Planning and Enacting Mathematical Tasks of High Cognitive Demand in the Primary Classroom

Kelly Georgius

University of Nebraska-Lincoln, kellygeorgius@yahoo.com

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This study offers an examination of two primary-grades teachers as they learn to transfer knowledge from professional development into their classrooms. I engaged in planning sessions with each teacher to help plan tasks of high cognitive demand, including anticipating and planning for classroom discourse that would occur around the task. A detailed description of the planning and teaching that took place during the study provides information about how a teacher can learn and what a teacher learns to consider in order to plan and implement meaningful mathematical lessons.

This design experiment describes the work of two teachers who participated in Primarily Math, a professional development program funded by the National Science Foundation. The overarching questions studied were about the transfer of knowledge from professional development to classroom practice and how teachers plan and implement tasks of high cognitive demand. Within the study, I examined the role of the curriculum and the understanding of student conceptions of mathematics in planning and teaching. As well as how a researcher can support teachers through the planning and task implementation.
The author found that weak mathematical knowledge for teaching can be overcome by learning to deeply understand students. Additionally, the intentional use of talk moves can help teachers improve classroom discourse, sustain press for justification and minimize the routinizing of math problems. The author also suggests guidelines for planning tasks of high cognitive demand and questions that teachers can use to reflect upon and learn from their implementation of tasks of high cognitive demand.

This study is funded in part by National Science Foundation, DUE-0831835. Any opinions, findings and conclusions or recommendations expressed in this document are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.
Acknowledgements

During my doctoral program, I have encountered many ups and downs. I persevered through it all, which only happened because of the support I received from so many people in my life. I have extremely grateful for the encouragement from so many.

I thank my committee members throughout these four years: Dr. Ruth Heaton, Dr. Jon Pedersen, Dr. Doug Kauffman, Dr. Lorraine Males, Dr. Meixia Ding, and Dr. David Fowler. They pushed me to meet high expectations and grow as a teacher educator and researcher. Their feedback gave me the guidance I needed to reflect on my learning and connect it to the bigger picture of research in mathematics education.

I thank Dr. Ruth Heaton for being my advisor, my mentor, and my supporter. You helped me think deeply about the teaching and learning of mathematics. When I struggled, you pushed me and continued to believe in me. When I was unsure of how I could finish, you knew that I could do it. Thank you for always encouraging me, through good and bad, and even through a few tears.

I also want to thank all of the people involved in the NebraskaMATH grant. I had an amazing opportunity to work for some of the greatest minds in mathematics education, for which I am forever grateful. To Dr. Jim Lewis, thank you for continuing to give me opportunities to learn from you and the NebraskaMATH grant. Thank you Wendy Smith for always offering your advice and feedback on all of my projects.
Thank you to the teachers in my study. Your willingness to have me in your classroom and study your practice, is much appreciated. Without you, this work would not have been possible. Your dedication to your students is unmatched.

Before beginning my doctoral journey, I enjoyed nine years of teaching for Lincoln Public Schools. During those years, I worked with countless amazing teachers and administrators that encouraged me on this path. The knowledge they shared with me is aplenty and built the core of my beliefs about education.

Above all, I am extremely grateful for the ongoing love, encouragement, and support of my family. To my parents, you always encouraged me to be a lifelong learner. You always pushed me to do my best, no matter what. I have done my best and I hope this achievement makes you proud.

Most importantly, to my children… It is likely that you do not know what it is like to have a mother who is not in school and constantly doing homework. Kiya and Maddox, you always encouraged me to keep working, even though I am sure you really did not want me to. I made many promises to you about the things that would happen when I graduated. So, Disneyworld, here we come!
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Chapter 1

Needs in Primary Mathematics Classrooms

Teaching mathematics in the primary grades requires teachers to possess knowledge and skills specific to the mathematical content they teach and to the students in their classroom. Teachers’ deep conceptual knowledge of early mathematical skills (Ball, Thames, & Phelps, 2008; Charalambous, 2010) helps them to understand young students’ conceptions of mathematical concepts and be able to advance students’ learning to prepare them for the future. To create deep conceptual learning\(^1\) opportunities for young students, teachers plan mathematical tasks, purposefully enact the tasks, and engage students in meaningful classroom discourse around the tasks. However, purposeful planning for and enactment of meaningful tasks is not always what occurs in primary grades mathematics classrooms.

For over 20 years, the mathematics education community has called for reform in the knowledge and skills that are required of effective teaching practice (Heck, Banilower, Weiss, & Rosenberg, 2008). According to the standards written by National Council of Teachers of Mathematics (NCTM), teachers need to have a better understanding of mathematics and a deeper understanding of how students think about mathematics (NCTM, 2000). NCTM describes its vision of an ideal classroom of mathematics teaching and learning using images focused on reasoning and sense making (Karp, Caldwell, Zbiek, & Bay-Williams, 2011). NCTM and Common Core State Words or phrases central to this study are italicized the first time used and defined at the end of the chapter. The italicized words or phrases are not always specific to mathematics education, but for the purpose of my study, I will define the words or phrases in the context of mathematics education and as they pertain to my study.
Standards Initiative (CCSSI) (2011) both state that teachers should not only spend time helping students understand mathematics, but also cultivate productive mathematical practices in students. CCSSI is a state-led initiative that created the Common Core Standards for School Mathematics (CCSSM), which is a framework of student learning (CCSSI, 2011). CCSSM also includes a list of standards for mathematical practice that students should be engaged in throughout their work in math class. The standards for mathematical practice are ways of doing mathematics that teachers should develop in their students (CCSSI, 2011), and are as follows. Students should:

1. make sense of problems and persevere in solving them,
2. reason abstractly and quantitatively,
3. construct viable arguments and critique the reasoning of others,
4. model with mathematics,
5. use appropriate tools strategically,
6. attend to precision,
7. look for and make use of structure, and
8. look for and express regularity in repeated reasoning. (CCSSI, 2011, pp. 6-8)

In addition to learning standard algorithms, the NCTM process standards of communication, problem solving, reasoning and proof, connections, and representations, should be taught and practiced in the classroom. NCTM correlated their process standards and CCSSM’s standards for mathematical practice in an attempt to integrate the two sets of standards (see Figure 1). The way in which students communicate and reason about mathematics is ideally the focus of the math classroom. Students should be able to flexibly think about mathematics and solve problems in multiple ways, which can be done through tasks of high cognitive demand. The enactment of tasks of high cognitive demand allows students to develop sense-making and reasoning skills, critique the reasoning of others, create viable arguments, and communicate ideas. These are ways of
Figure 1. Correlation of NCTM Process Standards and CCSSM Mathematical Practices (adapted from NCTM, 2012).

<table>
<thead>
<tr>
<th>NCTM Process Standards</th>
<th>CCSSM Standards for Mathematical Practice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem Solving</td>
<td>• Make sense of problems and persevere in solving them</td>
</tr>
<tr>
<td></td>
<td>• Use appropriate tools strategically</td>
</tr>
<tr>
<td>Reasoning and Proof</td>
<td>• Reason abstractly and quantitatively</td>
</tr>
<tr>
<td></td>
<td>• Critique the reasoning of others</td>
</tr>
<tr>
<td></td>
<td>• Look for and express regularity in repeated reasoning</td>
</tr>
<tr>
<td>Communication</td>
<td>• Construct viable arguments</td>
</tr>
<tr>
<td>Connections</td>
<td>• Attend to precision</td>
</tr>
<tr>
<td></td>
<td>• Look for and make use of structure</td>
</tr>
<tr>
<td>Representations</td>
<td>• Model with mathematics</td>
</tr>
</tbody>
</table>

Unfortunately, this ideal classroom described by NCTM “is not reality in the vast majority of classrooms, schools, and districts,” (Karp et al., 2011, p. 5). Berliner (2006) concluded that current school reform is simply not successful enough to increase achievement for all students. Based on additional recent studies, such as the 2011 National Assessment of Educational Progress (NAEP) and reported standardized testing, the ideal classroom is still not fully realized (National Center for Education Statistics, 2011). Some students are not given ample opportunities to learn important mathematics, while other students are not challenged by the curriculum, and yet other students do not actively engage in the mathematics (Karp et al., 2011; NCTM, 2000). Students are not achieving their fullest potential in mathematics for various reasons, but it is clear that
improvement in the quality of teaching mathematics needs to occur in order to combat low student achievement (Battista, 1994). The purpose of this study is to understand how the quality of mathematics instruction can be improved through teachers’ involvement in professional development which helps teachers attend to task selection, cognitive demand, and classroom discourse.

**Issues in Translating Professional Development into Classroom Practice**

One way that teachers can grow as professionals and learn new strategies for effective instruction is through professional development programs or sessions. Professional development is a catalyst for transforming theory and research into teaching practice, thereby producing more effective teachers (Kent, 2004; Telese, 2012). Mathematics teacher educators struggle with issues related to providing effective professional development opportunities to math teachers, such as how to engage teachers in an examination of their own teaching, in a non-threatening but effective way (Arbaugh & Brown, 2005). Reform in mathematics education has highlighted the need to understand and develop effective professional development for teachers. In order for professional development to be considered effective and successful, it needs to be sustained over time and the content must be directly related to everyday classroom teaching (Kent, 2004).

Professional development can help teachers learn instructional practices that aid in the growth of students’ conceptual understanding. However, the challenge for teacher educators to make professional development effective and for teachers to transfer their learning from professional development to the classroom is greater than ever (Loucks-
Historically, teacher professional development focused on additional strategies to add to teachers’ workload but this method often leaves teachers feeling overwhelmed by additional work. Gawande (2011) noted that professional development in the form of a one-day workshop led teachers to use newly learned skills only 10% of the time and with demonstrations and feedback that rose to less than 20%. Instead, professional development can be transformative, where teachers learn to integrate new ideas into their current practices or adapt their practices to better meet the needs of their students. Gawande (2011) also noted that when teachers were supported by a colleague or instructional coach, who provided teachers with suggestions and feedback, the use of new skills rose to over 90%.

Many delivery methods of teacher professional development take place but the extent to which programs are effective is still in question (Loucks-Horsley et al., 2010). Educational researchers have recently focused on professional development around content and pedagogical knowledge. Additionally, professional development is also focused on student learning, including case studies of real student understanding. The goal of much professional development in mathematics education is to deepen the teachers’ knowledge of mathematical content, pedagogy, and students (Heck et al., 2008; Telese, 2012).

Loucks-Horsley (2010) reported features of effective professional development including that it is based on student learning, builds content and pedagogical knowledge, and is collaborative. If teachers are aware of the areas in which students struggle and traditionally have low achievement, they are able to focus on students’ understanding of
the mathematical content. This allows teachers to better understand their students and they, therefore, are better able to teach them. Possessing deep content and pedagogical knowledge allows teachers to make better instructional decisions. With this knowledge, teachers are able to create learning experiences in which students gain meaningful conceptual knowledge rather than procedural knowledge.

Teachers gain new skills and strategies to incorporate into their teaching practice from their participation in professional development. When implementing new knowledge learned from professional development, teachers may find it helpful to have support. “When teachers feel supported, they are more willing to take professional risks by trying new things” (Hunzicker, 2011, p. 178). Support resources can come in many forms, including a math coach, a researcher or a professional development expert. Support can also be found through ongoing professional development, which takes place over a long period of time, possibly a year or more, as opposed to workshop-style professional development, which is often done in a one-day delivery model. Professional development that is ongoing and includes follow up to the sessions is most effective (Kent, 2004).

Effective professional development also has an element of discovery learning. Facilitators often help teachers create knowledge in the same way that teachers would expect students to construct knowledge. Learning in which teachers build their own understanding creates deeper understanding of the process that students go through, subsequently breaking teachers of traditional teaching methods (Little, 1993). Professional development for teachers also can include opportunities for reflection and
evaluation. The opportunity to self-evaluate can help teachers sustain continuous learning and professional growth. Teachers ask themselves questions about their practice and their students’ achievement to understand the effects of their teaching practices (Loucks-Horsley et al., 2010). Teachers reflect on how their planning and enactment of tasks enhanced or inhibited student learning, then use that information to plan subsequent lessons.

Elements of professional development work together to influence change in teachers’ beliefs and practices. In order for change to happen, teachers change their beliefs about learning and see the benefits to their students in action; changes and benefits often happen simultaneously (Arbaugh & Brown, 2005). “Change is a process that takes time and persistence” (Loucks-Horsely et al., 2010, p. 75). In order for change to take place, teachers need support (Hunzicker, 2011; Kent, 2004), specifically continuous support from their school district and building administration. However, Hunzicker (2011) notes that support teachers need may look different at difference stages of the change process.

**Issues in planning tasks of high cognitive demand.** The tasks a teacher selects and plans for are part of *planned instruction*, but how they play out in the classroom with real students, which may or may not be the way the teacher intended, is referred to as *enacted instruction*. Task planning and task enactment form a cycle of teaching practice that teachers perform everyday. It is possible that teachers do not adequately consider the importance of this cycle and progress through it too quickly, which ignores consideration of important elements of teaching practice, such as how students’ prior knowledge can be
the basis of new learning, how enactment of tasks changes the cognitive demand, and how purposeful planning can aid in student learning.

The tasks that a teacher plans to implement in the classroom are one of the most important pedagogical decisions that a teacher makes (Crespo, 2003). The mathematical tasks that students engage in convey a message about the nature of mathematics and influence students’ understanding of concepts and processes (Doyle, 1988; Resnick, 2006; Stein, Grover, & Henningsen, 1996). Therefore, how teachers go about planning tasks is of considerable importance but often overlooked by the teachers themselves. Pimm (2009) noted that task selection, modifications to the task, and the reasons for choosing the tasks were all important aspects of lesson planning.

Traditionally, teachers rarely focus on high cognitive demand tasks in daily lessons, rather real-life, meaningful connections are addressed through specific problem solving lessons which are spread throughout the curriculum. Traditional curricula often treat mathematics as a discrete series of lessons strung together, which does not push students to understand the mathematics deeply. In contrast, reform curricula often address mathematical content and problem solving together though daily lessons. In most reform curricula, important mathematical ideas are spread throughout multiple lessons, connected through a variety of problem solving tasks.

Within curricula, even reform curricula, there are tasks of high and low cognitive demand. Tasks of high cognitive demand enable students to make connections in and between mathematical concepts. Every curriculum has tasks that activate students’ prior knowledge and help students actively connect mathematical knowledge, as well as ones
that do not. Teachers need to learn to differentiate between tasks of high and low cognitive demand and select tasks that require students to make deep mathematical connections.

Implementing tasks of high cognitive demand can be a risk for teachers because the problems of high cognitive demand are more ambiguous and have multiple solution paths (Henningsen, & Stein, 1997). A teacher who is not confident in her own mathematical knowledge or her ability to teach mathematics is likely to follow the textbook more closely than a teacher who is confident (McClain, Zhao, Visnovska, & Bowen, 2009). A teacher who is not confident will likely have difficulty implementing tasks of high cognitive demand because students may have solution paths not included in the textbook. Unsure teachers often do not seek out additional solution strategies beyond the textbook, due to their own understanding of mathematics. Some teachers may avoid tasks of high cognitive demand because they can cause classroom management issues when students get frustrated and increase the complexity of the work of the teaching (Doyle, 1988; Henningsen & Stein, 1997). The alternative is that teachers may ask students to perform memorization tasks and tasks which promote procedural learning.

Crespo (2003) reported that teachers may choose to implement a particular task for many reasons. Specifically she noted that teachers may choose a task based on enjoyment for themselves and the students, a curriculum guide suggestion, or perhaps it may seem like an easy task for the students to do and the teacher to implement. However, Phillips and Cooney (2009) pointed out that recently published textbooks include an overabundance of mathematical tasks of varying levels of cognitive demand and it is the
job of the teacher to choose and plan tasks that are appropriate for her particular students. An overreliance on a textbook can result in rich mathematical experiences being missed and replaced by a higher quantity of superficial experiences, simply due to the high number of tasks included in a textbook. Additionally, although it may be helpful for a task to be perceived as fun by students or be a suggested part of the curricula, it is ultimately most important that the task be conceptually centered around a mathematical topic, so that students gain mathematical understanding through the implementation of the task.

There is, however, a fine line between overreliance on a textbook and sensible utilization of a resource. Curricula guides are designed for teachers to use to guide both their yearlong and daily plans (Remillard, 2005). As such, the curriculum guide is an important tool for a teacher to use for planning. However, teachers plan and adapt tasks to meet the needs of their students. Tasks within textbooks include a range of levels of cognitive demand (Charalambous, 2010), and so it is the work of the teacher to identify tasks that enable deep conceptual learning and meaningful classroom discourse.

When planning, the teachers take into account the needs of the students and select tasks that promote deep engagement and meaningful discussion in their classroom (Smith, Bill, & Hughes, 2008; Stein, Smith, Henningsen, & Silver, 2009). Teachers need to know how to purposefully plan for and enact tasks of high cognitive demand in order to help students engage in the mathematical content and develop a deep conceptual understanding of mathematical concepts. Engaging in mathematics through tasks of high cognitive demand allows students to develop a capacity for reasoning and sense making,
whereas tasks of low cognitive demand almost never do (Smith & Stein, 1998). Planning the implementation of a task is crucial to the student learning that occurs around that task. When planning, teachers need to consider students’ prior knowledge, the purpose of the task, methods students might use to work on the task, and how to implement a classroom discussion around the students’ work on the task (Smith et al., 2008).

Often teachers do not plan for the discourse that will happen during instruction. Teachers may plan questions to guide discussion, but fail to consider what student responses may occur. During lesson planning, teachers can anticipate the strategies students will use to solve math problems, which puts the teacher in a position to productively make use of student responses (Smith & Stein, 2011). Teachers can help students make meaningful mathematical connections by pressing students to discuss their own connections with classmates.

Too often teachers assume that students mastered particular skills and understand ideas from previous grade levels, however, this is not always the case. Teachers need to consider what students understand when planning. One way that a teacher can understand student knowledge is to examine students’ understanding through the lens of a particular learning trajectory. A learning trajectory helps teachers visualize a continuum of student understanding, where a student is on it, and how to help the student increase or deepen understanding (Clements & Sarama, 2009; Sarama & Clements, 2009). Learning trajectories give the teacher a lens in which to view student understanding relative to a particular mathematical concept. Understanding a sequence of student strategies allows
teachers to interpret why certain mathematical problems or concepts are difficult for students and help teachers contemplate what to do next (Franke & Kazemi, 2001).

**Issues in enacting tasks of high cognitive demand.** Even more than planning for a task, the enactment of a task determines student learning (Charalambous, 2010). According to Trends in International Mathematics and Science Study (TIMSS) study, teachers in the United States used the same percent of high cognitive demand tasks as did higher achieving countries (Institute of Education Sciences, 2007). However, when the tasks were enacted in the United States the cognitive demand lessened (Resnick, 2006). Furthermore, in an analysis of videos in the TIMSS study, researchers found that teachers in the United States have difficulty implementing tasks of high cognitive demand, even when they are planned as such (Boston & Smith, 2009). Based on the TIMMS video study, Stigler and Hiebert (1997) reported that in the United States, more mathematical concepts were stated by the teacher rather than developed by the teacher and the students together. Additionally, students in the United States spent more time at their seats practicing procedures rather than applying or thinking about concepts. According to the TIMMS study, the findings in relation to the way in which students learn mathematics in the United States are contrary to the findings in the way that students in Germany and Japan learn mathematics, both of which had higher achievement scores than the United States.

Unfortunately, even though reform curricula sets up opportunities for students to engage in doing meaningful mathematics, if the teacher does not know how to maintain high cognitive demand the students will not gain what was intended by a particular
curriculum (Manouchehri & Goodman, 1998). Traditionally, when students learn new mathematical concepts, they are presented with the material, shown a few examples, then expected to practice the procedure by solving many similar problems (Sweller, van Merrienboer, & Paas, 1998). In a traditional classroom, conceptual learning through reasoning and sense making is not promoted. Rather procedural knowledge is emphasized. Often in such traditional classrooms students are passive receivers of knowledge dispensed by the teacher or textbook (Brown, 1992).

While an adoption of reform curricula has had influence on teachers’ beliefs and approach to instruction, it is limited (Stein & Kim, 2009). Therefore, the adoption of reform curricula is a step towards a focus on problem solving and standards-based instruction, but it is not a final solution (Ball & Cohen, 1996; Charalambous, 2010). Additionally, planning for a task of high cognitive demand will not fix everything either; teachers must maintain high cognitive demand during the enactment of these tasks. Teachers can easily lower the cognitive demand of a task by helping students too much, which is especially likely when students get frustrated or teachers feel rushed for time (Stein et al., 2009).

Stein and Kim (2009) reported two major reasons that tasks of high cognitive demand are difficult for teachers to implement. First, teachers who were taught in a traditional classroom, using traditional curricula and traditional teaching methods can lack the conceptual knowledge necessary to teach mathematics meaningfully. A mathematical problem can arise when teachers are asking their students to gain the conceptual knowledge that they themselves may lack. Thus, teachers likely feel in greater
intellectual control when using traditional teaching methods (Smith et al., 2008). Second, the orchestration of mathematical tasks of high cognitive demand can be hard to manage (Smith et al., 2008). When students are discussing ideas and working in groups, it can lead to behaviors teachers may find more difficult to manage than when students are working quietly and independently (Stein, Engle, Smith, & Hughes, 2008).

If a teacher listens to the students to understand their mathematical conceptions, then the students can better lead a discussion because the teacher is in a position to help draw connections among students’ ideas (Doerr, 2006). The role of the teacher during classroom discourse can be to develop and build on the students’ sense-making abilities, rather than to deem answers correct and incorrect (Stein et al., 2008). Too often classroom discussions consist of the teacher attempting to dispel misunderstandings of the students by repeatedly explaining a mathematical procedure. An overabundance of this type of teacher-centered discourse does not benefit students. Rather, it is more likely for students to engage in and deeply learn from a discussion where student ideas are featured (Stein et al., 2008).

Teachers can maintain high cognitive demand during a task or raise the cognitive demand of a task by incorporating discourse into the lesson. Discourse is not traditional classroom talk, where the teacher quizzingly asks students questions as a means of evaluation (Crespo, 2003; Truxaw, Gorgievski, & DeFranco, 2008) but rather it is students constructing and sharing their conceptions of mathematics (Chapin, O’Connor, & Anderson, 2009; Stein et al., 2008). When students discuss a mathematical topic, including multiple solution paths, they can compare and critique their own work and the
work of other students, which incorporates sense-making skills and reasoning into the lesson (Varol & Farran, 2006). Teachers can prepare central questions to guide the discussion, as well as plan the connections they want students to make while anticipating the students’ solution strategies. However, low quality teacher questioning and lack of reasoning and sense making in the classroom lead to weak conceptual knowledge (Weiss, Banilower, & Shimkus, 2004). Additionally, teachers can develop questioning skills within themselves and their students to use during classroom discussion that help students make sense of the mathematics and move the discussion in mathematically productive ways. Chapin et al. (2009) call these talk moves.

During instruction, whether students are learning in a whole group, working in a small group or individually, teachers listen to their students in order to learn about their solution methods (Carpenter, Fennema, & Franke, 1996; Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). Being able to understand students is crucial for meaningful classroom discourse. Although knowing students’ strategies may seem like an easy task, it is rather complex. Too often teachers tend to evaluate what a student says, rather than truly listen to a student’s contribution to understanding a mathematical concept (Grant, Kline, Crumbaugh, Kim, & Cengiz, 2009). Teachers hear what a student says, then decide how they can best help that student progress in their understanding of the mathematical concept. Often this happens in a split second (Resnick, 2006). If a teacher does not know how a student thinks about a mathematical topic, it is unlikely that the teacher can help the student progress along a learning trajectory. It is more likely that the teacher will simply interject their own thoughts and ignore the student’s idea, which does
not help the student connect prior and new knowledge, creating meaningful and lasting mathematical understanding. Cognitive demand can be lessened due to multiple factors, such as lack of classroom discourse around the solving of the task, teachers’ inability to comprehend multiple solution paths, or the teachers’ desire to over-assist struggling students (Stein et al., 2009).

**Statement of the Problem**

The complex problem that is the focus of this study is helping primary-level teachers learn to attend to cognitive demand within the cycle of task planning and task enactment and helping primary-level teachers learn to plan and maintain high cognitive demand. These ideas were learned by the teachers in the abstract through professional development and in this study the teachers are learning to translate ideas learned in professional development into teaching practice. Specifically, this study will help teachers learn how to plan for tasks of high cognitive demand from their given curriculum, then implement the tasks maintaining high cognitive demand, leading to better conceptual understanding.

During instruction, teachers do not often receive feedback on their practice, however, it can enable teachers to gain understanding of their teaching practice, while in the midst of it. My study takes the form of a *design experiment*. The teachers plan purposeful lessons with my input. I also helped teachers to meaningfully reflect on their instruction.

I will examine the cycle of planning, teaching, and reflecting with two teachers through a design experiment (Cobb, Wood, & Yackel, 1990; Wood & Berry, 2003).
Together, we will focus on purposefully planning tasks of high cognitive demand and implementing meaningful classroom discourse to support the tasks. The focus of the research is not only on the teachers’ planning and enactment of high cognitive demand tasks, but also on the learning process of the teachers throughout this cycle. The teachers and I will consider the roles of curricula, discourse, and student understanding, while planning and enacting tasks. Additionally, I will examine how teachers can be supported throughout this process through a study of my own practice in the dual role of a researcher and teacher educator. Design experiment methodology will allow me to play an interactive role, as both a teacher educator and a researcher. As a teacher educator, I will work with the teachers to implement mathematical and pedagogical knowledge learned through the professional development program and, as a researcher, I will describe this process and identify ways that teachers can use curricula, student understanding, and discourse to implement meaningful lessons.

**Research Questions**

I will use design experiment methodology (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Peer Group, 2006) to study the cycle of planning and teaching mathematics in meaningful ways to primary grades students, through an examination of two teachers’ selection and implementation of tasks and efforts at maintaining or raising the cognitive demand of tasks through mathematical discourse. Additionally, I will address the issues that teachers overcome and what teachers learn, in order to create meaningful mathematical learning opportunities through tasks of high cognitive demand.
I will address the following questions through my research:

1. **How does a mathematics teacher transfer learning from professional development to classroom practices?**
   
   a. How does a researcher provide support to a teacher, when planning for learning that requires high cognitive demand?
   
   b. How does a researcher provide support to a teacher when enacting lessons of high cognitive demand?

2. **How does a teacher plan for mathematical learning that requires high cognitive demand?**
   
   a. What is the role of the curriculum in planning for high cognitive demand tasks?
   
   b. How can a teacher plan for discourse that supports high cognitive demand?
   
   c. How can a teacher use their knowledge of student understanding to plan subsequent learning?

3. **What happens to the cognitive demand as lesson plans are enacted?**
   
   a. How might the cognitive demand of a task change during the enactment?
   
   b. How does a teacher engage students in mathematical discourse to support high cognitive demand?
   
   c. How does a teacher use students’ understanding in the act of teaching?
Definitions

*Cognitive demand*—is the variation in the kind of thinking required of students engaging in a mathematical task. There are four levels of cognitive demand (a) memorization; (b) procedures without connections; (c) procedures with connections; and (d) doing mathematics. The level of cognitive demand determines the depth of understanding students will have the opportunity to learn (Stein, Smith, Henningsen, & Silver, 2009).

*Conceptual learning*—is “the knowledge of the underlying structure of math—the relationships and interconnections of ideas that explain and give meaning to procedures,” (Eisenhart et al., 1993, p. 9). Conceptual learning often involves the use of manipulatives, includes a discussion about the links between and among mathematical topics, and is intended to help students understand the mathematical processes related to a topic (Eisenhart et al., 1993).

*Classroom Discourse*—is a discussion among the teacher and her students or among students, around a mathematical concept, where students share multiple solution methods, compare, contrast, and evaluate each method to support sense making and conceptual understanding (Smith & Stein, 2011; Varol & Farran, 2006). Effective classroom discourse brings students’ misunderstandings to the surface, constructs understanding, pushes students to use reasoning and sense-making skills, motivates students, and teaches students to communicate about mathematics (Chapin et al., 2009). Within this study, the terms discourse and discussion will be used interchangeably.
Design experiment—is a research methodology in which the purpose is for the researcher to work with the teacher and collaboratively investigate the learning process (Cobb et al., 1990). In a mathematical design experiment, a specific mathematical learning process is targeted and theories are developed around that process. The work of a teacher is complex and has many elements (Wood & Berry, 2003). In a design experiment, the researcher focuses on how the multiple elements work together to support the teaching and learning process (Cobb et al., 2003). An important factor in a design experiment is the continual cycle of revision that occurs. The teacher and researcher work together to examine a chosen learning process or content topic (Cobb et al., 2003; Peer Group, 2006).

Enacted Instruction—is how teachers pose a mathematical task, how teachers work with students on a task, and how teachers engage in discourse around the task (Stein et al., 2009). Enacted instruction is the teaching and learning that actually take place during math class, including the active role of teaching and learning by both the teacher and students (Remillard, 2005).

Learning Trajectories—are a progression of students' thinking as they learn a mathematical topic (Clements & Sarama, 2009; Sarama & Clements, 2009). Learning trajectories serve as paths to help students develop skills and conceptual understanding that are mathematically sound and coherent. Learning trajectories have three parts (a) a learning goal; (b) a developmental progression; and (c) instruction that can help students move through the progression. Learning trajectories frame education as moving students
through levels of understanding, rather than moving through the curriculum (Sarama & Clements, 2009).

*Mathematical tasks*—are classroom activities, projects, questions, problems, constructions, applications or exercises focused on a specific mathematical concept (Cai, Moyer, Nie, & Wang, 2010; Stein et al., 1996). Mathematical tasks shape what students learn, how students think about mathematics, how students develop their understanding of mathematical concepts, and how students make sense of mathematics. A task is represented in multiple ways, including as it appears in the curriculum, how a teacher adapts the task, and how the task is carried out during instruction (Stein et al., 1996). Tasks can change through planning and implementation.

*Planned Instruction*—is how a teacher intends to implement a mathematical task, including consideration of the mathematical content, appropriate representations, materials, and assessment. Traditional planning includes what a teacher writes in her planbook, which can be page numbers, a list of activities, or even a few notes about the concepts to be covered (McCutcheon, 1980). Since not all of a teacher’s planned curriculum is written in her planbook (McCutcheon, 1980), this definition also includes the unwritten mental plans of how a teacher intends to spend time in math class.

*Procedural learning*—is the mastery of computational skills and memorization of procedures (Reys, Lindquist, Lambdin, & Smith, 2009). Procedural learning requires students to know how to use mathematical symbols, rules, and algorithms to complete mathematical problems (Eisenhart et al., 1993). However, what is lacking in procedural
learning is the understanding of the mathematical processes underlying the rules and algorithms in mathematics.

Reform curricula—are curricula based on principles and standards written by the National Council of Teachers of Mathematics (NCTM), with a goal of conceptual learning through meaningful mathematical tasks (Carpenter et al., 1996). Reform curricula focuses on conceptual understanding and big ideas within each grade level. Reform curricula are also referred to as standards-based curricula (Stein & Kim, 2009).
Chapter 2

Review of Literature Related to Mathematical Tasks

Teaching elementary mathematics for deep and meaningful understanding is a complex undertaking that requires specialized knowledge of both mathematics and pedagogy (Karp et al., 2011). Teachers may attend professional development sessions to increase their pedagogical knowledge and become more responsive to students. As students attempt to conceptualize mathematics, teachers are responsive to students by choosing worthwhile and meaningful mathematical tasks that challenge and build on students’ mathematical understanding.

An on-going discussion between the teacher and the students, and among the students themselves, can emerge as a main element of effective classroom instructional practices that enhance student understanding and is a means of being responsive to students’ needs. Students who are challenged to think through a problem and consider multiple solution paths for a problem have significantly different mathematical experiences than students who are asked to memorize facts and formulas. The Common Core State Standards Initiative (CCSSI) (2011) states, “One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student’s mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from.”

In this study I examined tasks and task selection, cognitive demand, and classroom discourse in the context of addition and subtraction instruction in first and second grades. Additionally, I examined the role of curriculum, teacher knowledge, and
student understanding, in the context of first and second grade mathematics. As well, I examine the role of a researcher in the design experiment process.

**Conceptual Learning and Teaching**

The National Research Council (NRC) (2001) outlined their research conclusions about what makes a student mathematically proficient in the following five strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Although specific proficiencies have been teased out into strands, the strands are not independent but compose a complex whole. The teacher’s role in the development of students’ mathematical proficiency necessitates planning, enactment, and continual improvement of instruction.

If a goal of teaching mathematics is conceptual understanding, then teacher educators and researchers must work together to educate practicing and pre-service teachers to teach in a way that allows students to gain a deep conceptual understanding of mathematics. Teachers must possess three crucial types of knowledge in order to best instruct students (a) knowledge of mathematics; (b) knowledge of students; and (c) knowledge of instructional practices (NRC, 2001). Teachers themselves must have conceptual knowledge of mathematics, knowing connections within and between concepts (Manouchehri & Goodman, 1998). Teachers must have a clear vision of the goals of the mathematics they are teaching and what it means for students to be proficient in the context of different mathematical concepts. Teachers must use their own mathematical knowledge flexibly through their use of instructional materials and representations. Additionally, teachers must plan and deliver instruction that represents
the mathematical concepts in ways that are accessible to students, and then assess what their students are learning.

Research by Sarama and Clements (2009) shows that the deeper conceptual understanding students have, the faster and more accurately they can solve problems. Students with deep conceptual understanding are also able to use their mathematical knowledge in a flexible way to solve different types of problems. In fact, a recent meta-analysis provided strong empirical evidence to support the claim that teaching mathematics conceptually improves student achievement (Rakes, Valentine, McGatha, & Ronau, 2010). Additionally, NRC (2001) stated that, “learning with understanding is more powerful than simply memorizing because the organization improves retention, promotes fluency, and facilitates learning related material” (p. 118). The conceptual interrelatedness of mathematical concepts is more meaningful than simply memorizing isolated facts and procedures. If a student forgets a procedure they were taught without connections, their ability to solve that type of problem later is unlikely. However, if a mathematical procedure that is connected to other known concepts is forgotten, it can be reconstructed based on related knowledge.

Based on the curriculum that a teacher uses, the teacher must make important decisions about how the concepts will be taught. “Conceptual learning refers to knowledge of the underlying structure of mathematics—the relationships and interconnections of ideas that explain and give meaning to mathematical procedures” (Eisenhart et al., 1993, p. 9). In contrast, procedural knowledge is the usual process used or standard algorithm used to get to a solution, but lacks important connections
Students must learn mathematics in a deep, meaningful way, which is done through an emphasis on conceptual rather than procedural learning. Students with conceptual knowledge of mathematics are able to connect new ideas to prior knowledge and see relationships among concepts (Stein & Kim, 2009). Both conceptual and procedural knowledge are essential to deep mathematical understanding, therefore a strong mathematical education must include both (Eisenhart et al., 1993; Stein et al., 2009).

Unfortunately, there is evidence that teachers of mathematics focus mainly on procedural knowledge, specifically rote memorization, rather than conceptual knowledge, (Eisenhart et al., 1993) which does not give students ample opportunity to actively engage in mathematical activities. Boesen, Lithner, and Palm (2010) reviewed extensive research by NCTM and reported that a main cause of low mathematics achievement is students’ reliance on rote, superficial memorization, instead of deep, meaningful understanding. A teacher’s choice of instructional strategies and activities plays a major role in whether students’ knowledge will be deep or superficial. In fact, whatever skills students spend time working on in class sets the boundary for whatever knowledge students will have the opportunity to acquire. That boundary is based on what mathematical concepts and mathematical practices teachers implement during classroom instruction.

Student learning depends partially on the mathematical knowledge of the teacher (NRC, 2001; Sherin & Drake, 2009). It is unlikely that the teacher will aid students in making deep, meaningful connections within the mathematics if teachers do not
understand the mathematics themselves. The knowledge required to teach mathematics well is specialized and also includes mathematical pedagogy (Ball et al., 2008). Teachers must also have considerable knowledge of how all students learn and which strategies are most effective for each student.

One major factor in the ability to teach conceptually is the appropriateness of the tasks that the teacher chooses to use in the classroom (NRC, 2001; Simon, Tzur, Heinz, & Kinzel, 2004). Diezmann, Watters, and English (2001) specifically studied seven- and eight-year-old students that were engaging in mathematical tasks and stressed the importance of representations and problem solving in daily life. Fennema et al. (1996) also studied primary grades students, with a focus on the changes in pedagogical beliefs of the teachers. They found that when teachers understood children’s mathematical thinking, they made fundamental changes in their teaching, as called for by reform recommendations.

Eisenhart et al. (1993) specifically studied a sixth grade student teacher and found that the teacher offered students more opportunities to learn procedurally than conceptually because of her limited knowledge of mathematics. The authors suggested that nationally there should be an increase in opportunities for teachers to learn how to teach conceptually, through observing conceptual teaching, writing lesson plans around conceptual knowledge, and receiving feedback on lessons taught. My study gave teachers opportunities to learn how to teach conceptually through task planning, observation, and feedback. The ethnographic study of Manochehri and Goodman (1998) examined 66 middle school teachers’ mathematical knowledge and pedagogical beliefs, as well as
barriers to developing deep conceptual understanding in their students. There is a lack of research in the primary grades involving the use of particular teaching strategies to support conceptual learning.

**Mathematical Tasks**

The definition of mathematical task is a classroom activity, project, question, problem, construction, application or exercise in which the purpose is to focus the students’ learning on a specific mathematical concept (Cai et al., 2010; Stein et al., 1996). Although the importance of the choice of task can be overlooked, it is crucial. Lappan (1993) stated:

> No other decision that teachers make has a greater impact on students’ opportunity to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages the students in studying mathematics. Here the teacher is the architect, the designer of the curriculum. (p. 524)

Not all mathematical tasks allow the same opportunities for learning (Stein et al., 2009). Some mathematical tasks focus on memorization, while richer tasks can engage students in reasoning and flexible problem solving. It is crucial that teachers match the tasks with the goal for learning. For example, if the goal of the lesson is procedural fluency, then tasks that encourage students to work quickly and efficiently are most appropriate. However, if the goal of the lesson is to deeply understand a concept, then the tasks that students engage in should activate problem solving and reasoning skills. Garrison (2011) noted that the primary characteristics of teachers related to choosing and enacting tasks of high cognitive demand are teacher experience, teacher knowledge, and student beliefs.
For a task to be mathematically worthwhile it should provide a challenge to students such that hard work is encouraged and explicitly linked to mathematical concepts that require deep ways of thinking (Cai et al., 2010; Hiebert, 2003). Tasks that are too easy for students do not encourage growth, while tasks that are too difficult for students often cause frustration (Van de Walle, 2003). A teacher must know the skills and knowledge that their students possess so they are able to alter or create tasks at a level appropriate for all students. “Even the best curriculum materials or resource books can only suggest tasks that may be effective for a particular grade level,” (Van de Walle, 2003, p. 68). Therefore, teachers must be cognizant of student knowledge so that each mathematical task creates an opportunity for growth of mathematical understanding by all students.

In order for students to engage in worthwhile mathematical tasks, the teacher must carefully select meaningful tasks with which students will interact. In fact, the process of selecting, altering, and constructing mathematical tasks is a crucial component of the teacher’s role in developing the curriculum (Remillard, 1999). Teachers’ choice of mathematical tasks for their students are influenced by several factors, namely their own ideas about mathematics and pedagogy, their students, the curriculum, and available resources (Remillard, 1999).

The tasks that a teacher chooses to implement convey a message to the students about what doing mathematics means and influences the students’ ways of processing information. Students’ thinking about mathematics can be limited or broadened, based on the tasks they are asked to solve. For example, when students engage in high-level
reasoning, such as problem solving, it involves increased ambiguity and increased risk which forces students to think deeply; whereas simple arithmetic problems allow superficial thinking (Henningsen & Stein, 1997). Teachers should choose tasks that allow students to obtain the educational outcomes aligned with curricular goals and societal expectations (Jaworski, 2009).

It is crucial for teachers to maintain students’ focus on the mathematics. Although additional resources, such as the use of children’s literature or manipulatives, can enhance a lesson, teachers must always be aware of what mathematical knowledge students should gain from each lesson and how the additional resources may or may not enhance learning. Research suggests that good mathematical tasks have a context within the students’ environment, are within students’ zone of proximal development, pique students’ curiosity, foster sense making and problem solving, and encourage multiple solution paths (NCTM, 2000; NRC, 2001).

Since every classroom has students with a range of mathematical ability, teachers must choose mathematical tasks that allow for multiple entry points. The most difficult tasks for teachers to implement are tasks that require students to use reasoning skills (Smith et al., 2008), however, those tasks are often the most meaningful for students. It can be difficult for teachers to anticipate student reasoning on difficult tasks. The lack of a clear solution path allows students to use their own method of solving and may lead to a meaningful classroom discussion, thereby, making the task worthwhile.

What students learn is related to the tasks that teachers choose and how the task is implemented. Students are exposed to and therefore have the opportunity to learn the
mathematical concepts and mathematical practices associated with the tasks put before them. Conversely, students cannot learn mathematical concepts and mathematical practices that are not presented to them (Boston & Smith, 2009). Students only have the opportunity to gain the skills embedded in the tasks the teacher chooses. The way in which a teacher poses a mathematical task either opens or closes the students’ opportunity to learn deep and meaningful mathematics. Teachers are challenged to pose mathematical tasks to students in ways such that the students are asked to think deeply about concepts and make connections to other concepts (Crespo, 2003).

Planning is a crucial part of building a classroom with tasks of high cognitive demand. Simply choosing the mathematical tasks does not predetermine the learning outcome, but rather it is the quality of the instruction associated with the tasks (Stylianides & Stylianides, 2007). When planning, teachers need to consider the mathematical content underlying a particular task, which representations appropriately highlight specific aspects of the mathematics, and the materials needed to help students perform a task in ways that lead to rich and meaningful learning. Planning should also give teachers the opportunity to reflect on mathematical concepts and students’ thinking and learning (NRC, 2001). Typical planning is seldom elaborate (O’Donnell & Taylor, 2007) or a map of student learning (Lampert, 2001). Unfortunately, some teachers plan by simply writing the page number from the textbook or lesson number in their plan book (O’Donnell & Taylor, 2007).

The way in which a task is planned may or may not be the way in which the task is actually implemented (Henningsen & Stein, 1997; Remillard, 1999; Stein et al., 2009).
Henningsen and Stein (1997) specifically studied three teachers and examined how classroom factors can shape engagement in mathematical tasks, whereas Remillard (1999) studied two teachers during their first year of reform curriculum to examine how the curriculum contributes to teaching. Not included in current research is how teachers actually go about choosing tasks for students to engage in. My study includes the use of curriculum, as well as how teachers go about choosing and implementing mathematical tasks.

As in most classrooms, the planned or intended curriculum does not always match the enacted curriculum. How a mathematical task is implemented may vary from the plan that the teacher intended. Teachers must alter instruction based on how students enact the task, adjusting the task or connecting to the understanding of the students, to accomplish the goal of the lesson. The enacted curriculum includes how teachers pose tasks, work on tasks with students, and engage students in discourse around the tasks (Stein et al., 2009). How a teacher manages the particulars of enacted curriculum determines students’ opportunities to learn meaningful mathematics. Altogether, the choice, the cognitive demand, and the enactment of the task, affect how much students learn and the quality of their learning (Charalambous, 2010).

Based on the findings of the Trends in International Mathematics and Science Study (TIMSS) (Institute of Education Sciences, 2007), Resnick (2006) reported that teachers in the highest achieving countries did not plan to implement a higher percentage of meaningful tasks than did the United States, however, in the United States, the enacted tasks were rarely as meaningful as they were intended. In the United States, the enacted
tasks were often reduced to procedural computational exercises. Tasks, even potentially mathematically rich ones, that are enacted with a focus on correct answers and a particular set of prescribed steps to solving result in lower levels of engagement and, therefore, often lower levels of retention of the mathematical concept. Conversely, tasks that are enacted with a focus on reasoning, where students justify and explain their work in ways that make sense to them, are likely to support deep engagement in mathematical concepts (Strayer & Brown, 2012). Resnick (2006) also reported that other research found that when meaningful tasks were planned and enacted as such, students performed better on assessments of problem solving and reasoning. The enactment of the task is what actually determines where and how student thinking and learning occurs (Charalambous, 2010).

One crucial piece of planning that is often left out of the process is how to address the specific needs of all students within a class. Teachers should consider what students already know and how to aid them in moving their learning along, both forward and deeper. During planning, teachers should solve the chosen tasks in as many ways as they possibly can, which allows the teacher to anticipate students’ varied ideas and prepare to aid students’ understanding (Stein et al., 2008). Grant et al. (2009) stated, “Teachers must be knowledgeable about the kinds of thinking that will likely be elicited and the mathematical rationale underlying that thinking” (p. 115). When teachers understand what strategies students may use while working on a task, they are better able to support student learning while teaching (Doerr, 2006). Additionally, teachers who have solved
tasks in multiple ways are better able to help students make judgments about, compare, and discuss the efficiency of strategies (Stein et al., 2008).

Even carefully thought out plans can go awry when enacted. Stein and Smith (1998) created a representation of how mathematics tasks are intended and enacted, highlighting the ways that a teacher can intervene on the task (Silver, Ghousseini, Charalambous, & Mills, 2009). This model is a basis of my study. In this model, three phases lead to students’ conceptions of mathematical concepts. Phase One is the examination of the tasks as they appear in the curriculum. For example, a problem may appear in the curriculum with a context that students may not understand, which could be altered by the teacher in Phase Two. In my study, I examined the textbook in Phase One. Phase Two is how the tasks are planned and set up by the teacher or how a teacher takes the curriculum from the textbook to implementation. A teacher may change the words or numbers to make the problems more accessible to students. I examined Phase Two through lesson planning. Phase Three is how the students carry out the tasks, which includes discourse around the mathematical tasks. In my study, I addressed Phase Three by examining classroom discourse and student understanding. Figure 2 is Smith and Stein’s (1998) model of the enactment of mathematical tasks.

Just as the research of Stein et al., (1996) analyzed 144 tasks from reform curricula for task features and cognitive demand, I analyzed tasks from the district curriculum materials. Additionally, Boston and Smith (2009), Crespo (2003), Diezmann et al. (2001), and Doerr (2006) all studied how teachers select tasks from their
curriculum, as I did, to gain an understanding of the materials with which teachers work. Boston and Smith (2009) examined secondary teachers before, during, and after involvement in professional development. Doerr (2006) also focused on secondary mathematical teaching and learning, but specifically student understanding, including how teachers listened to their students and responded to student knowledge. Crespo (2003) focused on progress pre-service teachers made posing tasks to students. Diezmann et al. (2001) concentrated mostly on seven- and eight-year-old students that were engaged in mathematical tasks that required students to investigate. Each of the mentioned studies included elements of mathematics teaching and learning that I incorporated into my study, such as examining curricula and student understanding.

Garrison (2011) and Imm and Stylianou (2012) both focused their studies on the cognitive demand of tasks for middle school students. Garrison (2011) examined the relationships between teacher, context, tasks, and cognitive demand, whereas Imm and Stylianou (2012) studied the relationships between discourse, tasks, and cognitive demand. Many current studies do not involve students from primary grades, as mine
does, rather they focus on upper elementary or middle school students. Additionally, there are a lack of research studies examining the cognitive demand of a task, from its appearance in the textbook through its enactment in a mathematics lesson.

**Cognitive Demand**

Variation in the difficulty in mathematical tasks is apparent because different tasks require different levels and kinds of student thinking (Stein et al., 2009). This variation in the types of thinking necessary to engage in a mathematical task is referred to as cognitive demand. The cognitive demand of a task can change at multiple times during a lesson, based on the implementation and enactment of the mathematical task (Henningsen & Stein, 1997; Resnick, 2006; Stein et al., 1996; Stein et al., 2009). The studies by Henningsen and Stein (1997) and Stein et al. (1996) both examined factors of tasks with high cognitive demand.

Stein et al. (2009) defined levels of cognitive demand as memorization, procedures without connections, procedures with connections, and doing mathematics. Memorization and procedures without connections are both categorized as having lower-level cognitive demand, while procedures with connections and doing mathematics are categorized as higher-level cognitive demand. Figure 3 is a snapshot of the levels of cognitive demand Stein et al. (2009). I added the column of examples to connect the levels of cognitive demand and first and second grade addition and subtraction content. Cognitive demand can be defined at each phase of the progression of mathematical tasks.
<table>
<thead>
<tr>
<th>Level of Cognitive Demand</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Memorization</td>
<td>Reproduction or memorization of facts, rules, formulas or definitions</td>
<td>Add.</td>
</tr>
<tr>
<td></td>
<td>No procedures</td>
<td>$1 + 2 = $</td>
</tr>
<tr>
<td></td>
<td>Exact reproductions, not ambiguous</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No connection to concepts or meaning that underlie the fact, rule, formula or definition</td>
<td></td>
</tr>
<tr>
<td>Procedures without connections</td>
<td>Algorithmic</td>
<td>$2 + _ = 5$</td>
</tr>
<tr>
<td></td>
<td>Little ambiguity about what needs to be done</td>
<td>$_ + 3 = 5$</td>
</tr>
<tr>
<td></td>
<td>No connections to the concepts of meaning that underlie the procedure</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Focused on producing correct answers, rather than mathematical understanding</td>
<td></td>
</tr>
<tr>
<td></td>
<td>No required explanations</td>
<td></td>
</tr>
<tr>
<td>Procedures with connections</td>
<td>Focused on procedure to develop deeper understanding of mathematical concepts and ideas</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Suggest solution paths that are broad but with a close connection to underlying ideas</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Usually represented in multiple ways and makes connections among representations</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Requires cognitive effort</td>
<td></td>
</tr>
<tr>
<td>Doing mathematics</td>
<td>Requires complex and nonalgorithmic thinking</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Requires students to explore and understand the mathematical concepts, processes, and relationships</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Demands self-monitoring or self-regulation of own cognitive process</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Requires students to analyze the task constraints and limits on solutions or strategies</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Could produce anxiety in students due to the unpredictable nature of the solution process</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 3. Levels of cognitive demand.*
Memorization tasks involve learning or reproducing facts, rules, formulas, or definitions. Tasks that require memorization have no connection to the mathematical meaning that underlies the concept. The directions for memorization tasks are specific and directly stated so that students create a reproduction of the material (Stein et al., 2009).

Procedures without connections often involve an algorithm. The cognitive demand called for is limited, even when the task is successfully completed. Just as with memorization, the procedure is not connected to the mathematical meaning that underlies the concept and the directions are straightforward. The focus of the task is on producing correct answers, rather than an emphasis on understanding the mathematical process that underlies the concept. Additionally, learning procedures without connections does not require the student to construct explanations for why a particular procedure was performed (Stein et al., 2009).

Procedures with connections are focused on the procedure used to solve a problem, but are connected to the mathematical concepts that are included in the task. Directions are broad and general, allowing students to interpret a math problem on their own, but directions may suggest a strategy for solving. Connections are made between multiple representations, thereby offering opportunities for deeper mathematical meaning. Effort is required when solving the task, which engages students in the meaning that underlies the mathematical concepts (Stein et al., 2009).

Doing mathematics is the highest level of cognitive demand, which requires complex and non-algorithmic thinking on the part of the students. Students must
understand the mathematical concept, as well as the processes and relationships involved in solving. Students may experience disequilibrium while doing mathematics, which encourages growth (Stein et al., 2009). Tasks with a high level of cognitive demand incorporate CCSSM’s mathematical practices and NCTM’s process standards. High cognitive demand tasks require students to use reasoning skills, representations, make connections, and communicate about their cognitive processes. Additionally, mathematical practices are interwoven within how students must do mathematics in order to sustain a task of high cognitive demand, such as construct a viable argument and reason abstractly and quantitatively.

The level of cognitive demand of a task is the foundation for students’ learning opportunities (Garrison, 2011) because it determines what the students have the opportunity to learn (Stein et al., 2009). However, while students are working on the task, the demand can rise, lower, or remain the same based on a variety of factors, such as questions asked and answered by the students or the teacher. The greatest gains in student learning occur when teachers assign tasks of high cognitive demand and the cognitive demand remains high throughout the enactment of the task (Boston & Smith, 2009; Garrison, 2011; Kotsopoulos, Lee, & Heide, 2011).

Several factors related to teacher instruction were identified by Stein et al. (2009) when tasks of high cognitive demand were enacted and the cognitive demand remained high. Factors included scaffolding student thinking and reasoning, providing students with a means of metacognition, sustaining press for justification, and opportunities to make conceptual connections. In contrast, Stein et al. (2009) identified factors that
correlate to lowering the cognitive demand of a task. Those factors include the teacher making a task procedural when it became problematic for students, an overemphasis on correct answers, and unclear or low expectations. I analyzed my data based on the aforementioned factors.

The study by Boston and Smith (2009) analyzed task selection and implementation before, during, and after secondary teachers participated in professional development, whereas I only examined post professional development. Garrison (2011) also examined the task choice, but in contrast, she framed her study around middle school teachers and the relationships among tasks, cognitive demand, the teacher, and the classroom context. Kotsopoulos et al. (2011) also examined middle school teachers, specifically the cognitive demand of classwork and homework.

Cognitive demand (Stein et al., 2009) is a relatively recent idea in the realm of mathematics education and thus minimal research has been done on the topic. Very little research is focused on the cognitive demand of students in primary grades, as my study is. In addition, there is a lack of research based on the cognitive demand of tasks from reform curricula for primary grade students, which is a potential contribution for some of the findings of my research.

**Classroom Discourse**

One avenue for increasing the cognitive demand of a task is through classroom discourse. Vygotsky, who originated the zone of proximal development (Culatta, 2013), believed that learning is a social experience and students should interact with others to challenge and make sense of ideas (Kostelnik, Sonderman, & Whiren, 2004). Students
first experience mathematics as a spoken language, and then once students can read and understand mathematical symbols, doing mathematics expands to include a written activity. Students are faced with the challenge of connecting their understanding of written and spoken mathematics in the primary grades.

One way to connect written and spoken mathematics is through classroom discourse. The purpose of discourse is to elicit student understanding through open-ended questions and acknowledge students through their contributions. Open-ended questions engage students in the mathematical concept and allow student contributions to be validated through multiple solution paths (Piccolo, Harbaugh, Carter, Capraro, & Capraro, 2008). During classroom discourse, the teacher is not the dispenser of knowledge, but the conductor of the discussion. The teacher’s role is to encourage students to engage in the discussion while moving the mathematics along (NRC, 2001). Teachers must make sense of the ideas that students present throughout the discussion, which will help teachers develop the ability to identify opportunities to move students forward in their mathematical understanding (Walshaw & Anthony, 2008). Teachers shape the discourse happening in their classroom not only by the way they allow the discourse to unfold, but also in the tasks they choose and the learning environment that is created for students (Kysh, Thompson, & Vicinus, 2007; Varol & Farran, 2006).

CCSSI (2011) stated that elementary school students should work to construct arguments, which can be referenced with diagrams, drawings or actions. Primary grades students can also listen to classmates’ arguments and evaluate them for validity, using
precise definitions and language. Young students can also ask questions to clarify one another’s thinking.

During a classroom discussion, students should share and compare their solution methods and critique the solution methods of others, which pushes students’ actions from simply listening, to sense making (Smith & Stein, 2011; Varol & Farran, 2006). Sense making through classroom discussion is significant to student understanding because it solidifies students’ conceptions of mathematics and therefore a feature of a quality mathematics education (Walshaw & Anthony, 2008), but is often difficult for teachers to implement (Grant et al., 2009). NCTM called for this shift in authority from the teacher and the textbook to students’ mathematical reasoning, through discourse (Herbel-Eisenmann, Drake, & Cirillo, 2009). Through a verbal exchange of ideas, students make their mathematical reasoning visible to others, which becomes a shared meaning for the class. The ideas of the class then become synonymous with reasoning and sense making done by individual students (Walshaw & Anthony, 2008; Yackel & Cobb, 1996).

Classroom discourse differs from traditional teacher questioning where teachers ask questions to evaluate student knowledge through a series of preplanned questions (van Zee & Minstrell, 1997). The goals of meaningful classroom discourse are to support students’ thinking while sharing and discussing multiple solutions, which draws students’ attention to important mathematical ideas targeted through the task (Silver et al., 2009). Teachers have difficulty carrying out meaningful whole class discussions that advance the mathematical needs of all students (Stein et al., 2008). Students have differing conceptions of mathematics, which complicates the teacher’s planning and
implementation of classroom discourse. Asking students to share a solution is often the beginning of a classroom discussion, however, what happens after that determines if the discussion is deep and meaningful (Smith & Stein, 2011). The teacher’s role in classroom discourse is not to rule students’ strategies as correct or incorrect, nor to instruct the steps of a procedure, but rather to develop and build the sense making and reasoning skills of the students (Stein et al., 2008). How teachers connect student understanding to each other and to the mathematical content separate “show and tell” discussions from fruitful learning opportunities (Smith & Stein, 2011).

One primary way that teachers can support students’ deep mathematical understanding is by guiding discourse in a way that initiates a shift toward connected and conceptual understanding. Teachers often find it easy to pose questions and ask students to describe their strategies, but find it difficult to engage students in productive mathematical inquiry (Kazemi & Stipek, 2001). Teachers must have deep understanding of the mathematical content in order to help students make connections and generalizations. Teachers also must be able to understand students’ strategies for problem solving, including students’ invented strategies. When a teacher understands the mathematics and students’ strategies, they are better able to engage the students in productive mathematical discourse.

Chapin et al. (2009) outlined productive talk moves to help teachers engage students in a meaningful discussion about mathematics. Through their research, they found that over time students were able to increase their ability to provide a clear explanation, as well as use precise language. The use of talk moves brings students’
understandings into the discussions, which allows the teacher to aid those students who do not understand the mathematical concepts clearly (Chapin et al., 2009). In the course of classroom discussions, students are able to talk about their mathematical ideas with other students and bring differences in understandings to light. The goal of talk moves is not necessarily to increase the amount of classroom discourse, but rather to increase the quality of the classroom talk (Chapin et al., 2009).

Revoicing, repeating, reasoning, adding on, and waiting are specific talk moves, outlined by Chapin et al. (2009), intended to generate meaningful math talk (see Figure 4). Revoicing is when the teacher repeats what a student has said, with the aim of verifying whether or not what the teacher understood is what the student intended. Repeating is when the teacher asks students to repeat what another student said. The goal of repeating is for the class to hear the mathematical idea multiple times. Reasoning is when the teacher asks students whether or not they agree with a student’s explanation. It gives other students an opportunity to explain why their idea differs from or is similar to other students’ ideas. Adding on is when the teacher asks students to add their own ideas onto ideas that have already been explained. Waiting is when the teacher pauses after asking a question, which allows students to gather and organize their thoughts about the mathematical topic before being expected to answer.

Just as Chapin et al. (2009) described the talk moves as actions teachers can make to improve the quality of mathematical discourse, Sfard, Nesher, Streefland, Cobb, and Mason (1998) described the quality of mathematical discourse that occurs in a classroom as expounding, exploring, expressing, explaining, examining or exercising (see Figure 5).
<table>
<thead>
<tr>
<th>Talk Move</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revoicing</td>
<td>So you’re saying that it’s an odd number?</td>
</tr>
<tr>
<td>Repeating</td>
<td>Can you repeat what he just said in your own words?</td>
</tr>
<tr>
<td>Reasoning</td>
<td>Do you agree or disagree and why?</td>
</tr>
<tr>
<td>Adding On</td>
<td>Would someone like to add something more to this?</td>
</tr>
<tr>
<td>Waiting</td>
<td>Take your time… we will wait…</td>
</tr>
</tbody>
</table>

*Figure 4.* Talk moves (Chapin et al., 2009).

<table>
<thead>
<tr>
<th>Type of Discourse</th>
<th>Description</th>
<th>Center of Discourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expounding</td>
<td>Teacher in conversation with the content while students are present</td>
<td>Teacher</td>
</tr>
<tr>
<td>Explaining</td>
<td>Teacher tries to “enter” the students’ world through the content</td>
<td>Teacher</td>
</tr>
<tr>
<td>Exploring</td>
<td>Student is in conversation with the content in the presence of the teacher</td>
<td>Student</td>
</tr>
<tr>
<td>Examining</td>
<td>Focus is on assessment of student knowledge</td>
<td>Student</td>
</tr>
<tr>
<td>Expressing</td>
<td>Focus is on the content, students engaged in content</td>
<td>Content</td>
</tr>
<tr>
<td>Exercising</td>
<td>Students repeat actions until automaticity is created</td>
<td>Student</td>
</tr>
</tbody>
</table>

*Figure 5.* Types of discourse (Sfard et al., 1998).

Expounding is characterized by no interaction between students and the teacher. The teacher engages in a lecture-like conversation with the mathematical content in the presence of the students. Explaining occurs when the teacher attempts to interact with the students by means of the mathematical content. When a misunderstanding arises, it is likely that the teacher will transition into expounding. Exploring is when the students are interacting with the mathematical content in the presence of the teacher. The students
would likely be guided by the text or other materials designed to make sense of the mathematical content. Examining is an assessment of student knowledge such that the student is attempting to validate their conceptions. This is not typical paper and pencil assessment, but rather an on-going discussion. Expressing is characterized by the content “bursting forth” to help students make deep mathematical connections and strengthen their conceptions of mathematics, which may look like brainstorming. Exercising is when the student engages in rehearsal to gain mastery. The student performs a procedure repeatedly until it is made automatic.

Much of the research on classroom discourse focuses on students in middle school and high school (Herbel-Eisenmann, Drake, & Cirillo, 2008; Imm & Stylianou, 2012; Piccolo et al., 2008; van Zee & Minstrell, 1997). The study by Herbel-Eisenmann et al. (2008) was situated around teachers who did action research in their classrooms based on inquiries into understanding discourse. Imm and Stylianou (2012) and Piccolo et al. (2008) all studied the role of mathematical discourse in the classroom and how that effected learning. Similarly, the research of van Zee and Mistrell (1997) was centered on discourse, specifically teacher questioning strategies. My study encompasses aspects of discourse from of each of these research projects.

In contrast, the studies of Cobb, Boufi, McClain, and Whitenack (1997) and Hufferd-Ackles, Fuson, and Sherin (2004) focused on studies of discourse in elementary classrooms, as mine did. Cobb et al. (1997) studied the relationship between mathematical discourse and mathematical development in first grade students and Hufferd-Ackles et al. (2004) studied how a teacher can establish a classroom with
mathematical discourse with third grade students. In general, studies of classroom discourse focused mainly on implementation and effects of classroom discourse. There is a lack of attention to what types of discourse are actually happening in classroom and how teachers can use talk moves to enhance classroom discussions. Minimal research exists as to how classroom discourse can raise lessons of low cognitive demand, or maintain lessons with high cognitive demand, as I examined.

**Reform Curriculum**

In many countries, including the United States, traditional mathematics education consists of memorization of calculation procedures and lacks emphasis on the base-ten place value system, subsequently resulting in long term errors in calculations (Fuson et al., 1997). Cooney (2009) goes farther by saying that traditional curriculum historically features teacher-centered talk, where teachers only cover homework, explain new material, and assign homework. Consequently, if traditional mathematics education is unable to meet the needs of students, and ultimately society, then it must be altered so that less emphasis is placed on procedures, and more emphasis is placed on deep understanding of mathematical concepts generated and sustained through discussion about rich mathematical tasks.

TIMSS shows the unequal achievement in mathematics, where the United States was the lowest performing country (TIMSS & PIRLS International Study Center, 2014). Based on results of TIMSS (IES, 2007) and the NAEP (National Center for Education Statistics, 2011), Resnik (2006) concluded that there are two major challenges for educators: (a) raising the scores of the low achievers in order to raise the achievement of
all students; and (b) preparing high achievers to be better prepared to be mathematical leaders of the future.

Due to the underperformance of students that has occurred in the United States (Battista, 1994), the mathematics education community called for a new and reformed curriculum rooted in the goals of NCTM and aligned with its standard, to address the problem. Curriculum is the underlying framework that directs mathematical instruction and provides a basic outline of potential educational opportunities in the classroom (Kolovou, van den Heuvel, & Bakker, 2009). The goal of reform curricula is to give students the opportunity to learn mathematics conceptually and engage in meaningful mathematical tasks, by moving teachers beyond using traditional teaching methods (Carpenter et al., 1996; Stein & Kim, 2009). Reform curricula, especially curricula whose development has been funded by the National Science Foundation (NSF), aim to develop students’ mathematical reasoning by organizing curricular materials around important mathematical topics (McDuffie & Mather, 2009; NSF, 2013).

Since conceptual understanding is critically important to begin developing in primary grades, reform curricula have begun to be widely used for young students (Carpenter et al., 1996; Resnick, 2006; Sarama & Clements, 2009). Reform curricula offers students opportunities to solve different types of problems in context, not simply number problems, such as 5+2, with no real-world context. Mathematical problems given in context encourage students to be flexible in their mathematical thinking and develop sense-making skills (NCTM, 2000) because students must consider the reasonableness of their answer. Fuson et al. (1997) noted that as more schools implement reform curricula
as their primary source of information (Bryant et al., 2008; Jitendra, Sczesniak, & Deatline-Buchman, 2005; Kolovou et al., 2009), teachers and students use a wider range of strategies to solve addition and subtraction problems.

Park (2011) compared traditional and reform texts for Algebra. She found that traditional texts mostly focused on procedural knowledge, whereas reform curricula challenged students to reason and communicate about mathematics. Reform curricula often emphasize group work and communication. Park also found that reform curricula include a higher percentage of problems with higher demand, than do traditional curricula. Park’s study focused on middle school textbooks, whereas I only examined select units from first and second grade *Math Expressions* (Fuson, 2011) curricula.

Reform curricula differ from traditional curricula in both mathematical topics and pedagogical approaches (Sherin & Drake, 2009). Sherin and Drake (2009) focused on a study of the interactions between the teacher and students and the teacher and the text, for ten elementary teachers during their first year using reform textbooks. A change in curricula, especially from traditional to reform, often requires a pedagogical shift, which requires that teachers change the way they teach the mathematical concepts, and in turn change how students view learning mathematics (Moyer, Cai, Wang, & Nie, 2011). Sherin and Drake (2009) also noted that when using reform curricula, teachers are expected to adapt instruction while listening to students’ ideas, thus determining the direction of the lesson in the act of teaching. This ability to adapt to students’ ideas during instruction requires that teachers remain flexible in their thinking about student understanding and mathematical ideas throughout the lesson. Teachers can understand
and adapt the lesson to students’ ideas through classroom discourse. The opportunity for students to share their strategies also allows teachers to understand students’ conceptions of mathematics.

Various studies done by multiple researchers have found that the crucial factor in mathematical instruction is the teacher. How teachers interpret and use curriculum materials is an important factor in the overall effectiveness of reform curricula (Ball & Cohen, 1996; Cai et al., 2010; Manouchehri & Goodman, 1998; Remillard, 1999, 2005).

In Remillard’s (2005) review of the past 25 years of research in mathematics curriculum, she concluded the importance of the teacher. Cai et al. (2010) examined the instructional differences when teachers used reform curricula versus traditional curricula and found that reform curricula allowed for more conceptual understanding than did traditional curricula. Manouchehri and Goodman (1998) only studied teachers using reform curricula and found mathematical content knowledge, pedagogical practices, and personal beliefs about teaching and learning influence how a teacher implements the curriculum. They also found that the major problems teachers face in curriculum implementation are lack of time for planning, lack of conceptual understanding, and lack of professional support. Remillard (1999) also focused on teachers using reform curricula, however, the subjects were two elementary teachers and their relationship to the reform text. Through the use of curriculum maps, she found that, “students’ encounters with new curriculum are mediated by a variety of teachers’ decisions” (p. 339). Remillard also noted that curriculum developers need to attend to the teachers’ role in curriculum implementation. Ball and Cohen (1996) also explored the role of the
teacher in the use of reform curriculum and suggested that better curriculum could be
designed to help teachers be more thoughtful and effective through lesson enactment.

Much research in mathematics education has been done on the subject of reform
curricula, focusing on textbooks. The studies of Cai et al. (2010), Moyer et al. (2011),
Bryant et al. (2008), Kolovou et al. (2009), Remillard (1999), Remillard (2000),
Remillard (2005), and Zhou and Peverly (2005) examined reform curricula in general,
and compared reform and traditional curriculum. In contrast, my study is not comparing
curricula, rather I examined a specific set of reform curriculum materials. Both Kolovou
et al. (2009) and Zhou and Peverly (2005) did not specifically focus on a grade level, but
rather analyzed broad use of reform curricula in the Netherlands and China, respectively.
The studies of Park (2011) and Garrison, (2011) focused on the cognitive demand of
problems in reform curricula. Park (2011) specifically compared the tasks in reform and
traditional curricula and Garrison (2011) studied middle school curriculum and the
relationship to teachers, tasks, and cognitive demand.

Additionally, some studies of reform curricula focus on the teacher and the
teacher’s implementation of instructional resources. The studies of Cai et al. (2010),
Moyer et al. (2011) both focused on middle school students and the impact of curriculum
on teacher instruction. In contrast, the study of Bryant et al. (2008) focused on students in
grades Kindergarten through second grade, examining lessons to determine critical
features of instruction. The studies of Remillard (1999) and Sherin and Drake (2009)
focused specifically on teachers who were using reform curricula for the first time.
Additionally, the studies of Charalambous (2010) and Manouchehri and Goodman (1998)
studied mathematical knowledge for teaching (MKT) and how that relates to the implementation of reform curricula. Charalambous (2010) compared lessons of two teachers with differences in their mathematical knowledge, whereas Manouchehri and Goodman (1998) studied the relationship between mathematical knowledge and pedagogical beliefs among 66 middle school teachers. The studies by Sherin and Drake (2009) and Drake and Sherin (2006) both considered how teachers interact and adapt reform curricula. My study incorporates both a look at a textbook and teacher implementation. While many research studies involve middle school students, missing from the current research are studies of the implementation of reform curricula in the primary grades.

**Learning Trajectories**

Of the many methods to solve problems, some methods are more advanced than others, that is, they are further along on a learning trajectory. Learning trajectories describe children’s thinking as they learn mathematical concepts and grow into abstract thinkers. Each trajectory has three parts, the goal, the developmental progression, and instruction (Clements & Sarama, 2009; Sarama & Clements, 2009). Due to the fact that learning trajectories express a spectrum of development, using them affords teachers the opportunity to identify ways to reach all students.

Learning trajectories are a framework for teachers to understand their students’ learning of mathematics. In order for a teacher to enact quality mathematical lessons they must understand both the mathematics to be taught and the learning process of children. Learning trajectories can enable teachers to plan effective lessons for students with
different conceptions of mathematics because they include multiple points of understanding (Clements, Sarama, Spitler, Lange, & Wolfe, 2011).

For addition and subtraction, the developmental progression of understanding is the following: Pre-Explicit, Nonverbal, Small Number, Find Result, Make It N, Find Change, Counting Strategies, Part Whole, Numbers in Numbers, Deriver, and Problem Solver (Clements & Sarama, 2009; Sarama & Clements, 2009). Figure 6 describes children’s developmental progression of each part of the learning trajectory for addition and subtraction, including an example problem and solution method (Adapted from Clements & Sarama, 2009; Sarama & Clements, 2009).

For multi-digit addition and subtraction, the developmental progression of understanding is the following: Pre-Part Whole Recognizer, Inexact Part Whole Recognizer, Composer to four then five, Composer to seven, Composer with tens and ones, Deriver, Problem Solver, and Multidigit (Clements & Sarama, 2009; Sarama & Clements, 2009). Figure 7 describes the developmental progression of each part of the learning trajectory for multidigit addition and subtraction, including an example problem and method for solving (Adapted from Clements & Sarama, 2009; Sarama & Clements, 2009).

The studies of Carpenter et al. (1989) and Clements et al. (2011) both incorporated the learning trajectories of addition and subtraction into their research. Both studies focused on young children’s understanding of addition and subtraction. Carpenter et al. (1989) investigated teachers’ use of knowledge about students’ mathematical
<table>
<thead>
<tr>
<th>Age</th>
<th>Learning Trajectory</th>
<th>Developmental Progression</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Pre-Explicit</td>
<td>Sensitivity to adding and subtracting perceptually combined groups; no formal adding</td>
<td>Shows no signs of understanding adding or subtracting</td>
</tr>
<tr>
<td>2-3</td>
<td>Nonverbal</td>
<td>Adds and subtracts very small collections nonverbally</td>
<td>Shown 2 objects then 1 object going under a napkin, identifies or makes a set of 3 objects to “match”</td>
</tr>
<tr>
<td>4</td>
<td>Small Number</td>
<td>Finds sums for joining problems up to 3+2 by counting-all with objects</td>
<td>Asked, “You have 2 toys and get 1 more. How many do you have now?” Counts out 2, then counts 1 more, then counts all 3.</td>
</tr>
<tr>
<td>4-5</td>
<td>Find Result</td>
<td>Finds sums for joining and part whole problems by direct modeling, counting-all, with objects</td>
<td>Solves take-away problems by separating objects</td>
</tr>
<tr>
<td>4-5</td>
<td>Make It N</td>
<td>Adds on objects to make one number into another without needing to count from one</td>
<td>Asked, “This coat has 4 buttons but should have 6. Make it 6.” Puts up 4 fingers on one hand, immediately counts up from 4 while putting up 2 more fingers, saying, “5, 6.”</td>
</tr>
<tr>
<td>4-5</td>
<td>Find Change</td>
<td>Finds the missing addend by adding on objects</td>
<td>Join-to/Count all groups ⇒ Asked, “You have 5 balloons and then you get some more. Now you have 7 in all. How many more did you get?” Counts out 5, then counts those 5 again starting at 1, then adds more counting, “6, 7,” then counts the balloons added to find the answer. Separate-to/Count all groups ⇒ Asked, “You had 8 stickers and gave some to your sister, now you have 5 stickers. How many did you give to your sister?” Counts 8 objects, separates until 5 remain, counts those taken away.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Compares by matching in simple situations</td>
<td>Match/Count rest ⇒ Asked, “Here are 6 dogs and 4 dog treats, how many dogs won’t get a treat?” Counts out 6 dogs, matches 4 treats to 4 dogs, then counts the 2 dogs that have no treat.</td>
</tr>
</tbody>
</table>

Figure 6 continues
<table>
<thead>
<tr>
<th>Age</th>
<th>Learning Trajectory</th>
<th>Developmental Progression</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>5-6</td>
<td>Counting Strategies</td>
<td>Finds sums for joining and part whole problems with finger patterns and/or by counting on</td>
<td>Asked, “How much is 4 and 3 more?” Starts with 4 and counts up.</td>
</tr>
<tr>
<td>6</td>
<td>Part Whole</td>
<td>Has initial part whole understanding; solves all previous problem types using flexible strategies</td>
<td>Asked, “You have some flowers. Then you get 6 more. Now you have 10 flowers. How many did you start with?” Lays out 6, counts up from 6 to 10, then recounts the added group.</td>
</tr>
<tr>
<td>6-7</td>
<td>Numbers in Numbers</td>
<td>Recognizes when a number is part of a whole and can keep the part and whole in mind simultaneously; solves start unknown problems with counting strategies</td>
<td>Asked, “You have some pencils, then you get 4 more pencils, now you have 9. How many did you start with?” Counts up, possibly putting up fingers.</td>
</tr>
<tr>
<td>6-7</td>
<td>Deriver</td>
<td>Uses flexible strategies and derived combinations to solve all types of problems; includes Break-Apart-to-Make-Ten; can simultaneously think of three numbers within a sum, and can move part of a number to another, aware of the increase in one and the decrease in another</td>
<td>Asked, “What is seven plus eight?” Thinks ( 7 + 8 \rightarrow 7 + (7+1) \rightarrow (7+7) + 1 = 14 + 1 = 15 ) or thinks ( 8 + 2 = 10 ), so separate 7 into 2 and 5, add 2 to 8 to make 10, then add 5 more, 15.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Solves simple cases of multidigit addition and sometimes subtraction by incrementing tens and/or ones</td>
<td>Asked, “What is 20+34?” Uses manipulatives to count up to 20, 30, 40, 50, plus 4, 54.</td>
</tr>
<tr>
<td>7</td>
<td>Problem Solver</td>
<td>Solves all types of problems, with flexible strategies and known combinations</td>
<td>Asked, “If I have 13 and you have 9, how could we have the same number?” Says, “9 and 1 is 10, then 3 more to make 13, 1 and 3 is 4, so I need 4 more.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Multidigit may be solved by incrementing or combining tens and ones</td>
<td>Asked, “What is 28+35?” Thinks 20+30=50 ( \rightarrow 50 + 8 = 58 \rightarrow 2 ) more is 60 ( \rightarrow 3 ) more is 63. (8+5 is like 8+2+3)</td>
</tr>
</tbody>
</table>

*Figure 6. Learning trajectories for addition and subtraction.*
<table>
<thead>
<tr>
<th>Age</th>
<th>Learning Trajectory</th>
<th>Developmental Progression</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-2</td>
<td>Pre-Part-Whole Recognizer</td>
<td>Only nonverbally recognizes parts and wholes; recognizes sets can be combined in different orders, but may not explicitly recognize that groups are additively composed of smaller groups</td>
<td>When shown 4 red blocks and 2 blue blocks, intuitively appreciates that all the blocks includes the red and blue blocks, but when asked how many there are in all, may name a smaller number than 6.</td>
</tr>
<tr>
<td>3-4</td>
<td>Inexact Part-Whole Recognizer</td>
<td>Knows that a whole is bigger than parts, but may not accurately quantify</td>
<td>When shown 4 red blocks and 2 blue blocks and asked how many there are in all, names a large number, but may not be correct.</td>
</tr>
<tr>
<td>4-5</td>
<td>Composer to 4, then 5</td>
<td>Knows number combinations; quickly names part of any whole, or the whole given the parts.</td>
<td>Shown 4 blocks, then 1 is secretly hidden, and then is shown the 3 remaining blocks, quickly says 1 block is hidden.</td>
</tr>
<tr>
<td>4-5</td>
<td>Composer to 7</td>
<td>Knows number combinations to totals of 7; quickly names parts of any whole, or the whole given parts; doubles to 10</td>
<td>Shown 6 blocks, then 4 are secretly hidden, and shown the 2 remaining blocks, quickly says 4 blocks are hidden.</td>
</tr>
<tr>
<td>4-5</td>
<td>Composer to 10</td>
<td>Knows number combinations to totals of 10; quickly names parts of any whole, or the whole given parts; doubles to 20.</td>
<td>Given 9 blocks and asked to double, quickly says 18.</td>
</tr>
<tr>
<td>7</td>
<td>Composer with tens and ones</td>
<td>Understands 2-digit numbers as tens and ones; count with dimes and pennies; 2-digit addition with regrouping</td>
<td>Asked, “17+36?” Thinks 17+36 → 17+3=20 → add 33, which is 53.</td>
</tr>
<tr>
<td>6-7</td>
<td>Deriver</td>
<td>Uses flexible strategies and derived combinations to solve all types of problems; includes Break-Apart-to-Make-Ten; can simultaneously think of 3 numbers within a sum, and can move part of a number to another; aware of the increase in one and the decrease in another; solves simple cases of multidigit addition by incrementing tens and/or ones</td>
<td>Asked, “What is 7+8?” Thinks 8+2=10, so separate 7 into 2 and 5, add 2 and 8 to make 10, then add 5 more, 15. Or thinks 8+2=10, so separate 7 into 2 and 5, add 2 to 8 to make 10, then add 5 more, 15.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Asked, “What is 20+34?” Uses manipulatives to count up 20, 30, 40, 50, plus 4, is 54.</td>
</tr>
</tbody>
</table>

Figure 7 continues
Table 7. Learning Trajectories for Multidigit Addition and Subtraction.

<table>
<thead>
<tr>
<th>Age</th>
<th>Learning Trajectory</th>
<th>Developmental Progression</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>Problem Solver</td>
<td>Solves all types of problems, with flexible strategies and known combinations</td>
<td>Asked, “If I have 13 and you have 9, how could we have the same number?” Says, “Nine and one is ten, then three more to make 13. One and three is four, so I need four more.” Multidigit problems may be solved by incrementing or combining tens and ones</td>
</tr>
<tr>
<td>7-8</td>
<td>Multidigit</td>
<td>Uses composition of tens and all previous strategies to solve multidigit addition and subtraction problems.</td>
<td>Asked, “What is 28+35?” Thinks 20+30=50 (\rightarrow) 50+8=58 (\rightarrow) 2 more is 60 (\rightarrow) 3 more is 63. Or thinks 20+30=50 (\rightarrow) 8+5 is like 8 plus 2 and 3 more, so it’s 13. 50 and 13 is 63. Asked, “What is 37-18?” Says, “I think 1 ten from the 3 tens, that’s 2 tens. I take 7 off the 7, that’s 2 tens and 0, which is 20. I have one more to take off, that’s 19. Asked, “What is 28+35?” Thinks 30+35 would be 65, but it’s 28, so it’s 2 less, 63.</td>
</tr>
</tbody>
</table>

*Figure 7. Learning Trajectories for Multidigit Addition and Subtraction.*

thinking, after attending professional development on the subject. Additionally, the study by Clements et al. (2011) evaluated the effectiveness of an intervention curriculum for preschool-aged children by studying how students progressed in their mathematical knowledge when the curriculum was based on learning trajectories. Most research about learning trajectories is about the development of them, not the use of the trajectories. There is a lack of research on how teachers can use learning trajectories to understand their students’ thinking and adapt lessons to meet the needs of students.

**Addition and Subtraction**

Number and operations, especially addition and subtraction, are a main staple of the mathematics curricula in primary grades (Common Core State Standards Initiative, 2011; Karp et al., 2011). Although addition and subtraction may seem like simple
concepts, they are essential to teach in a deep and meaningful way because they are building blocks of mathematics. When students learn such important mathematical concepts, it is crucial that they gain conceptual understanding through the use of tasks with high cognitive demand and the use of classroom discourse. Implementing tasks of high cognitive demand and having meaningful classroom discussions can help teachers create a classroom where students work together to learn, discuss, and justify mathematics, which is of interest in mathematics education.

In order for students to really understand addition and subtraction, students need to know when to use each operation, how to solve different problem structures, and how to perform the computations. If students know these relationships, it will make learning new addition and subtraction combinations easier because they are generating related knowledge rather than just relying on rote memorization (NRC, 2001). Henry and Brown (2008) suggest that students who learn basic facts through strategic acquisition, rather than memorization, have an advantage. For example, students who learn to group by fives and tens often have a better understanding of the base-ten system, which aids students in regrouping when adding and subtracting multi-digit numbers. Additionally, even young students should be able to explain why addition and subtraction work and justify their method of solving.

Karp et al. (2011) said that first grade instruction should focus on the development of students’ understanding of addition and subtraction, and related facts and strategies. Teachers should use a variety of models, such as objects and number lines, and a variety of strategies, such as part whole and adding on, to help students see meaningful
relationships between addition and subtraction. Second grade instruction should focus on quick recall of addition and subtraction facts, as well as develop fluency with multi-digit addition and subtraction. Students should be able to apply their understanding of the relationships and properties of numbers to solve problems in context. The inverse relationship between addition and subtraction is a powerful foundation in mathematics that teachers must emphasize (Nunes, Bryant, Hallett, Bell, & Evans, 2009).

In primary grades, it is especially important for students to connect formal and informal knowledge. Students come to school with some mathematical knowledge base, which should be acknowledged and connected to the formal knowledge taught in schools. Referred to by Selter (1998) as progressive mathematisation, students use their newly integrated formal knowledge to discover generalities and connections. Tasks with a familiar context can also help students connect informal solution paths to formal operations.

Several research studies and organizations have described four types of addition and subtraction problems that students should be exposed to and have experience solving (Carpenter et al., 1996; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Common Core State Standards Initiative, 2011; Karp et al., 2011; Sarama & Clements, 2009). Although different researchers have called problem structures by different names, I will refer to them as the following: joining, separating, part whole, and comparison. Figure 8 describes each type of problem structure, including an example.
<table>
<thead>
<tr>
<th>Problem Structure</th>
<th>Description</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joining</td>
<td>Putting together two groups of items</td>
<td>Kiya has 2 dolls and gets 3 more. How many does she have now?</td>
</tr>
<tr>
<td>Separating</td>
<td>Group of items is being taken away</td>
<td>Maddox had 6 cars, but he lost 2. How many does he have now?</td>
</tr>
<tr>
<td>Part Whole</td>
<td>Different parts of one whole</td>
<td>Jim has 4 red apples and 5 green apples. How many does he have altogether?</td>
</tr>
<tr>
<td>Comparison</td>
<td>Compare how many more or less of an item</td>
<td>Rex has 3 toys and Bode has 7 toys. How many more toys does Bode have than Rex?</td>
</tr>
</tbody>
</table>

Figure 8. Addition and subtraction problem structures.

According to Sarama and Clements (2009), categorization of problems into types can help teachers make decisions about the nature of students’ misunderstandings. It is important that teachers also teach multiple methods of solving each problem structure. By exposing students to several methods it is likely that at least one of the methods will connect with prior knowledge and the student will retain it. In fact, much time in primary grades should be devoted to students working out problems with their own method followed by sharing (Fuson et al., 1997).

Many of the studies of teaching addition and subtraction either examine the procedural learning of basic facts or the conceptual understanding of the operations. The studies of Carr, Taasoobshirazi, Stroud, and Rover (2011) and Henry and Brown (2008) both focused on the procedural learning of basic mathematical facts. Whereas the studies of Carpenter et al. (1989), Nunes et al. (2009), and Selter (1998) focused on the conceptual development of students’ understanding of addition and subtraction, but not in the context of reform curricula.
Gaps in Current Research

A research gap is present in primary grades mathematics, with regard to task selection, cognitive demand, and discourse. These features of mathematics teaching and learning may be thought of as more easily being applicable to students’ learning mathematics in upper grades. My study will examine what it takes for teachers to apply these important features in the mathematics teaching and learning of children in the primary grades. Additionally, there seems to be a lack of research at the primary level involving teaching strategies to teach students conceptually. With the current push of reform curricula, this area has now begun to be explored. In addition, there is a lack of research based on the cognitive demand of tasks from reform curricula for primary grades students. There is a theme of lack of research with regard to how teachers learn to promote deep understanding of mathematics by students in primary grades.

Although research is growing in the field of teachers’ use of curriculum materials, it is still underdeveloped (Lloyd, Remillard, & Herbel-Eisenmann, 2009). Remillard (2009) states, “We know little about how teachers interact with curriculum resources” (p. 89). Additionally, there are certainly a lack of theories to guide the analysis of the relationship between teaching and curriculum (McClain et al., 2009; Remillard, 2009). The research that has been done on the teacher-curriculum relationship has not been consolidated in a way to produce theoretically grounded knowledge by which to guide researchers, policy-makers or curriculum designers (Remillard, 2005, 2009). Minimal research exists that examines the cognitive demand of tasks as they appear in a written curriculum and through the enactment of the tasks. There is also a gap in research in what
types of discourse happens in primary mathematics classrooms, and how teachers use talk moves to enhance classroom discussions in ways that raise and maintain high cognitive demand and support students’ learning.
Chapter 3

Design Experiment Methodology

My study was conducted as a design experiment (Cobb et al., 2003) which allowed me, as a researcher, to collaborate with the teachers involved in the study. Cobb et al. (1990) state that the purpose of a teaching experiment is to investigate the process that allows children to construct mathematical knowledge. Throughout my study, each of two teachers and I collaborated on our understanding of student knowledge and lesson planning in their classrooms, in order for us to collectively examine the process of students’ learning addition and subtraction and for me to study the processes of teachers learning how to plan and implement tasks of high cognitive demand. We planned daily lessons together, specifically focusing on cognitive demand and discourse. I observed daily lessons and reflected with each teacher at subsequent planning sessions.

Design experiments are generally conducted to target a specific student learning process, with the goal of developing theories, often directed at a specific mathematical concept. Cobb et al. (2003) described the complex learning that occurs within a design experiment as a learning ecology, which is an interactive system of factors, including multiple elements related to teaching and learning that work together, rather than a collection of separate activities. The cooperative elements of teacher planning, lesson implementation, and student and teacher learning examined in my research are mathematical tasks, cognitive demand, and classroom discourse. Researchers who carry out design experiments focus on how the specific elements work together to support learning; they seek to understand, change, and improve educational practices. Wood and
Berry (2003) stated that a design experiment attempts to identify approaches to teaching that ensure teachers develop skills to address the complex nature of teaching mathematics. Researchers can aid teachers in digging deeply into their own practices. I helped the teachers I studied delve into the mathematics of the curriculum, examining their understanding of students’ mathematical conceptions. At the same time, I was able to examine the beliefs of the teachers (Davenport, Phillips, & Cooney, 2009). During planning sessions, we discussed student understanding, ways to generate discourse, and mathematical tasks.

Researchers involved in design experiments are able to help teachers manage the complexities and limitations of real-time classroom teaching (McClain et al., 2009) whereas traditional research methods may only be an observer’s view of teachers’ decision making, planning, and implementation. Multiple dependent variables exist in the complex setting of a classroom, not all are examined in traditional research (Peer Group, 2006). Researchers use design research to address the complex setting by employing multiple and mixed methods, in order to build up a body of evidence to improve the status quo. My goal was not to just study the teacher and learning of my two subjects. I also wanted to help them improve their practices and their students’ learning.

As Cobb, Stephan, McClain, and Gravemeijer (2001) emphasized, the overall goal of a design experiment is to effectively develop instruction that supports the mathematical learning of students. The process of design research allows researchers to continually modify work with teachers to get closer to the goals of research (McClain et al., 2009), rather than adhere to a prescribed plan. The use of a design experiment as
my methodology allowed me to be involved in the heart of the study, rather than remain an observing bystander. I had the unique opportunity to intervene during planning and influence the factors being examined, including task selection, cognitive demand, and classroom discourse, and in the process influence what teachers in my study attended to during planning and noticed during instruction.

My teaching and research experiences, together with my work as a teacher educator, equip me to work alongside teachers. As a graduate student, I have researched the foci of the study and have taught both pre-service and in-service teachers about cognitive demand, planning, discourse, and student understanding. The total of these experiences enable me to engage teachers in learning around these topics, through meaningful discussion and reflection on these topics.

**Purpose of Research**

My study was situated within the progression of implementation of mathematical tasks, using a figure from Stein and Smith (1998). Stein and Smith (1998) explain their task progression as a way to capture how a task changes throughout implementation. I included cognitive demand, problem structure, learning trajectories, and classroom discourse within the progression (see Figure 9) of Stein and Smith. This model enabled me to focus specifically on the critical topics of my study. Phase One considers how the task appears in the curriculum, before planning or teaching. Teachers must consider the cognitive demand as the task appears in the text, as well as the problem structure of the task. During Phase Two, the planning phase, teachers must take into account how the
cognitive demand of the task may change, based on how it is presented to students. Additionally, while planning, teachers should think through their students’ current understanding of the mathematical concept and how through the task they can help the students move forward and deepen their understanding. In Phase Three, which takes place during instruction, the task is altered by the students through the strategies they use to solve a math problem, as well as through the discussion they have around the task. Together, these three phases of mathematical tasks leads to student conceptions of the mathematical concept.

This qualitative study allowed me to explore the learning of two teachers in the context of their efforts to teach mathematics in meaningful ways in the primary grades. An underlying assumption in the study was that teachers play a central role in the process of curriculum implementation, selection of mathematical tasks, and lesson planning (Lloyd et al., 2009). The study included an examination of two teachers’ selection and implementation of mathematical tasks, including their efforts to maintain or raise the

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**Figure 9.** Progression of mathematical tasks (Stein & Smith, 1998).
cognitive demand of the selected tasks, and the nature of the resulting mathematical discourse within their primary math classes.

**Participants**

Garrison (2011) noted that the primary teacher characteristics that play a role in achieving a level of cognitive demand of mathematical tasks are years of teaching experience and depth of mathematical knowledge. Teachers with more teaching experience, and those who had deep conceptual mathematical knowledge tended to plan and implement tasks of high cognitive demand more often than did their less experienced and knowledgeable counterparts. Garrison’s finding is helpful to my research because the teachers in my study were participants in Primarily Math, a professional development program funded by the National Science Foundation, that focuses on helping primary teachers increase and deepen mathematical and pedagogical knowledge (University of Nebraska-Lincoln, 2013). I researched and taught for the project as well.

Throughout the Primarily Math program, the participants take three mathematics classes. The courses are designed to strengthen teachers’ conceptual knowledge of the kindergarten through third grade mathematics curriculum. The mathematics courses focus on the base ten place value system, arithmetic operations and relationships, and spatial reasoning, aligned with the NCTM Standards (2000) and Focal Points (Schielack, et al., 2006). The courses also focus on developing the teachers’ mathematical practices (CCSSI, 2011) as well as how to develop mathematical practices within the teachers’ students.
During Primarily Math, teachers take three courses that focus on mathematical pedagogy. Specifically, the courses engage teachers in understanding students’ learning trajectories, developing strategies for promoting conceptual and procedural understanding, and understanding and applying current research on mathematics teaching and learning. The courses also address the needs of diverse learners and how to teach in ways that are responsive to students’ mathematical understandings. Together, the courses help teachers see the connection between teaching and learning mathematics.

An overall focus of the Primarily Math program is to help teachers become more intentional, planful, observant, and reflective in their teaching practice. Teachers who are intentional, plan experiences that focus on the development of their students’ learning. Planful teachers prepare activities that support students’ understanding of the mathematical content (Lampert, 2001). Teachers who are observant take notice of students’ thinking and learning, without judging correctness, but rather to better understand students’ ideas in mathematics in order to deepen knowledge (Prediger, 2010). Lastly, teachers who are intentional constantly question and alter their teaching practice, specifically how knowledge is constructed, evaluated, and used, and the effects of teaching on that process (Cochran-Smith & Lytle, 2009). Additionally, Primarily Math focuses on building teachers’ MKT. A deep understanding of elementary mathematics concepts helps teachers better understand the thinking of their students. With a focus on learning how to be more intentional, planful, observant, and reflective, Primarily Math helps teachers of young children develop mathematical and pedagogical knowledge necessary to deepen students’ conceptual knowledge of mathematics.
I studied one first- and one second-grade teacher with varied years of teaching experience. Both teachers finished the Primarily Math program in June 2012, just prior to when I began my research in September 2012. The teachers were introduced to task selection, cognitive demand, and classroom discourse through their Primarily Math coursework. I used purposeful sampling to choose the teachers. I wanted to work with two teachers from the same Primarily Math cohort, specifically the group who had just finished the program. I hoped this would enable me to focus on helping teachers implement the newly acquired knowledge from Primarily Math in their classrooms. I specifically chose these two teachers because they both wanted to improve their planning and teaching, however, they had different teaching experiences and taught at different schools. Additionally, both teachers were recommended by course instructors and school administrator, as being eager to learn and reading and willing to improve their practice. I hoped to learn from both teachers how to implement tasks at different grade levels and at contrasting schools (see Figure 10). The teachers’ flexibility and desire to improve were crucial to my research, based on the time and thought I asked them to put into planning and teaching. Additionally, the teachers’ school district implemented a new reform curriculum, *Math Expressions* (Fuson, 2011), which is of interest in my research because reform curricula often offer more opportunities for conceptual teaching and learning.

Hillary taught first grade at the same school for two years. She earned a Bachelor of Science degree at a large local university and is nearly finished with a Master of Science degree in Curriculum and Instruction at a small liberal arts college in the area. In her application to be accepted into Primarily Math, the professional development
program, she noted that the biggest challenges in teaching mathematics for her are teaching abstract concepts and teaching students of varying abilities. Hillary’s school is generally lower socioeconomic status, as noted in Figure 10.

<table>
<thead>
<tr>
<th></th>
<th>Hillary’s School</th>
<th>Dina’s School</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of students</td>
<td>496</td>
<td>869</td>
</tr>
<tr>
<td>Number of teachers</td>
<td>54</td>
<td>55</td>
</tr>
<tr>
<td>Percent of minority students</td>
<td>70%</td>
<td>13%</td>
</tr>
<tr>
<td>Percent of gifted students</td>
<td>2%</td>
<td>7%</td>
</tr>
<tr>
<td>Percent of special education students</td>
<td>17%</td>
<td>8%</td>
</tr>
<tr>
<td>Mobility Rate</td>
<td>27%</td>
<td>6%</td>
</tr>
<tr>
<td>Percent eligible for free/reduced meals</td>
<td>94%</td>
<td>5%</td>
</tr>
<tr>
<td>Average daily attendance rates</td>
<td>95%</td>
<td>96%</td>
</tr>
</tbody>
</table>

Figure 10. A comparative description of the schools of Dina and Hillary.

Dina has been teaching second grade at the same school for 16 years, where she has been team leader for the past 12 years. She earned both a Bachelor of Arts degree and Graduate degree from a small liberal arts college in the area. In her application to Primarily Math, she noted that the biggest challenges in teaching mathematics for her are having enough time to plan and address student needs, as well as simultaneously teaching students who excel and students who struggle. She specifically noted that she struggles to find ideas other than flash cards to teach students addition and subtraction facts. Dina’s school has a generally high socioeconomic status, as noted in Figure 10.
During my study, both teachers were in their second year of implementing a new reform curriculum. *Math Expressions*, a curriculum developed in part by funding through the National Science Foundation, is based on ten years of classroom research (Fuson, 2011). *Math Expressions* (Fuson, 2011) is a comprehensive standards-based, reform curriculum that uses objects, drawings, conceptual language, and real-world situations to help students learn mathematics in a deep and meaningful way. *Math Expressions* focuses on gaining conceptual knowledge of core concepts at each grade level, based on the recommendations of NCTM (2000) and NRC (2001), a main focus of reform curricula (Fuson, 2011). According to Fuson (2011), *Math Expressions* helps teachers create classrooms in which inquiry into mathematics and classroom discourse is encouraged. *Math Expressions* was designed to promote students’ natural solution methods, while introducing students to research-based strategies to help students become reflective and resourceful problem solvers (Fuson, 2011).

**Data Sources**

I collected some data before working with the teachers, such as teacher coursework and samples of the teachers’ textbook. Then while working with the teachers, I collected additional data in the forms of videos, field notes, and journals. Refer to Figure 11 for the various types of data I collected. Each source of data was a piece of the puzzle that helped me better understand the process of planning and teaching primary grades mathematics using tasks of high cognitive demand and the discourse that surrounds the tasks.
I analyzed data from teacher coursework, planning session videos, researcher field notes, and researcher and teacher journals, to answer questions related to transferring knowledge from professional development to classroom practice. Whereas to address questions regarding how teachers plan for learning that requires high cognitive demand, I examined the textbook, planning session videos, and researcher field notes. Lastly, to answer questions related to the implementation of mathematical tasks, I analyzed teaching session videos, researcher field notes, and researcher and teacher journals.

### Data Collection

Before working with the teachers, I examined the textbook and the project that the teachers completed at the end of the course; all other data were collected while working with teachers. Each source of data served a different purpose, but came together and complimented each other for an overall picture of teaching primary mathematics. Data collection and analysis occurred as a cycle, repeating daily, see Figure 12. The work done

![Table](data_sources.png)

*Figure 11. Data sources.*
by the teacher and researcher in the planning and reflecting time influenced the work done by the teacher and students during the teaching time.

My examination of teacher coursework and the textbook was done in a single session. However, since the collection of planning and teaching session videos and research and teacher journals were collected in the midst of planning and teaching, I used daily data to continually inform the discussions during planning. This was a direct influence of the design experiment methodology I chose to use in the study. The cyclic nature of design experiment allowed me to collect data that influenced subsequent data collected.

Figure 12. Cycle of design experiment.
Teacher coursework. Through Primarily Math, both teachers completed coursework around the issues of task selection, cognitive demand, and classroom discourse. During the summer prior to my study, they learned about cognitive demand and task selection. They planned a unit, including tasks of high cognitive demand, and anticipated the discourse that could happen around those tasks. I examined their coursework before and after I observed in their classrooms to give myself a picture of their understanding and struggles as well as other noteworthy aspects of their mathematical and pedagogical knowledge. I examined their coursework before working with them in order to better understand their philosophy underlying teaching and learning. When I examined their coursework after completing the work in their classroom, I analyzed their written work for themes related to what I was finding in my analysis of their planning and teaching.

Within their coursework, I looked for themes that both teachers had in common, in terms of what they had plans to improve. In my examination of what teachers needed to learn, I looked for what these two specific teachers noted as struggles when attempting to obtain specific knowledge about overall opportunities to learn about selecting and enacting tasks of high cognitive demand. The teachers’ coursework gave me insight into their knowledge before my actual work in the classroom with them began. I used the knowledge I gained through the examination of their coursework as a springboard to our work together. I knew what some of their struggles were, which enabled me to create a plan to support them with those issues prior to beginning interactions as a researcher with each of them.
Textbook. I analyzed the textbook, *Math Expressions* (Fuson, 2011), for two purposes: (a) to examine what types of addition and subtraction problem structures are represented in the text, and (b) to examine the cognitive demand of the problems in the text, based on framework for cognitive demand of Stein et al. (2009). This information showed what the curriculum offered the teachers and, eventually, which types of tasks the teachers chose to implement in the classroom. Textbook analysis offered a picture of the teachers’ opportunities to implement tasks of high cognitive demand and include multiple problem structures.

Field notes. As each teacher and I planned together, I took notes on the teacher’s actions in relation to her planning and in response to her students. I used the field notes to reference the conversation at a later date and to complement the information within the video recordings of the planning sessions. I also took field notes while watching the teacher during instruction. These notes enabled me to notate specific occurrences in each teacher’s practice, such as moments when the teacher used a talk move, a type of discourse that occurred, or something of significance was said during classroom discussions.

Planning/reflecting sessions. I purposefully planned with the teachers during the units on addition and subtraction. Addition and subtraction are a major emphasis in the primary grades, and a building block upon which subsequent mathematical knowledge is constructed. Students’ understanding of addition and subtraction is crucial to their understanding of mathematics as a whole. Each unit lasted approximately three weeks
during the first semester of the 2012-13 school year. Figure 13 shows the dates and concepts on which data was collected.

I prepared for the planning sessions by reviewing student work done in class during a previous lesson and pre-reading the textbook. I came prepared with ideas of how I thought the teacher could help students gain deeper conceptual understanding, based on the connections students had made previously and appeared ready to make. I videotaped

<table>
<thead>
<tr>
<th>Grade and Teacher</th>
<th>Unit</th>
<th>Big Idea</th>
<th>Lesson and Content</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st grade</td>
<td>3</td>
<td>Addition Stories</td>
<td>2 Stories with Unknown Partners</td>
<td>Oct 18</td>
</tr>
<tr>
<td>Hillary</td>
<td></td>
<td>with Unknown Partners</td>
<td>3 Solve Equations with Unknown Partners</td>
<td>Oct 19</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>4 Addition Game: Unknown Partners</td>
<td>Oct 22</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5 Practice with Unknown Partners</td>
<td>Oct 23</td>
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<tr>
<td></td>
<td>6</td>
<td>Subtraction Stories</td>
<td>6 Subtraction Strategies</td>
<td>Oct 24</td>
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<td>7</td>
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<td>7 Subtraction Stories</td>
<td>Oct 25</td>
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<td>8</td>
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<td>8 Subtraction Game: Unknown Partner</td>
<td>Oct 26</td>
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<td>9</td>
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<td>9 Practice with Subtraction Stories</td>
<td>Oct 29</td>
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<td></td>
<td>10</td>
<td>Addition Stories</td>
<td>10 Addition Stories with Unknown Totals</td>
<td>Oct 31</td>
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<tr>
<td></td>
<td>11</td>
<td>with Unknown Totals</td>
<td>11 Addition Practice: Unknown Totals</td>
<td>Nov 1</td>
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<tr>
<td></td>
<td>12</td>
<td>Mixed Practice</td>
<td>12 Stories with Mixed Unknowns</td>
<td>Nov 2</td>
</tr>
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<td></td>
<td>13</td>
<td>with Different Unknowns</td>
<td>13 Addition and Subtraction Game</td>
<td>Nov 5</td>
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<tr>
<td></td>
<td>14</td>
<td></td>
<td>14 More Practice: Mixed Unknowns</td>
<td>Nov 6</td>
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<td></td>
<td>15</td>
<td></td>
<td>15 Use Mathematical Processes</td>
<td>Nov 7</td>
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<tr>
<td>2nd grade</td>
<td>1</td>
<td>Relate Addition and Subtraction</td>
<td>17 Use a Number Line to Add or Subtract</td>
<td>Sept 6</td>
</tr>
<tr>
<td>Dina</td>
<td></td>
<td></td>
<td>18 Equations and Equation Chains</td>
<td>Sept 7</td>
</tr>
<tr>
<td>Equations and Inequalities</td>
<td>19</td>
<td>Equations from Math Mountains</td>
<td>Sept 10</td>
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<tr>
<td></td>
<td>20</td>
<td>Stories from Math Mountains</td>
<td>Sept 11</td>
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<tr>
<td></td>
<td>21</td>
<td>Compare and Order Numbers</td>
<td>Sept 12</td>
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<tr>
<td></td>
<td>22</td>
<td>Add Three Numbers</td>
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Figure 13 continues
<table>
<thead>
<tr>
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<th>Lesson and Content</th>
<th>Date</th>
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<td>2nd grade</td>
<td>3</td>
<td>Addition and Subtraction</td>
<td>Change Plus and Change Minus Story Problems</td>
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<td>Dina (cont’d)</td>
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<td>Situations</td>
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<td></td>
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<td></td>
<td>2</td>
<td></td>
<td>More Change Plus and Change Minus Story Problems</td>
<td>Oct 3</td>
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<td></td>
<td>3</td>
<td></td>
<td>Collection Problems</td>
<td>Oct 4</td>
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<td>4</td>
<td></td>
<td>Story Problems with Group Names</td>
<td>Oct 5</td>
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<td></td>
<td>5</td>
<td></td>
<td>Comparison with Story Problems</td>
<td>Oct 8</td>
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<tr>
<td></td>
<td>6</td>
<td></td>
<td>More Comparison Story Problems</td>
<td>Oct 9</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td></td>
<td>Mixed Story Problems</td>
<td>Oct 10</td>
</tr>
</tbody>
</table>

*Figure 13. Overview of mathematical concepts and dates of data collection.*

each planning session, which included reflections on the previous lesson. The teacher and me examined the varied strategies students were using to solve problems, which helped the teacher and I gain a better understanding of the students’ mathematical conceptions and guided us in planning subsequent instruction. I led the planning sessions by offering my suggestions, but then tried to hand over the leadership for planning to the teachers, waiting for them to offer suggestions. The planning sessions took approximately two hours per week.

During the planning sessions, the teacher and I used an adaptation of Thinking Through a Lesson Protocol (TTLP) (see Figure 14) from the work of Smith et al. (2008). TTLP is a framework that provides a set of questions related to selecting a mathematical task, supporting students exploration of the task, and planning questions to engage
### Pre-planning

What are the mathematical goals of the lesson?
What do students already know about the topic?
What vocabulary do students need to know to understand the lesson?

### Instructional Planning

What tasks will students work on?
Why are you choosing those specific tasks? What is the purpose of each task?
What are all the ways it can be solved?
Which of those methods do you think students will use?
What misunderstandings might students have about the topic?
What errors might students make on these tasks?
What is the best way for students to work on the tasks? (whole group, small group, partner, individual)
How will you keep students engaged in the tasks?
How will you alter tasks for students who are struggling, while maintaining the cognitive demand of the task?

### Planning for discourse

What questions will you ask students to prompt a meaningful mathematical discussion?
Which of the strategies will you highlight?
What connections do you want students to make and how will you help this happen?
What generalizations do you want students to make and how will you help this happen?
How will you use the talk moves during the discussion?

*Figure 14.* Planning questions from TTLP (Smith et al., 2008).

Students in discussion, which supports teachers when planning tasks of high cognitive demand (Silver et al., 2009). When teachers focus on TTLP, they are able to develop instructional routines that support students’ engagement in cognitively demanding tasks.

Before planning a task, the teacher and I discussed the mathematical goals of the lesson based on the textbook and our collective knowledge of students and mathematics.
We discussed what students may already know about the topic and any vocabulary students may need to know to understand the topic. The teacher and I discussed why the teacher chose specific tasks, as not all of the tasks were implemented, and the cognitive demand of each of the chosen tasks. I used the work of Stein et al. (2009) to guide the discussion about cognitive demand with each teacher by planning questions for students that would maintain or raise the cognitive demand. We examined the cognitive demand of the task and how we might alter it to raise the cognitive demand of low-level tasks. We also talked about how to maintain a high level of cognitive demand when the task was actually implemented.

After choosing a task, the teacher and I discussed multiple ways it could be solved, which of those methods students would be likely to use, what misunderstandings students may have about the task, and what errors students may make, all based on our collective pedagogical knowledge. Next, we discussed multiple ways students could work on the task, including independently or in pairs, small groups or as a large group. We also discussed how students should engage in the task and what questions the teacher could ask to prompt learning. Lastly, we discussed how the teacher could orchestrate a classroom discussion around the chosen task. We considered what questions the teacher could ask, which solution strategies would be useful to highlight in such a discussion and why, and what connections the teacher hoped would be made, and how this might lead to forming generalizations. We also talked about how the talk moves could be used to further the students’ understanding of the mathematical topic. While it was difficult to specifically plan times to use the talk moves, the teacher and I discussed how to
recognize opportunities for each talk move. Figure 14 shows the questions that the teacher and I discussed during each planning period.

I asked the teacher to reflect on the choice of tasks within the textbook, the maintenance of cognitive demand, and the nature of classroom discourse. We also reflected on instruction during the planning sessions, focusing on students’ work, my observation notes, and the teachers’ reflections on the lesson. This cyclic nature of planning, teaching, and reflecting aided the teachers and me in making ongoing adaptations to instruction.

**Classroom discourse.** As I videotaped instructional time, I wrote in my field notes which types of discourse and talk moves occurred. I categorized the nature of classroom discourse using the categories of expounding, exploring, expressing, explaining, examining, and exercising, based on the work of Sfard et al. (1998). Frequently, the nature of the discussion changed throughout the course of one class time, based on the task, the students, and the teacher. As these changes occurred during instruction, I made notes each time the type of discourse changed, which enabled me to later examine my notes for patterns in classroom interactions across the duration of the class period.

When analyzing the data, I focused on how the teacher attempted to raise or maintain the cognitive demand, and whether or not she was successful. I looked for specific factors (see Figure 15) that maintain or lower cognitive demand, as noted by Stein et al. (2009). In addition, I looked at the teachers’ use of the talk moves, types of discourse that occurred, and how the teachers maintained or lowered cognitive demand (Stein et al., 2009).
Factors associated with the decline of high level cognitive demand
Factors associated with the maintenance of high level cognitive demand

<table>
<thead>
<tr>
<th>Task becomes routinized</th>
<th>Scaffolding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher reduces complexity of task</td>
<td>Students able to monitor their own progress</td>
</tr>
<tr>
<td>Teacher does the problem solving instead of the students</td>
<td>Teacher helps make conceptual connections</td>
</tr>
<tr>
<td>Correctness is emphasized</td>
<td>Sustained press for explanations</td>
</tr>
<tr>
<td>Not enough time</td>
<td>Tasks build on students’ prior knowledge</td>
</tr>
<tr>
<td>Classroom management problems</td>
<td>Models of high level performance</td>
</tr>
<tr>
<td>Inappropriate task for students</td>
<td>Sufficient time</td>
</tr>
<tr>
<td>Students not hold accountable for process</td>
<td></td>
</tr>
</tbody>
</table>

*Figure 15.* Factors associated with decline and maintenance of high cognitive demand.

I watched the videos of instruction to examine the teachers’ use of talk moves and how that affected the classroom discourse. I looked to see how often the teacher was able to use the talk moves and if and how the students responded. I made note of which and how the talk moves increased or maintained cognitive demand.

**Journals.** Both the teachers and I kept journals of our reflection and additional thoughts. My journal was used for my reflections on what happened during instruction and planning sessions. I also asked the teacher to complete a journal. This helped me analyze what each were thinking after a lesson was competed. We specifically answered the questions in Figure 16. Each question was designed so that it was related directly to a research question. The researcher and teacher questions mirrored each other, except for the last researcher question, which was solely based on researcher observations.
Researcher Questions | Teacher Questions
---|---
What did the teacher base her instructional decision on? What helped her decide which tasks to implement? | What did you base your instructional decision on? How did you decide which tasks to use?
How did the teacher think about planning for the maintenance of high cognitive demand? | How did you think about planning for the maintenance of high cognitive demand?
How did the teacher use student understanding/learning trajectories to aid her in planning? | How did you use the student understanding/learning trajectories to help you plan?
What did the cognitive demand look like throughout the lesson? Illustration? | What happened to the cognitive demand throughout the lesson?
How did the teachers’ use of talk moves maintain or raise cognitive demand? | How did your use of talk moves change the cognitive demand?
What types of discourse occurred throughout the class period? |  

*Figure 16*. Researcher and teacher journal questions.

**Methods of Data Analysis**

I analyzed data in multiple ways. I used quantitative methods in a minimal way when analyzing the textbook. The majority of my data analysis consisted of qualitative methods (Creswell, 2007), looking for themes and factors related to my research questions within the teacher coursework, videotaped planning and teaching sessions, as well as field notes and journals. I triangulated the analysis of individual sources to collectively address the research questions.

I analyzed teachers’ planning in several ways. First, I looked at what was being discussed during the planning sessions. I coded the discussions with the following labels: mathematical content, mathematical task, solution strategies, students, objective, materials, discourse, reflection or other. The same segment could be categorized by
several codes. For example, during planning, the teacher and I could be discussing the mathematical task and the objective at the same time. I also analyzed the planning session based on research by Westerman (1991), who made distinctions between how expert teachers and novice teachers plan lessons. I used Westerman’s characteristics to examine the planning of the teachers with whom I worked. Additionally, I looked at my role and the role of the teachers during the planning sessions. I compared and contrasted the planning of the two teachers.

**Textbook.** I analyzed the textbook quantitatively (Creswell, 2007) by tallying the levels of cognitive demand of the tasks as memorization, procedures without connections, procedures with connections, and doing mathematics. I only analyzed the lessons that were part of my study. My goal was to find out what types of tasks were in *Math Expressions* for the first and second grades. Since the textbook is considered reform based, in which the goal is to learn conceptually through meaningful tasks, I wondered if it contained a large amount of tasks of high cognitive demand. The cognitive demand of the tasks offered in the textbook has a major impact on the cognitive demand of the tasks planned by the teacher.

**Field notes.** I used the field notes to anchor my videotaped sessions. I made specific notes when teachers used talk moves, when the type of discourse changed, and when students made noteworthy connections and generalizations. I noted when the content of the lesson converged to or diverged from the decisions we made during our planning session. I used field notes as a source of information and as a way to locate video references while analyzing data.
Planning/reflecting sessions. I took field notes and videotaped the teaching sessions, then coded both for themes of discourse and cognitive demand, as is the general process for qualitative research (Creswell, 2007). I looked at how the teachers used the talk moves, what types of discussions were going on in the classroom, and what the teacher did to lower or maintain the cognitive demand of the tasks.

I kept records that enabled me to analyze how my planning of the research project ended up being different than how I carried out my research. For example, the way in which I planned to use the learning trajectories was different than the way I actually used them. I triangulated my data with regard to my own practices as a researcher through multiple data sources, such as video taped planning sessions and teaching sessions, researcher and teacher journals, student work, teacher work, and the textbook. The textbook was the primary source for the analysis of cognitive demand of tasks as they appeared initially. A mix of video, notes, and journals served as sources to examine what teachers needed to learn to successfully implement tasks of high cognitive demand and maintain a high level of cognitive demand throughout the lesson. All of these sources together painted a picture of the teaching that was going on and the thought processes behind it. I attempted to make the planning sessions transparent so that I could thoroughly examine the work of teachers, including their intentions and their actions as well as my own.

Limitations and Benefits of Design Experiment Methodology

One limitation of a design experiment in mathematics education could be that, as the researcher, I was a central part of the study. As Wood and Berry (2003) put it,
researchers using design experiments are trying to, “fly the airplane and fix it at the same time” (p. 197). Although it is the case that I was in the middle of the research, it does not mean that the study is tainted. An advantage of being involved in the experiment process is that I was able to influence the work of the teacher. It benefited each of the teachers for me to work closely with them because I have deep knowledge of mathematical content and pedagogical reasoning. I separated my own view from the views of the teachers, through the use of journals.
Chapter 4

Findings in Planning and Implementation of Mathematical Tasks

Overall, I found that both teachers transferred knowledge from professional development to their classroom. They both used talk moves, planned tasks of high cognitive demand, and reform curriculum. However, I found varying levels of cognitive demand during the lesson and specific factors that both maintained and lowered high cognitive demand. This chapter is organized around my research questions; the transfer of learning from professional development to classroom practice and how teachers plan and enact tasks of high cognitive demand. Around each question, I will describe the work of each teacher, as well as similarities and differences between them.

Transfer of Learning from Professional Development to Classroom Practice

In June 2012, I taught both Dina and Hillary in TEAC907, which is the capstone course to Primarily Math. Dina did not actively participate and appeared less willing to adapt her teaching to new ideas she was learning. I was concerned that she did not see the reasoning behind the big mathematical ideas of professional development and that she would be unwilling to implement ideas that did not strictly follow the district-prescribed curriculum. However, when I began working with her in the context of this study, it appeared that she was dedicated to sound instructional practices and student learning. She viewed planning as simply pacing, which did not provide enough time and thought to plan teaching and learning deeply. When we worked together, we certainly afforded ourselves enough time and I helped each teacher to think through how to plan sound lessons, including ideas for teaching, learned in Primarily Math. Even if Dina did not
spend as much time planning as we did planning together, she did have the drive to continue using the talk moves and teaching in a conceptual manner.

In contrast, during TEAC907, Hillary was outspoken and dominated classroom discussion with new ideas. For example, during the class, Hillary liked an idea that a presenter showed and met with her principal before class the next morning to discuss implementing it at her school. Based on this observation, I thought Hillary would jump into new ideas during my research project, however, she was actually fairly rigid when it came to trying new instructional ideas. Although Hillary saw the application to the classroom and we had adequate planning time, she struggled to apply the knowledge from professional development to her classroom. She was not flexible in using new strategies and thinking about her students from a different perspective. Hillary reported after our work together that she finally saw how important our work together was, although she did not see it well at the time. The following is her description of how she thought about planning before and after our work.

My planning with cognitive demand, I mean, before was more of a differentiation thing, and kind of differentiating on the fly, and planning reteaching and planning enrichment. And now, just by preplanning that higher cognitive demand, you go into it with a different mentality that doesn't necessarily require planning for reteaching and enrichment later. (Hillary Journal, 10/31/12)

As an end of course assignment for TEAC907, Dina and Hillary planned a series of lessons, using their curriculum, but also incorporating ideas from Primarily Math. In Dina’s assignment, she reported that she wanted to engage in many practices developed through the program, such as building math talk, including using the talk moves, asking justifying questions, discussing strategies, and listening to and learning from peers, while
encouraging participation and using wait time. The following are some comments from her End of Course assignment.

- My first goal is to build upon the Math Talk in my classroom.
- My goal is for students to connect the idea of making tens columns to Quick Tens. This becomes important in the next lesson because they will be given several numbers and they will have to represent it in various ways. I am hoping that my students will be able to generalize the fact that ten of anything makes a ten.
- This will become important as we move into addition and subtraction because students will be expected to show how they got the answer with a drawing before they move in the actual algorithm. (Dina End of Course Assignment, 07/01/12)

Notice that she attends to the conceptual knowledge that she wants students to gain. She wanted to help students construct that conceptual knowledge by examining error patterns, creating multiple representations, making connections and generalizations, connecting to prior knowledge, and explaining the meaning behind procedures. Dina wanted to help students reason, compare strategies, and be efficient, while maintaining high cognitive demand. Additionally, Dina mentioned that she wanted to simplify steps for students and look for correctness, which routinizes the problem for students.

Hillary noted in her assignment that she planned to help students make connections, such as the relationship between addition and subtraction, by using multiple representations, math talk, and formative assessment. She wanted to maintain high cognitive demand through having students justify their strategies, show proof, and use reasoning. The following are some comments from her End of Course assignment.

- My goal is for our math talk to take our reasoning and proof from the representational stage of learning to more abstract discussions and conjectures.
- Through math talk, we can discuss students’ proofs and determine if they are true or not true and false.
• I will modify the talk move of agreeing or disagreeing to ask students if an equation is reasonable. (Hillary End of Course Assignment, 07/01/12)

However, Hillary also wrote that she did not want to make any changes to the teacher’s editions plans, even though the cognitive demand may decline as a result. She said that she would not try to raise the low cognitive demand of a lesson, which is worrisome because she may plan low cognitive demand tasks often.

Both Dina and Hillary indicated that they learned from the professional development program. In their end of course assignment, they both planned to use math talk, multiple representations, and justifications. They wanted to maintain high cognitive demand while making connections and generalizations, however, Hillary was less willing to alter the lessons from the teacher’s edition in order to implement the new strategies she learned from professional development. Hillary seemed to think there was benefit in maintaining low cognitive demand during introductory activities.

Support while Planning Lessons of High Cognitive Demand

Through my work with Dina, I made many suggestions, such as how discourse might occur throughout the lesson, how to carry one activity over a few days, and how to raise the cognitive demand of an activity. For example, the following discussion on equation chains occurred during planning.

Dina: I’m going to give them a number. They are going to put it on a link. I’ll put it together. And then, I think we could just put them on the board. Put the number at the top and maybe we just have a yes or a no.

Kelly: Well I was debating, would you want have, I know time is an issue, but would you want to like, pick a kid, not necessarily their ring. You would say, oh, Tanner, this one says 6+4=10, Tanner can you prove it? Does everyone agree? And do a few of them that way. Then see if you can get all the partners.

Dina: Oh, we could. (Dina Planning Session, 09/06/12)
As Dina and I planned together, she began to understand how to plan purposefully with a focus on cognitive demand and discourse, and therefore took over the lead in the planning sessions, and consequently did not need my help as much. In the following example, we discussed students writing their own story problems.

Dina: If we map these out with the kids anyway. I wonder if I even put on sticky notes, one of these kinds of problems. They have to write one to match it and see if the class can say which one they wrote. That way we can for sure cover all of the different types. Otherwise everybody is just going to go with the easiest one. Give partners a number sentence equation and have them write the story problem.

Kelly: Would you want to just do one equation for all of them?

Dina: Yes. Then we could do addition first, and practice a few subtraction ones then. (Dina Planning Session, 09/28/12)

I hope her lead role in the planning allowed her to continue to plan in a meaningful way after I was done collecting data in her classroom.

I helped Dina think about what it meant to implement curriculum with fidelity and integrity. During the previous school year, teachers in the district were asked to implement the curriculum with complete fidelity and not veer from the teacher’s edition at all, because the instructional materials were new. With a year of implementation under her belt, Dina was able to see the teacher’s edition as the skeleton of the curriculum and her ability to alter the plans in order to better meet the needs of her students.

In working with Hillary, I also made many comments subtly suggesting that she plan activities that are of higher cognitive demand than the teacher’s edition and engage students in meaningful discourse. I also noted benefits of talk moves for English Language Learners, which is a concept she struggled with. The following example shows
how Hillary did not want to have a discussion with me about a problem that students had solved the previous day, but rather, she preferred to just skip to solving another problem.

Kelly: Okay, so tell me what you were thinking.
Hillary: For today?
Kelly: Yeah. You have new problems?
Hillary: I mean, it’s still a partner of ten. They have to find both partners.
Kelly: So, what were you, I noticed you had hung these ones up. What were you going to do with the other ones that they did, the candy and sticker one?
Hillary: It’s on the back of this.
Kelly: Oh, okay. Were you planning on having them share that one? The candy and sticker one?
Hillary: I just thought it was so long ago, I mean, by now. I was just going to do this, and then share this one. Does that make sense?
Kelly: Yeah, I just wonder if it might.
Hillary: Cause I think this was almost so easy for them. After they got five and five, they got a ton of the different. And I think today.
Kelly: Except like nine and one. Sometimes there’s not a difference in nine and one and one and nine, but in the case of candy and stickers, there would be.
Hillary: They would be different. (Hillary Planning Session, 10/24/12)

Unlike Dina, Hillary did not take the lead in the planning. She was more reluctant to change, which did not enable me to release the lead to her during planning. She was unable to plan lessons of high cognitive demand and stick to the set plans. I struggled to keep Hillary focused on the lesson that she was teaching and how to maintain high cognitive demand. I helped Hillary release some control of discourse to her students, but we struggled in that process as well.

Although my conversations with Dina and Hillary were similar in content, they differed in depth. I preplanned our planning sessions with what I wanted to focus on. In the example below, you can see that I felt Dina was ready to plan with much less
guidance than Hillary. Hillary’s struggle was to implement the talk moves to help
students better engage in meaningful mathematical discussions.

What am I helping Dina learn? I want to help her learn when the cd is low, how she can alter plans ahead of time to raise it. (Kelly Journal, 09/28/12)

Hillary basically has said that she thought her students could already repeat, but they cannot. They really struggled to be able to listen carefully and repeat another student’s thoughts. Hillary ended up making it very procedural. (He did this, then this, then this). (Kelly Journal, 10/28/12)

During the study, Dina was able to plan, teach, reflect and learn, and cycle back to plan to continue the growth process. She was much farther along in her understanding and implementation of the talk moves and meaningful discussion than Hillary. Hillary seemed to plan, teach, reflect, but missed learning about her own practice, which rendered her stagnant in the professional growth process.

A researcher can help a teacher by discussing cognitive demand and discussion during lesson planning. Since my work with each teacher was personalized, I was able to reflect with each of them to improve their own practice. Teachers may simply pace their lesson and follow the teacher’s edition directly, rather than consider how to use it to create meaningful classroom discussions. It is likely that if a teacher learns to complete the cycle of planning, teaching, reflecting and learning, they will develop skills to allow them to continue this process without the researcher present.

**Support during Teaching Lessons of High Cognitive Demand**

Although my role during classroom instruction was only to observe, Dina said that my presence in her classroom prompted her to use the talk moves more often, until
they became automatic. She tried to implement the talk moves before we worked together, but often forgot to use them while she was teaching. The following excerpt shows how Dina wanted to use the talk moves and was able to recognize opportunities to use them, but was not fully able to implement them yet.

Kelly: So when I was thinking, sort of reflecting on it afterwards with those questions, one thing that I was thinking was that you did a couple of revoicings.
Dina: Uh huh.
Kelly: But when the kids were sitting on the floor and you were having them share, like Gabriel got up a shared, that would have been an opportunity to do repeating. Like somebody repeat what he said. I was just thinking of other opportunities.
Dina: I know. I think of it when I’m up there, but I was so flustered about how the lesson was going, that I thought, let’s just speed this along.
Kelly: But when it’s you, sometimes it’s hard to see the opportunities. When it’s me sitting in the back of the room, I can see them a little easier.
Dina: Yeah. (Dina Planning Session, 09/06/12)

My role in Hillary’s classroom was also just to observe, but during one lesson I stepped in to help Hillary see the talk moves in action because she struggled to implement the talk moves during classroom discussion. I wanted Hillary to see what her students were capable of and how she could help them discuss mathematics. Unfortunately I found that her students did not know how to talk about mathematics, although she had reportedly used the talk moves before we worked together. They were unable to repeat what each other said, which made it difficult to having deeper, meaningful conversations. The following excerpt is from that classroom discussion. A student has just stated that when you start with a total and subtract a partner, you are left with the other partner, and students are being asked to repeat.
Damon: He said that five is the total.
Kelly: And what else? What else did he say? Five is the total, and he knew that if he did what?
Damon: He put the two there.
Kelly: Why did he say he put the two there?
Damon: Cause that’s his partner.
Kelly: That was a partner and he knew if he started with the total and took away a partner, what would he be left with?
Damon: Partner?
Kelly: The other partner, the box, right. The one he didn’t know. Who else can repeat and say what Gary had just said?
Danika: He said that, ummm.
Kelly: What did he start with?
Danika: The total.
Kelly: And he knew if he started with the total, what would happen if he subtracted?
Danika: He.
Kelly: What did he subtract from the total?
Danika: Partner.
Kelly: And what did he end up with?
Danika: Partner.
Kelly: The other partner. Good job. (Hillary Teaching Session, 11/01/12)

This example illustrates how the students struggled to simply listen and repeat classmates. I had to prompt the students in order to help them repeat.

**Planning for High Cognitive Demand**

Throughout our work together, Dina learned the importance of how whatever current lesson she is teaching relates to previous and future lesson planning. During our planning session on August 17, 2012, Dina used the directions given in the teacher’s edition and did not want to stray much from the textbook. She did not consider what students knew when planning. However, on September 28, 2012, she said that she would need to focus more on vocabulary, based on formative assessments she had given. She thought about how her students were currently solving addition and subtraction problems and how to move their thinking forward. She considered the cognitive demand of the
lesson from the teacher’s edition, including different alterations she could make to it to raise the cognitive demand.

The planning process took more time than I originally thought when I planned this design experiment, however it required less time throughout the duration of the project. We planned for approximately one hour per day. It took longer at first because we spent a lot of time discussing ideas. As we worked together, the planning began to take less time because Dina seemed to understand the main concepts within my study cognitive demand and discourse. She was able to quickly determine the level of cognitive demand of tasks from the textbook and consider ways to increase the cognitive demand. She also was able to foreshadow the discussion that the class would have around tasks, which allowed her to anticipate her questioning strategy.

Hillary struggled to see the reasons for the purposeful planning sessions that we were having. She did not see how planning so in depth could really effect her teaching and in turn, student learning. I saw her struggle to implement the curriculum with fidelity, while implementing new strategies she learned from professional development. The following excerpt illustrates Hillary’s perception of her own planning and lesson implementation.

Hillary: I’m writing down my talk moves on every page.
Kelly: I think it’ll come more natural in a couple of days as you get used to it.
Hillary: I think so too. I’m just, I’m a bad planner. I don’t always go in the order I should. (Hillary Planning Session, 10/22/12)

Later she said that if she had something specifically written in her plans, she was more apt to do it, but still only wrote page numbers and lesson numbers in her plan book, a typical planning practice of many teachers, as noted by O’Donnell and Taylor (2007). I
suggested making a Power Point presentation of the problems that she wanted students to solve and use the Power Point slides with students during her math class, so that she would have a quick reference regarding what she had planning and she could more easily refer to her plans while teaching, which helped. She was able to consider the numbers in the problem that she was using prior to the lesson, rather than in the moment, which proved to be beneficial for her.

Hillary did try to plan tasks that built on the difficulty of previous lessons, but struggled to help students make meaningful, long-term connections. She recognized when tasks were repeated in the textbook and tried to alter instruction as to not repeat, but when she saw that students would be bored with a lesson due to the cognitive demand being memorization, she still planned the lesson as it was written in the textbook. She said she wanted to allow students to discuss mathematical ideas in order to raise the cognitive demand, but she struggled to enact a discussion according to her plans. On October 24, 2012, she also suggested that she type up her lesson plan as a way of implementing the plans with fidelity. She thought writing up plans and having them in front of her while teaching would help her stick to the plans more closely. During lessons she often forgot what we discussed during planning, however she hoped that having the plans right in front of her would remind her how she wanted to enact the tasks.

Both Dina and Hillary viewed planning as simply pacing out the lessons, as they stated in our first preplanning session. They both said they used the suggested pacing guide given to them by the district as a framework for planning daily lessons. Both teachers planned daily lessons by writing down the lesson and page number in their
planbook, which was all their lesson planning consisted of. Through our work together, both Dina and Hillary began to see that what they were doing was pacing and that planning involves much deeper thought about the students and the mathematics. I could see this change through what they wrote in their plan book. During the first planning session, both teachers wrote notes in their plan book, but as they realized how much it would help them stick to their plans, they wrote more for each day, for the rest of the planning sessions.

Dina and Hillary displayed different levels of understanding of mathematics and teaching mathematics during planning. They both wanted to help their students make connections and understand mathematics. Dina’s successful implementation of tasks of high cognitive demand could be related to her depth of knowledge of previous and future mathematical standards, possibly due to her sixteen years of teaching experience. Hillary only had three years of experience, so it is possible that she did not have the same depth of horizon knowledge as Dina.

Westerman (1991) identified characteristics that novice and expert teachers display when planning, see Figure 17. Westerman studied how teachers planned and found a list of factors that separated the ways in which teachers plan. Based on that list of factors, I inferred that Dina would be considered an expert and Hillary would be considered a novice, however both teachers displayed characteristics of both types of planner. It is important to note that experience does not mean expert and inexperience does not mean novice. The factors are solely based on the way in which teachers plan, not necessarily the way in which they carry out the plans during instructional time.
Dina looks at lessons from the perspective of the students. For example, on September 6, 2012, while planning a lesson, she told me that it would be easier for her class this year to learn addition strategies because they were exposed to them the previous year.

<table>
<thead>
<tr>
<th>Expert teachers…</th>
<th>Novice teachers…</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Think about learning from the perspective of the student</td>
<td>• Use specific lesson objectives to form structured lesson plans</td>
</tr>
<tr>
<td>• Perform a cognitive analysis of each learning task during planning</td>
<td>• Do not adapt to meet student needs during teaching</td>
</tr>
<tr>
<td>• Adapt to the needs of the students during teaching</td>
<td>• Lack metacognitive and monitoring skills</td>
</tr>
<tr>
<td>• Have well-elaborated schemas that allow them to monitor classroom situations, recognize problems, and make decision to solve problems</td>
<td>• Have difficulty with discipline and classroom management than with delivery of subject content matter</td>
</tr>
<tr>
<td>• Attend to a larger number of instructional goals in making interactive decisions</td>
<td>• Form mental representations of their lessons that are too narrow or incorrect and that therefore lead to problems during teaching</td>
</tr>
<tr>
<td>• Use a larger range of instructional strategies</td>
<td>• Have less elaborate, less interconnected, and less accessible cognitive schemas</td>
</tr>
<tr>
<td>• Link their actions to student cues in complex ways</td>
<td>• Do not have enough knowledge about the overall curriculum nor sufficient awareness of student characteristics to allow them to perform an adequate cognitive analysis of the lessons they are planning</td>
</tr>
<tr>
<td>• Have an understanding of what to expect in the classroom and therefore set up procedures and rules for student behavior</td>
<td>• Rely on the curriculum objectives as prescribed by their superiors</td>
</tr>
<tr>
<td>• Use curriculum guidelines as a foundation for building lessons and making them uniquely their own by changing, combining, and adding them according to their students needs and their own goals</td>
<td>• Often start their lessons without recalling students’ prior knowledge</td>
</tr>
<tr>
<td>• Often double back to repeat a point, relate new information to prior knowledge, and assess their students’ understanding</td>
<td>• Do little to relate present learning to past or future learning</td>
</tr>
<tr>
<td>• Summarize the discussion and set the stage for what is coming next</td>
<td>• Do not adapt their lessons even when the children become restless from being too long at a task</td>
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<tr>
<td>• Monitor lessons and adapt accordingly</td>
<td>• Stick closely to their lesson plans, sometimes ignoring students who bring up interesting points for discussion</td>
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<tr>
<td>• Use well-practiced classroom strategies or routines</td>
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*Figure 17. Factors of expert and novice teachers as planners (adapted from Westerman, 1991).*
year through the implementation of *Math Expressions*, whereas her classes in previous years used a traditional curriculum. During planning sessions, she referred to her experience in previous years and offers suggestions of how students might solve a problem. This reflective nature allows her to anticipate student confusion and consider how students will perceive problems, which enabled us to plan lessons of high cognitive demand including classroom discourse.

Additionally, Dina was able to cognitively analyze the tasks within each lesson by considering the cognitive demand, student errors, and how she might teach the lesson in order to maximize student understanding. On both September 10 and 28, 2012, she analyzed the connections that students would make between equation representations. She considered where gaps might exist if the curriculum was taught straight from the teacher’s edition with no alterations. Specifically, on September 28, 2012, Dina noticed that the teacher’s edition shows ways for students to write equations, but does not connect the representations. She stated that the connections are up to the teacher, which can be difficult for some. She sees the goal of a task and how it is connected to other tasks and assessment. Although her first inclination was to follow the teacher’s edition to a fault, she began to see the meaningful opportunities that could be afforded through altering the lesson even in minimal ways.

Dina also displayed a few characteristics of a novice planner. Initially, she wanted to plan concepts in the way she had before; she wanted to rely on non-mathematical strategies for teaching math, rather than considering deeper more meaningful conceptual understanding. For example, on September 10, 2012, she explained how she had taught
inequalities in the past; she drew the greater than and less than signs into a goldfish, which is not mathematical, see Figure 18. However, with time, I was able to open her eyes to the benefits of strategies rooted in mathematics, such as teaching students to read inequalities as a number sentence. Dina does not have strong mathematical skills, which may account for her initial use of cute strategies, rather than sound mathematical strategies.

Figure 18. An example of Dina’s non-mathematical teaching strategy.

In contrast, Hillary had a narrow view of the individual lessons. She struggled to see the mathematical connections between each lesson, as I noted during our planning session on October 17, 2012. She said she wanted to try new ideas, but struggled to implement them. Due to her narrow view of a lesson, she often left herself to “wing it.” On October 22, 2012, she specifically said that she does not like to decide what she will actually do with students before the lesson, rather she prefers to see how the lesson goes and make split-second decision. She comes up with instruction while she is teaching, which in her view is being responsive to the students. If she had a broader view of the lesson and how it can connect to previous and subsequent lessons, she would likely be able to plan for multiple options, rather than one, which would allow her to be more
purposeful and not “wing it.” Hillary viewed the lessons as independent and struggled to help students make connections between strategies.

However, Hillary does display characteristics of an expert teacher too. She looks for connections that students need to make and tries to incorporate reasonableness into her lessons, as I specifically noted in my field notes.

Hillary includes the +-- signs in the math mountain. She introduces the connection between the math mountain and the subtraction equation. This is a strong link! (Kelly Field Notes, 10/24/12)

Hillary has strong mathematical skills and could plan more purposefully to increase student understanding. At times, Hillary’s planning seems to be all for nothing because she does not follow through with implementing the planning. She has a vision of excellence, but has several roadblocks implementing ideas. She forgets or loses track of what the goal of the lesson is and gets sidetracked easily.

Both Dina and Hillary recognized that the teacher’s edition gives strategies to teach, but the teachers are responsible for making connections between the lessons and strategies. Specifically, the teacher’s edition gives one strategy per lesson, but does not tell the teacher how to connect the strategies to each other. The realization that connecting the strategies is the work of the teacher was an important understanding that helped Dina and Hillary see the importance of purposeful planning. They both said that in order for students to make important mathematical connections and generalizations, as the teacher, they need to plan ahead and be purposeful about each part of their lesson.

Dina and Hillary each have characteristics of both expert and novice teacher planning. The major characteristic that separated them as planners was their perspective
on the lessons. Dina was able to view problems from the students’ perspective, while Hillary viewed the problems from only the teacher’s point of view. Dina was open and willing to think about new strategies and consider different student perspectives, which was evident by her suggestions in our planning sessions and found to be a crucial piece of effective planning by other researchers (Grant, Kline, Crumbaugh, Kim, & Cengiz, 2009; Stein, Engle, Smith, & Hughes, 2008). She was open to my suggestions. Conversely, Hillary thought she already knew what worked and what did not work. This became evident in our planning sessions because she was not always willing to change the plans she came up with. Hillary, although a fairly inexperienced teacher, felt that she had a good grasp of what her students knew and needed to learn. For example, she struggled to allow students to share their own ideas, but rather she wanted to make sure the exact language was used so she shared the students’ ideas herself. This closed-mindedness did not allow Hillary to give control of the learning over to her students.

I found that in order for teachers to plan activities with high cognitive demand, they needed to spend time examining each task to determine cognitive demand and what meaningful discussion students could have around the task. In this study, both of the teachers considered how the mathematical concept would be presented and how students would be asked to demonstrate understanding. We also considered what connections the students needed to make with the mathematical concepts. This deeper thought process while planning allowed the teachers to utilize tasks of high cognitive demand and maintain high levels of cognitive demand while teaching.
The role of curriculum in planning. Although, by definition, *Math Expressions* (Fuson, 2011) is considered to be a reform curriculum, I found that the text gives little meaning to procedures. The book does not always provide the teacher with the underlying mathematical knowledge necessary for them to help students make connections and generalizations. If the teacher follows the text, students will be allowed to explore and think on their own, but not with much conceptual depth. The teacher’s edition leaves connections and generalizations up to the classroom teachers to make, some of whom may not have the mathematical knowledge to do so. Teachers end up using their, sometimes limited, teaching experience to fill what they perceive as gaps in the curriculum. Uninformed alterations to the curriculum are not ideal. With help from myself as the researcher, Dina and Hillary made informed alterations to the curriculum during planning sessions to improve student understanding.

*Math Expressions* (Fuson, 2011) includes tasks of varying levels of cognitive demand. See Figure 19. I examined all of the tasks within the lessons we planned to determine the cognitive demand of the tasks as they appeared in the actual text. I followed the guidelines of Stein, Smith, Henningsen, and Silver (2009) when labeling each task as Level One: Memorization, Level Two: Procedures Without Connections, Level Three: Procedures With Connections, and Level Four: Doing Mathematics. Notice that the cognitive demand of most lessons hovers around level 2, procedures without connections. This is reinforced by the teachers’ view that the text does not connect ideas, rather the teachers are responsible for doing so. Also notice that no lesson has a mean
**Figure 19.** Mean cognitive demand of first grade tasks.

**Figure 20.** Mean cognitive demand of second grade tasks.
cognitive demand over level three. This data is only the cognitive demand as the lesson was written in the teacher’s edition, not as it was planned and implemented in this research study.

At the beginning of my work with Dina, I asked how she planned and how our work could fit into her current planning. I did not intend on having her spend much more time planning than her current practice, but rather to help her use the plan time in a different way.

Kelly: When you plan out the unit for the team are you just pacing or actually planning?
Dina: I am pacing. Everyone kind of does their own plans or we have in the past anyway because there were 7 on the team. This year we only have 4 teachers on our team. (Dina Planning Session, 08/17/12)

She perceived planning as writing the lesson number in the plan book and glancing at the pages to skim the content. She did not plan to read through the teacher’s edition to connect to previous and future lessons. Essentially what Dina referred to initially as planning was really pacing the objectives.

I also asked Hillary about her planning habits before working with her. She informed me that she stuck to the teacher’s edition for planning, but then did not always follow the plan. She believed that she needed to follow the lesson pacing of the teacher’s edition because that is what is expected of teachers in the district. She reported that a directive had been given by the district leaders to implement the instructional resources with fidelity, meaning no variance from the teacher’s edition. However, she did want to be responsive to students’ needs while implementing the textbook with fidelity. However,
in an attempt to be responsive to students she did not always stick exactly to the teacher’s edition lesson plans.

Through my work with both teachers, I saw changes in their planning and their use of the teacher’s edition. Both teachers began to use the teacher’s edition as a resource, rather than a plan book. I found that the teachers learned to use the curriculum as the basis for planning and pacing, rather than simply copying page numbers from it. Planning with the teacher’s edition is a crucial step in meaningful planning, but certainly not the only planning activity. The teachers realized that the curriculum should not be used as the sole document for pacing and planning. The teachers had additional knowledge and skills that they could capitalize on while planning to maximize student learning.

**Planning for discourse that supports high cognitive demand.** Initially, Dina and I focused on her use of the talk moves, but before long, they became automatic for her. Specifically, when we were planning the discussions, we talked about ways she could incorporate the talk moves into the discussions. During a planning session, we had the following conversation.

Kelly: So what I was thinking, sort of reflecting on it afterwards with those questions. One thing that I was thinking is that you did a couple of revoicing. Dina nods. But when the kids were sitting on the floor, and you were having them share. Like when Gabriel got up and shared his or whatever, that would have been an opportunity to have like, to do repeating.

Dina: Yeah, I know.

Kelly: Somebody repeat what he said, or. I mean I was just thinking of other opportunities.

Dina: I was thinking of it the whole time I was up there, but I was so flustered with how the lesson was going.

Kelly: Well, and sometimes.
Dina: Let’s just speed this along.
Kelly: Well but when it’s you, sometimes it’s hard to see the opportunities. Me sitting in the back of the room, I can see it a little easier.
Dina: Yeah.
Kelly: Or even in that instance to agree or disagree. Does everyone agree with this? Dina nods.
Dina: We did do that the other day.
Kelly: So I was just thinking, because you mentioned before that sometimes you forget to use them.
Dina: Yeah. (Dina Planning Session, 09/06/12)

Additionally, she asked students how they solved a problem, then either revoiced or had another student repeat, to reinforce the said strategy. She often asked students to agree or disagree with the student who was explaining his or her strategy, and then explain why they agreed or disagreed. Her students were attentive and willing to participate, which helped create meaningful classroom discussions.

Once the talk moves became automatic for Dina, we focused on what connections and generalizations she wanted students to make. We then designed discussion questions to help students think about the mathematical concepts. During planning, we discussed what students might say in response to her questioning so that she would be prepared for most responses that students might have. We talked about not only the tasks and the discussion questions, but also how the lesson might play out, especially what students might struggle with and which students might struggle. The following discussion took place when we were planning a lesson about different kinds of problem types.

Dina: I think they are not going to pay attention to, I think they are going to say, this is a change plus or a change minus because they are going to look at, what do I need to add or subtract. I don’t know if they are going to pay attention to, am I going to…
Kelly: Yeah.
Dina: Am I joining just horses or am I? Cause like there, 13 people on a bike, 8 are on top, the rest are on the bottom. They’re going to look at
that as 13 equals 8 plus what? And they are going to solve. I don’t think they are going to say like, oh this is a collection. It’s all people. I just don’t think they are going to do that.

Kelly: Although if you…
Dina: I don’t know. Maybe I’m wrong.
Kelly: But it’s together so you could say, we are putting things together that means adding. Separating things is taking away.
Dina: Yeah, that’s true.
Kelly: Or is there’s no action. So on the first one they’re going to say, it’s all people and some are at the bottom. But like I would think that’s subtraction. You know what I mean? 13 minus 8.
Dina: And some kids will do that. (Dina Planning Session, 09/28/12)

The discussions Hillary and I had about the talk moves, were much the same as the ones Dina and I had, however, Hillary had significant struggles implementing the talk moves. When asked to, Hillary’s students could not repeat what another student said, which was frustrating for Hillary. She did not see the benefit in having her students repeat when it was such a struggle for them to do so. In my pre-observation notes I wrote about this topic.

I was concerned that yesterday’s planned use of the talk moves was too much for Hillary and the kids. I am going to talk to her prior to the lesson to propose that she only focus on revoicing. Then she only has to focus on one move and the kids will see proper ways of revoicing, then when it is their turn, they will be able to do it better. (Kelly Pre-observation Notes, 10/23/12)

We discussed the rationale of repeating, for students to hear an idea more than once and explained by more than one person, both of which can deepen understanding (Chapin, O’Connor, & Anderson, 2009). After that, she seemed to be on board to try one talk move at a time.

We concluded that her students had a difficult time repeating because they had a difficult time listening to other students. In the following excerpt, Hillary talked to the students about the importance of listening and how to be a better listener.
Hillary: We need to understand. And what do we do so we can understand? A lot of times we talk. I’m going to tell you something. And actually, me and Miss Kelly talked about this this weekend. *You cannot be good at math talk* (writing this on white board), which is what we do a lot. Do you remember how we do that in the morning during daily math routines?

Students: Yes.

Hillary: But what we talked about was, cause I said my kids are so good at math talk. But you know what, *you cannot be good at math talk until you*…

Student: Understand.

Hillary: Well yes, you do need to understand. This helps you understand. *You are good at math*…

Student: Learning. (other inaudible answers)

Hillary: Close. Does anyone know what this word is?

Students: (inaudible)

Hillary: Not learning. Listening. And that is where I’ve been having some trouble. You guys know how to tell me your answers, but you don’t always know how to listen to each other. Cause even like right now, you’re really good at talking, but are you all listening to me?

Students: (inaudible)

Hillary: Some of you even have your eyes on me. But if you’re playing with your hair or your shoes, are you really listening to me?

Students: No.

Hillary: No. So today and tomorrow we are going to work really hard at math listening, cause that’s how we are going to get better at math talk. (Hillary Teaching Session, 10/29/12)

In this conversation with her students, she goes on to discuss small strategies that students can use to listen, such as keeping one’s eyes on the speaker and facing the speaker. She also changed where the speaker stood so that the students’ eyes and ears could focus on the same location. This seemed to improve the students’ listening skills because they were better able to repeat another student.

During planning, Hillary and I created essential questions for each lesson that she should ask students about the mathematical concept. She often wrote them down in her
planbook, but rarely asked the question during instructional time. She struggled to both plan and carry out meaningful classroom discussions.

I think at times she strays from the lesson when she thinks she needs to. However, staying with the planned lesson is often what the students need in. (Kelly Field Notes, 10/26/12)

It is helpful that teachers implement their planned lessons in order to make progress through the learning trajectories. Although she had her planbook at the front of the room with her when teaching, she rarely referenced it. She was used to just having page numbers and not needing to reference it. Through our work, she began to write more down, but still rarely referenced it.

It was helpful for both Dina and Hillary to plan big idea questions for their discussions. It helped them see the connections and generalizations they wanted students to make, prior to teaching the lesson. For example, Hillary and I were discussing how to review the unit.

Kelly: If you did one of each, you know and spent like 15 minutes. Like okay, solve it. Let’s talk about different ways to solve it. How did you know? Like on, Ann had 8 crayons, she lost 2 of them, how many crayons does Ann have now? How did you know that 8 was the total? What were the clues that made you know that 8 wasn’t a partner?

Hillary: That’s good.
Kelly: Because that’s where they get tripped up.
Hillary: Right.
Kelly: And it’s not a keyword strategy. It’s more of like, look at this problem…
Hillary: Like the context.
Kelly: Yes. (Hillary Planning Session, 11/2/12)

Both teachers wrote the questions in their planbook and had it near where they were during the lesson. Even though the specific questions may not have been ultimately asked
by either teacher, at least they had thought about the mathematical ideas prior to teaching
and had a target of what they wanted students to know at the end of the lesson or unit.

Dina’s automatic use of the talk moves allowed her to think about the big
discussion questions and helped her students develop a deeper conceptual understanding
of the mathematics. She reported that she was not bogged down with thinking about the
talk moves, rather, she was using the talk moves to get at the big conceptual ideas. In
contrast, Hillary’s struggle to use the talk moves rendered her unable to focus on the big
questions while she was engaged in classroom discussion. Hillary’s main focus during
classroom discussions was implementing the talk moves. The following excerpt
illustrates her focus on using the talk moves.

I really scaffolded my use of talk moves. Students explained, I repeated using
explicit vocabulary, then students repeated me. We did not move past the
repeated talk move. (Hillary Journal, 10/27/12)

Teachers can plan questions that prompt meaningful mathematical discussions,
which can raise or maintain high cognitive demand. Knowing the essential questions that
the class is working toward gives the teacher a target. Teachers can also familiarize
themselves with the talk moves and use them to help enhance mathematical discussions.
The teacher can use the talk moves to prompt discussion to work toward answering the
essential questions.

**Using student understanding to plan.** Even though I planned to explicitly use
the Sarama and Clements (2009) learning trajectories during planning with the teachers,
we did not. Initially I thought the teachers and I would examine student work in light of
the learning trajectories. However, it did not play out that way in practice. *Math*
Expressions (Fuson, 2011) is set up to teach strategies for addition and subtraction along their own trajectory that sequences strategies in a way that students’ knowledge builds from concrete to abstract. For example, count all is taught before count on, which is before make a ten, etc. See Figure 21 for the addition and subtraction learning trajectories in Math Expressions. When the teachers and I looked at student work, we discussed what strategies students were using and how we might move them along the Math Expressions learning trajectories for addition and subtraction, to use more sophisticated strategies.

These learning trajectories are different from the learning trajectories of Clements and Sarama (2009) in that they are a progression through strategies, rather than a progression through the development of the strategies. Clements and Sarama offered ways in which teachers could help students move along the trajectory, whereas Math Expressions only offers teachers suggested instructional strategies.

Dina simultaneously considered what struggles and successes students from previous years experienced in the curriculum and knowledge of her current students, to determine the tasks in her lessons. I wrote the following in my journal.

What did the teacher base her instructional decisions on? What helped her decide which tasks to implement?
Based on our discussion and the teacher’s knowledge from last year, she decided to skip the number path for now. She will address it during math routines. Also based on our discussion, she adapted the challenge card at the end of the lesson to be a game for all of the students. (Kelly Journal, 09/06/12)

Additionally, she took into account her students’ individual needs. She had several students with special education needs in class and always made accommodations to help all students succeed. On September 6, 7, 8 and October 3 and 4, 2012, I wrote in my field notes that Dina specifically worked with her special education students. Dina used her
<table>
<thead>
<tr>
<th>1st Grade</th>
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<th>Subtraction</th>
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<tbody>
<tr>
<td>Addition</td>
<td>Break Aparts</td>
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<td>Simple Pictures</td>
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<td>Circle Drawings</td>
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<td>Number Sentences</td>
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<td></td>
<td>Count On</td>
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<td></td>
<td>Unknown Partner</td>
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<td></td>
<td>Mixed Unknowns</td>
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<table>
<thead>
<tr>
<th>2nd Grade</th>
<th>Addition</th>
<th>Subtraction</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Make a Ten</td>
<td>Make a Ten with Teen Totals</td>
</tr>
<tr>
<td></td>
<td>Count On</td>
<td>and Subtraction</td>
</tr>
<tr>
<td></td>
<td>Make a Ten with Teen Totals</td>
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<tr>
<td></td>
<td>Relate Addition and Subtraction</td>
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<td></td>
<td>Add and Subtract on a Number Line</td>
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<td></td>
<td>Equation Chains</td>
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<td></td>
<td>Add 3 numbers</td>
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<td></td>
<td>Solve Different Problem Types</td>
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*Figure 21. Learning Trajectories based on Math Expressions.*
knowledge of her students and the curriculum to anticipate confusion, consider how students would solve a problem, and group students to work on math tasks. This was helpful to those students who would have otherwise struggled.

Hillary did not take the same perspective as Dina on how well students need to understand the mathematical concepts at the end of the class time. Hillary felt that if she taught the lesson and students did not understand, the concept could just be readdressed during intervention time. Hillary’s school had planned intervention time to address student needs, such as English Language Learners and Special Education, whereas Dina’s school did not. For this reason, Hillary probably did not know as much about her students’ depth of knowledge as Dina did, simply because she did not pay as much attention to it. Hillary did not spend as much time listening to her students explaining their thinking. Therefore, she had less understanding of why students were struggling with a concept. Hillary did try to anticipate confusion and think about how students would solve a problem, however, she often wanted them to solve a math problem in a specific prescribed way, and looked for divergences from that. The following excerpt is from our planning session. Hillary describes a struggle that she perceives the students to have. Notice that she has described understanding a procedure, rather than understanding the relationships between numbers.

Here’s the problem, it seems like we almost have to because what’s happening with my kids is like right now we are doing missing partner or missing total. Now we are going to switch over to subtraction. They get so confused on, okay if it’s missing total I do this, if it’s missing partner I do this. (Hillary Journal, 10/24/12)

Both teachers attempted to anticipate students’ confusion and solution strategies, however they used the information they learned differently. Dina used what she knew
about students to build onto their existing knowledge. For example, I wrote the following in my field notes about Dina.

Does first problem with students. She reads the problem and asks what they already know. She relates the problem to the students and says it in terms that the students understand. Has the students solve and move back to the floor. We have discussed that the students are more engaged and participate more when they are on the floor. (Dina Field Notes, 10/02/12)

Dina used the students’ understanding to help all students understand and learn the mathematical concepts. In contrast, Hillary struggled to understand her students and even used the students’ understanding to alter how students were solving in order to match her solution method. The following excerpt shows Hillary trying to allow students to share their solution strategies. She and the students just read the story problem aloud together.

Hillary: Freeze. What did he do?
Class: (Several different answers being shouted out.) Take away.
Hillary: He took away how much?
Class: Two.
Hillary: Why?
Class: Because he wanted to buy a car.
Hillary: Cause he’s buying a car. So where are we going to put two? Down here. That is a partner.
Student: And then what’s the other partner?
Hillary: How are you going to find this other partner?
Class: Drawing dots.
Hillary: Count on. So start with two.
Hillary and her class: Two, three, four, five, six, seven, eight.
Hillary: But Gary had a different way. Gary, what was your way?
Gary: Cross out the two pennies.
Hillary: I spend one, two pennies on a car. But now look. Some people tell me. (Stops discussion to discipline student.) How many coins do I have left? Two coins. (Hillary Teaching Session, 10/25/12)

Notice that when the students suggest drawing dots to solve, she skims over that and works on counting on. She also abandons Gary’s method and takes over the explanation of his work. Hillary was aware that she did not always understand her students well.
Student understanding/learning trajectories, in this case, actually challenged and confused me as the teacher. I was unsure of the focus of my lesson, because it involved story problems, math mountains, and coin. All of these concepts were not firmly understood by my students, so I struggled with how to integrate these concepts but pull from them the most important conceptual understanding. (Hillary Journal, 10/26/12)

Cognitive Demand of Enacted Lessons

Dina implemented her lessons as we planned, therefore, her enactment of the plans had a high cognitive demand. See Figure 22 for the cognitive demand of lessons as planned and enacted. All of Dina’s lessons were implemented as procedures with connections or doing mathematics or somewhere between them. The numbers in the graph are the mean of one to three observed tasks. The days in which the cognitive demand lowered from planning to enactment may be due to me giving one overall score for cognitive demand, rather than separate scores for each task in the lesson. If I had labeled each task separately, some tasks within the lesson may have maintained cognitive demand while other tasks may have decreased in cognitive demand.

![Cognitive Demand of 2nd Grade Lessons](image)

*Figure 22. Cognitive demand of planned and enacted lessons.*
In contrast, Hillary often varied from the plans that she and I made for the lesson. At times, this lowered the cognitive demand, and at times the cognitive demand stayed the same. See Figure 23 for the cognitive demand of both planned and enacted lessons. It seemed that when she got off track according to her plans, she still tried to have the students work on challenging mathematical tasks, but ones that were not necessarily the original focus of the lesson.

![Cognitive Demand of 1st Grade Lessons](image)

**Figure 23.** Cognitive demand of planned and enacted lessons.

She planned an activity that did not get carried out. It was high cd. What actually happened in today’s lesson was high cd, but so much time was wasted. (Kelly Journal, 10/29/12)

In these instances she seemed to lose sight of the goal of the lesson. She admitted did not follow her plans, but did not see the problem in doing that. However, she failed to recognize that the plans she made were meaningful and when she varied from the plans, her lessons often lost the connections that she intended for students to make.
For both teachers, high cognitive demand was often maintained when the teacher implemented purposeful lessons as planned. They kept the learning goals and essential questions in mind while teaching. However, if the teacher varied from the plans, cognitive demand often decreased. For different reasons, each teacher lost sight of the goals and questions essential to the lesson.

**Cognitive demand alterations during enactment.** Stein, Smith, Henningsen, and Silver (2009) researched factors that increase, maintain or decline the cognitive demand during the enactment of lessons. See Figure 24 for that list of factors. They found that the extent to which the teacher supports the students’ thinking and reasoning is a crucial part of determining the cognitive demand of the enactment of a lesson. In contrast, the extent to which the teacher allows the students to struggle with a task also determines the cognitive demand. For example, if a teacher cuts off an opportunity for students to make sense and reason through the task, the cognitive demand lessens.

I watched all of the videotaped lessons and recorded how often Dina and Hillary displayed each of the factors associated with decline and maintenance of high cognitive demand. See Figure 25 for the graph of how often Dina and Hillary displayed factors that decline or maintain cognitive demand (Stein, Smith, Henningsen, & Silver, 2009). The table shows the percent of days each teacher displayed each factor. Notice that Dina often displayed scaffolding, metacognition strategies, sustained press for justification, and access to prior knowledge, whereas Hillary often displayed routinizing, issues with classroom management, and having low standards. I found these factors to represent a major difference between Dina and Hillary’s teaching.
<table>
<thead>
<tr>
<th>Factors Associated with the Decline of High Cognitive Demand</th>
<th>Factors Associated with the Maintenance of High Cognitive Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Problems become routinized</td>
<td>• Scaffolding of students thinking and reasoning</td>
</tr>
<tr>
<td>• Emphasis is on the correct answer rather than meaning, concepts or understanding</td>
<td>• Students are provided with a means of metacognition</td>
</tr>
<tr>
<td>• Not enough time or too much time is provided for students to wrestle with the demanding aspects of the task</td>
<td>• Teacher or students model high-level performance</td>
</tr>
<tr>
<td>• Classroom management problems prevent sustained engagement</td>
<td>• Sustained press for justifications, explanations or meaning through teacher questioning, comments feedback</td>
</tr>
<tr>
<td>• Inappropriate task for the students</td>
<td>• Tasks build on prior knowledge</td>
</tr>
<tr>
<td>• Students not held accountable for high-level products or processes</td>
<td>• Teacher draws conceptual connections</td>
</tr>
<tr>
<td></td>
<td>• Sufficient time to explore</td>
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</tbody>
</table>

*Figure 24*. Factors associated with decline and maintenance of high cognitive demand (Stein et al., 2009).
Figure 25. Percent of days teachers displayed factors of declining and maintaining high cognitive demand.
Dina displayed most of the factors that maintain high cognitive demand throughout the enactment of a task. Dina often scaffolds her lessons by asking questions, which helps students clarify. I noted on several days that Dina began a discussion with the question, “What do you know?” The following is an example of this practice.

Dina: Let’s take a look at number one. This is an example of a change plus problem. So I’m going to read it out loud and I want someone to tell me again, tell me back what it’s asking or what kind of information we know. Okay? So number one says, last year, Breck and Skyler, our school had five computers in the library. They bought some more over the summer. Now there are twelve. How many computers did they buy over the summer? So, first of all tell me, what do you know from this story problem? What information do we already know? Tanner, what do we know?

Tanner: They had five computers.

Dina: Okay, stop there. So they got five, they started with five. Okay. What else do we know? Tai?

Tai: They bought some more over the summer.

Dina: Okay, do we know how many yet?

Tai: No.

Dina: No. So we had five and then we got some more. That’s kind of pretty cool in our library when that happens, isn’t it? When Mrs. Kristen has some computers and then she gets some more computers because then there is more space for us, or more computers for people to work on. Okay, now what else do we know? So we know how many we started with. We know we got some more, but we don’t know how many. And Jenna, what else do we know?

Jenna: That the total is twelve.

Dina: That the total what? It is twelve. Twelve what, I guess I’m asking?

Jenna: Twelve computers.

Dina: I ended up with twelve computers total. So we started with five. We know we have twelve now, but we don’t know how many in the middle, how many we got. So what is our question? What are we trying to figure out? What do we want to know? (Dina Teaching Sessions, 10/02/12)

This allowed students to focus on what they do know before they begin to solve and potentially connect it to prior knowledge. She also asked specific students questions to help them to solve the problem.
Dina helped her students think about their own learning by sharing different strategies and comparing them. For example, in the following excerpt, the class was working on addition problems with three addends. In class, Dina shared two strategies, make a ten and doubles, that could help them solve, however she allowed students to choose their strategy. The following excerpt illustrates how she discussed strategies with students.

Dina: Underneath that I had six and nine and four. Why did you circle six and nine?
Brynn: They were the highest numbers.
Dina: Okay, did anyone else circle something different on that one? Look at yours. Everybody look down. This is the third row down on the last row. Keegan, what did you circle?
Keegan: I circled six and four.
Dina: Why did you circle six and four?
Keegan: It makes a ten.
Dina: It makes a ten. So you could have done ten plus nine on this one and you would have gotten the same total that Brynn did on hers. (Dina, Teaching Session, 09/13/12)

She allowed students to use strategies that worked best for them, instead of a teacher-dictated best strategy. She used the talk moves to help students see that many different strategies can be used to solve the same problem. She allowed students to share their strategy and validated their strategy.

One of Dina’s best assets as a teacher was her sustained press for students to justify their solution strategy during classroom discussion. Often when a student gives a correct answer, the teacher praises the child and moves on, however, Dina questions the student in order to further clarify the concept for that student, and to build further understanding for the rest of the class. In the following example, students have written different ways to make the number thirteen. Dina had students share their solution and
proof and then had students agree or disagree, followed by another student proving the answer.

Dina: So she says thirteen equals six plus one, plus one, plus one, plus one, plus one, plus one. (Kayleigh points to her work and says something inaudible.) Oh and plus one down here. I’m sorry. Alright, take a look at that. Six, one more, one more, one more, one more, one more, one more, one more. Think about what that total is. See if that equals thirteen. Halle and Kylie I need you focused up here. When I say three, I will count to three, go ahead and keep your thumb to yourself. When I count to three you are going to do thumbs up or thumbs down. Remember how we have to prove everything? We are going to have to prove it to make sure it is right. One, two, three. (Dina looks around to see student responses. Students thumbs are up.) Okay, Jenna, how do you know?

Jenna: Because I counted up six, seven, eight, nine, ten, eleven, twelve, thirteen. (Shows using hands)

She told students that just getting the answer is not enough, but they needed to be able to show how they got the answer. She pushed students’ conceptions by asking “why” questions and asking questions in different ways.

The students in Dina’s class made conceptual connections because the basis of many classroom discussions was how different strategies could be used to solve the same problem, and how those strategies were related. She allowed students to use strategies that make sense to them, while connecting those strategies to what students already learned. She reminded students of strategies and concepts they had learned and how those were connected to the current strategies they are using. I specifically noted this in my field notes, when Dina and the students were discussing a difficult story problem.

Students are working on a problem they didn’t get to yesterday. It is a comparison problem, but involves 3 people, which is more challenging than yesterday. Dina gives the direction that students need to show how they figured it out, writing the numbers isn’t enough. She walks around and monitors students working and helps. She also chooses students to share. Christos does a good job explaining.
You can see that he has been taught to share. Dina prompts with clarifying questions to help Christos and the other students understand the process. Dina highlights yesterday’s strategy of making connecting lines. *Connection.* (Kelly Field Notes, 10/09/12)

Dina’s teaching is also minimally sprinkled with factors that decline the cognitive demand. During the first lesson I observed, Dina was confused between the counting on and counting back strategies. This confusion could have occurred because of her lack of mathematical knowledge for teaching as measured by the MKT test given in Primarily Math. She was trying to teach students when they should solve subtraction by counting on and counting back.

Do you have kids count on or count back? Dina shows counting back vs counting on – for 9-6 counting on is faster but on 8-2 counting back is faster. The discussion on efficiency is difficult now. This is why the problems a teacher chooses to present to students is so important. (Kelly Field Notes, 09/06/12)

The explanation that Dina gave students confused both her and the students. They were not able to distinguish when either strategy was the most efficient choice.

The following excerpt shows how Dina gave very prescribed directions on how to use a number line.

Dina: Let’s try another one and then I am going to have you practice some. So let’s say I have five plus three. Raise your hand and tell me where I am going to start. Tristan, where am I going to start?

Tristan: On five.

Dina: On five. So I am just going to put a dot there, so I know where I’m going to start. How many hops am I going to make Tristan?

Tristan: Three.

Dina: I want you to count on as I make my hops. Ready? Five, six, seven, eight. So five plus three is eight. (Dina Field Notes, on 09/09/12)
She perceived the detailed directions as simply being specific, but she was routinizing the problems for the students, taking away the problematic nature of the mathematical work, which is an opportunity for the students to learn.

Hillary displayed many factors associated with declining cognitive demand while implementing tasks, with sprinkles of maintaining factors. For most tasks, Hillary taught students a procedure that they were to follow. Her lessons were often step-by-step directions for the procedure. She had students copy what she was writing and asked fill-in-the-blank questions about the procedure. The following example illustrates her questioning strategy when discussing a combination of ten problem

Hillary: I had a couple of people tell me that two partners for ten would be ten and zero, and zero and ten. Are these partners of ten?
Class: Yes.
Hillary: Do we concur on that?
Class: Yes, we concur.
Hillary: But, can I use them for this?
Class: (Mixed yes and no)
Hillary: Thumbs up if you think yes. Thumbs down if you think no. Pause. Who thinks I can use these? Devin, why can I use those?
Devin: Because they are opposite.
Hillary: Okay, because they are opposite and they are partners of ten. But I got to tell you something. I can’t use these. If I spend ten dollars on stickers, would I have any money to buy candy?
Class: No.
Hillary: And what does this say?
Hillary and Class: We buy stickers and candy.
Hillary: So I can’t spend zero dollars on stickers and all of it on candy. Can I spend zero dollars on stickers and all of it on candy?
Class: No.
Hillary: Does that work with this?
Class: No. (Hillary Teaching Session, 10/14/12)

This way of teaching limited the students’ thinking and reasoning about the mathematics.

A majority of the questions she asked could be answered by yes or no, or were fill-in-the-
blank questions with only one correct answer. Throughout the lessons I observed, she often asked questions such as, “What do you do next?” which impresses upon students that the mathematics to be learned is simply a series of steps. This type of questioning emphasizes a correct answer. When students gave a correct answer, she praised them and moved on, whereas when a students gave an incorrect answer, she asked questions to help lead them in the right direction. A correct answer is not the end of the students’ work; they should be pressed to explain how they arrived at their answer.

Another factor that often lowered the cognitive demand was Hillary’s classroom management. Although she believed that she was a good classroom manager, her students were obviously disruptive during class. Her students often shouted out while working at their seats and sitting on the carpet for discussion. Although the following example only took one minute of class time, it was a reoccurring issue that continued to disrupt instructional time. This happened multiple times through the math time. Additionally, the behaviors that Hillary stopped for over and over were behaviors that Hillary should have been able to nearly eliminate. It was difficult for Hillary to carry on a meaningful mathematical conversation with such constant interruptions.

Hillary: What we are going to do today is we are going to do a bunch of word problems. Whispers. Can I have your eyes right here? (Does hand gestures to gain students’ attention.) This is actually not a time to mess with your red ribbon. It is not a time to tie your shoe. What are we working on right now?

Students: Learning.
Hillary: Learning what?
Students: Math.
Hillary: So where should your eyes be?
Students: On you.
Hillary: And Angel and Damon, if they are not on me, are you going to be able to understand these directions?

Students: No. (Hillary Teaching Session, 10/26/12)

She spent a lot of instructional time redirecting students, which interrupted the flow of the discussion, and often times required her to go back and repeat directions or questions related to the mathematics. At times her directions were unclear, so she had to repeat them multiple times, which lengthened transition time, thereby losing instructional time.

Hillary did attempt to engage students in meaningful discussions about mathematics, but this most often occurred when students were at their seats working with partners. She asked students what they knew about the problem in order to get them started working on a math problem. She tried to help students avoid mistakes by pointing out traditional mistakes that students typically make.

Beginning on October 24, 2012, Hillary improved her attempts to help students make conceptual connections. The following excerpt describes Hillary helping students make connections between a story problem and a math mountain after they have just read the problem aloud.

Hillary: What would my equation be?
Raina: Eight minus four equals four.
Hillary: Does that work with my math mountain friends? Inaudible response. Look. Eight minus four equals four. What if I go this way? Help me again. (Stops to redirect student.) Eight minus four equals four. It is a double. Now what other equations. Now we did this for homework and lots of my friends could get one or two. (Stops to redirect student.) What is another equation I could get with this math mountain? There’s only two since it’s a double. Damon?
Damon: Four plus four equals eight.
Hillary: Four plus four equals what? Eight. Does that work?
Students: Yes.
Hillary: Four plus four equals eight. (Hillary Teaching Session, 10/29/12)
She tried to help students connect story problems to equations and reason why they match. She also pushed students to see the connections between a math mountain and the four equations she asked students to write based on it. With more time to practice good sense-making strategies, Hillary could have improved her teaching and her students’ understanding.

Both Dina and Hillary tried to focus on conceptual connections during our planning sessions. The focus on conceptual connections was likely due to the work we did identifying questions that would help students make connections and generalizations. Hillary focuses on connecting the math mountain to the equations in the following segment. The students started with the math mountain, 3, 7, and 10, and were asked to write four equations to go with it.

Flower: Ten take away seven equals three.
Hillary: Why? Why does that work?
Student: Inaudible.
Hillary: Okay, because all those numbers are on the math mountain, yes. Karla, why does this work? (Teachers comes in and interrupts.)
Karla: Because it, it works cause if you start with the total you can take away, uh, you can start with the total and take away, inaudible.
Hillary: Did anyone hear what she said? Well, a couple people did so if you are listening you might be able to hear. Alayah, what did she say?
Alayah: She said ten equals
Hillary: What did she say? What did Karla say?
Flower: She said if you start with the total, you have to take away.
Hillary: Yes, she said if you start with the total you have to take away. Now what did she take away?
Student: A partner.
Hillary: Oh my gosh, that was smart. (Hillary Teaching Session, 11/7/12)

She continued by having students repeat this idea in order to try to reinforce student understanding. Our focus on big-idea questions helped both teachers see the target at the end of the lesson or unit. Since the teachers had a clear target of where they wanted
students to be at the end of the unit, they were able to be more focused in their daily interactions with students.

I see Dina as being farther along in her understanding of cognitive demand and how to maintain high cognitive demand while teaching. Below is an example of their journal responses to the same question.

**Question:** How did you think about planning for the maintenance of high cognitive demand?

My goal was to provide examples of story problems that made all students hold accountable for solving the problem independently and then be able to share it with the class. All students were expected to be engaged through working the problem on their own or listening so they could repeat the strategy of another student if called upon. (Dina Journal, 10/08/12)

I thought most about the vocabulary that I needed to use in order for my students to be able to work with the materials and wording of mathematical situations. (Hillary Journal, 10/18/12)

Hillary has less understanding of what students needed to know to be successful in subsequent grade levels, and therefore struggled to see the mathematical connections and generalizations that need to be made. Hillary spent more time focusing on correct answers and how well students followed procedures, rather than the fruitful discussion that comes from students using and sharing multiple strategies.

For a teacher to be highly successful at engaging their students in tasks of high cognitive demand, they need to plan in a purposeful manner, then implement those plans with fidelity. The more a teacher varies from their plans, the less likely it is that they will help students make connections and generalizations, and therefore the less likely the students are to conceptually understand the mathematical idea. Teachers must teach the
curriculum with fidelity while being responsive at the same time. This requires that teachers be familiar with the curriculum; they must know what comes before and after at their specific grade level as well as previous and subsequent grade levels. Then teachers must spend time learning what their students know. Teachers must know what strategies students understand and what misunderstandings students might have. Thus, knowing the curriculum and the students is key to engaging students successfully in tasks of high cognitive demand.

**Mathematical Discourse to Support High Cognitive Demand.** Dina used the talk moves to generate meaningful discussion around the mathematical content. I watched the videotaped teaching sessions and logged each time the teachers used the talk moves, revoicing, repeating, adding on, and agree/disagree. Figure 26 shows the percentage of days Dina used each talk move, as well as the number of times she used them each day that the talk moves were present. Most days Dina used revoicing, repeating, and agree/disagree. On October 9 and 10, 2012, she did not use the talk moves as much as on other days because the students were reviewing the mathematical concepts to prepare for testing. On average, she used the talk moves at least once per day, however on September 11 and October 2, 2012, she used the talk moves over ten times. By the beginning of October 2012, Dina used the talk moves flawlessly. The following is an excerpt from my field notes and journal on.

At this point the talk moves seem like second nature. Dina seems to embody the talk moves! 😊 (Kelly Field Notes, 10/02/12)

How did the teachers’ use of talk moves maintain or raise cognitive demand? Dina flawlessly implemented the talk moves. She continually asks students to prove, repeat, and agree/disagree. This is wonderful. (Kelly Journal, 10/02/12)
At this point, Dina and I agree that her use of the talk moves enabled her to engage students in fruitful mathematical discussions.

<table>
<thead>
<tr>
<th>Percentage of days talk move used</th>
<th>Revoice</th>
<th>Repeat</th>
<th>Adding on</th>
<th>Agree/disagree</th>
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<table>
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<tr>
<th>Number of times used per day</th>
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<th>Repeat</th>
<th>Adding on</th>
<th>Agree/disagree</th>
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Figure 26. Dina’s use of the “talk moves” during each of the 13 observed lessons.

Hillary’s data varies from Dina’s data in that she did not use the talk moves as often. As previously stated, at first, on October 25 and 26, 2012, Hillary tried to incorporate all of the talk moves into her discussion all at once, however, that proved to be too difficult, so we focused solely on repeating. As the Figure 27 shows, Hillary used repeating more days and more times per day than the other talk moves. On October 26, 2012, Hillary solely focused on using repeating. She asked students to repeat six times.

<table>
<thead>
<tr>
<th>Percentage of days talk move used</th>
<th>Revoice</th>
<th>Repeat</th>
<th>Adding on</th>
<th>Agree/disagree</th>
</tr>
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<td>53.33%</td>
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<td>40.00%</td>
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<table>
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<th>Number of times used per day</th>
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<th>Repeat</th>
<th>Adding on</th>
<th>Agree/disagree</th>
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Figure 27. Hillary’s use of the “talk moves” during each of the 14 observed lessons.

It is evident from Figures 26 and 27 that Dina used the talk moves more frequently than Hillary. Revoicing was a strategy that Dina used to clarify what a student
said, which helped not only the student presenting information, but also the other students in the class. In the following example, Dina is having students prove solutions from an equation chain.

Dina: Christos, why do you say it is ten? How could you prove it?
Christos: Because if you count on, from eight you, and you count on, and eight and count on two more.
Dina: Can you demonstrate for us?
Christos: Eight, nine, ten.
Dina: Okay, Eight, nine, ten. You got it. It works. (Dina Teaching Session, 09/13/12)

Hillary did not have students explain their strategies often, so there was less opportunity for her to revoice. Additionally, Dina used agree/disagree over twice as much as Hillary. Dina used agree/disagree as an opportunity for students to evaluate the strategies of others. This talk move set Dina up to discuss efficiency and to compare and contrast strategies, which leads to conceptual understanding (Eisenhart et al., 1993). Dina used the talk moves to promote critical thinking and conceptual understanding, which Hillary often failed to do. In the following excerpt, students created and proved equation chains and then discussed the definition of an equation. The use of correct mathematical vocabulary can help students understand and make important mathematical connection.

Dina: I see lots and lots of different ones compared to yesterday. Raise your hand and tell me some of the things you could have included. Not an example, but when we think of what an equation is, what are some things that we need to remember? Skyler?
Skyler: The equals sign.
Dina: The equals sign. And what is on one side has to equal what is on the other side. And we know nine is on one side, so everything we put on the other side has to equal nine. Tanner, what else do you know?
Tanner: Numbers and letters.
Dina: We can use numbers and letters. What else do we know? Brynn?
Brynn: You can have more than two partners.
Dina:  Okay, you can have more than two partners. Do I have to have, can I do less than two?
Students:  Yes.
Dina:  Yes, I could say nine equals nine. So you can have as many partners as you want. What else? Skyler?
Skyler:  You can have plus and minuses, inaudible.
Dina:  That’s right, you can have plus, minus, multiplication, division, any of those, that’s right. (Dina Teaching Session, 09/11/12)

This discussion, as an extension of the previous excerpt, illustrates Dina’s ability to help students understand the mathematics in a meaningful way.

Dina took on the role of discourse facilitator quite well. She often stood at the side or back of the classroom, conveying the message to students that she was not the center of the discussion. She had students stand in front of the class eleven out of twelve videotaped days and explain how they solved. She asked questions to clarify their own thinking, as well as help them explain it to the other students. She used the talk moves fluidly to help with clarity and understanding.

In contrast, Hillary was at the front of the classroom or at the document camera eleven out of fifteen videotaped days. Although she did show student work, she was the one who was explaining the concepts thirteen out of fifteen videotaped days, rather than the students. She did not often or willingly allow students to explain their thinking to the class. Classroom discourse was often a struggle for her and due to that, she tended to give up on it and move on with the lesson.

I always struggle between using explicit vocabulary for my ELL students and incorporating discussion for the whole group. After talking with Kelly, she suggested I repeat what students were doing and explaining, then maybe have the students repeat. I thought this was a little silly, because my students repeat all the time. Really, she suggested this on Thursday, and I halfway tried :) But understanding of my students suffered from a lack of language unorganization! On Friday, I really scaffolded my use of talk moves. Students explained, I
repeated using explicit vocabulary, then students repeated me. We did not move past the repeated talk move, but I realized that student understanding of concepts and vocabulary was much stronger sticking to one method of discussion. (Hillary Journal, 10/27/12)

Her lack of effort to use the talk moves effectively lowered the overall cognitive demand of the lesson because her expectations for the students were lowered. She gave up on expecting the students to be able to successfully justify their solution strategies and discuss those strategies with other students. This lack of sustained press and low expectations ultimately lowered the cognitive demand.

Since I helped both teachers plan the lessons, I made sure the lessons were planned with high cognitive demand. See Figures 19 and 20 for the cognitive demand of the tasks as planned. Additionally, most often the cognitive demand also stayed high. The students in both classes most often worked on tasks that required them to think deeply about the mathematics. The discussions in Dina’s classroom enhanced the lesson and helped students make connections and generalizations, whereas in Hillary’s classroom discussion did not. Notice the contradicting comments by Dina and Hillary about the use of discussion in their classroom.

It seemed as though my students were just barely hanging on to the concept, so I explained the different strategies so that our vocabulary could be more clear. This lowered the cognitive demand, but I think it brought it to a successful level for my students. (Hillary Journal, 10/18/12)

I felt the most successful with talk moves in this lesson than I have all year. One student would share their way of solving a problem and another child had to repeat what they heard. The first child then had to prove how they solved the problem. (Dina Journal, 10/02/12)

Hillary felt that she needed to lower the cognitive demand so that her students could understand, whereas Dina felt that her students would understand when they were
explaining their strategies and learning from their peers. This crucial difference in philosophies was a factor in the successful implementation of tasks of high cognitive demand.

Sfard, Nesher, Streefland, Cobb, and Mason (1998) described types of classroom discourse that occur as expounding, exploring, expressing, explaining, examining, and exercising. See Figure 5 for a description of each type of discourse. Expounding is teacher-centered discourse where the teacher is talking about the mathematics while the students are listening. This is typical of lecture-based teaching and due to the nature of primary grades mathematics, not present in my study. Explaining is when the teacher tries to help students understand the mathematical content by giving the students her own explanation. Exploring allows students to dig into the mathematics and create their own meaning, while the teacher facilitates the exploration and aids in learning. When teachers examine their students, the focus is on assessing student knowledge, rather than learning. Although both teachers questioned students to better understand student knowledge, that was not often the goal of the discourse and therefore was not coded as examining. For example, when students were expressing their ideas, the teacher gained knowledge about the students, but since that was not the goal of the discourse, I coded it as expressing. Students can express their ideas about mathematics to their peers and the teacher. The focus is on the mathematics and how students think about the content. Exercising is when students repeat the same action or procedure until it becomes automatic. Repetition is not a feature of Math Expressions and was therefore minimally found in both classrooms.
After each teaching session, I noted in my field notes which of these categories happened during the class period. I considered who was speaking, the teacher or the students, and what the focus and goal of the discourse was. These factors helped me determine which of these types of discourse was occurring. Since primary mathematics lessons often consist of multiple tasks, sometimes on somewhat disconnected topics, more than one type of discourse often occurred. I found that overall both teachers used mostly exploring, expressing, and explaining. See Figure 28 for the types of discourse that occurred in both classrooms.

<table>
<thead>
<tr>
<th>Number of days each teacher displayed categories of classroom discourse</th>
<th>expounding</th>
<th>explaining</th>
<th>exploring</th>
<th>examining</th>
<th>expressing</th>
<th>exercising</th>
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<td>10</td>
<td>8</td>
<td>1</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Hillary</td>
<td>0</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>10</td>
<td>2</td>
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*Figure 28. Types of classroom discourse that occurred.*
At first glance, the teachers appeared to be fairly similar, however, a main
difference is the category of expressing. When the classroom discussion was focused on
expressing, in Dina’s class that meant that the students were expressing their ideas about
a mathematical concept, whereas in Hillary’s class, expressing was often her expressing
her idea about the mathematical concept. This distinction of the speaker is important
because allowing students to express their own ideas helps them solidify their
mathematical understanding. The following are sample excerpts from my journal on days
that I saw expressing in each classroom.

**Hillary**
- Hillary was explaining and expressing. (Kelly Journal, 10/24/12)
- Hillary did some explaining and expressing. (Kelly Journal, 11/06/12)

**Dina**
- Students were exploring and expressing. They were the ones
talking throughout class and sharing ideas. They were engaged
in the mathematics and manipulating it. (Kelly Journal, 10/02/12)
- Students were also expressing their ideas when sharing strategies. (Kelly Journal, 10/04/12)
- Then the students were expressing their ideas when solving the problems. (Kelly Journal, 10/10/12)

Dina had students compare strategies and share their thoughts, while Hillary compared
them herself and shared her own ideas. Notice that Dina’s class was allowed to explore
twice as much as Hillary’s class. This links to the previous point in that Dina allowed her
students to do much of the thinking and sharing, while in contrast, Hillary did much of
that for her students.

With meaningful planned questions, teachers are able to implement the talk
moves to engage students in productive classroom discussions. Often when the teachers
failed to use the talk moves and ask planned big-idea questions, the engagement in and learning from classroom discussion suffers greatly. Planning lessons of high cognitive demand, including generalizations, questions, and/or connections, and implementing them with fidelity was a key to success for the teachers.

**Use of Student Understanding while Teaching.** As part of the Primarily Math program, Dina and Hillary were both tested on their mathematical knowledge for teaching (MKT). The teachers were given the MKT test each spring after they were accepted into the professional development program. The test of MKT consists of questions that get at the meaning of mathematics and are applicable to teaching. See Figure 29 for a sample question from the MKT. Questions include mistakes made by students, representations of numbers and operations, and multiple solution methods.

![Figure 29: Example question from MKT test](http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf)
Overall, Dina’s MKT scores were much lower than Hillary’s MKT scores. See Figure 30 for Dina MKT scores and Figure 31 for Hillary’s MKT scores. The test has a mean score of zero and a standard deviation of one. See Figure 31 for the conversion of z-scores to percentile. Notice that Dina only had one score above the mean, whereas all of Hillary’s scores were above. Interestingly, this data did not match what I was observing in the classroom.

**Figure 30.** Dina’s MKT scores from 2009-2012.
I found that although Dina had less mathematical knowledge for teaching, she listened to students well. Dina spent much of the class times genuinely listening to students’ thoughts and strategies. She understood the basis of *Math Expressions* (2009).
and how students could move through the strategies. The following example shows how Dina thought about the curriculum and her students.

I don’t anticipate the kids will have difficulty with this because we’ve done so much with story problems. This was really hard for our kids last year, but I don’t think, with just the background and the stuff that we’ve done already, I think that it’s going to. Now, they won’t necessarily be aware of the vocabulary, so that will be something we will have to. (Dina Planning Session, 09/28/12)

On the one hand, with Dina’ understanding of her students and of the mathematics curriculum, she was, in a way, able to overcome her lack of mathematical knowledge. She was about to listen to student strategies and use those ideas to forward the classroom discussion and students’ understanding of the mathematics.

On the other hand, Hillary had more mathematical knowledge, but was unable to apply that to her teaching. Hillary did not spend time listening to student strategies, and, therefore, did not understand their thinking well enough to use the strategies to move their thinking forward. Instead of trying to help students move through a learning trajectory, Hillary tried to force all students to use a prescribed strategy and follow her procedure. For example, on October 25, 2012, students were working on writing math mountains and writing equations, from subtraction story problems. She told students they could solve using any strategy, but proceeded to make students draw a math mountain and count on using circles. She was not open to the new ideas of the students, which limited the connections students could make.

The teacher must understand the students’ ideas before being able to utilize them in the overall understanding of the class. When teachers had students explain their reasoning and/or strategy, the teacher often understood and was able to use the ideas to
forward the mathematical discussion. However, when the teacher did not have the students explain their own work, the teacher seemed to have less of a grasp on what students knew and therefore was limited in her use of student understanding to increase the knowledge of the class collectively.

For a researcher, much can be learned from observing in a classroom, but somewhat surprisingly, the teacher can also benefit just from the researcher’s presence, as noted previously. I helped the teachers plan lessons of high cognitive demand and watched them try to implement the lesson with fidelity, which was sometimes successful, while sometimes not. Many factors played a part in the implementation of the lessons of high cognitive demand, such as ability to implement the talk moves and ability to listen and understand student solution methods.
Chapter 5

Discussion of Planning and Implementing Mathematical Tasks

Through my analysis of two teachers’ planning and implementation of tasks of high cognitive demand, I saw several similarities as well as several differences. Both teachers tried to implement tasks of high cognitive demand and engage students in meaningful mathematical discussions and achieved varying levels of success. Additionally, I considered the role of the researcher and how the design research process contributed to this study.

Transfer Knowledge from Professional Development to Classroom Practice

For the teachers, the transfer of knowledge from the professional development setting to their classroom was difficult. They needed to see an explicit application of their learning in professional development to their classroom, which I helped lay out through my suggestions about cognitive demand and classroom discourse. I was surprised at how difficult this process was for the teachers. At times, they struggled to make changes in their planning and implementation of lessons. For example, Dina was able to use open questions easily, whereas Hillary continued to ask many closed questions. I helped the teachers think deeply about the mathematics of the tasks they were asking students to do and how they could help students make connections within the mathematics. They needed adequate time to plan implementation of the new learning. Through their commitment to my research, we had a specified amount of time to plan meaningful mathematical tasks and I provided the support necessary to do so.
My work with both teachers began the same, but changed throughout the study. My work with Dina progressed as I thought it would; she learned from her teaching and our discussions and continually improved. She mastered the use of the talk moves quickly and focused on big ideas, connections, and generalizations. Hillary was unable to take away as much learning as Dina, so her process to improve was much slower. She was less likely to implement plans with fidelity, which made it more difficult to reflect on how she implemented the plans we made. We were not able to evaluate how well the plans we made worked out because of the digressions she made from the plans. Hillary and I spent more plan time justifying the talk moves and how to help the students learn the talk moves. For example, Dina and I discussed using the talk moves during the first planning session, then reflected on her use of the talk moves during the second planning session. In subsequent planning sessions, we did not discuss the talk moves, but rather connections and generalizations for the mathematical tasks. In contrast, Hillary and I discussed implementing the talk moves during all of the planning sessions. She was not able to successfully implement the talk moves consistently, so in each planning session we reflected and planned how she would use them.

As a researcher, I helped both teachers plan meaningful lessons by discussing topics such as cognitive demand and classroom discourse. During planning, I gave each of the teachers suggestions as to how they could increase the cognitive demand of tasks, how to implement the talk moves, and questions they could ask students to spark meaningful discourse. The planning sessions looked different for each of the teachers I worked with, but contained the same basic elements of cognitive demand and classroom
discourse. These planning and reflecting sessions were the most important part of our work together. It allowed the teacher and I an opportunity to discuss how the lesson implementation and the instructional decisions that were made during the lesson played out. We were also able to take time to look at student work to see how their understanding was progressing. We were able to use all of that information to make instructional decisions about subsequent lessons.

I played a minimal role in the enactment of the lesson. It was important that I did not help the teachers much during the implementation of the lesson. I did not want them to come to rely on my assistance, because at the conclusion of the study, I wanted them to continue planning and enacting tasks of high cognitive demand without me. I observed the planned lessons to learn about how the teacher implements the lessons. The student learning that occurs during a lesson can be influenced by a researcher, but is ultimately up to the teacher during enactment.

Originally I thought a limitation of my research, in terms of understanding how the teachers planned and implemented tasks of high cognitive demand, would be the fact that I was inside the research project, rather than strictly an observer, but it did not turn out to be a limitation at all. I was concerned that the data could be skewed if I intervened during lessons too much because it would not show the work of the teacher, rather it would reflect my work. I needed to study the work that the teachers were doing, so if I intervened and influenced too much, I would not be able to critically examine the work of the teacher. However, even though I really wanted to jump in many times to help the teachers, with one exception, I did not. I pushed myself to sit back and take notes, rather
than take over discussions or instruction. The choice to mainly watch the enactment of the lessons was crucial to the teacher being able to continue planning and teaching in this manner. I allowed the teachers to rely on my mathematical and pedagogical knowledge during planning at the beginning, but I gradually released my lead in planning in order to help them learn how to plan in a similar manner without me.

Being on the inside of the study and working closely with the teachers enabled me to better understand the teachers’ decision making. During our planning sessions, we discussed issues that would come up during the implementation of the lessons and how the teacher might deal with the issues. Then while I was observing the lessons, I was able to make note of when the teachers followed the plan we discussed and when they diverged from it. Had I been strictly an observer, I would not have known when the teachers were not following the lesson plan. During the next reflection session I was able to ask the teachers questions about their decision to vary from the plans.

I noted when the teachers made moves that helped to maintain or lessen the cognitive demand, such as using the talk moves, having students share their solutions or making connections between solution strategies. Since each teacher and I had discussed the tasks during a planning session, we had planned how the tasks would be implemented and what connections and generalizations could be make from the task. Since I had also previously examined the tasks from the textbook, I was able to offer suggestions to the teachers about how they could make alterations to engage students and increase the cognitive demand. At subsequent planning sessions, the teachers and I discussed my notes and how the teachers’ moves affected the lesson.
I allowed the teachers to make mistakes and learn from them, rather than for me to make corrections during the lesson. For example, when Dina confused the counting up and counting back strategies, I did not jump in to fix it. I allowed Dina to struggle through the explanation and then we discussed it at the next reflection and planning session. This gave Dina the opportunity to reflect on her own and then talk with me about what the confusion was and how that issue could be remedied. Dina often recognized a mistake as it was happening, whereas Hillary did not. Hillary did not often see what I noted as a mistake really being a mistake. For example, as mentioned in chapter 4, Hillary did not think it was important to help students learn to repeat each other, however when we further discussed it, she was able to understand the benefits and push herself to have students repeat.

**Planning for Tasks of High Cognitive Demand**

Both teachers redefined their notion of planning, based on the work we did. The teachers had to go beyond the textbook pages in order to plan activities with high cognitive demand, as the textbook did not always have lessons of high cognitive demand. See Figures 19 and 20. They also learned to be critical of the textbook instead of simply following the directions. While planning, the teachers learned to consider the students’ knowledge and how that influences the lesson. The opportunity to think about and discuss cognitive demand, student understanding, and discourse with an expert while planning really allowed the teachers to take risks that they may not have on their own.

Both teachers I worked with used the teacher’s edition to pace their lesson, but called that planning. I attempted to implement the TTLP with both teachers, but I found it
to be too cumbersome and time consuming. Instead, I developed a guideline for planning tasks of high cognitive demand. See Figure 33. The guidelines address the essential questions that a teacher needs to consider in order to plan deep and meaningful mathematical tasks. Although following the guidelines does not ensure that a task is implemented with high cognitive demand, it does help teachers plan high cognitive demand tasks.

Guideline for Planning Tasks of High Cognitive Demand

- What is the essential understanding of the lesson?
- What do students already know about the topic?
- What tasks should students do in order to gain the essential understanding?
  - What numbers should be used?
  - What directions should be given?
- What strategies will students use to solve?
- What errors might students make and how will you remediate them?
- What connections and generalizations should students make as a result of solving?
- What questions can you ask students to help them make connections and generalizations about the essential understanding?

Figure 33. Guideline for planning tasks of high cognitive demand.

I helped the teachers learn to alter directions and numbers in order to increase the level of cognitive demand, and in turn deepen students’ understanding. In order to plan tasks of high cognitive demand, the teachers and I examined how the mathematical concept was presented in the textbook, what they asked students to do with their mathematical understanding, and what connections students should make. Both teachers learned to plan questions in order to prompt meaningful mathematical discussions, in
addition to familiarizing themselves with the talk moves to enhance mathematical discussions.

While working with the teachers, I learned how to help teachers grow professionally and recognize that there is no one prescribed method to work with teachers. The teachers I worked with were very different and I played a bit of a different role in work with each teacher. Dina understood the reasons for engaging students in meaningful discourse, specifically using the talk moves, whereas Hillary did not. I learned how to work with different types of teachers by starting with what they understand and building from there.

I found, just as O’Donnell and Taylor (2007) did, that too often teachers’ ideas of planning are simply writing down the lesson and page number in their plan book, which does not suffice as planning. In response, during this research project, I wanted teachers to analyze the tasks in the teacher’s edition and plan tasks that allowed for conceptual understanding and meaningful classroom discourse. Many researchers advocate this type of deep text analysis because it creates opportunities for growth (Cai, Moyer, Nie & Wang, 2010; Hiebert, 2003; Van de Walle, 2003). Both teachers considered the mathematical content as well as the students’ knowledge of the topic when planning, just as NRC (2001) suggested. However, this is not an easy task. Teachers must have both deep MKT and understanding of their students. Beyond the knowledge, considering all of these elements while planning takes time, which is not always aplenty for teachers. As a researcher working closely with these two teachers, I saw firsthand as the teachers struggled at times to get a grasp on the mathematics and the students’ understanding. In
order for a teacher to learn to plan in this manner, they need support, which may come in the form of a researcher, a coach or another teacher.

Many researchers have concluded that the key to sound classroom instruction is the teacher, specifically how the teacher interprets and uses the curriculum and instructional materials (Ball & Cohen, 1996; Cai, Moyer, Nie, & Wang, 2010; Manouchehri & Goodman, 1998; Remillard, 1999; Remillard, 2005). My findings were certainly in line with theirs; although the teachers had similarities, the instructional results were different. I learned that teachers tend to follow the curriculum directly, believing that is teaching the curriculum with fidelity. Too often teachers do not consider student understanding when planning.

Together the teachers and I worked to implement the curriculum while tending to the needs of the students. Through this process, the teachers discovered that the curriculum is the backbone of instruction, but other factors are part of the instructional decision making process. In my study, the other factors were cognitive demand, student understanding, and classroom discourse. Teachers must learn to recognize the cognitive demand when planning and then what is happening to the cognitive demand during enactment. This is not easy to see in a teacher’s own teaching; it takes support and reflection to help the teacher see this. Teachers also must learn to listen to students’ justification of their strategy and quickly evaluate its effectiveness and efficiency. They must decide if it is a strategy they want to highlight or not. If they have already considered student strategies during planning, the in-the-moment decision is much easier because it has already been rehearsed. Additionally, the teacher has to plan for the
discussion that will occur around the task. The teacher must consider what connections and generalizations they want students to make and design questions to help them get there. For a teacher to learn how to manage all of this during their plan time is a monumental task that all starts with examining the textbook. As a teacher learns to question the textbook and make sense of the tasks for her specific class, she will need to consider all of these factors to truly engage her students in tasks of high cognitive demand.

**Enacting Tasks of High Cognitive Demand**

My findings were inconsistent with those of Manoucherhri and Goodman (1998), Sherin and Drake (2009) and NRC (2001) who found that teachers’ mathematical content knowledge played a major role in their planning and implementation of reform curriculum. Of the two teachers I worked with, the one with lower mathematical content knowledge, Dina, planned and implemented more meaningful lessons. She carried out her lessons as planned and engaged her students in deep mathematical discourse. In contrast, the teacher with higher mathematical knowledge, Hillary, tended to plan lessons of high cognitive demand, but not implement as she planned or have meaningful mathematical discussions. Her lessons lacked mathematical meaning, due to the fact that she did not follow her lesson plans. When she got sidetracked on some part of the lessons, she lost sight of the big ideas. The plans we made consisted of rich tasks with multiple solution methods and even multiple solutions. As well, we planned rich questions that would engage students in meaningful discussions. Being part of the planning sessions and
knowing what was planned gave me insight into the teachers’ practices and the intentions of the plans, which was an advantage over just observing.

An important, yet not necessarily surprising finding, in my research was that how a task is planned may or may not be how it is actually implemented in the classroom, as confirmed by Henningsen and Stein (1997), Remillard (1999), and Stein et al. (2009). Even though the teachers planned lessons of high cognitive demand, the level of cognitive demand could change throughout the lesson. If the teacher implemented the lesson as planned, the cognitive demand was often maintained, whereas if the teacher varied from the set plans, there was often a decrease in cognitive demand. Veering from her plans caused Hillary to lose sight of the lesson goals and the questions meant to help students make connections and generalizations. She was frequently distracted by a part of the problem and got off on a tangent, which derailed the lesson. If the teacher asked the meaningful questions as planned, she was able to engage students in productive classroom discourse.

Smith et al. (2009) found factors that effect task implementation (see Figure 15). I further considered that list in the context of planning and teaching (see Figure 34). Since these are the factors that I found to separate the teachers in my study, I think it is important to introduce teachers to these factors. If teachers are aware of these factors during planning and teaching, they may be more likely to attend to them.
Additionally, it is important that teachers reflect on their own practice as a means of professional growth. The teachers I worked with reflected individually in a journal and also with me during planning sessions. If teachers are aware of the factors that effect cognitive demand, from Figure 34, they may be more likely to attend to them. I created a list of reflection questions that relate to the factors that effect high cognitive demand (see Figure 35). Reflecting on these questions can help teachers become more planful and intentional.

The key to asking meaningful questions and being responsive to students is planning. If a teacher can predict students’ strategies, they can plan their response and the connections students need to make. Understanding the mathematics deeply in order to see connections between different strategies is crucial for teachers. Teachers should
Reflection on Factors that Effect Cognitive Demand

- How did I alter the task for students who were struggling?
- How did I alter the task for students who were excelling?
- How did I help students monitor their own learning?
- How did I connect this task to previous and future tasks?
- Did I give students enough time to productively struggle with the task?
- Did I have students model high level thinking?
- Did I push students to justify their solutions?

Figure 35. Reflection on factors that effect cognitive demand.

understand the meaning behind each strategy and what implications using that strategy might have. For example, if students use the make a ten strategy, that indicates that they understand place value and regrouping. A teacher who knows that can help students make generalizations about problems in which the make a ten strategy is efficient.

**Meaningful mathematical discussions.** One way to increase the cognitive demand of a task is by having students engage in meaningful classroom discourse. However, as I was watching the teachers try to implement discourse, I realized what a challenging task this could be, as did Crespo (2003). As I was working with the teachers, I was reminded that teacher change takes time and effort. Both Dina and Hillary had time and put forth effort and grew professionally through my study. Although meaningful classroom discussions can be difficult to implement, I encouraged teachers to allow students to share their solution methods with the class, and use those ideas to forward mathematical understanding for the whole class, which was found to be an important instructional feature by many researchers (Grant et al., 2009; Smith & Stein, 2011; Varol & Farran, 2006).
This study helps to explain how teachers learn to plan and implement productive classroom discussions and how they can be supported during the learning process. One major key to the teachers’ learning is support. Teachers are often in an isolated profession where they do not have adequate time to plan discourse and get feedback on the implementation. Given structured support to plan, the teachers were able to make progress learning to plan and implement meaningful classroom discourse.

The manner in which Dina questioned her students did not deem their answer correct or incorrect, but rather engaged students in the sense-making process. Dina’s questioning also engaged the other students in the act of critiquing the work of others. Whereas Hillary struggled to carry out a classroom discussion that advanced the understanding of all students, as researchers have found to be difficult for many teachers (Kazemi & Stipek, 2001; Stein et al., 2008). She tended to treat classroom discussion as a way to deem students’ answers as correct or incorrect, which opposes the view of discourse of Smith and Stein (2011) and Stein et al. (2008).

The tasks teachers choose for students to work on and the classroom environment play roles in the discourse that occurs in a classroom (Kysh, Thompson, & Vicinus, 2007; Varol & Farran, 2006). My findings aligned with this notion; both teachers chose high-level tasks, but one had a classroom environment more conducive to meaningful classroom discussion. In the end, that classroom had more productive results from the discussions. The classroom environment must be such that students feel comfortable taking risks and learning from their peers. Using the talk moves is one way to help students learn to talk about mathematics and produce meaningful mathematical
discussions. The goal of using the talk moves is not necessarily to increase the quantity of discussion, but rather to increase the quality of the discussion (Chapin et al., 2009). Both teachers certainly enhanced the quality of classroom discourse by incorporating the talk moves.

Before my study, Dina was already implementing classroom discussions, however, with the addition of the talk moves and guiding questions, the classroom discussions became more meaningful and helped students understand addition and subtraction more deeply. Hillary had attempted to implement classroom discussions, however, during the discussions she did a majority of the talking. During my study we worked on releasing some of the control and having students share their solution strategies more often. This was a very difficult task for Hillary. She preferred to have complete control over the discussion and, as reported in Chapter 4, often explained the students’ strategies for them, rather than letting the students explain their own strategy. I encouraged Hillary to allow the students to explain, although she resisted at times. I also encouraged her to be near the back of the students and have one student stand in front and talk. This forced the students’ attention to be on their peer, instead of their teacher.

**Understand students’ knowledge of mathematics.** The better a teacher understands the students’ solution methods, the better they were able to use the methods to forward the understanding of the entire class. The teacher should understand both the mathematical meaning of the problem and how the students’ methods connect to the meaning. The teacher should be able to diagnose misunderstandings based on the solution methods that students use to solve. However, when the teacher had less understanding of
what students knew, she was limited in her use of students’ understanding to increase overall knowledge of the class. Students’ ability to make sense of their own work and the work of others is crucial to understanding mathematics. Although meaningful classroom discourse is a feature of quality mathematics education, it can be difficult for teachers to begin to engage students in discussions (Grant et al., 2009; NRC, 2001; Piccolo et al., 2008; Silver et al., 2009; Smith & Stein, 2011; Varol & Farran, 2006; Walshaw & Anthony, 2008).

In order to engage students in meaningful classroom discourse, the teacher must reconsider the meaning of control. The teacher need not lead the discussion in words, but rather needs to facilitate the discussion. The teacher must anticipate strategies, connections, and generalizations. They must decide which strategies they want to highlight, and in what order, to be able to make those connections and generalizations. Learning to plan in this way, instead of writing down page numbers, takes time and support.

Dina was particularly skilled at listening to students’ strategies. She questioned students to help herself, the student presenting, and the rest of the class’ understanding, then used that information to further the mathematical understanding of the whole class, which syncs with the findings of Sherin and Drake (2009). They found that questioning students helped clarify ideas and further the understanding of the whole class. In contrast, Hillary struggled to listen to her students and understand their strategies without pushing them to use her preconceived strategy. When listening to a student explain, she often listened superficially and moved on, without clarifying and questioning further to better
understand. It is crucial that teachers be able to listen and understand each student’s strategy in order to connect them and further students’ thinking.

I found planning to be the most important part of teachers being able to implement tasks of high cognitive demand. Without planning, teachers would rely on the textbook, which does not suffice when trying to implement meaningful tasks and discussion. The textbook offers many problems, some solution strategies, and some pedagogical explanations. However, it does not include an adequate amount of mathematical knowledge for teaching, connected to the pedagogical reasoning. Teachers need to take the time and put in the effort to plan meaningful tasks and questions around those tasks.

If teachers keep the questions, connections, and generalizations they want students to make in mind, they are more likely to be successful in the implementation of high cognitive demand tasks. Often, these essential questions, connections, and generalizations are not specified in the textbook. For example, if the teacher wants students to understand when counting up and counting back are more efficient, they must have the students solve specific planned problems that illustrate the efficiency of each strategy. The teacher must plan these outcomes using their own mathematical and pedagogical knowledge. It is important that teachers have support while learning to plan this way.

**Implications**

Although much can be learned from my study, it also raises many more questions. It points out that more research can be done to learn more about the planning and
implementation of tasks of high cognitive demand and engaging students in productive mathematical discourse. Additionally, my study points at implications for teachers and teacher education. How teachers learn to plan is a key in the study and is crucial to being able to plan and implement the lessons as planned.

**Further research.** Much research is done in the field of math education, however, design research needs to be emphasized. Although test scores are important, understanding students’ knowledge is more important. Researchers need to immerse themselves in classrooms to examine learning qualitatively, rather than rely solely on quantitative data related to achievement. The work that researchers can do with teachers in the setting of a design experiment can mine information about teaching and learning. I have learned about the work a teacher needs to do in order to plan and implement tasks of high cognitive demand and engage students in productive mathematical discussions. Additionally, when a researcher is solely on the outside of the study, watching as a bystander, the reflection and instructional decisions of the teacher can be missed. When I immersed myself in the study and worked with each teacher individually, I was able to pinpoint what each teacher needed to work on and how to help them grow professionally, which could not have been done by only observing. For example, Hillary needed help implementing the talk moves and I was able to support her doing that. In contrast, Dina needed support increasing her mathematical knowledge, which I could also provide.

One limitation that I found through my research is the fact that I only worked with two teachers. Since the two teachers had different findings, it would be interesting to study more teachers to see how having a larger group to study would affect the findings.
If I could extend this research project I would focus on the factors that Stein et al. (2009) found that lower or maintain cognitive demand. The factors from Figure 24 were the characteristics that truly separated Dina and Hillary.

My study contributes to the work of Stein et al. (2009) because I used their factors and applied them to the classroom setting. Their work uncovered factors that maintained cognitive demand, such as sustained press and conceptual connections, as well as factors that lowered cognitive demand, such as an emphasis on correct answers and problems being routinized. I took those factors, from Figure 24 and applied them to actual teachers. The presence of these factors separated effective and ineffective teacher practices. The method of planning that the teachers and I practiced can help get at the factors that maintain cognitive demand, such as scaffolding tasks, maintaining sustained press even when students struggle, and helping students make conceptual connections. If the teacher plans using mathematical understanding and student knowledge, they are more likely to be able to maintain that high cognitive demand, due to the substantive plans they have created.

Another area that is intriguing to continue to research is the implementation and analysis of the guidelines of planning lessons of high cognitive demand (see Figure 33). Although I used this model with two teachers, it would be interesting to study how teachers use this guideline at different grade levels. Planning is the first step to actually implementing tasks of high cognitive demand, so helping teachers learn to plan tasks of high cognitive demand is crucial.
When I chose Dina and Hillary, I wanted to learn from two teachers who had different experiences and taught in different schools. I had a preconceived idea about what it would be like to work with each of the teachers, which turned out to be completely incorrect. Based on the work I had done with Dina and Hillary in the Primarily Math program, I expected Hillary to be more successful than Dina at implementing tasks of high cognitive demand and engaging students in meaningful classroom discourse. As I worked with the teachers, I clearly saw that my work with them was different. I was able to understand their mathematical and pedagogical knowledge and work with it to help each grow. The factors of Stein et al. (2009), see Figure 24, enabled me to make specific distinctions between the teachers and their ability to implement tasks of high cognitive demand. In the future, I would like to spend time working with more teachers analyzing the factors of Stein et al. (2009) to determine if these factors in fact do correlate with planning and implementing tasks of high cognitive demand.

Another recommendation for further research is to delve deeper into the implementation of the talk moves. The talk moves were the other instructional strategies that set the two teachers apart. Dina learned quickly to use the talk moves to help clarify students’ methods and engage students in meaningful discourse, whereas Hillary struggled to use the talk moves. I spent much more time working with Hillary on how and why to implement the talk moves. It would be interesting to better understand how a teacher learns to use the talk moves and the process in making the talk moves an automatic part of their instructional strategies. I would like to further understand if there
are talk moves that teachers use that actually lower the cognitive demand and do not engage students in meaningful classroom discourse. For example, Hillary often asked fill-in-the-blank type of questions. She would explain a procedure then ask questions such as, “What does this say?” or “Does this work?” This did not allow students to expand upon their answer and explain their method. I think learning what teacher moves lower the cognitive demand and making teachers aware of them will actually help teachers learn to maintain or raise the cognitive demand.

**Teaching.** I found that teachers often spend far less time planning than necessary to produce meaningful mathematical lessons. Too often planning is simply pacing the lessons throughout the unit, rather than focusing directly on the mathematical content. Teachers need to understand the mathematical pedagogy underlying the tasks before they can truly plan for meaningful mathematical tasks. The more effort a teacher puts into purposefully planning, the more likely she is to follow through with those plans, and ultimately engage herself and her students in a meaningful lesson where students make connections and generalizations. This type of purposeful planning allows the teacher to really consider the big ideas of the lesson and generate questions to help students understand and make connections.

My study illustrates what spending time on planning can really accomplish for the teacher. Teachers may think that planning can be done quickly because they are busy, but planning for tasks of high cognitive demand require deeper thinking and understanding, which takes time and effort. If a teacher really wants to help students make mathematical connections and generalizations, they must take the time to plan how that will happen.
That requires choosing problems with specific values and highlighting specific strategies in a set order. Knowing those values and strategies must be attended to and why during the planning phase in order to be most effective is an important part of what needs to be learned.

However, it is not enough to plan meaningful tasks. It is crucial that teachers learn to implement their own lessons as planned. Teachers must keep the end goal of their plans in mind when implementing the lesson or they may lose sight of the goal. Additionally, in order to maintain high cognitive demand during the lesson implementation, teachers must sustain press for justification, implement tasks that build on prior knowledge and connect strategies, and help students make conceptual connections. These factors work together to maintain cognitive demand and help students solidify their understanding of the mathematical concept.

**Teacher education.** Although both of the teachers I worked with had experience teaching, much of what I learned through this study can be applied to pre-service teachers. Both Dina and Hillary used their teaching experience to help them understand student solution methods, but pre-service teachers can supplement experience with knowledge. For example, a first year teacher lacks teaching experience, but they can have deep MKT, which will help them be successful while they are gaining teaching experience. Teacher educators can help pre-service teachers understand possible solution methods and their link to mathematical understanding. Pre-service teachers often rely on their own experience as a learner to pattern their own teaching after. This practice often
creates a slew of traditional teachers, rather than teachers that are focused on the students and the mathematical content.

Teacher education programs should be focused on helping pre-service teachers learn to plan mathematical tasks of high cognitive demand, including planning for rich mathematical discussions. Pre-service teachers should have opportunities to examine student work and learn how to recognize strategies and make connections between strategies. They should learn how to choose specific mathematical strategies to highlight, and in what order to highlight them. Learning how to pose open-ended questions that allow pre-service teachers to reason and think through mathematical problems themselves is also key in a good mathematics teacher education program. Teacher educators need to prepare pre-service teachers for the difficult work that effective teachers do.

Mathematics teacher preparation programs should include teaching pre-service teachers about cognitive demand, talk moves, and understanding students. Teachers need to be able to recognize rich mathematical tasks and know how to integrate those into the classroom. Additionally, instead of asking questions in which the answer does not require an explanation, teachers should learn to use the talk moves while in their preparation program. The talk moves are an extremely useful tool that can be used in all curricular areas. They enhance the classroom discussions and therefore should be part of the curriculum in teacher education programs. Also, teachers need to learn how to focus on students’ conceptions of mathematics, rather than correct answers. Pre-service teachers need to learn how to understand students’ justifications, which helps them to understand what students know about that particular mathematics topic.
Conclusion

In summary, I found that when teachers purposefully planned meaningful math lessons and implemented them with fidelity, the cognitive demand was maintained. At times, it is okay for teachers to veer from the plans. If a teacher has high MKT they may see an opportunity to highlight and connect a strategy they did not previously consider, while still achieving the same mathematical goal. However, if a teacher with low MKT saw the same strategy, they may not be able to connect it to the mathematical goal and therefore may derail the lesson. High cognitive demand was maintained when teachers used the talk moves to help students understand their own strategies and the strategies of other students. Teachers can use the talk moves to help clarify and connect the students’ strategies. The use of the talk moves and planned questions that help students make connections and generalize helped guide the teacher and maintain the cognitive demand. I found that it is complex and difficult work to plan and implement lessons of high cognitive demand because there are many factors that influence both planning and implementation.
References


