 37\ \text{cm}^2$, $\theta = 160°$. $\vec{A} \times \vec{B} = -35\hat{i} + 13\hat{j}$.

MASTERY TEST GRADING KEY - Form C

What To Look For	Solutions
1.(a) Answer should be dimensionless with two significant figures; remind student of this if necessary, but do not grade question incorrect for this. Numbers must be correct.	1.(a) $\vec{A}\cdot\vec{B} = -3.0$.
(b) If student gives 180° - θ instead of θ it should be marked incorrect, and the grader should point out how to pick quadrant when cos θ is negative.	(b) $\cos\phi = \vec{A}\cdot\vec{B}/\lvert\vec{A}\rvert\,\lvert\vec{B}\rvert = -3.0/(\sqrt{5})(5) = -0.27$; $\phi = 74°$. $\theta = 180° - \phi = 106°$.
2. Same as 1(a) above.	2. $\vec{A}\times\vec{B} = 6.0\hat{i} + 4.0\hat{j} + 8.0\hat{k}$.