1975

Wave Properties of Light

WAVE PROPERTIES OF LIGHT

INTRODUCTION

How do Polaroid sunglasses reduce glare? What evidence is there for an expanding universe? You will learn the answer to these two questions by considering some properties of light. Gamma rays, x rays, light (ultraviolet, visible, and infrared) and radio waves are all forms of electromagnetic radiation and therefore share the same basic properties as mentioned in this module and the module Maxwell's Predictions. The basic difference among these types of electromagnetic radiation is their wavelengths or frequencies.

PREREQUISITES

Before you begin this module, you should be able to:	Location of Prerequisite Content
*Calculate the cross product of two vectors (needed for Objective 1 of this module)	Vector Multiplication Module
*Define energy and power (needed for Objective 1 of this module)	Work and Energy Module
*Define momentum (needed for Objective 2 of this module)	Impulse and Momentum Module
*Solve problems using Newton's second law (needed for Objective 2 of this module)	Newton's Laws Module
*Solve problems using the relations among the properties of electromagnetic waves (needed for Objectives 1 through 3 of this module)	Maxwell's Predictions Module

LEARNING OBJECTIVES

After you have mastered the content of this module, you will be able to:

1. Poynting vector - Write the defining equation for the Poynting vector $\vec{S} = (1/\mu_0)(\vec{E} \times \vec{B})$, and apply this definition with $|\vec{E}| = c|\vec{B}|$ and the fact that $\vec{E}$ is perpendicular to $\vec{B}$ to determine the power, Poynting vector, electric field, and magnetic field of an electromagnetic wave.

2. Radiation momentum - Write the equation that relates radiation momentum to the radiation energy, and use this equation to calculate the momentum trans-

ferred or forced on an object for cases of total absorption or total reflection.

3. Polarization - Write the polarization equation $I = I_0 \cos^2 \theta$, and apply it to solve problems involving the intensity of light transmitted by a polarizing material.

4. Doppler effect - Write and apply the equation that describes the observed frequency when a source of electromagnetic radiation and a observer are either approaching or receding from each other.

GENERAL COMMENTS

This module attempts to relate some apparently abstract properties of electromagnetic radiation to situations that you have experienced. The warmth of the Sun on your skin is the result of energy that has been released from the nuclear "burning" of hydrogen in the Sun and transported across 1.50×10^{11} m to Earth in the form of electromagnetic radiation. This experience can be quantified by the use of the Poynting vector, which gives the magnitude and direction of this energy transfer. The Poynting vector is the power per unit area transmitted by the electromagnetic field. We do not feel the momentum associated with radiation because its impulse is too small.

Sunlight reflected from objects tends to be plane polarized. Your Polaroid sunglasses have their polarization axis perpendicular to the polarization axis of the reflected light and thereby reduce glare. Police radar uses the frequency change, or Doppler effect, produced by reflection from a moving object. One must be careful to distinguish between the Doppler effects for sound and for light. Only one equation is necessary for the Doppler effect for light, namely,

$$\nu = \nu_s\left(\frac{1 + v/c}{1 - v/c}\right)^{1/2} = \nu_s\left(\frac{1 + v/c}{(1 - v^2/c^2)^{1/2}}\right),$$

where ν_s is the frequency of the source; ν is the observed frequency; v is the relative speed of approach between the source and the observer, where v is positive if the source and the observer are approaching each other and negative if they are receding; and c is the speed of light.

The order of the subject material in this module is not important. Texts differ in the order and emphasis given each subject. Therefore, it is not necessary to study the objectives in the listed order, except that the radiation momentum is usually defined in terms of the Poynting vector.

TEXT: Frederick J. Bueche, Introduction to Physics for Scientists and Engineers (McGraw-Hill, New York, 1975), second edition

SUGGESTED STUDY PROCEDURE

First read the General Comments. Your text describes the concepts in the objectives in various chapters. Section 29.3 in Chapter 29 treats the average value of the Poynting vector; its instantaneous value can be written in vector form as $\vec{S} = (\vec{E} \times \vec{B})/\mu_0$. The text briefly treats radiation momentum on p. 626, and therefore it is essential to work Problems B and F. The concept of radiation pressure follows directly from the Poynting vector.

Bueche lets you derive the polarization expression $I = I_0 \cos^2 \theta$ in Problem 21 of Chapter 33. You should work Problems C and G. Your text only treats the Doppler effect for sound. Interestingly, the expression for the Doppler effect for light is simpler than that for sound. Only one equation is necessary for light:

$$\nu = \nu_s\left(\frac{1 + v/c}{1 - v/c}\right)^{1/2}$$

(see General Comments), and this depends only on the relative velocity of approach or recession. When you have worked Problems E through H and 7 through 9 in Chapter 29, and 21 in Chapter 33, take the Practice Test.

BUECHE

Objective Number	Readings	Problems with Solutions		Assigned Problems	
		Study Guide	Text	Study Guide	Text
1	Sec. 29.3	A	Illus.[a] 29.1	E	Chap. 29, Probs. 7, 8, 9
2	p. 626	B		F	Chap. 29, Quest.[a] 5
3	Secs. 33.8, 33.9, 33.10	C		G	Chap. 33, Prob. 21
4	General Comments	D		H	

[a]Illus. = Illustration(s). Quest. = Question(s).

TEXT: David Halliday and Robert Resnick, Fundamentals of Physics (Wiley, New York, 1970; revised printing, 1974)

SUGGESTED STUDY PROCEDURE

First read the General Comments. Your text treats the concepts in the same order as the objectives. Read all of Chapter 35, work Problems A through H and the text problems, and take the Practice Test before attempting a Mastery Test.

HALLIDAY AND RESNICK

Objective Number	Readings	Problems with Solutions		Assigned Problems	
		Study Guide	Text (Chap. 35)	Study Guide	Text (Chap. 35)
1	Sec. 35-4	A	Ex.[a] 1	E	11, 13, 17
2	Sec. 35-5	B	Ex. 2	F	27, 28, 30
3	Sec. 35-6 [Eq. (35-15)]	C		G	35, 36
4	Secs. 35-9, 35-10 [Eqs. (35-17), (35-20)]	D	Ex. 6	H	44, 45, 47, 48

[a]Ex. = Example(s).

TEXT: Francis Weston Sears and Mark W. Zemansky, University Physics (Addison-Wesley, Reading, Mass., 1970), fourth edition

SUGGESTED STUDY PROCEDURE

First read the General Comments. For Objective 1, the Poynting vector, Eq. (36-9), may be written as $\vec{S} = (1/\mu_0)(\vec{E} \times \vec{B})$ using $\vec{H} = \vec{B}/\mu_0$. Your text defines a momentum density, which can be shown to be equivalent to momentum p = (energy per unit area × unit time)/c = U/c (for total absorption).

Your text has a very extensive treatment on polarization, but Sections 42-1 and 42-5 alone cover Objective 3. Sears and Zemansky treat the Doppler effect for both sound and light, but this module is concerned only with the Doppler effect in light.

Study Problems A through D before working the Assigned Problems and taking the Practice Test.

SEARS AND ZEMANSKY

Objective Number	Readings	Problems with Solutions		Assigned Problems		Additional Problems
		Study Guide	Text	Study Guide	Text	
1	Sec. 36-3	A	Example on p. 522	E	36-5, 36-6	
2	Sec. 36-3	B		F		
3	Secs. 42-1 to 42-11	C		G	42-7, 42-12	
4	Sec. 23-9	D		H		23-12,[a] 23-13,[a] 23-15[a]

[a]These problems are for sound only.

TEXT: Richard T. Weidner and Robert L. Sells, Elementary Classical Physics (Allyn and Bacon, Boston, 1973), second edition, Vol. 2

SUGGESTED STUDY PROCEDURE

First read the General Comments. Your text uses the symbol $\vec{I}$ for the Poynting vector, but since most texts use the symbol $\vec{S}$ our Problems and Tests use the symbol $\vec{S}$. Section 40-3 with the Examples 40-1 and 40-2 is the most important part of the reading for Objective 3.

Study Problems A through D before working the Assigned Problems and taking the Practice Test.

WEIDNER AND SELLS

Objective Number	Readings	Problems with Solutions		Assigned Problems	
		Study Guide	Text	Study Guide	Text
1	Sec. 35-4 [Eqs. (35-26), (35-18)]	A		E	35-3, 35-5
2	Sec. 35-5	B		F	35-5, 35-8
3	Secs. 40-1 to 40-5	C	Ex.[a] 40-1, 40-2	G	40-3, 40-4
4	Sec. 35-10	D		H	35-21, 35-23, 35-24

[a]Ex. = Example(s).

PROBLEM SET WITH SOLUTIONS

A(1). A plane radio wave is propagating in free space along the positive z axis with $\vec{E} = E_0 \sin(\omega t - kz)\hat{i}$. Find (a) the expression for the magnetic field as a function of t and z; and (b) the Poynting vector as a function of t and z.

Solution

(a) $|\vec{E}| = c|\vec{B}|$, $|\vec{B}| = |\vec{E}|/c = (E_0/c)\sin(\omega t - kz)$ with $\vec{B}$ along $\hat{j}$,

$\vec{B} = (E_0/c)\sin(\omega t - kz)\hat{j}$.

(b) $\vec{S} = (1/\mu_0)(\vec{E} \times \vec{B}) = (1/\mu_0)E_0 \sin(\omega t - kz)\hat{i} \times (E_0/c)\sin(\omega t - kz)\hat{j}$

$= (1/\mu_0)cE^2 \sin^2(\omega t - kz)\hat{k}$.

B(2). A parallel beam of electromagnetic radiation with energy flux S = 10.0 W/cm^2 is incident normally on a perfectly reflecting mirror of 1.00 cm^2 area for 1000 s.
(a) What is the momentum transferred to the mirror?
(b) What is the average force acting on the mirror?

Solution

(a) p = 2U/c, where U is energy = S × t × A.

$$p = \frac{2StA}{c} = \frac{2(10.0\ \text{W/cm}^2)(1000\ \text{s})(1.00\ \text{cm}^2)}{c} = \frac{2.00 \times 10^4\ \text{W s}}{3.00 \times 10^8\ \text{m/s}}$$

$$= 6.7 \times 10^{-5}\ \frac{\text{J}}{\text{m/s}} = 6.7 \times 10^{-5}\ \text{kg m/s}.$$

(b) $F = p/t = (6.7 \times 10^{-5})/(1000) = 6.7 \times 10^{-8}$ N (small force).

C(3). A beam of unpolarized light shines on the three polarizers in Figure 1. The lines indicate the axes of transmission of the polarizers. Find the intensity of light (in terms of I_0) at points A, B, and C.

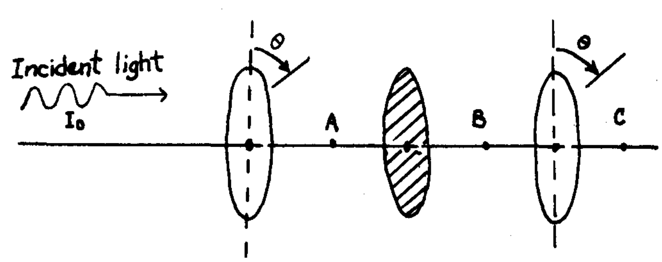

Figure 1

Solution

At point A there is a reduction in intensity caused by the transmission of only the components of E at θ and the light is plane polarized:

$$I_A = I_0/2.$$

At point B there is a reduction of the intensity by

$$I_B = (I_0 \cos^2 \theta)/2.$$

At point C the intensity is

$$I_C = I_B \cos^2 \theta = I_0(\cos^2 \theta)(\cos^2 \theta)/2 = (I_0 \cos^4 \theta)/2.$$

D(4). A characteristic hydrogen line from a distant galaxy is measured on Earth to have a reduction in frequency of 25% compared to the same hydrogen line on Earth. What is the radial velocity of this galaxy with respect to Earth?

Solution

Doppler effect:

$$\nu = \nu_s \frac{1 + v/c}{[1 - (v/c)^2]^{1/2}}, \qquad \frac{\nu}{\nu_s} = \frac{3}{4} = \frac{1 + v/c}{[1 - (v/c)^2]^{1/2}}.$$

Let $v/c = x$. Then

$$\frac{3}{4} = \frac{1 + x}{(1 - x^2)^{1/2}}, \qquad \frac{9}{16} = \frac{(1 + x)^2}{1 - x^2} = \frac{(1 + x)^2}{(1 - x)(1 + x)} = \frac{1 + x}{1 - x},$$

$$\frac{9}{16} - \left(\frac{9}{16}\right)x = 1 + x, \qquad -\frac{7}{16} = \left(\frac{25}{16}\right)x, \qquad x = -\frac{7}{25} = \frac{v}{c} = -0.280.$$

Thus $v = -8.4 \times 10^7$ m/s away from Earth. (Galaxy is receding.)

Problems

E(1). The time-average value of the Poynting vector at $y = 0$ is $\vec{S}_{av} = S_{av}\hat{j}$. Find
(a) the maximum values of the electric and magnetic fields; and
(b) their plane of oscillation.

F(2). Calculate the force due to solar radiation on a 10.0-m^2, perfectly absorbing panel above Earth's atmosphere. The plane of the panel is perpendicular to the Sun's radiation, and the solar constant is 1400 W/m^2. (The solar constant is the power per unit area from the Sun above Earth's atmosphere.)

G(3). A beam of unpolarized light is incident on two crossed polarizers, i.e., the first one has its direction of polarization vertical, the second one has its direction horizontal. A third polarizer is placed between the other two. Neglecting partial absorption of the transmitted component, what fraction of an incident unpolarized beam is transmitted through the whole system if the middle polarizer has its direction of polarization:
(a) vertical?
(b) 45° to the vertical?
(c) horizontal?

H(4). Police use radar to measure the speed of automobiles. What is the relative difference between the transmitted and received frequencies $(\nu - \nu_s)/\nu_s$ for an automobile approaching at a speed of 30.0 m/s?

Solutions

E(1). (a) Since $[\int_0^T \sin^2(\omega t)\,dt]/\int_0^T dt = 1/2$,

$$S_{av} = (E_0B_0/\mu_0)\{[\int_0^T \sin^2(\omega t)\,dt]/\int_0^T dt\} = E_0B_0/2\mu_0.$$

But $B_0 = E_0/c$ and $E_0 = \sqrt{2c\mu_0S_{av}}$, thus

$$B_0 = \sqrt{2\mu_0S_{av}/c}.$$

(b) $\vec{E}$ and $\vec{B}$ are in the xz plane perpendicular to $\vec{S}$.

F(2). 4.7×10^{-5} N. Total reflection would give twice this amount.

G(3). (a) 0. (b) 1/8. (c) 0.

H(4). $(\nu - \nu_s)/\nu_s = 2 \times 10^{-7}$.

PRACTICE TEST

A laser emits a narrow beam of light with an average power of 10^{-2} W over a cross-sectional area of 5.0×10^{-6} m^2.

1. Find the maximum electric and magnetic fields in the beam.

2. What force would this beam exert on a completely absorbing surface perpendicular to the beam?

3. If the laser is plane polarized, what must be the angle between the polarization axes of a polarizing sheet and the polarization axes of the laser if the transmitted light is to be 25% of the incident intensity?

4. What would be the relative frequency difference $(\nu - \nu_s)/\nu_s$ of this laser as measured by a spaceship receding from the laser at 3.00×10^4 m/s?

Practice Test Answers

1. $E = 1.20 \times 10^3$ V/m. $B = 4.1 \times 10^{-6}$ T.
2. $F_r = (1/3) \times 10^{-10}$ N.
3. $\theta = 60°$.
4. -10^{-4}.