2004

Recent Literature on Taxable-Income Elasticities

Seth H. Giertz
Congressional Budget Office, giertz2@unl.edu

Technical Paper Series
Congressional Budget Office
Washington, D.C.

Recent Literature on Taxable-Income Elasticities

December 2004
2004-16

Seth H. Giertz
Tax Analysis Division
Congressional Budget Office
Washington, D.C.
E-mail: Seth.Giertz@cbo.gov

The author would like to thank Joel Slemrod, Jon Bakija, Jim Poterba, Emmanuel Saez, Bob Williams, David Weiner, Bob Dennis, and Roger Hitchner for helpful comments.

Table of Contents

Recent Literature On Taxable-Income Elasticities

by Seth H. Giertz

Abstract

This paper reviews the literature on taxable-income elasticities, focusing primarily on empirical studies examining the U.S. tax changes of 1981, 1986, 1990, and 1993 and the bracket creep of the late 1970s and early 1980s. The paper first provides background on the importance of the elasticity of taxable income both for forecasting income tax revenue and for assessing the efficiency implications of tax rate changes. It then discusses the major methodological issues and obstacles before delving into the literature. The paper emphasizes the different methodologies employed and the sensitivity of estimates to an array of factors, including sample selection, the tax reform under examination, and econometric techniques (or model specification). Although it recognizes advances in the literature in recent years, the paper concludes that responses to tax rate changes are far from fully understood and that there is much to be gained from continued research on this topic.

I. Introduction

Taxes can alter incentives for economic activity, in turn affecting income and reporting behavior. From the government's perspective, it is important to understand the relationship between taxes and taxable income, because that relationship affects income tax revenue. The elasticity of taxable income (ETI) measures the responsiveness of reported taxable income to tax rate changes. The ETI equals the percentage change in reported taxable income associated with a one-percent increase in the net-of-tax rate, where the net-of-tax rate equals the share of the next dollar of reported taxable income that is not taxed, or one minus the tax rate. The ETI is also key to evaluating the efficiency implications of a tax change. Feldstein (1999) shows that, if accurately estimated, the ETI can be used to calculate the deadweight loss from income taxation.[1] Although the variable is important, accurately estimating it is daunting. The tax code provides opportunities to shift income between different tax bases (or to alter the timing of taxation), but reporting requirements do not always allow analysts to observe the full spectrum of behavior. Likewise, underlying economic conditions are complex and impact reported taxable income in ways that are not fully understood. Separating true responses to tax policy from responses to non-tax factors is difficult.

The first modern studies to examine the broad range of responses to income taxation (Lindsey,

1. Feldstein shows that $deadweight\ loss = -0.5 \cdot tax\_rate^2 \cdot (1 - tax\_rate)^{-1} \epsilon_{taxable\_income} \cdot taxable\_income$. However, there are exceptions when a breakdown of the response can add insight into the efficiency implications. For example, suppose tax rates rise and, in response, taxable income falls, but a portion of that drop in taxable income is due to increased charitable contributions (and suppose those charities produce positive externalities). Or, suppose that a tax increase is used to finance an underprovided public good. In instances such as those, where external costs or benefits are present, assessing efficiency implications is more complex.

1987, and Feldstein, 1995) found reported taxable income to be extremely responsive to the across-the-board rate cuts of 1981 and 1986. Those findings were supported by anecdotal evidence that some responses to taxation are large (and encompass more than labor supply responses). In 1981 the top marginal individual income tax rate was 70 percent; five years later the top rate had been more than cut in half, to 28 percent. Yet, despite substantial across-the-board rate cuts, economic growth and base-broadening pushed revenues from the corporate and individual income tax up by nearly 15 percent in real terms from 1981 to 1990.[2] And, despite a much less progressive rate structure, the share of federal taxes paid by the highest income groups increased substantially.

Since those seminal studies, much has been learned about the experience of the 1980s. It is now recognized that factors other than tax policy played a role in those early estimates. And, contrary to forecasts based on the 1980s experience, taxable income continued to grow rapidly for the upper-income groups after their tax rates were raised in 1990 and 1993. That brought to the fore the complexities inherent in measuring responses to taxation. Subsequent research, by examining the experience of the 1990s and by applying more sophisticated methods to the 1980s, has concluded that the response of taxable income to tax rates, while sometimes substantial, is much smaller than that found in the earliest studies.

Given the inherent complexities, the similarity in some of the recent estimates is surprising. For example, several studies, using different methodologies and examining different tax changes, cite

2. Based on IRS data available at www.irs.gov/pub/irs-soi/03db07co.xls.

a "best estimate" of the ETI of around 0.40. That might suggest that responses, if not perfectly estimated, have at least been narrowed to a tight range. A closer examination of the literature suggests that that may not be the case. In fact, some evidence suggests that responses may be particularly sensitive to sample restrictions, econometric techniques, the period under examination, and the past history of tax rate changes. Accurately estimating the ETI is especially important because mis-estimation can profoundly affect both the estimated efficiency impact and the tax revenue forecasts of a proposed change in tax rates. For example, suppose overall taxable income were $5 trillion with a flat rate tax of 20 percent, yielding income tax revenue of $1 trillion.[3] Now suppose the tax rate were raised by 10 percent (from 20 to 22 percent). If the ETI were 0.40, that rate increase would cause reported taxable income to fall by $50 billion (= 0.40 * 10 percent * $5 trillion). The rate increase would also cause an additional deadweight loss of $11.4 billion, while income tax revenues would increase by $89 billion. If, for comparison, the ETI were 1.0, the same tax rate increase would cause taxable income to fall by $125 billion, or 2.5 times as much. Deadweight loss would rise by $26 billion, also 2.5 times as much. In addition, the rate increase would cause tax revenues to increase by $72.5 billion, or 20 percent less. If rates were raised more, revenues would rise at a decreasing rate, while the deadweight loss would rise at an increasing rate.

Over the past two decades, the ETI has emerged as one of the key variables for assessing the budgetary and efficiency implications of tax policy. But, because responses to taxation are so complex and never occur in isolation, even estimates using the most sophisticated methods and

3. Current annual taxable income and income tax revenue numbers are somewhat smaller, but roughly correspond to those in the example.

the best available data are often inexact.

II. Why the Elasticity of Taxable Income?

Because of its revenue and efficiency implications, the elasticity of labor supply is an important variable in any examination of the impact of taxation. In a simplified model—where labor is the only source of income and all income is taxed when earned—taxable income equals labor income, and the ETI with respect to the tax rate is the same as the labor-supply elasticity.

But the real world is much more complex. Slemrod (1998) notes that taxable income encompasses more than just labor income, and that small (compensated) labor-supply elasticities do not necessarily imply small efficiency costs.[4] Income comes from many sources and those sources are taxed differently, or sometimes not at all. Taxpayers have many opportunities for tax deductions, exclusions, and credits; some of those opportunities allow income to escape the tax base (that is, to go untaxed). Others allow taxpayers to shift when and under what base (for example, individual versus corporate) income is reported and taxed. In that more realistic setting, taxable income and labor income (and their corresponding tax elasticities) can differ substantially.

As Slemrod (1998) and Slemrod and Yitzhaki (2000) report, people respond to income taxation in many ways:

1) **Real behavior** involves individuals changing their consumption or the amount they work,

4. The term "compensated responses to tax rate changes" refers to the portion of the overall response attributable to changes in relative prices (as opposed to the portion of the response due to changes in income). It is the distortion in relative prices that leads to losses in efficiency.

moving away from taxed goods or activities toward those that are untaxed or more lightly taxed. The labor supply elasticity (which measures the trade-off between time spent on labor and leisure) captures only a portion of that response.

2) **Circumvention** includes bypassing the tax system both illegally (by evasion) and legally (by avoidance). In the case of evasion, income is concealed or at least is not reported to the tax authorities. In the case of avoidance, income is shifted (intertemporally or between sources) in order to receive more favorable tax treatment. Diverting income into a tax-deferred retirement account is an example of avoidance. Higher tax rates generally increase the benefits from evasion and avoidance.[5]

3) **Administration and compliance** policy affect how people respond to taxes. Rigorous enforcement and low compliance costs should limit evasion and lead to smaller income responses of reportable taxable income to tax changes. In contrast, lax enforcement and high compliance costs will tempt taxpayers to hide income, and thus will result in larger changes in taxable income for a given change in tax rates. That implies that, instead of structural parameters, taxable-income elasticities are endogenous and a function of institutions.

Behavioral changes have efficiency implications. To assess them accurately requires

5. Following Slemrod and Yitzhaki (2000), "avoidance" refers to avoiding the tax but not avoiding the activity. For example, choosing leisure is one way to avoid paying income tax, but that decision falls under real substitution and not avoidance, because the consumption bundle has changed as a result of the tax.

differentiating between real behavioral changes that affect resource allocation and accounting maneuvers that simply re-label income. For example following the Tax Reform Act of 1986, Subchapter S income increased nearly threefold as income was shifted from Subchapter C corporations to Subchapter S corporations (because the 1986 act set the tax rate on Subchapter S income below that on Subchapter C income). That shift of income was simply a transfer from one tax base to another, but, because individuals do not report Subchapter C income, only half of the picture was in view: the increase in Subchapter S income. Without information on the drop in Subchapter C income, the relationship between marginal income tax rates and taxable personal income is somewhat specious.

In addition to behavioral responses, the government's costs of administering and enforcing the tax system and compliance costs borne directly by taxpayers and employers affect tax system efficiency. According to Slemrod and Yitzhaki (2000), tax administration and compliance costs are nontrivial, and institutional factors both at home and abroad can be key in determining both the efficiency costs of taxation and the size of elasticities.

III. Methodological Issues and Obstacles

The primary methodological objective is to devise a method for parsing the response of taxable income to changes in taxes from the many other factors that also affect taxable income. This section focuses on those complicating factors. Tax changes take place in a changing economic environment; changes to that environment affect income growth. Adequately controlling for non-tax-induced trends in taxable income poses a major challenge to estimating elasticities. In

addition, a sound methodology must address several other important issues, including mean reversion, tax-rate endogeneity, institutional changes (which often coincide with changes in the rate structure), and the distinction between transitory (or temporary) and permanent (or longer-term) responses. Those complicating issues are discussed in more detail below. The following section discusses methods for addressing them.

Exogenous Income Shifts and Mean Reversion

The distribution of reported income has widened over the past 30 years. That trend accelerated in the 1980s, especially at the very top of the distribution. According to Piketty and Saez (2003), the share of income reported by the top 10 percent of filers rose by more than one-third from, 32.9 percent in 1979 to 41.4 percent in 1988, but two-thirds of that increase went to the top 1 percent of taxpayers. The share of income reported by the top one-half of one percent more than doubled, the share reported by the top one-tenth of one percent nearly tripled, and the share reported by the top one-hundredth of one percent more than quadrupled. Because people with the highest income pay a disproportionate share of taxes—the top 1 percent pay approximately one-third of all federal income taxes—their behavior is especially important.[6] Not fully accounting for the portion of that income growth that was unrelated to tax policy can result in large biases. For example, the 1980s cuts in marginal tax rates were greatest at the top of the income distribution and thus inversely correlated with the great income growth at the very top of the distribution. If the exogenous (non-tax-related) portion of that income growth is not fully accounted for, that trend will bias ETI estimates in a positive direction.

6. See www.irs.ustreas.gov/pub/irs-soi/01in01ts.xls.

Mean reversion is another issue that complicates estimation. Over a person's lifetime, income often follows a general path with many fluctuations. After a period when income is particularly high or low it will often revert to a more normal path. That phenomenon is especially pronounced at the tails of the distribution. Those at the extreme top of the income distribution are often not there for long, and will likely have a substantial drop in income—a drop that is unrelated to tax policy. At the other extreme, those in school (or not employed) will often have large increases in income upon entering the workforce. Not accounting for that mean reversion at the tails of the distribution can substantially bias estimated elasticities. More specifically, not fully controlling for mean reversion will erroneously count non-tax-related increases by those below their lifetime path and decreases in taxable income by those above their lifetime path as responses to changes in tax rates. Those factors will bias ETI estimates in opposite directions, depending on whether tax rates are raised or lowered, but there is no reason to believe the biases will cancel each other out. Partly for that reason, many studies exclude those with very low earnings. Those at the high end cannot be so easily discarded, because they are responsible for a large share of both taxable income and tax revenues.

One of the keys to successfully identifying taxable-income elasticities rests on the approach used to control for non-tax-related changes to taxable income, including both exogenous shifts to the income distribution and mean reversion. Mean reversion is certainly an important and complex issue, but divergence in the income distribution seems to be more complicated and less well understood. The fact that the exogenous income trend has persisted through periods of both increases and decreases in the level and progressivity of tax rates suggests that it is, in large part,

not a direct response to tax changes. In addition, because the trend has been irregular, distributional changes in years without tax changes may not provide useful measures of exogenous shifts that occur during periods with tax changes.

It is also likely that the size of taxable income elasticities varies across the income distribution. In general, estimated ETIs are larger for higher-income groups. People with higher incomes generally have more opportunities to respond to tax changes. They generally rely less on wage and salary income and have more control over the timing and source of their income than do other groups. People with more modest incomes can alter their labor supply, but may have few other alternatives altering their taxable income. If the ETI does in fact vary with income, a single overall elasticity will not be applicable when considering the impact of rate changes that target only part of the income distribution or that differ in magnitude across the distribution.

Endogeneity of the Tax Rate

Because of the federal tax system's progressivity, it is almost axiomatic that a simple cross-section regression will show a direct relationship between tax rates and taxable income. Even with longitudinal data, an individual's tax rate rises with taxable income. In order to isolate the impact of taxes on taxable income, tax rates should be imputed based on an instrumented (or exogenous) measure of taxable income. After instrumenting, the correct relationship between taxable income and the tax rate should be achieved for each individual, but that method does not address the cross-sectional correlation between taxable income and tax rates. Studies using cross-sectional

variation for identification generally must also include differencing methods (which transform the key dependent variable to the change in the tax rate).

Institutional Factors: Contemporaneous Tax Policy Changes

Institutional changes to the tax system, taking effect contemporaneously with rate changes, could affect reported taxable income, biasing estimated elasticities or at least complicating the estimation. In fact, Slemrod (1996) shows that changes to the underlying tax base may result in pre- and post-tax-change elasticities that differ substantially. Most regression techniques yield a weighted average of the two elasticities. For more detail, see Section IV.

Most elasticity measures assume policies toward tax evasion and avoidance as given, when in fact those are choices that can change. But, in order to achieve a given objective (whether a revenue goal, an improvement in the system's efficiency, or altering the distribution of the tax burden), it may be preferable to consider not simply changing tax rates (given the expected response), but to alter both the rates and institutional factors.

The definition of taxable income itself may influence results. Changes to the tax system may alter that definition. Using the concurrent definition for taxable income (that is, the definition that was in effect when the income was received) will confound responses to tax rates with statutory changes to the tax base. But even if a consistent measure is chosen, Slemrod (1998) shows that estimates may depend on the definition used and that even a constant-law definition can yield biased results.

Transitory versus Permanent Responses and Income Shifting

Permanent, or longer-term, behavioral responses to tax changes are of primary importance; transitory responses are a lesser concern. For illustration, suppose that in 1986 it was well known that the tax rates were set to fall beginning in 1987. That change could induce both short- and long-term responses. In the short term, some may have delayed the receipt of income from December of 1986 to January of 1987. That response would not have affected real economic behavior and would not have impacted long-term taxable income. By contrast, a persistent change in investment or labor market behavior would have represented a long-term response, affecting the allocation of resources and taxable income for years to come. That is not to say that transitory behavior is always small or trivial. For example, capital gains realizations rose by over 96 percent from 1985 to 1986 in anticipation of less-favorable treatment of capital gains set to begin in 1987.

Separating transitory from permanent responses is often difficult. Measuring changes in taxable income in the year prior to and the years succeeding a tax change will likely yield a combination of permanent and transitory responses. Phase-in periods and taxpayer expectations about future tax legislation also matter. For example, if rate cuts phase in, people not only divert income (on paper) to the future, but may also substitute leisure in the short term (while the rate cuts are in progress) for work in the future (after the rates have been fully lowered). In that instance, intertemporal substitution could result in a short-term understatement and a longer-term overstatement of the ETI.

A related issue is the relationship between tax policy and long-term career and investment decisions. Tax policy can affect investment in both human and physical capital, which over time could influence taxable income. That long-run response is important in measuring the true response to tax changes, but may not be fully observed for many years following a tax change, leading to an understatement of the ETI.

In addition to intertemporal shifting of income, people may shift revenue from one tax base to another or from outside the tax base to taxable income (or vice versa). Income shifting can occur between different tax bases for an individual or, as Carroll (1998) notes, between individuals. For example, a highly paid lawyer may reduce his workload in response to a tax increase targeted at high earners, but his reduction may shift business to lawyers in lower tax brackets. That point is especially important when an analysis uses data that exclude portions of the income distribution. In terms of tax revenue, it is important to account for gains and losses throughout the tax system, as opposed to focusing on a limited set of sources.

IV. Recent Empirical Work

With a couple of exceptions, the ETI literature divides into two strains, one focusing on the tax cuts of the 1980s (the Economic Recovery Act of 1981 and the Tax Reform Act of 1986) and the other looking at the tax increases of the early 1990s (the Omnibus Budget Reconciliation Acts of 1990 and 1993). Over time, that literature has led to a clearer understanding of the fundamental issues and to more sophisticated methods and better data for addressing them. Most of the papers use individual-level panel data and employ a first-differences or differences-in-differences

methodology, while several use aggregated data and examine the relationship between tax rates and the income (or share of income) accruing to various segments of the taxable income distribution. Despite a good deal of research, the ETI is a far from settled issue. In many instances estimates are not robust and the process underlying (or sources driving) the taxable income response is not fully understood.

The remainder of this section is organized into four parts. The first lays out the rationale for grouping the papers by the time period examined. The next part discusses a key point drawn from the literature—namely, that it is premature to conclude that the ETI has been precisely estimated (and hovers around 0.40). The final two parts discuss the literature covering the 1980s and 1990s, respectively.

Do Responses Depend on the Tax Change Examined?

The nature and circumstances surrounding a tax reform may influence the true elasticities, and may also tend to impart upward bias in some instances and downward bias in others. Comparing studies focusing on the same tax change makes it easier to evaluate the impact of other study-specific factors (such as data and methodology) on the results.

The reform under consideration (or unique economic conditions) can be a key factor driving results. For example, as discussed later in this section, Slemrod (1996) found that simply including the TRA 86 experience increased measured responses for some income sources many times compared with examining only the period from 1954 to 1985. In addition, the coefficient

on Slemrod's control for exogenous changes in wage inequality often changes signs (as well as changing greatly in absolute magnitude) when data from 1986 to 1990 are included.

The decision to group the papers by the reform examined is supported by two factors: First, the actual elasticities may differ from one reform to the next and, second, the key non-tax factors that affect taxable income, discussed in the previous section, may also depend on the reform. With respect to the first point, identical changes in tax rates elicit smaller responses under a tough enforcement regime with a broad tax base (and thus few opportunities to shelter income) than under a regime with lax enforcement and a small base. With respect to the second point, suppose that a law change affects both marginal rates and other aspects of the tax system, as typified by TRA 86. That scenario may call for methods that would not be relevant to a case where only the rates changed.

Is There a Consensus on the Size of the Elasticity?

Although the ETI literature appears to be moving toward a consensus, a deeper look at the research suggests little agreement. Despite the complexities inherent in the estimation, several recent articles suggest a consensus value of about 0.40 (*see Table 1*). Carroll (1998), Saez (2003) and Gruber and Saez (2000) all find an overall elasticity of 0.40, even though they examine different tax changes and use alternative methodologies. A closer examination of the studies suggests, however, that focusing on that single number masks considerable variation in the estimates. In addition, Saez (2004) and Kopczuk (2003) suggest a great deal of uncertainty surround their ETI estimates. In some cases the overall elasticities vary greatly depending on

specification. In other instances, breaking down the response by factors such as income source, filing status, and income group paints a different picture.

Saez (2003), for example, finds elasticities ranging from -0.14 to 0.71 by separately analyzing four income groups that are further divided by marital status and the decision to itemize deductions. An elasticity of 0.40 applies only to the overall response for married taxpayers or to the overall response for married and single taxpayers who itemize. The estimated overall elasticity is more than 20 percent smaller: 0.31. The 0.38 overall elasticity found by Carroll (1998, for OBRA 90 and OBRA 93) falls to 0.33 when he includes taxpayers on the AMT and rises to 0.56 when he restricts his comparison group, a sample of taxpayers facing no change in rates, to taxpayers with incomes above $75,000 (instead of above $50,000) and below the threshold where rates begin to change. Gruber and Saez (2002) come to 0.40 by including a 10-piece spline of logged income to control for heterogeneous movements in the income distribution (assumed to be unrelated to tax changes). But Kopczuk (2003), using nearly the same data and methodology, finds the elasticity falls by nearly 50 percent, to 0.21, when he uses the full sample instead of only taxpayers with incomes greater than $10,000. Kopczuk provides additional evidence on the sensitivity of estimates to sample selection and, furthermore, uses elaborate controls for exogenous variations in income, which yield a wide range of taxable-income elasticities.

ERTA and TRA 86

ERTA (1981) and TRA 86 both substantially reduced marginal tax rates. ERTA reduced the top rate from 70 percent to 50 percent, and TRA 86 dropped it further, to 28 percent. Both acts also

cut rates across income distribution. Those two acts are sometimes examined together.

Lindsey (1987) examines ERTA, treating the act as a natural experiment. He uses IRS tax return data from 1979 in conjunction with National Bureau of Economic Research's (NBER) TAXSIM model to make post-ERTA projections of the income distribution, assuming both pre-ERTA law and the macroeconomic conditions of the intervening years.[7] He then compares the counterfactual projections against the actual post-ERTA distribution of income and finds large taxable-income responses to the tax cuts. The consequent estimate of the overall ETI between 1.60 and 1.80[8] suggests that behavioral responses (to lower marginal tax rates) sharply reduce revenue losses, more than the labor supply literature predicted.[9] One important methodological issue is that Lindsey does not control for exogenous shifts to the income distribution (which were not widely recognized at the time) and thus attributes the secular income trend (especially at the high end) to ERTA. That method yields estimated elasticities that are almost certainly overstated.

Feldstein (1995) follows a sample of married taxpayers over time, and, instead of running a simulation to produce counterfactuals, uses a difference-in-differences approach to identify responses to TRA 86. Feldstein uses data from the Statistics of Income's (SOI) Continuous Work History Survey (CWHS) for years from 1985 and 1988. The sample includes married couples

7. Specifically, in creating the counterfactual baseline, TAXSIM "ages" the data by adjusting the various income sources by the corresponding changes to the National Income and Product Accounts.
8. For perspective, under the assumption that taxable income is an inverse function of the tax rate, under a single-rate tax system tax revenue is maximized when the net-of-tax elasticity equals (1 – *tax_rate*)/*tax_rate*), which is equivalent to an elasticity with respect to the tax rate of -1. Under that highly simplified model, an ETI of 0.40 would imply a revenue-maximizing income tax rate of 70 percent, whereas an ETI of 3.0 would imply a revenue-maximizing tax rate of 25 percent. (See Appendix I.)
9. Although most recent studies find small labor-supply elasticities (close to zero for prime-age males), Prescott (2004) finds very large responses when examining the impact of marginal tax rates across countries over time.

who were sampled in both 1985 and 1988 and who were under age 65 in 1988. Feldstein further excludes filers who began reporting Subchapter S income between 1985 and 1988, as well as those with 1985 marginal income tax rates below 22 percent.[10]

Feldstein groups taxpayers by their 1985 marginal tax rates and inflates 1985 AGI to 1988 by using the increase in personal income (17.4 percent) over the three years.[11] He then uses a difference-in-differences approach. For the first difference, he calculates both the percentage change in mean taxable income and the percentage change in the net-of-tax rate from 1985 to 1988. The result of that first step shows the relationship between change in tax rate and change in taxable income for each group. But, there may be other, non-tax, factors affecting taxable income that are not taken into account. In an attempt to remove the influence of those non-tax factors, Feldstein employs a second difference. For taxable income changes and tax rate changes respectively, he subtracts the percentage change for one of the two lower-income groups from the change for one of the two higher. If the influence of the non-tax factors is proportional across the groups, that second difference will remove the non-tax component from the income measure, leaving the portion that is a true response to the tax change. For the final step, Feldstein calculates the elasticities by dividing the second difference for taxable income by the second difference for the net-of-tax rates.

---

10. The chief difference between Subchapter S and Subchapter C small businesses is that Subchapter S allows shareholders to pass profits and losses through to their individual tax returns, whereas Subchapter C profits are taxed at the corporate level and are often retained with the firm, thus not appearing on the stockholder's individual tax return. As mentioned earlier, after TRA 86, many who had used Subchapter C corporations switched to Subchapter S, which were now taxed more favorably.

11. The sample is divided into three groups. Filers in the medium-income group have taxable income ranging from $21,020 to $62,450. That is followed by the high-income group, which includes those with taxable incomes up to $113,860. The highest-income group includes filers with taxable incomes above $113,860. Those ranges are expressed in 1985 dollars.

Feldstein's taxable-income elasticities for the two highest-income groups range from 1.10 to 3.05 for AGI and 1.04 to 1.48 for AGI plus gross losses. The difference-in-differences approach controls for unobserved factors that may have influenced the change in taxable income over the period. But, as mentioned above, it implicitly assumes that those unobserved factors have a proportional (or homogeneous) impact across the income distribution. If, as is now generally believed, the secular upward trend in taxable income is much more pronounced at the upper end of the distribution, then that approach leads to overestimates of the ETI (for periods when tax rates are falling).

Sample size and the influence of outliers should not be overlooked. Feldstein's estimates for the top income group derive from just 57 observations. Slemrod (1996) reports that one filer in the Feldstein's study reported Subchapter S income of $5000 in 1985 and Subchapter S income of more than $3 million in 1988.[12] Slemrod claims that that one filer was responsible for close to 50 percent of the growth in net partnership and Subchapter S income after TRA 86—and that just four filers accounted for close to 90 percent of the growth.

Auten and Carroll (1995) begin their empirical analysis by employing Feldstein's methodology, but compare CWHS data for 1985 against those for 1989 rather than 1988, and generally find large elasticities (from 0.46 to 3.04, but usually 2 or greater) that are consistent with Feldstein's. To test the sensitivity of their results, they explore several specifications, adding variables such

12. An individual in 1985 with nominal Subchapter S income and substantial Subchapter C income would not be excluded from the sample, but could bias results if he or she converted Subchapter C income to Subchapter S after TRA 86 took effect.

as age, age squared, and wealth to the model. They also expand the panel to include a large number of high-income filers (from the SOI) and add state tax rates (as well as demographic information) to the model. They find estimated ETIs that are quite sensitive to both income definition and regression weights. Auten and Carroll seem most confident in an ETI estimate of 0.74, which is based on a constant law definition of taxable income and employs regression weights (the analogous unweighted ETI estimate is 0.96) to control for the SOI's nonrandom sampling.[13] With the SOI, people from the 1985 sample that have higher income are more likely to be sampled in 1989 than those whose income declines. Auten and Carroll conclude by noting that their results are "highly sensitive to specification decisions" and that results are conditional on factors such as weighting, the definition of taxable income, and sample construction.[14]

Auten and Carroll (1999). The most recent research on taxable income in the 1980s not only uses panel data, following the same people over time, but also pays special attention to issues such as mean reversion and secular income trends, which likely plagued some of the earlier studies. Auten and Carroll focus on TRA 86 and draw a distinction between tax and non-tax factors that have contributed to the rise in income inequality in recent decades. Their key dependent variable is pre-tax taxable-income net of capital gains. They use SOI data and include an especially large

13. By contrast, the CWHS is a subsample of the SOI and represents a random sample of tax filers.
14. Moffitt and Wilhelm (2000), using data from 1983 and 1989 Survey of Consumer Finances (SCF), also find large responses to TRA 86, with nearly all of the action at the high end. Using Feldstein's (1995) methodology, they report an elasticity of 1.83. Because the SCF does not provide information on taxable income, their focus is on AGI. Moffitt and Wilhelm explore alternative specifications, which include adding independent variables to the model and various instruments for the net-of-tax rate. Results from those techniques vary greatly. Estimated elasticities are often large, but are generally smaller than those found using Feldstein's (1995) methodology. For example, when including asset values in the first-stage equation, they report a weighted elasticity of 0.65.

number of high-income filers.[15] Filers whose marital status changes between 1985 and 1989 are dropped, as are those for whom the secondary social security number changes. The sample includes only those ages 25 to 55 who were at or above the 22 percent marginal tax bracket ($21,020 joint, $15,610 single) in 1985. Also excluded are AMT filers and those who report Subchapter S income in 1989 but not in 1985.

Both constant-law gross and taxable income are calculated based on post-TRA 86 net of capital gains. The income variables (after first-differencing) are regressed against changes in instrumented marginal tax rates and against a set of demographic variables. The instrumented change in tax rates is calculated by inflating 1985 income to 1989 and calculating the corresponding tax rates under 1989 law. The actual change in tax rates is regressed against the instrumented change in rates and some exogenous variables. Fitted values from that regression are used in the final estimation. Non-tax control variables include a 1985-based wealth proxy, the number of children, age and age-squared, and dummies for children away from home, entrepreneurship, census region, and occupation (as a proxy for human capital).

Auten and Carroll are most confident in a weighted least-squares model that includes both tax and non-tax factors (including occupational dummies), which yields a 0.55 ETI with respect to the net-of-tax rate.[16] Excluding occupation information reduces the elasticity only slightly, to 0.51, and excluding all non-tax factors raises the elasticity to 0.75. (The unweighted models yield

15. Due to the SOI's sampling methods, those with rising incomes are more likely to be included in the sample than those with declining incomes. To account for this non-random attrition, a weighted regression model is employed.
16. Auten and Carroll emphasize results for the gross-income models, which are generally quite similar to those in the taxable-income models.

much higher elasticities, but are most certainly biased by nonrandom attrition.)

Gruber and Saez (2000) use data from the SOI's Continuous Work History File for years 1979 through 1990 to examine taxable income responses to ERTA and TRA 86. They calculate constant-law income using 1990 law excluding capital gains and use NBER's TAXSIM model to estimate federal and state tax rates. Behavioral changes are measured between pairs of years over three-year intervals. They use three-year intervals to focus on long-term responses instead of year-to-year fluctuations.[17] The natural log of the income growth is regressed against the instrumented change in the after-tax rate (and year fixed effects and dummies for marital status).[18]

Gruber and Saez control for tax rate endogeneity by using instrumented tax rates calculated first by inflating beginning-year income by the growth in broad income over the three-year interval.[19] Counterfactual tax rates are then calculated by applying the future tax law (three years hence) to the grossed-up beginning-year income. Next, the instrumented rates are imputed via two-stage least squares (where the actual change in tax rates is regressed against the counterfactual change in rates along with the other independent variables). Recognizing the possibility of mean reversion and secular trends in income, they explore three specifications using:

1) No controls other than scaling the data by the growth in broad income,
2) Log of initial period income as an independent variable, and

18. It is not clear that that procedure will eliminate transitory influences on the elasticity. Nearly every year of the 1980s is likely to have some transitory responses, in part because of multiyear phase-in periods. Thus, comparing years three years apart is unlikely to avoid transitory fluctuations.
19. An income effect variable is also discussed, but is left out of their most-preferred specification.
17. Broad income is defined as total income less capital gains and Social Security benefits. Taxable income is closest to the 1990 definition of taxable income. The measures are consistent for all years.

3) A 10-piece spline based on the log of initial-period income.

Regression results are dollar weighted. With dollar weighting, responses by high-income people are the driving factor in determining the overall elasticity. Scaling income by the growth in broad income is intended to control for inflation and exogenous increases in income. That assumes that overall income growth is independent of tax rates (or at least that that response is trivial for most of the distribution) and that exogenous growth impacts all taxpayers in the same manner.

Gruber and Saez control for mean reversion and exogenous income trends with independent variables that significantly affect elasticity estimates. They are most confident in the estimated ETI of 0.40 from the model that includes a 10-piece spline based on the natural log of initial-period income. The spline allows the functional relationship between the dependent variable and the independent variables to vary by decile,[20] but assumes that the relationship between initial income and income changes follows the same functional form within each decile.[21]

Gruber and Saez's corresponding elasticity for broad income is much smaller, 0.12, suggesting that much of the taxable-income response comes through deductions, exemptions, and exclusions, rather than through changes in labor supply. Elasticities vary greatly over the different income groups—from 0.57 for people with incomes over $100,000 to 0.11 for those from $50,000 to $100,000 and 0.18 for those from $10,000 to $50,000 range.

---

20. Including the log of base-year income in place of the spline imposes a single functional form between that variable and the dependent variable. The spline allows for more flexibility, which should improve the model fit by accounting, at least to some degree, for unobserved factors that have a heterogeneous impact on taxable-income growth across the reported taxable-income distribution.

21. Within the 1980s, the income pattern of the top one percent is often quite different from the pattern of the top 5 percent or top 10 percent.

Gruber and Saez use their preferred specification to estimate separate models with state rates and with only federal rates. The authors find no statistically significant difference between the coefficients on the state rate (0.63) and the federal rate (0.41).[22]

Kopczuk (2003) also uses CWHS data from 1979 to 1990 and tax rates from NBER's TAXSIM to examine the tax changes of the 1980s. He assumes that the ETI is not a structural parameter but rather a function of the tax system's structure: The ETI is a function of the tax base—or, more precisely, of taxable income divided by total income. Available deductions lower the cost of shifting income outside the tax base; as the tax base becomes smaller, responses to changes in tax rates increase. Kopczuk's model treats mean reversion and divergence within the income distribution as separate issues, each requiring its own variables to identify responses properly.

Kopczuk replicates Gruber and Saez's work showing the sensitivity of their results to sample selection. The chief difference between the two datasets is that Kopczuk uses the full sample, while Gruber and Saez exclude taxpayers with less than $10,000 of income. (Also, Kopczuk deflates his data by the CPI, whereas Gruber and Saez deflate by growth in broad income.) Using the log of initial income to control for mean reversion, Kopczuk reports an elasticity of 1.44 compared with 0.61 for Gruber and Saez. Using Gruber and Saez's preferred specification with a 10-piece spline, Kopczuk finds an elasticity of 0.21, versus 0.40 for Gruber and Saez.

22. That finding can be compared with Long (1999), who uses 1991 public use SOI data and exploits cross-state differences in marginal tax rates to produce taxable-income elasticities. Long reports overall elasticities between 0.19 and 0.82. Estimates are somewhat lower for married filers. The study contributes to the literature by using an identification strategy that relies solely on cross-sectional variation in tax rates, and not tax changes, for people with similar incomes. Like some of the other studies, Long shows that responses are quite sensitive to the model's specification. (The data for the study are restricted to filers with incomes between $0 and $200,000.)

To explore the sample selection issue further, Kopczuk performs a sensitivity analysis using data from 1985 through 1988. He finds that, at least in that instance, relatively modest changes to the sample can produce a wide range of reasonable values. He notes that Gruber and Saez restrict their sample to those earning more than $10,000 and that Auten and Carroll (as well as Feldstein) exclude those with marginal tax rates below 22 percent.[23] Kopczuk finds that, with no other restriction, excluding taxpayers with incomes below $10,000 results in an elasticity of 0.347. Excluding those with tax rates below 22 percent yields an elasticity of 0.85. Making no sample restrictions gives an elasticity of 2.72. Increasing the exclusion to filers earning less than $30,000 lowers the elasticity to essentially 0, suggesting that mean reversion is important at the bottom of the distribution. Those results show elasticities that are highly sensitive to sample selection. As discussed in the previous section, mean reversion can bias ETI estimates if a portion of the non-tax-related increase in income is mistakenly attributed to a response to the tax change. In addition, nonrandom attrition at the low end of the income distribution may further bias ETI estimates because people with rising incomes are likely to remain in the panel, while some with falling incomes may disappear from the sample if they no longer file a tax return. One interpretation of Kopczuk's finding is that it is not possible to accurately estimate the true overall ETI because estimates are highly sensitive to modest changes to the sample and because there are no sound criteria for restricting the sample. Another, more optimistic interpretation is that better methods are needed to control for mean reversion and nonrandom attrition—or that improvements could result from the development of sound criteria for restricting the sample.

23. Kopczuk wants to compare sample restrictions used by Feldstein (1995), Auten and Carroll (1999), and Gruber and Saez (2002). He restricts the analysis to TRA 86, a period covered in the other three papers.

Kopczuk experiments with alternative controls, including specifications that control separately for mean reversion and divergence in the income distribution. Controlling separately for those two factors results in elasticities between 0.55 and 1.37.[24] Kopczuk's preferred specification, which includes both a spline based on the log of 1979 income and a spline based on the difference between the log of taxable income in the beginning year of each three-year period and the log of 1979 taxable income, yields an elasticity of 0.57.

Kopczuk repeats the analysis after adding information on the size of the tax base interacted with the net-of-tax rate, and finds the coefficient on that variable to be statistically significant for all specifications. As predicted, the elasticity falls as the tax base increases. Coefficients on the direct tax elasticity still vary greatly depending on the other controls. His most preferred specification, including the tax base effect, yields a direct tax elasticity of 0.44. Evaluated at the average tax base, that is consistent with an elasticity of 0.53.[25] Again, estimates are much larger for single filers than for married filers: The corresponding elasticity for married filers is just 0.09. Kopczuk discounts his results for single filers, noting the lack of demographic information; he suggests that younger filers entering the workforce may be biasing those estimates upward.

Slemrod (1996) addresses some of the non-tax factors, including changes in wage inequality, that may have biased earlier studies. In addition, he highlights the importance of maintaining a

24. Corresponding elasticities for married filers are much lower. That finding contrasts with other studies, such as Saez (2003), which find larger elasticities for married filers.
25. Kopczuk adds the direct coefficient on the log of the net-of-tax rate (0.44) to the product of one minus the tax base (0.09) and the coefficient on the tax base control (1.04). That yields 0.53. The tax base control variable is constructed by multiplying one minus the tax base times the log of the net-of-tax rate.

constant definition of taxable income—but also shows that the definition chosen can affect the results. Slemrod focuses on the share of taxable income accruing to the top 1 percent of the distribution, building on the research of Feenberg and Poterba (1993).[26]

Slemrod's use of aggregated time-series data (which includes a share approach that focuses on the share of income reported by various segments of the taxable income distribution) differs greatly from the differencing approaches used with panel data. Using time-series data to examine the impact of tax rates on taxable income (or taxable income shares) avoids problems associated with mean reversion. In addition, the problem of transitory shifting, which can confuse short-term responses with the permanent response, are less severe. The share approach has another advantage over differencing techniques that treat the behavior of one income group as a counterfactual in estimating the ETI. With panel data, if the ETIs or the exogenous taxable income trends differ by income, differencing alone will result in biased ETI estimates, since differencing implicitly assumes that the unobserved factors influencing taxable income are very similar and affect taxable income in the same manner for both groups.

The use of aggregated time-series data does, however, have some drawbacks. The issue of exogenous income trends that differ across the income distribution is not necessarily overcome by using aggregated time-series data. In addition, panel data are superior for capturing shifting

26. Using aggregated time-series data from 1954 to 1990, Feenberg and Poterba find the share of taxable income going to the top one-half of one percent increasing, and find that that increase accelerated in the 1980s after ERTA and TRA 86, an apparent response to those tax cuts. Feenberg and Poterba, however, do not use a constant-law definition of income. Slemrod (1996) builds directly on Feenberg and Poterba, using the same data (with adjustments to account for changes to the definition of taxable income) and beginning with a sensitivity analysis of their results.

that occurs within the income distribution. For example, suppose that, in response to a change in tax rates, people at the top of the income distribution substitute taxable bonds for tax-exempt bonds and those in lower tax brackets do the reverse, substituting taxable bonds for tax-exempt. The net effect of that change on taxable income may be zero, but a share approach focusing on the top portion of the income distribution may capture the drop in income but miss the corresponding rise in other segments of the distribution. On top of that, rank reversal, in which the composition of the segment of the income distribution under examination changes over time, can potentially bias ETIs estimated using aggregated time-series data. (Rank reversal is discussed in more detail below.)

Slemrod illustrates the complicating factors inherent in estimating taxable-income elasticities, showing that apparently innocuous assumptions can have an important impact on the estimates. He uses both pre- and post-TRA 86 taxable-income definitions to show that constant-law definitions yield positive but smaller growth in the income share for the top one-half of one percent. For example, using concurrent taxable-income definitions, the share accruing to the top one-half of one percent increases by 3.4 percentage points from 1984 to 1990. Using the 1990 definition, that increase falls to 1.9 percentage points.

Next, Slemrod regresses the share of taxable income received by the top one percent of filers from each of several different sources (AGI, wages and salaries, capital gains, interest, and dividends) against both tax and non-tax variables. The key explanatory tax variables are based on current, lagged, and prospective tax rates for both realized capital gains and labor income. The non-tax

factors include a measure of wage inequality,[27] the real level of stock prices as a proxy for non-tax influences on capital-gains realizations, and the nominal AAA bond interest rate.

Slemrod isolates the effects of TRA 86 by comparing regressions based on data for 1954 through 1985 and those based on 1954 through 1990. He finds that adding the years surrounding TRA 86 fundamentally alters the results. In addition to marked changes to the ETI, the coefficient on the wage inequality variable is vastly different when the post-1985 data are included. With data from 1954 to 1985, the coefficient on the wage inequality variable is positive and often large; for that same period, the "permanent" response to rate changes is extremely small. That suggests that demand factors, not taxes, are the driving force behind the growth in taxable income. When the 1985 through 1990 data are added, the opposite is found: Taxes appear to be the driving force behind the taxable income growth at the top of the distribution. With the addition of the 1985 through 1990 data, the coefficient on the concurrent labor tax variable is much larger (or more negative, since Slemrod uses tax rates as opposed to net-of-tax rates).[28]

Slemrod concludes that even estimates based on a constant-law income definition may overstate the taxable income gains accruing to the top 1 percent in response to TRA 86. Although real responses may well be large, a significant portion of the increases is likely because of changes in the tax structure that lead people to shift income and change their reporting behavior, and not

27. The measure is calculated as the difference between the log of the 90th and the 10th percentiles of male weekly wages.
28. The increase from 1985 to 1990 in the share of wages earned by the top 0.5 percent and top 1 percent exceeded the increases from 1973 to 1985, but the difference in earnings of the 90th and 10th percentiles actually fell between 1985 and 1990. So, with the post-TRA 86 data, an increase in wage inequality is actually associated with a decrease in the concentration of taxable income held by the top 0.5 percent of the distribution.

because of traditional behavioral responses to lower marginal tax rates. Shifting income from corporate to individual sources plays an important role. Other factors include changing demographics at the top of the distribution. Post-1986, the wealthiest taxpayers were younger and thus more likely to have increasing incomes in the absence of rate changes.

Rank reversal, where, in the context of Slemrod's paper, the composition of people with reported taxable incomes in the top 1 percent changes over time and taxable-income sources are cross-sectionally correlated, could also contribute to the results. For example, a positive correlation exists between Subchapter S income and wage and salary income. As income is shifted to Subchapter S after TRA 86, more filers with Subchapter S earnings move into the top 1 percent. That is, the shifting of income from Subchapter C, which is not observed, to Subchapter S, which is observed, results in a re-ranking of tax filers, thus altering the composition of the top 1 percent. If those filers that move into the top 1 percent as a result of the re-ranking also have higher wages and salaries than the filers they replace, it may appear that the wage and salary increase is a behavioral response. Insight into the impact of that correlation on the estimated elasticities could be gained from repeating the analysis, while omitting Subchapter S income.[29]

Saez (2004) also employs a share analysis to aggregated time-series data. Saez uses aggregated SOI data from 1960 to 2000 and a consistent definition of taxable income that excludes realized

29.Slemrod cites another example of rank reversal that could result from not using a constant law definition of taxable income. Slemrod notes that pre-TRA 86, 40 percent of realized capital gains were counted as taxable income, while post-TRA 86 100 percent were included. Use of concurrent definitions of taxable income likely overstates the growth in the share of income held by the top 1 percent, because those in the top 1 percent likely have a greater share of capital gains than the rest of the distribution. In addition, some who derive a large fraction of their income from capital gains would move into the top 1 percent. Thus, the cross-sectional correlation between realized capital gains and other sources of income would also be spuriously correlated with the change in tax rates.

capital gains. Average marginal tax rates for each group are estimated via TAXSIM, and ETIs are generally weighted by income. Saez's study is unique in that it does not focus primarily on the tax cuts of 1980s or the tax increases of the 1990s, but examines the responses to all of the tax changes over the past four decades.

Using a simple model where the log of taxable income is regressed against the log of the net-of-tax rate with no controls for non-tax related trends in taxable income, Saez reports a statistically insignificant estimated ETI of -0.44 for the entire distribution of tax filers. Adding a time trend raises the overall estimate ETI to 0. Including the time trend and the square of the time trend raises the overall estimated ETI to a statistically insignificant 0.20. For the top 1 percent of the taxable income distribution, the results are quite different. All three specifications yield much larger estimated ETIs and all show strong statistical significance. With no controls, the estimated ETI for the top 1 percent is 1.83. With the time trend, that number falls to 0.71. Adding the square of the time trend lowers the ETI estimate further to 0.50. (For the bottom 99 percent of the distribution, ETI estimates are negative, and not statistically different from 0, for each specification.)

Saez reports estimated ETIs that vary greatly over some subsets of the 40 years examined. For example, dividing the change in log income from 1981 to 1984 by the change in logged net-of-tax rates between the same years yields an estimated ETI of 0.77. The same analysis, comparing 1985 to 1988, yields a much larger estimated ETI of 1.7. Comparing 1991 with 1994 yields an estimated ETI of about 0. The variation in ETIs over time (and within the distribution) may be

consistent with Kopczuk's (2003) finding that the ETI depends on the size of the tax base (or the availability of deductions).

Next, Saez employs a regression framework that uses taxable income shares to estimate ETIs for different segments of the taxable-income distribution. Special attention is paid to the top 1 percent of filers. For the various taxable-income groups, Saez regresses the log of the group's share of taxable income against the log of the net-of-tax rate.[30] Without any time trends, that yields an estimated ETI of 1.58 for the top 1 percent. Including both the time trend and square of the time trend yields an estimated ETI of 0.62. Saez expresses confidence in the 0.62 estimate because that regression has an adjusted coefficient of determination of 0.98 and the fitted values do an excellent job of tracking the trend in the share of income reported by the top 1 percent. (By contrast, predicted values from the model with no time trends fit the data poorly.) By further division of the taxable-income distribution, Saez shows that, even among the top 1 percent of the distribution, estimated ETIs vary greatly by income. In fact, the same approach that yields 0.62 for the top 1 percent yields an estimated ETI of 1.09 for the top one-hundredth of one percent. For those in the 90th to 95th percentiles, the same approach yields a negative, although statistically insignificant, estimated ETI.

The analyses of the tax cuts in the 1980s developed methodologies to estimate taxable-income elasticities and advanced the understanding of the key issues, but did not fully resolve the issue.

30. Saez also tries a two-stage approach, where the top net-of-tax rate is used as an instrument for the tax rate variable. For the most part, he concludes that the two-stage approach does not affect the results and, in the case where it does, he prefers the OLS result because the instrument in that case is weak.

The large elasticities estimated by the earliest studies have been supplanted by lower estimates in later studies that included more elaborate controls for important non-tax factors. The newer estimates, however, are sensitive to sample selection and the choice of control variables.

OBRA 90 and OBRA 93

In contrast with the rising incomes and declining tax rates of the 1980s, income and tax rates both rose in the early 1990s. In the 1980s, high-income taxpayers, whose tax rates fell most, experienced the greatest income gains. In the 1990s, the same group again experienced the greatest income gains even though their tax rates increased.

The tax increases of the early 1990s provided an opportunity to use methodological advances in a new context. The tax changes of OBRA 90 and 93 were smaller and more focused on high-income filers than those of the 1980s, potentially focusing the effects more narrowly. At the same time, the ETI cannot be estimated for lower-income taxpayers because their rates did not change. Furthermore, any potential bias from the secular trend at the top of the distribution is reversed from the 1980s, when the exogenous growth of income in the 1980s potentially biased estimates of tax elasticities upward. In contrast, exogenous growth of incomes in the 1990s may have hidden any drops in income induced by increased tax rates and thus may have biased elasticity estimates downward.

Carroll (1998) uses SOI data for a group of taxpayers from 1989 through 1995 to analyze the

responsiveness of taxable income to the tax increases of 1990 and 1993.[31] The analysis includes only taxpayers between ages 25 and 55 in 1989. People with income below $50,000 were not affected by the two tax increases and were excluded, in part, because they were deemed a poor control group in identifying the response of higher-income filers.

To control for tax-rate endogeneity and mean reversion, Carroll uses average income over the seven years to generate a "synthetic" or instrumented tax rate that varies only with exogenous tax-law changes.[32] His model includes year and individual fixed effects, age, proxies for industry and region, financial wealth, and information on children.

Carroll is most confident in the specification yielding a ETI of 0.38.[33] (The corresponding elasticity with respect to gross income is 0.32.) Excluding more returns at the low end of the distribution by raising the exclusion to $75,000 increases the estimated elasticity to 0.56. Restricting the sample probably raises Carroll's estimate because he examines tax increases (as opposed to decreases). That is consistent with mean reversion at the low end.[34]

Goolsbee (2000) examines high-income responses to OBRA 93 using panel data on compensation for the five highest-paid employees for each firm in the Standard & Poor's 500, Standard &

31. As with Auten and Carroll, a weighting procedure is used in the regression analysis to control for attrition bias.
32. Carroll uses an instrumental-variables approach to impute the tax rate (because the tax rate is a function of income, which is not exogenous). The approach assigns tax rates based on the filer's permanent income (defined as the average income over the seven-year period).
33. Simulations suggest that the revenue gain from the 1993 tax increase may have been as much as 39 percent or $6.8 billion (or as little as 12 percent or $2.1 billion), less than what would have occurred absent behavioral responses.
34. As mentioned earlier, estimates from OBRA 90 and OBRA 93 correspond only to higher-income filers because rates were raised only for that group.

Poor's Mid Cap 400, and the Standard & Poor's Small Cap 600 from 1991 through 1995. The sample includes 21,299 observations for executives with at least four years of observations. Included variables are salary, bonuses, Long-Term Incentive Plan payouts, stock options exercised, and other income. (Capital gains, other family income, and deductions are not available. The estimation generally assumes that earners are married filing jointly and that the spouses are not in the labor force.) The panel has a mean of $852,000, a median of $451,000, and 25th and 5th percentile values of $250,000 and $150,000, respectively.

In his standard specification, Goolsbee regresses income against both the net-of-tax rate and the future year's net-of-tax rate, along with year dummies and firm-specific information. Sources of executive compensation vary greatly from year to year (and in some years, no income is reported for some sources), which complicates matters. To address those issues, Goolsbee explores various approaches, including regressions with linear fixed effects, first-differencing, and split samples. He also divides responses into short-term (such as the timing of transactions) and more permanent behavioral changes.

In general, Goolsbee finds large responses to the current net of tax rate—ETIs often well above one.[35] But, significantly, much of that response is transitory—that is, a temporary shifting of income into the relatively low-tax period. Longer-term (or permanent) elasticities are much smaller—often close to 0, but as high as 0.40. Using the specification yielding the latter result,

35. Goolsbee focuses on actual tax rates after concluding (from running a regression with tax rates based on permanent income) that tax-rate endogeneity is a minor issue with this sample because nearly everyone is always in the top bracket.

Goolsbee divides his sample into income groups and finds the largest permanent responses (0.56) for those earning over $1 million. Among income sources, stock options are the most responsive to tax rates.

Goolsbee uses few controls for exogenous changes to the income distribution, which could bias responses down. However, his focus on high-income filers and the inclusion of firm-specific variables may control for some of the exogenous trends in the income distribution. Also, the dataset includes only observations appearing four or more times. If attrition is non-random, and is more likely for those whose income falls, the estimates would be biased downward, because tax rates are rising.

Sammartino and Weiner (1997) use data from the SOI's Sales of Capital Assets (SOCA) panel, a subset of the SOI that follows the same (mostly high-income) taxpayers from 1989 through 1994, to examine responses to the tax increases associated with OBRA 90 and OBRA 93. They do not generate taxable income elasticities, but do examine income patterns surrounding OBRA 90 and OBRA 93. For the top 1 percent of the distribution, AGI plus gains fluctuates greatly from year-to-year, at least in part because of tax changes, but the responses appear to be transitory, and are much more modest when examined over several years. For example, average AGI less capital gains for the top 1 percent rises by over $40,000 in 1992 (preceding OBRA 93), falls by nearly $26,000 in 1993, but rises by close to $24,000 in 1994 and 1995 combined.

Saez (2003) examines responses to tax increases resulting from bracket creep (as opposed to

legislated tax changes) between 1979 and 1981, using data from the University of Michigan Panel, a publicly available SOI-based dataset. Over those years, inflation was around 10 percent a year, but the tax brackets were fixed in nominal terms (that is, not indexed). Saez compares taxpayers with similar incomes (and thus the same tax rate) in period 1, but whose predicted incomes (that year's income grown at the inflation rate) put them into different brackets in a subsequent year.

That approach has the advantage of examining tax changes that result from inflation, and not from explicit legislation that often also alters the structure of the tax system, obfuscating true responses to the rate changes. However, because the tax changes were not legislated, many filers may not have realized that their marginal tax rate had risen. That could be especially problematic when applying those estimates to cases where rate changes are explicitly legislated. Furthermore, the study cannot examine taxpayers in the top bracket because inflation cannot move them to a higher tax rate.

Saez finds a statistically insignificant ETI of 0.311 for all earners, which he decomposes into values of 0.42 for itemizers and about 0 for non-itemizers. The elasticities are much larger and are statistically significant for married taxpayers: 0.39 overall and 0.65 for itemizers. Saez argues that itemized deductions are likely a key factor driving the taxable income estimates, which is consistent with Kopczuk's findings on the tax base. He also emphasizes that the estimates capture only the short-term response to the rate changes.

Studies examining responses to the tax increases of the early 1990s generally find smaller ETIs than the earlier work looking at tax changes in the 1980s. Transitory changes are often large, but permanent responses are much smaller. Unlike the 1980s, responses to the tax changes of the 1990s only apply to the top brackets, which were the only groups experiencing a tax change. The 1990s tax changes involved few changes to the structure of the tax system (in contrast to TRA 86), which make the estimation process somewhat less complex.

V. Conclusion

The ETI, which measures the responsiveness of reported income to changes to marginal tax rates, is a key parameter for estimating the impact of income-tax-rate changes on tax revenues and for measuring the efficiency implications of those rate changes. Studies have examined the tax changes of 1981, 1986, 1990, and 1993, as well as the bracket creep of the late 1970s and early 1980s, to estimate an overall ETI with respect to the net-of-tax rate. In addition, some studies have separately estimated ETIs for different income groups. Estimated elasticities appear to vary by income, with the highest-income households being the most responsive. In terms of overall findings, several of the more recent studies have reached similar conclusions, finding overall elasticities of about 0.40. A closer examination of the literature, however, suggests a great deal of variation. Estimated elasticities are often quite sensitive to sample selection, methodology, and the reform under consideration.

Research over the past two decades has greatly improved the understanding of the relationship between taxes and income, but has also highlighted many complicating factors such as tax-rate

endogeneity, mean reversion, and exogenous distributional changes that differ greatly across income classes. Those factors make parsing taxable-income elasticities from the array of non-tax factors that also affect taxable income difficult, and current methods may still fail to control fully for them.

The logical next steps, some of which are already under way, include a deeper examination into the importance of the tax base, a decomposition of responses by income source and type of deduction, extending the years of data to include both tax increases and decreases, more work on the non-tax factors driving the heterogeneous income trends across the distribution, and a closer examination of income shifting between the corporate and individual tax bases. One promising approach (see Kopczuk, 2003) involves including the tax base (or the availability of deductions) in the model to see whether it is a determinant of the elasticity. Decomposing responses found in earlier research by the different sources of income would improve our understanding of the process underlying the overall response. Evidence suggests that particular income sources are driving the overall estimates. Separately calculating elasticities for each component of taxable income would reveal which factors have the greatest influence on overall responses. Conducting the analysis for different tax changes or over longer periods would show whether the same factors always dominate or whether the determining factors vary, depending on the nature of the tax change or other factors. Both the direction of response and the robustness of estimates may differ, depending on whether tax rates rise or fall. In addition to those issues, the exogenous income trends that differ across the income distribution demand further investigation to determine, for example, whether controls

for the trends can explain income changes over years without tax changes or, perhaps more accurately, over years when people seem to have adjusted to previous or anticipated tax changes. Finally, the shifting of income between the corporate and individual tax bases is a response to changing tax rates that is only partially understood and demands further research. Clearly, much work is still needed in order to better understand the process by which incomes respond to tax changes.

Table 1. Empirical Studies on the Elasticity of Taxable Income

Paper	Data	Approach[1]	Focus	Tax Change (Years)	Permanent v. Transitory	Best Estimate	Comments
Auten & Carroll (1995)	CWHS	PS	All income groups	TRA 86 (1985, 1989)	Permanent	0.46 to 3.04	Replicates Feldstein (1995) and explores additional specifications.
Auten & Carroll (1999)	SOI	PS	Joint > $21k Single> $15.6k	TRA 86 (1985, 1989)	Permanent	0.45 to 1.13 0.57 best estimate	Uses first-differences with year fixed effects and individual characteristics, such as occupation.
Carroll (1998)	SOI	PS	> $50k	OBRA 90 & 93 (1989-1995)	Permanent	0.4	Uses weighted least squares year and individual FE, and demographic/occupational information.
Feldstein (1995)	SOI/CWHS	PS	All income groups	TRA 86 (1985, 1988)	Permanent	1.04 to 3.05	Uses difference-in-differences.
Goolsbee (2000)	S&P EXECUCOMP	PS	Upper Income	OBRA 93 (1991-1995)	Both	0 to 0.40	Includes variables for current and future tax rates. Tries to distinguish between the transitory and permanent responses.
Gruber & Saez (2002)	SOI/CWHS	PS	All income groups	ERTA & TRA 86 (1979-1990)	Permanent	0.4	ln(incomet/incomet+3) is regressed against year dummies, marital status, and a 10-piece spline based on ln(incomet) breakdown by income.
Kopczuk (2003)	SOI/CWHS	PS	All income groups	ERTA & TRA 86 (1979-1990)	Permanent	0.21 to 0.57 w/o tax base effect 0.53 overall	Builds on Gruber and Saez. Presents a model that separates mean reversion and exogenous distributional changes into two variables.
Lindsey (1987)	SOI	CS	All income groups	ERTA (1979)	Not clear	1.6 to 1.8	Compares simulated counterfactual to actual outcomes.
Long (1999)	SOI Public Use File	CS	$0k to $200k	State variation (1991)	More transitory	0.193 to 0.819	Uses differences in tax rates across states for identification.
Moffitt & Wilhelm (2000)	SCF	PS	All income groups	TRA 86 (1983, 1989)	Permanent	0 to 1.83	Uses SCF data and explores several specifications, including a replication of Feldstein's methodology.
Saez (2003)[3]	SOI/Michigan Panel	PS	All income groups	Bracket creep (1979-1981)	Transitory	0.311	Uses bracket creep from the non-indexation of tax rates. Uses a natural experiment approach.
Saez (2004)	SOI	TS	All income groups	1960-2000	Permanent	0 to 1.7 (pairs of years) 0.62 for top 1%	Uses aggregated time-series data from 1960 through 2000. Special attention is paid to the top of the distribution. Time trends are used to control for non-tax-related income trends.
Sammartino & Weiner (1997)	SOI/SOCA	TS	Top 1%	OBRA 90 & 93 (1989-1994)	Both	large transitory small permanent	Calculates shares of income (by type) accruing to the top 1%. Also calculates the shares for cohorts grouped by MTRs.
Slemrod (1996)[2]	SOI	TS	Top 1%	TRA 86 (1954-1990)	Both		Regresses shares of income against tax variables and other controls.

1. PS = panel following the same people; TS = time-series; CS = cross-sectional.
2. This study builds on Feenberg and Poterba (1993) and uses essentially the same core dataset.
3. Those starting in the top bracket are not impacted by bracket creep. For joint filers, this group has income of greater than $162, 400; for singles, the highest bracket begins at $81,800.

Primary Articles

[1] Auten, Gerald and Robert Carroll, "Behavior of the Affluent and the 1986 Tax Reform Act," in *Proceedings of the 87th Annual Conference on Taxation of the National Tax Association*, Columbus, Ohio, 1995, 7-12.

[2] Auten, Gerald and Robert Carroll, "The Effect of Income Taxes on Household Behavior," *Review of Economics and Statistics*, November 1999, 81 (4), 681-693.

[3] Carroll, Robert, "Do Taxpayers Really Respond to Changes in Tax Rates? Evidence from the 1993 Act," Office of Tax Analysis Working Paper 78, U.S. Department of Treasury, 1998.

[4] Feenberg, Daniel and James Poterba, "Income Inequality and the Incomes of Very High Income Taxpayers: Evidence from Tax Returns," *Tax Policy and the Economy,* J. Poterba, ed., MIT Press: Cambridge, 1993, 7, 145-177.

[5] Feldstein, Martin S., "Tax Avoidance and the Deadweight Loss of the Income Tax," *Review of Economics and Statistics*, November 1999, 81 (4), 674-680.

[6] Feldstein, Martin S., "The Effect of Marginal Tax Rates on Taxable Income: A Panel Study of the 1986 Tax Reform Act," *Journal of Political Economy*, June 1995, 103 (3), 551-572.

[7] Goolsbee, Austan, "What Happens When You Tax the Rich? Evidence from Executive Compensation," *Journal of Political Economy*, 2000, 108 (2), 352-378.

[8] Gruber, Jon and Emmanuel Saez, "The Elasticity of Taxable Income: Evidence and Implications," *Journal of Public Economics*, April 2002, 84 (1), 1-32.

[9] Kopczuk, Wojciech, "Tax Bases, Tax Rates and the Elasticity of Reported Income," National Bureau of Economic Research Working Paper 10044, www.nber.org/papers/w10044, October 2003.

[10] Lindsey, Lawrence, "Individual Taxpayer Response to Tax Cuts: 1982-1984, with Implications for the Revenue Maximizing Tax Rate," *Journal of Public Economics*, July 1987, 33 (2), 173-206.

[11] Long, James E., "The Impact of Marginal Tax Rates on Taxable Income: Evidence from State Income Tax Differentials," *Southern Economic Journal*, April 1999, 65 (4), 855-869.

[12] Moffitt, Robert and Mark Wilhelm, "Taxation and the Labor Supply Decisions of the Affluent," *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, Joel Slemrod, ed., New York: Harvard University Press and Russell Sage Foundation, 2000.

[13] Piketty, Thomas and Emmanuel Saez, "Income Inequality in the United States, 1913-1998," *Quarterly Journal of Economics*, February 2003, 118 (1), 1-39.

[14] Saez, Emmanuel, "Reported Incomes and Marginal Tax Rates, 1960-2000: Evidence and Policy Implications," National Bureau of Economic Research Working Paper 10273, www.nber.org/papers/w10273, January 2004.

[15] Saez, Emmanuel, "The Effect of Marginal Tax Rates on Income: A Panel Study of 'Bracket Creep,'" *Journal of Public Economics*, 2003, 87, 1231-1258.

[16] Sammartino, Frank and David Weiner, "Recent Evidence on Taxpayers' Response to the Rate Increases in the 1990s," *National Tax Journal*, September 1997, 50 (3), 683-705.

[17] Slemrod, Joel, "Income Creation or Income Shifting? Behavioral Responses to the Tax Reform Act of 1986," *American Economic Review*, 1995, 85 (2), 175-180.

[18] Slemrod, Joel, "High Income Families and the Tax Changes of the 1980s: the Anatomy of Behavioral Response," *Empirical Foundations of Household Taxation*, M. Feldstein and J. Poterba, eds., University of Chicago, 1996.

[19] Slemrod, Joel, "Methodological Issues in Measuring and Interpreting Taxable Income Elasticities," *National Tax Journal*, December 1998, 51 (4), 773-788.

[20] Slemrod, Joel and Shlomo Yitzhaki, "Tax Avoidance, Evasion, and Administration," National Bureau of Economic Research Working Paper No. w7473, January 2000.

Ancillary Material

[1] Aarbu, Karl O. and Thor O. Thoreson, "Income Responses to Tax Changes—Evidence from the Norwegian Tax Reform," *National Tax Journal*, June 2001, 54 (2), 319-334.

[2] Auten, Gerald E., Holger Sieg, and Charles T. Clotfelter, "Charitable Giving, Income and Taxes: An Analysis of Panel Data," *American Economic Review*, March 2002, 92 (1), 371-382.

[3] Blundell, R. and T. MaCurdy, "Labor Supply: A Review of Alternative Approaches," *Handbook of Labor Economics*, O. Ashenfelter and D. Card, eds., North-Holland, Amsterdam, 1999, 3A.

[4] Brownlee, W. Elliot, "Historical Perspective on U.S. Tax Policy toward the Rich,." *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, Joel Slemrod, ed., 2000.

[5] Buchanan, James M., "The Political Efficiency of General Taxation" *National Tax Journal*, December 1993, 46, 401-410.

[6] DiNardo, John, Nicole Fortin, and Thomas Lemieux, "Labor Market Institutions and the Distribution of Wages, 1973-1992: A Semiparametric Approach." *Econometrica*, 1996, 64 (5), 1001-1044.

[7] Feenberg, Daniel R. and Elizabeth Coutts, "An Introduction to the TAXSIM Model," *Journal of Policy Analysis and Management*, Winter 1993, 12 (1), 189-94.

[8] Feenberg, Daniel and James Poterba, "The Income and Tax Share of Very High Income Households, 1960-1995," *American Economic Review Papers & Proceedings*, 2000, 90 (2), 264-270.

[9] Feldstein, Martin S., "Tax Avoidance and the Deadweight Loss of the Income Tax," *Review of Economics and Statistics*, November 1999, 4 (81), 674-680.

[10] Feldstein, Martin and Daniel R. Feenberg, "Higher Tax Rates with Little Revenue Gain: An Empirical Analysis of the Clinton Tax Plan." *Tax Notes*, March 22, 1993, 58, 1653-1657.

[11] Goldin, Claudia and Lawrence Katz, "The Returns to Skill across the Twentieth Century United States," National Bureau of Economic Research Working Paper No. 7126, 1999.

[12] Goldin, Claudia and Robert Margo, "The Great Compression: The Wage Structure in the United States at Mid-Century," *Quarterly Journal of Economics*, 1992, 107 (1), 1-34.

[13] Goldsmith, Selma, George Jaszi, Hyman Kaitz, and Maurice Liebenberg, "Size Distribution of Income Since the Mid-Thirties," *Review of Economics and Statistics*, 1954, 36 (1), 1-32.

[14] Goolsbee, Austan, "It's Not About the Money: Why Natural Experiments Don't Work on the Rich," *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, Joel Slemrod, ed., New York: Harvard University Press and Russell Sage Foundation, 2000.

[15] Goolsbee, Austan, "Evidence on the High-Income Laffer Curve from Six Decades of Tax Reforms," *Brookings Papers on Economic Activity*, 1999 (2), 1-47.

[16] Gordon, Roger and Joel Slemrod, "Are 'Real' Responses to Taxes Simply Income Shifting Between Corporate and Personal Tax Bases," *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, Joel Slemrod, ed., Cambridge University Press, 2000.

[17] Kopczuk, Wojciech and Emmanuel Saez, "Top Wealth Shares in the United States, 1916-2000: Evidence from Estate Tax Returns," *National Tax Journal*, 2004, 57 Part 2, 445.

[18] Hall, Brian and Jeffrey Liebman, "The Taxation of Executive Compensation," *Tax Policy and the Economy*, J. Poterba, ed., MIT Press: Cambridge, 2000.

[19] Katz, Lawrence and David Autor, "Changes in the Wage Structure and Earnings Inequality," *Handbook of Labor Economics*, O. Ashenfelter and D. Card, eds., North-Holland, 1999, 3A.

[20] Katz, Lawrence and Kevin Murphy, "Changes in Relative Wages, 1963-1987: Supply and Demand Factors." *Quarterly Journal of Economics*, 1992, 107 (1), 35-78.

[21] Keat, Paul, "Longrun Changes in Occupational Wage Structure, 1900-1956," *Journal of Political Economy*, 1960, 68, 584-600.

[22] Kopczuk, Wojciech, "Redistribution When Avoidance Behavior is Heterogeneous," *Journal of Public Economics*, July 2001, 81 (1), 51-71.

[23] Kuznets, Simon, *Shares of Upper Income Groups in Income and Savings*, National Bureau of Economic Research, 1953.

[24] Lindert, Peter, "Three Centuries of Inequality in Britain and America," *Handbook of Income Distribution*, A. Atkinson and F. Bourguignon, eds., North-Holland, 2000, 167-216.

[25] Mayshar, Joram, "Taxation with Costly Administration," *Scandinavian Journal of Economics*, 1991, 93 (1), 75-88.

[26] Mellon, Andrew, *Taxation: The People's Business*, New York, 1924.

[27] Murphy, Kevin R., "Executive Compensation," *Handbook of Labor Economics*, O. Ashenfelter and D. Card, eds., North-Holland, 1999, 3B.

[28] Navratil, J., "The Economic Recovery Tax Act of 1981: Evidence on Individual Taxpayer Behavior from Panel Tax Return Data," unpublished thesis, 1995.

[29] Park, Thae S., "Comparison of BEA Estimates of Personal Income and IRS Estimates of Adjusted Gross Income," *Survey of Current Business*, November 2000, 7-13.

[30] Prescott, Edward C., "Why Do Americans Work So Much More Than Europeans?," *Federal Reserve Bank of Minneapolis Quarterly Review*, 28 (1), July 2004, 2-13.

[31] Rotemberg, Julio, "Perceptions of Equity and the Distribution of Income," National Bureau of Economic Research Working Paper No. 5624, 1996.

[32]Sillamaa, Mary Anne and Michael R. Veall, "The Effect of Marginal Tax Rates on Taxable Income: a Panel Study of the 1988 Tax Flattening in Canada," *Journal of Public Economics*, June 2001, 80 (3), 341-356.

[33] Saez, Emmanuel, "The Optimal Treatment of Tax Expenditures," *Journal of Public Economics*, 88 (12), 2004, 2657-2684.

[34] Saez, Emmanuel, "Do Taxpayers Bunch at Kink Points?," National Bureau of Economic Research Working Paper No. 7366, 1999.

[35] Saez, Emmanuel, "Using Elasticities to Derive Optimal Income Tax Rates," *Review of Economic Studies*, 2001, 68, 205-229.

[36] Slemrod, Joel, "A General Model of the Behavioral Response to Taxation," *International Tax and Public Finance*, March 2001, 8 (2), 119-28.

[37] Slemrod, Joel, "The Economics of Taxing the Rich," National Bureau of Economic Research Working Paper No. w6584, May 1998.

[38] Slemrod, Joel, "Fixing the Leak in Okun's Bucket: Optimal Progressivity When Avoidance Can Be Controlled," *Journal of Public Economics*, September 1994, 55 (1), 41-51.

[39] Slemrod, Joel and Jon Bakija, "Does Growing Inequality Reduce Tax Progressivity? Should It?," National Bureau of Economic Research Working Paper No. 7576, 2000.

[40] Slemrod, Joel and Wojciech Kopczuk, "The Optimal Elasticity of Taxable Income," *Journal of Public Economics*, April 2002, 84 (1), 91-112.

[41] Slemrod, Joel and Wojciech Kopczuk, "The Impact of the Estate Tax on the Wealth Accumulation and Avoidance Behavior of Donors," National Bureau of Economic Research Working Paper No. 7960, 2000.

[42] Steuerle, C.E., *The Tax Decade: How Taxes Came to Dominate the Public Agenda*, The Urban Institute Press, Washington, D.C., 1991.

[43] Williamson, Jeffrey and Peter Lindert, *American Inequality—A Macroeconomic History*, Academic Press, 1980.

[44] Wilson, John Douglas, "On the Optimal Tax Base for Commodity Taxation," *American Economic Review*, December 1989, 79 (5), 1196-1206.

[45] Wolff, Edward, "Top Heavy—The Increasing Inequality of Wealth in America," The Twentieth Century Fund, 1994.

[46] Yitzhaki, Shlomo, "A Note on Optimal Taxation and Administrative Costs," *American Economic Review*, June 1979, 69 (3), 475-480.

Appendix I

Highly Simplified Model for the Revenue-Maximizing Tax Rate

maximize $tax\_revenue = tax\_rate \cdot taxable\_income$

$$\frac{dtax\_revenue}{dtax\_rate} = taxable\_income + tax\_rate \cdot \frac{dtaxable\_income^{set}}{dtax\_rate} = 0$$

$$\Rightarrow \frac{tax\_rate}{taxable\_income} \cdot \frac{dtaxable\_income}{dtax\_rate} = \varepsilon_{tax\_rate} = -1$$

Note that dtax_rate = –1 ·d(1 – tax_rate) = –1 ·d$net_of_tax_rate$.

$$\Rightarrow -1 \cdot (1 - tax\_rate) \left(\frac{tax\_rate}{taxable\_income} \right) \left(\frac{\mathrm{d}taxable\_income}{\mathrm{d}net\_of\_tax\_rate} \right) = -1 \cdot (1 - tax\_rate)$$

$$\Rightarrow tax\_rate \cdot \left(\frac{(1 - tax\_rate)}{taxable\_income} \cdot \frac{\mathrm{d}taxable\_income}{\mathrm{d}net\_of\_tax\_rate} \right) = (1 - tax\_rate)$$

$$\Rightarrow \varepsilon_{ntr}^{*} = \left(\frac{1 - tax\_rate}{tax\_rate} \right)$$